Proceedings of the 8th European Conference of the Prognostics and Health Management Society 2024 - ISBN – 978-1-936263-40-0

Residual Selection for Observer-Based Fault Detection and Isolation in a Multi-Engine Propulsion Cluster

Renato Murata[1,2], Julien Marzat[1], Hélène Piet-Lahanier[1], Sandra Boujnah[2], and Pierre Belleoud [2]

[1] *DTIS ONERA, Université Paris-Saclay, Palaiseau, 91120, France*
firstName.lastName@onera.fr

[2] *CNES, Sous-Direction Techniques Systèmes de Transport Spatial, Paris, 75012, France*
firstName.lastName@cnes.fr

Abstract

For complex systems, the number of residual candidates generated by Structural Analysis could be in the order of tens of thousands, and implementing all candidates is infeasible. This paper addresses the residual generator candidate selection problem from a state-observer perspective. First, the most suitable candidates to derive state-observers are selected based on two criteria related to the state-space form and a low number of equations. Then, a novel algorithm finds the minimal subset of residual generator candidates capable of detecting and isolating all faults. A procedure is introduced to compare the fault sensitivity of the selected candidates. This residual selection method is applied to the multi-engine propulsion cluster of a reusable launcher to illustrate its benefits.

1. Introduction

A classical model-based approach for fault detection and isolation usually comprises two main steps: the residual generation and the residual evaluation (Simani et al., 2003). The first step relies on the mathematical model of the system to generate signals, called residuals, that contain fault information. Then, the presence of the faults is inferred by a residual evaluation method. Structural Analysis (SA) has been proven to be a powerful tool for developing model-based fault diagnosis systems (Escobet, Bregon, Pulido, & Puig, 2019). It is a graph-based tool that uses the model equations to build a structural model. From the structural model, efficient algorithms (Krysander, Åslund, & Nyberg, 2007) can be applied to find residual generator candidates automatically. However, the number of candidates increases exponentially with the number of sensors. For large-scale systems, the number of residual generator candidates can be in the order of tens of thousands. This brings a new problem to be solved: the selection of the best subset of residuals that meets both fault detectability and isolability requirements.

The residual selection problem is addressed in (Svärd, Nyberg, & Frisk, 2013) where algorithms are proposed to find a minimal subset of residuals to meet the isolability constraints. In (Jung & Frisk, 2018), the residual selection problem is solved using convex optimization. In this case, the optimization problem depends on recorded data to find the minimal and most effective subset of residuals. In (Jung & Sundström, 2017), the residual selection problem is addressed by combining the fault sensitivity information of the residuals with machine learning methods. However, in all of those works, the fault isolability constraint used is very restrictive, leading to sub-optimal solutions with more residuals than necessary to isolate all faults.

Here, it is proposed to use a different fault isolability constraint based on the fault signature. This less restrictive constraint is able to lead to an optimal subset of residuals with minimal cardinality. Such an isolability constraint based on the fault signature has been used previously in (Zhang & Rizzoni, 2017) for residual selection. However, the objective was to find a subset of residuals that would produce the most "unique" fault signature for robustness purposes.

This paper proposes a new algorithm to find the minimal subset of residual generators able to detect and isolate predefined faults. The algorithm is adapted for an observer-based residual generation technique. State observers are more robust to modeling errors and parameter uncertainty when compared with other model-based residual generation techniques, such as Sequential Residual Generation (Isermann, 2005). The idea is to select the residual generator candidates based on two main criteria: the candidates that can be easily written into the state-space form and the residual generators with the lowest number of equations. The state-space form is required to implement the majority of observers, such as the Kalman filter (Kalman, 1960) or Luenberger observers (Luenberger, 1964). It is preferred to have fewer equations because each one has a degree of uncertainty and modeling errors.

Depending on the number of residual generator candidates, it is possible to find many subsets of residuals with minimal cardinality. In order to choose the most suitable residuals in terms of fault sensitivity, a procedure based on the equations of the residual generator candidates is proposed. It quantifies the impact that a fault will have on the measured variables.

