

Time-Varying Mesh Stiffness Modelling for Multi-Fault Spur and Helical Gear Diagnostics: An Integrated Analytical, Numerical, and Experimental Framework

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ABSTRACT

Gear Time-Varying Mesh Stiffness (TVMS) is the primary internal excitation governing gear dynamic response. It is the physical bridge between fault severity and measurable vibration signatures. That is why its accurate modeling is important. An accurate model of TVMS may lead to accurate prediction of the gearbox system's response. Most existing TVMS formulations are restricted to spur gears with single, idealized faults and limited experimental validation. This research aims to develop a physics-consistent, slice-coupled analytical TVMS framework for spur and helical gears, incorporating compound multi-location faults (pitting, tooth cracking), along with elastohydrodynamic (EHL) lubrication corrections, profile shift, and tooth lead deviations to accurately represent real gears used in practice. A three-phase methodology, comprising improved analytical model development, multi-source validation (Finite Element Analysis (FEA), Multibody simulations (MBS), precision test rig), and signal processing for fault diagnosis, establishes an experimentally grounded framework for vibration-based gear health monitoring, directly relevant to PHM practice.

1. INTRODUCTION AND PROBLEM STATEMENT

Gearbox failures are among the most frequent and costly in industrial rotating machinery. In wind turbines, each gearbox failure causes an average of over 250 hours of downtime (Santelo et al., 2022); in aerospace and high-value industrial applications, undetected progressive tooth damage poses critical safety risks. Root crack propagation and surface pitting are the dominant gear tooth failure modes, progressively reducing local tooth stiffness and modifying the TVMS, thus influencing the fault signatures. Characteristic TVMS alterations include stiffness amplitude reduction, changes in single-to-double tooth contact

transition, and impulse-like excitations at mesh frequency, providing the physical basis for condition monitoring and diagnosis. Despite decades of development, existing TVMS formulations fail to capture these dynamics under realistic operating conditions, and this research seeks to address this limitation.

2. RESEARCH GAPS

Three modelling gaps undermine current TVMS fidelity. First, helical gear TVMS requires the slice method with structural inter-slice coupling to correctly capture the 3D contact line; independent slice approaches, widely used in the literature, overestimate total mesh stiffness relative to FEA benchmarks (Wang et al., 2018) because they violate structural continuity across the face width. Second, published fault models are limited to isolated single-tooth defects with idealized geometries, whereas real gear systems exhibit compound degradation, that is, simultaneous root crack and surface pitting at multiple tooth locations, whose nonlinear stiffness interaction cannot be captured by superposition of individual fault models (Chen and Shao, 2011; Ma et al., 2014). Third, the dry-contact assumption of standard Potential Energy Method (PEM) systematically overestimates Hertzian contact stiffness relative to lubricated operating conditions, while profile shifts and tooth lead deviations are routinely excluded, limiting direct applicability to lubricated, manufactured gear assemblies.

Two validation gaps further constrain the field. The main validation approach relies only on quasi-static FEA benchmarks, providing no assurance that stiffness model accuracy leads to correct system-level dynamic response. Direct experimental correlation, linking TVMS-derived MBS vibration predictions to measured signals from physical test rigs under controlled compound fault conditions, is rare, especially for helical gear configurations.

3. PROPOSED RESEARCH METHODOLOGY

The research follows a three-phase architecture illustrated in Figure 1: (i) TVMS model development and FEA calibration

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(ii) System-level MBS & experimental validation on a precision test rig and (iii) Signal processing and model-based fault diagnosis. Each phase builds directly on the preceding one, ensuring diagnostic outputs are grounded in experimentally verified physics.

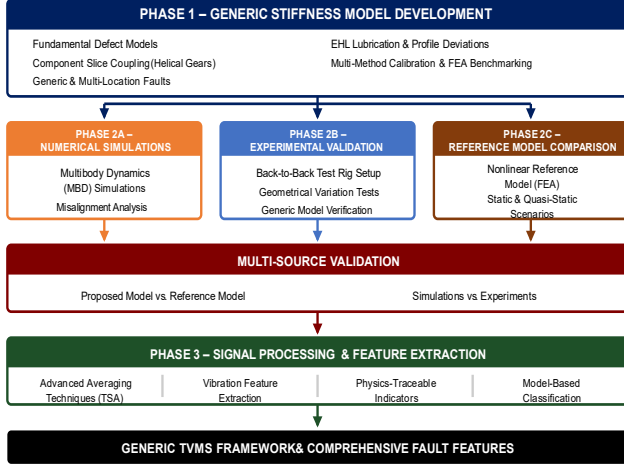


Figure 1. Three-phase research methodology overview.