The main contributions of this paper are as follows. First, an algorithm to find the minimal subset of residuals to detect and isolate all faults. Second, a procedure based on the equations of the residual generators is proposed to compare the sensitivity of the residuals for one specific fault. The paper is organized as follows. In Section 2, basic notions of model-based diagnosis are recalled. In Section 3, the minimal residual selection problem is described and an algorithm to solve this problem is proposed in Section 4. Section 5 describes the procedure that uses the residual generator equations to compare the sensitivity of two residual generator candidates for a given fault. In Section 6, the proposed algorithm is applied in a multi-engine propulsion cluster of a reusable launcher. Conclusions are presented in Section 7.

2. Preliminaries on Model-Based Diagnosis

This section recalls some model-based diagnosis notions needed to formulate the residual selection problem formally introduced in (Svärd et al., 2013). Those notions are used to define necessary conditions to meet detectability and isolability constraints. Consider a model defined as

$$M = (E, X, Z, F) \tag{1}$$

where E is the vector of n_e system equations, X the vector of unknown variables in $\mathbb{R}^{n_x}$, Z the vector of known variables in $\mathbb{R}^{n_z}$ and F the vector of fault variables in $\mathbb{R}^{n_f}$. It is assumed that each fault $f \in F$ affects only one equation $e \in E$. This basic assumption is not as limiting as it may initially appear, as the equation e affected by the fault can propagate its effect through other equations. If a fault affects simultaneously more than one equation in the system, the system may be poorly modeled. Given the model (1), an ideal residual generator is defined as

Definition 2.1 (Ideal residual generator) *Consider a model M such as (1). A system R with input Z and output r is a residual generator for M, and r is a residual if $f = 0$ implies $r = 0$ for all $f \in F$.*

In reality, residuals slightly deviate from zero even when no fault is present in the system due to unmodeled dynamics such as measurement noise and parameter uncertainty. One important property of residuals is their fault sensitivity, which defines the subset of faults that will affect this residual:

Definition 2.2 (Fault sensitivity) *Let R_i be a residual generator for model M. Then R_i is sensitive to fault $f \in F$ if $f \neq 0$ implies $r_i \neq 0$.*

With a set of residual generators $R \supseteq R_i, i \in \mathbb{N}$, the fault signature S_f of a fault f can be defined. The fault signature describes the subset of residuals that are sensitive to this fault:

Definition 2.3 (Fault signature) *For a set of residual generators R, the fault signature S_f of a fault f contains all the residuals $R_f \subseteq R$ sensitive to f.*

Using the fault signature, the fault isolability can be defined. If the fault has a unique signature, i.e., a unique subset of residuals is sensitive to it, the fault can be isolated from the others.

Definition 2.4 (Fault signature isolability) *A fault f is isolable using a set of residual generators R if its fault signature S_f is unique when compared to the other fault signatures.*

3. Minimal Residual Selection Problem

The minimal residual selection problem is formally defined as an optimization problem. Considering all residual generators available R_{all} to detect and isolate n_f faults, the objective is to find a minimal subset of R_{all} that respects the fault signature isolability property presented in def. 2.4, i.e., that generates unique fault signatures S_i for each fault $f_i, i = 1, 2..., n_f$. The optimization problem is formulated as

$$\begin{aligned} \min_{R \subseteq R_{all}} \quad & |R| \\ \text{s.t.} \quad & S = \{S_1, S_2, ..., S_{n_f}\}_{\neq} \\ & S \neq 0 \end{aligned} \tag{2}$$

where $|R|$ is the cardinality of the subset R. An equivalent optimization problem, using the fault signature, is introduced in (Zhang & Rizzoni, 2017), but a solution to this problem is not addressed.

The fault signature isolability concept is a key notion of finding the minimal subset of residuals to isolate all the faults. In previous works, such as (Svärd et al., 2013) and (Jung & Frisk, 2018), a different fault isolability definition was used. For instance, a fault $f_i \in F$ is considered to be isolable from another fault $f_j \in f$ if there exists a residual $R_k \in R$ that is sensitive to f_i but not to f_j. Due to the fact that the isolability is defined by pair of two faults, to isolate n_f faults, it is necessary to meet $\frac{n_f!}{(n_f-2)!}$ isolability requirements. This notion of isolability is thus more restrictive compared to the proposed definition 2.4.

The difference between the two notions of fault isolability is illustrated on the following simple example. Consider a set of three residual generators and three faults with different sensitivities defined in Tab. 1. The symbol $*$ indicates that a given residual r_i is sensitive to a fault f_j.

$$R_{all} = \{r_1, r_2, r_3\} \quad F = \{f_1, f_2, f_3\}. \tag{3}$$

Table 1. Fault signature matrix.

	r_1	r_2	r_3
f_1	0	$*$	$*$
f_2	$*$	$*$	0
f_3	$*$	0	$*$

To isolate all three faults using the fault isolability requirements employed in previous works, the set of residuals should respect six different constraints:

$$\begin{aligned} &c_1 : f_1 \times f_2 \;\; c_2 : f_1 \times f_3 \;\; c_3 : f_2 \times f_1 \\ &c_4 : f_2 \times f_3 \;\; c_5 : f_3 \times f_1 \;\; c_6 : f_3 \times f_2 \end{aligned} \tag{4}$$

where $f_i \times f_j$ denotes a constraint that requires a residual sensitive to f_i but not to f_j.

Analysing the fault signature from Tab. 1, all three residuals are thus required to meet the six fault isolability constraints. However, it is possible to find smaller subsets of R_{all} capable of detecting and isolating all faults (3) using the fault signature isolability concept. The number of isolability constraints is then divided by two:

$$c_1 : S_1 \neq S_2 \;\; c_2 : S_1 \neq S_3 \;\; c_3 : S_2 \neq S_3. \tag{5}$$

It can be checked that any pair of residuals generates a unique fault signature for each fault, respecting the constraints 5, and is, therefore, a solution to the optimization problem (2).

4. Minimal Residual Selection Algorithm

A new algorithm to solve the optimization problem (2) is proposed. First, the minimal number of residuals needed to isolate n_f faults is calculated. Assuming that each residual r_i has only two states: $r_i = 0$ when $F_i = 0$ and $r_i \neq 0$ when $F_i \neq 0$, where F_i is the vector of faults that affects r_i. The lowest number of residuals n_{min} necessary to isolate n_f faults must follow the inequality:

$$2^{n_{min}} \geq n_f + 1. \tag{6}$$

It must be highlighted that n_{min} represents the theoretical lowest number of residuals necessary to generate n_f different fault signatures. The existence of such subset will depend on the sensitivity of each residual.

For instance, to isolate the three faults from Eq. (3), at least two residuals are required to generate three different fault signatures, considering that the fault signature $(0, 0)$ is excluded because it is equivalent to the fault-free state. For comparison, the solution proposed in (Jung & Frisk, 2018) based on optimization finds a subset of six residuals to isolate four faults.

The main idea behind the proposed algorithm consists in taking all possible combinations of n_{min} at a time of residual generators in R and checking if this subset of R generates a different fault signature for each residual. Assuming that the total number of residual generators is n_R, the number of all possible combinations is defined as

$$n_c = \frac{n_R!}{(n_R - n_{min})! n_{min}!} \tag{7}$$

From Eq. (7), if the number of residual generators is too big, it would be impossible to test the isolability properties of all possible subsets of R. For example, to isolate thirty faults ($n_f = 30$) using sixty residual generators ($n_R = 60$), it is necessary to have at least five residuals ($n_{min} = 5$), and there are more than five million possible combinations of residuals to be tested ($n_c > 5 \times 10^6$).

To restrict the number of residual generators, two new concepts are introduced:

- **Detectability class:** for each fault $f \in F$, list every residual sensitive to this fault $R_{df} \in R_{all}$;
- **Undetectability class:** for each fault $f \in F$, list every residual that is not sensitive to this fault $R_{uf} \in R_{all}$.

The idea is to select the most suitable residual generator for each detectability and undetectability class. The criterion for selecting that residual generator will depend on the residual generator method. In this work, the observer-based residual generation method is used. Two criteria are defined to choose the most suitable residual generator from a state-observer point of view:

1. Choose the residual generators composed of Ordinary Differential Equations (ODEs) or Differential Algebraic system of Equations (DAE) of index 1.
2. Select the residual generators with minimal "state cardinality," which means the residual with a minimal number of equations, which is equivalent to the state dimension of the corresponding observer.