3.1 Phase 1: Generic Stiffness Model Development

The analytical TVMS model is built upon the PEM (Yang and Lin, 1987; Tian, 2004), extended from spur to helical gears using a slice coupled formulation incorporating inter-slice coupling effects. The total mesh stiffness is estimated as:

$$K(t) = \sum_i \left[\left(\frac{1}{k_{h,i}} + \frac{1}{k_{b,i}} + \frac{1}{k_{s,i}} + \frac{1}{k_{a,i}} + \frac{1}{k_{f,i}} \right)^{-1} + \Delta k_{c,i} \right] \quad (1)$$

where $k_{h,i}$, $k_{b,i}$, $k_{s,i}$, $k_{a,i}$ and $k_{f,i}$ denote the Hertzian contact, bending, shear, axial, and fillet-foundation stiffness components of the i -th slice, respectively. The fillet-foundation stiffness formulation is adopted from Sainsot et al. (2004). $\Delta k_{c,i}$ represents the additional stiffness contribution associated with inter-slice coupling effects and structural continuity across the face width. Root crack degradation is modelled by modifying bending and shear integration limits as a function of crack depth and propagation angle; surface pitting is modelled through a reduction in local Hertzian contact stiffness based on the effective loss of contact area. Compound fault configurations are represented through the combined influence of multiple faults induced stiffness degradations. EHL lubrication will be incorporated via a contact stiffness correction factor parameterised by lubricant viscosity, entrainment velocity, and contact curvature radius. Profile shift and tooth lead deviations are incorporated as geometric inputs, extending applicability beyond idealized reference geometry. The analytical model is calibrated and validated against nonlinear FEA, with agreement assessed using mean mesh stiffness and TVMS trends over the full mesh cycle.

3.2 Phase 2: Multi-Source Validation

The validated TVMS model will be embedded in a multi-degree of freedom MBS model

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K(t)]\{q\} = \{F(t)\} \quad (2)$$

incorporating shaft misalignment and gear form deviation errors. Standard diagnostic indicators will be computed from predicted vibration signals before experimental comparison. Experimental validation is conducted on a precision back-to-back gear test rig covering helix angles $\beta = 30/10, 15/30$ deg, normal modules $m_n = 1$ to 3 mm, pressure angle 20 deg, and rotational speeds up to 2700 rpm. Tests will cover healthy baseline operation and controlled fault introduction (machined root notches and artificially/real seeded surface pitting). Multi-source validation convergence ensures the models accuracy at both stiffness and dynamic-response levels.

3.3 Phase 3: Signal Processing and Feature Extraction

Simulated and measured vibration signals will be processed and correlated with generated TVMS degradation signatures for systematically varied fault types, locations, and severities. A model-based fault classification framework will distinguish healthy from faulty gear states and identify fault type and severity, validated on both MBS-generated synthetic and real test-rig vibration data.

4. EXPECTED CONTRIBUTIONS

1. A slice-coupled analytical TVMS model to represent realistic spur and helical gears with compound multi-location faults (simultaneous root crack and surface pitting), calibrated and validated against nonlinear FEA and experimental measurements.
2. An EHL-corrected Hertzian contact stiffness formulation for more realistic TVMS prediction under lubricated conditions, linking dry-contact analytical models with test-rig conditions.
3. A modelling framework validated using analytical TVMS, MBS simulation, nonlinear FEA, and test-rig experimental data, providing a reproducible method for physics-based gear diagnostics.
4. A set of vibration diagnostic features linked to fault type, location, and severity, enabling model-based gear condition monitoring.

5. PRELIMINARY RESULTS AND RESEARCH PLAN

Doctoral research commenced in January 2026, placing it at an early stage with more than three years remaining before expected completion. A nonlinear FEA model of a healthy spur gear pair has been implemented, and the TVMS has been successfully extracted. The computed stiffness profile, exhibiting the characteristic alternation between single and double tooth pair contact zones, has been validated against

published benchmarks (Yang and Lin, 1987; Ma et al., 2014). This confirms correct model setup and mesh convergence and establishes the numerical reference baseline for progressive analytical model calibration.

The analytical PEM model for the healthy spur gear is under active development. Quantitative agreement with the FEA reference is being refined before faults and helical gear extension are introduced. Research phases are scheduled as: Phase 1 (Months 1–24), Phase 2 (Months 20–36), and Phase 3 (Months 32–48).

ACKNOWLEDGEMENT

The authors gratefully acknowledge the European Commission for its support of the Marie Skłodowska Curie program through the Horizon Europe DN PATRON project (GA 101120172). Views and opinions expressed are, however, those of the authors only and do not necessarily reflect those of the European Union. The European Union cannot be held responsible for them.

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BIOGRAPHIES



Maruti Patil received his Bachelor of Engineering degree from Visvesvaraya Technological University, India, and his Master of Engineering degree in Machine Design from the Indian Institute of Technology Madras, India. He joined the LMSD group in the Department of Mechanical Engineering at KU Leuven, Belgium, as a PhD researcher in 2026. His PhD research focuses on modelling gear time-varying mesh stiffness while accounting for gear degradation, with the objective of linking physical damage evolution to drivetrain dynamics and condition monitoring indicators.



Konstantinos Gryllias received the Diploma and Ph.D. degrees in Mechanical Engineering from the National Technical University of Athens, Greece, in 2004 and 2010, respectively. He is currently a Professor of Vibro-Acoustics of Machines and Transportation Systems with the Department of Mechanical Engineering, KU Leuven, Belgium. He also serves as the Manager of the University Core Lab Flanders Make@KU Leuven Motion Products, Belgium. His research interests include condition monitoring, signal processing, prognostics, and health management of mechanical and mechatronic systems.