The first criterion is related to the observer theory, which is mostly based on ODE systems. DAE systems of index one are also included because they can be easily transformed into an ODE by taking the derivative of the algebraic equations (Campbell, Linh, & Petzold, 2008).

The second criterion is related to model uncertainty. Each equation has a level of uncertainty due to modeling errors. It is thus suitable to choose the residuals with fewer equations to minimize the combined level of uncertainty.

Finally, the union of all residuals that meet both criteria for each detectability and undetectability class is used to test all possible combinations to verify the fault isolability requirements. The formal description of the process to find the minimal subset of residual generators is described in Algorithm 1. It can be divided into two main loops. The first loop takes the set of residual generator R and filters it using the two criteria defined above. The isolability properties of the filtered subset of residuals R_f are inspected. If the isolability properties are not met, a flag to relax the filtering constraints ($rCons$) is activated. The second loop tests all possible subsets of R_f based on the minimal number of residuals (n_{min}) needed. If no subset of R_f containing n_{min} residuals is capable of detecting and isolating the faults F, the minimum number of residuals n_{min} is increased, and the search restarts. The procedure returns a list R_{min} containing all subsets with n_{min} residuals that can detect and isolate all faults. The other procedures used in Algorithm 1 are described below.

- DETECTABILITYCLASS(R, F) for each fault $f \in F$, lists all residuals from R that are sensitive to this fault. Returns n_f subsets of residuals corresponding to each fault.
- UNDETECTABILITYCLASS(R, F) for each fault $f \in F$, lists all residuals from R that are not sensitive to this fault. Returns n_f subsets of residuals corresponding to each fault.
- FILTERRESIDUALS($d, u, rCons$) for each detectability class d and undetectability class u, filter the residuals considering cardinality and equations structure criteria. If the flag $rCons$ is activated, the cardinality criteria are relaxed. Returns the list of residuals R_f that fits all filtering criteria.
- CHECKISOLABILITY(R, F) checks if a group of residuals R generates unique fault signatures for each fault $f \in F$. Returns 1 if true and 0 if false.
- COMPUTESUBSETS(R, n_{min}) compute all possible combinations of residuals from R separated into groups of n_{min} residuals. Returns a list containing all possible combinations.

Algorithm 1 Residual Selection Algorithm

```
Inputs: Set of residual generators R, List of faults F
Output: Subsets of R with minimal cardinality R_min
  procedure RESIDUALSELECTION(R,F)
      d ← DETECTABILITYCLASS(R,F)
      u ← UNDETECTABILITYCLASS(R,F)
      rCons ← 0
      isol ← 0
      while isol = 0 do
          R_f ← FILTERRESIDUALS(d, u, rCons)
          if CHECKISOLABILITY(R_f,F) then
              isol ← 1
          else
              rCons ← rCons + 1
      n_min ← COMPUTENUMMINRES(F)
      while R_min = ∅ do
          R_S ← COMPUTESUBSETS(R_f,n_min)
          k ← 0
          for all R_i ∈ R_S do
              if CHECKISOLABILITY(R_i, F) then
                  R_min(k) ← R_i
                  k ← k + 1
              if R_min = ∅ then
                  n_min ← n_min + 1
      return R_min
```

5. Residual Evaluation Problem

The Algorithm 1 presented in Sec. 4 returns all possible combinations of residuals with minimal cardinality capable of isolating the predefined faults. In example (3), there are three different pairs or residuals that could be used to isolate the faults. A method to compare the residual generators is presented here. The objective is to quantitatively measure whether one residual is more sensitive than another.

Assuming that state observers will generate the residual by measuring the difference between the output estimated by the state observer $\hat{y}$ and the measured output y, the idea is to quantify the "innovation" that the fault will have on the measured states of the residual generator. Taking y_{bf} as the measured output before fault and y_{af} the measured output after fault, the innovation is defined as

$$In = y_{bf} - y_{af}. \tag{8}$$

In theory, the innovation brought by the fault is important to better the sensitivity of the residual generator to this fault. This procedure is illustrated on the same simple example used in (3). Consider a linear time-invariant system composed of a chain of integrators:

$$\begin{aligned} &e_1 : \dot{x_1} = k_1 x_2 \;\; e_2 : \dot{x_2} = k_2 x_3 \;\; e_3 : \dot{x_3} = k_3(u + f_3) \\ &e_4 : y_1 = x_1 + f_1 \;\; e_5 : y_2 = x_2 + f_2 \;\; e_6 : y_3 = x_3 \end{aligned} \tag{9}$$

where the unknown variables are $x = \{x_1, x_2, x_3\}^T$, the outputs are $y = \{y_1, y_2, y_3\}^T$, the input is u, the fault vector is $F = \{f_1, f_2, f_3\}^T$, and $k_i, i \in [1, 3]$ are known constants.

Thee residual generators candidates can be extracted from (9) using structural analysis, where three MSOs are computed and taken as residual generators. They are composed of the following equations

$$\begin{aligned} &r_1 = \{e_2, e_3, e_5\} \;\; r_2 = \{e_1, e_4, e_5, e_6\} \\ &r_3 = \{e_1, e_3, e_4, e_6\} \end{aligned} \tag{10}$$

the fault signature of the residuals (10) are illustrated in Tab. 1. It has been shown previously that any pair of (10) is enough to isolate the three faults. However, the impact of the faults is different for each residual.

For instance, let us compare the sensitivity of r_1 and r_2 when f_2 is injected using the procedure described above. For r_1, the relation between the fault f_2 and the output is direct because the output of r_1 is y_2. For r_2, the measurement of y_2 is first used to estimate $\dot{x_1}$, which is then used to estimate the output y_1. The innovation brought by f_2 in r_1 and r_2 can be summarized as

$$In_{r_1,f_2} = f_2 \quad In_{r_2,f_2} = k_1 f_2. \tag{11}$$

If the known constant k_1 is bigger than one, this empirical analysis indicates that r_2 will be more sensitive to r_1 to detect f_2. Repeating this analysis to the other faults and residuals, an efficient subset of residual generators concerning fault sensitivity can be found.

6. Application Example

The algorithm presented in Section 4 is used to find a list of minimal subsets of residuals capable of detecting and isolating a predefined list of faults in a multi-engine propulsion cluster of a reusable launcher.

6.1. Multi-Engine Propulsion Cluster Description

The propulsion cluster considered here is composed of three main parts: propellant tanks, feeding lines, and liquid-propellant rocket engines. The tanks are where the propellant is stored, and the feeding lines connect the propellant tanks with the rocket engines, where the thrust is generated. The propulsion cluster considered is composed of three rocket engines. The rocket engines use liquid oxygen (LOX) and liquid hydrogen H_2 as propellants (Pérez Roca, 2020). A simplified scheme of the LOX part of the multi-engine cluster is illustrated in Fig. 1. The feeding lines architecture with one main line splitting into secondary lines is optimal for minimum mass and pressure drop values (Miquel, 2020).

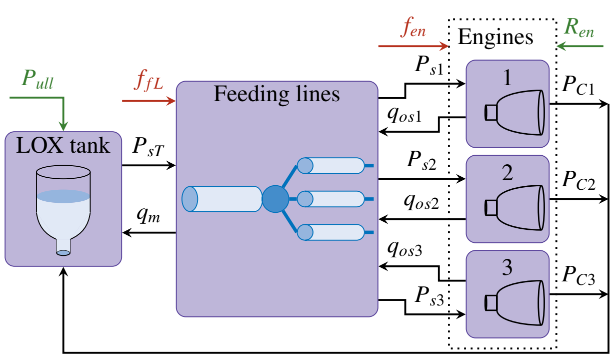

Figure 1. Multi-engine propulsion cluster scheme

The cluster operation can be summarized as follows: the rocket must follow a predefined trajectory. The trajectory is converted into thrust reference (R_{en}) and then given to the engines. Each engine has its control law that uses the control valves to meet the references. The valve position defines how much mass flow (q_{oi}) is used by the engine and, therefore, defines the mass flow that goes out of the tank through the feeding lines. The outlet pressure of the feeding lines (P_{si}) is imposed as the input pressure of the engines. P_{si} also depends on the tank's outlet pressure P_{sT}. The tank's outlet pressure is defined by the tank's ullage pressure (P_{ull}) and the rocket acceleration a_L. The acceleration depends on the thrust generated by each motor, which is directly related to the engines' combustion chamber pressure P_{Ci}. The index $\cdot_i \in \{1, 2, 3\}$ denotes the respective rocket engines.

The state vector is composed of the following variables

$$\begin{aligned} x =\{&P_{sT}, P_m, P_{si}, q_m, q_{si}, q_{GHi}, q_{CHi}, q_{THi}, q_{TOi},\\ &P_{Ci}, P_{Gi}, P_{THi}, P_{TOi}, \omega_{Hi}, q_{GOi}, q_{COi}, \omega_{Oi}\}^T \end{aligned} \tag{12}$$

where P_{sT} is the output pressure of the LOX tank, P_m/q_m are the output pressure/mass flow of the main line, P_{si}/q_{si} are the output pressure/mass flow of the i-th secondary line, q_{GHi}/q_{GOi} are the gas generator H_2/LOX mass flow, q_{CHi}/q_{COi} are the combustion chamber H_2/LOX mass flow, q_{THi}/q_{TOi} are the turbine H_2/LOX mass flow, P_{Ci}/P_{Gi} are the combustion chamber/gas generator pressure, P_{THi}/P_{TOi} are the H_2/LOX turbine intake pressure, and ω_{Hi}/ω_{Oi} are the H_2/LOX pump rotating speed.

The vector of known variables z is composed of input variables u and output measurements y, $z = \{u, y\}$

$$\begin{aligned} &u = \{V_{GOi}, V_{GHi}, V_{COi}, V_{CHi}, V_{Zi}\}\\ &y = \{P_H, P_{ull}, P_m, P_{si}, P_{Ci}, P_{Gi}, \omega_{Hi}, \omega_{Oi},\\ &RMC_i, RMG_i\}^T \end{aligned} \tag{13}$$

where VG_{Hi}/V_{GOi} are the gas generator H_2/LOX control valves, V_{CHi}/V_{COi} are the combustion chamber H_2/LOX control valves, V_{Zi} is the valve that directs the gas generated by the gas generator to the turbines, P_H is the outlet pressure of the H_2 line, P_{ull} is the ullage pressure at the LOX tank, and RMC_i/RMG_i are the combustion chamber/gas generator mixture ratios.

The mixture ratios are the relation between the LOX and H_2 mass flows:

$$RMC_i = \frac{q_{COi}}{q_{CHi}} \quad RMG_i = \frac{q_{GOi}}{q_{GHi}}. \tag{14}$$

The fault vector f is composed of the following faults:

$$f =\{f_{qCOi},\ f_{VCHi},\ f_{VGHi},\ f_{\omega Hi},\ f_{RMCi}\}^T \tag{15}$$

where f_{qCOi} is a external LOX leakage in the combustion chamber, f_{VGHi} is a blockage in V_{GHi}, f_{VCHi} is a blockage in V_{CHi}, and $f_{\omega Hi}/f_{RMC}$ are bias faults in the sensors of ω_{Hi}/RMC_i respectively.

All the equations that describe the relations between states, inputs, and faults are listed in Appendix 7.

Considering that each engine is identical, only the measurements and faults from engine one are considered to simplify the implementation of the algorithm and avoid unnecessary computational. However, all results obtained for engine one can be automatically extended to the other two engines.

The first step is to find all residual generator candidates R. This step is performed using the Fault Diagnosis Toolbox (Frisk, Krysander, & Jung, 2017). In total, considering the equations and measurements of only engine one, the system is composed of fifty-three equations, and the degree of redundancy is eight. The residual generator candidates are obtained by computing all the Minimally Structurally Overdetermined (MSO) sets. Each MSO is a subsystem with a degree of redundancy one, i.e. it has one more equation than the number of unknown variables. All MSOs can be solved independently and are, therefore, residual generator candidates.

6.2. Algorithm implementation

The computation of all residual generators results in 24433 candidates that can possibly be used to detect and isolate five faults. From (6), at least three residuals are used to detect and isolate five faults.

The first loop of Algorithm 1 finds a subset of 16 residuals that meet both cardinality and state-observer criteria.

$$R_f = \{r_{166}, r_{167}, r_{170}, r_{710}, r_{711}, r_{713}, r_{1000}, r_{1001}, \\ r_{1006}, r_{1085}, r_{1320}, r_{1321}, r_{1326}, r_{1408}, r_{1593}, r_{1838}, \}. \tag{16}$$

The fault sensitivity of the selected residuals is expressed in Table 2. It shows that the selected residuals 16 are enough to detect and isolate all faults considered.

Table 2. Fault signature matrix.

	f_{VGH1}	f_{VCH1}	f_{qCOi}	$f_{\omega Hi}$	f_{RMCi}
r_{166}	0	0	0	$*$	0
r_{167}	0	0	$*$	$*$	$*$
r_{170}	0	0	$*$	$*$	$*$
r_{710}	$*$	$*$	$*$	$*$	$*$
r_{711}	$*$	$*$	0	0	0
r_{713}	$*$	$*$	0	0	$*$
r_{1000}	0	$*$	$*$	$*$	$*$
r_{1001}	0	$*$	$*$	$*$	$*$
r_{1006}	0	$*$	$*$	$*$	$*$
r_{1085}	0	$*$	$*$	$*$	$*$
r_{1320}	$*$	0	$*$	$*$	$*$
r_{1321}	$*$	0	$*$	$*$	$*$
r_{1326}	$*$	0	$*$	$*$	$*$
r_{1408}	$*$	$*$	$*$	$*$	$*$
r_{1593}	$*$	0	$*$	$*$	$*$
r_{1838}	$*$	$*$	$*$	$*$	$*$

The second loop of Algorithm 1 takes the selected residuals R_f from equation (16) and tests all possible combinations using the minimum number of residuals and selects the combination that generates unique fault signatures for each fault. In the first iteration of the loop, the minimum number of residual generators is three, and from (6), there are 560 possible combinations to be tested. However, there are no subsets of three residuals capable of isolating all faults. In the second iteration, the minimum number of residuals is increased by one, resulting in 1820 possible combinations of four residuals. The Algorithm 1 returns 40 combinations, each one containing four residuals from (16) that can isolate all faults. For comparison, the algorithm proposed in (Svärd et al., 2013) returns a subset of at least seven residuals to isolate the same five faults.

Two possible subsets of residuals are chosen for further analysis:

$$\begin{aligned} R_1 &= \{r_{166}, r_{170}, r_{713}, r_{1001}\} \\ R_2 &= \{r_{166}, r_{170}, r_{713}, r_{1321}\} \end{aligned} \tag{17}$$

both subsets in (17) have almost the same structure; the only difference is the last residual generator. To compare those residual generators, the empirical residual evaluation method presented in Section 5 is used. The residuals are composed of the following variables:

- r_{1001}

$$\begin{aligned} &x_{1001} = \{q_{GO1}, q_{CH1}, \omega_{H1}\} \\ &z_{1001} = \{P_{s1}, P_H, P_{G1}, P_{C1}, RMC_1, RMG_1, \\ &\omega_{O1}, \omega_{H1}, V_{CH1}, , V_{GO1}\} \end{aligned} \tag{18}$$

- r_{1321}

$$\begin{aligned} &x_{1321} = \{q_{GH1}, q_{CO1}, \omega_{H1}\} \\ &z_{1321} = \{P_{s1}, P_H, P_{G1}, P_{C1}, RMC_1, RMG_1, \\ &\omega_{O1}, \omega_{H1}, V_{CO1}, , V_{GH1}\} \end{aligned} \tag{19}$$

the residuals have a very similar structure, having three states and the same output ω_H. One difference is when the fault f_{RMC1} is injected. In r_{1001}, RMC_1 is used to estimate q_{CO1}, on the other hand, in r_{1321}, RMC_1 is used to estimate q_{CH1}. When fault f_{RMC1} is injected, the estimation of the mass flows will be given by

$$\begin{aligned} r_{1001}: &\quad q_{CO1} = (RMC_1 + f_{RMC1}) q_{CH1} \\ r_{1321}: &\quad q_{HC1} = \frac{q_{CO1}}{RMC_1 + f_{RMC1}}. \end{aligned} \tag{20}$$

For residual r_{1321}, the influence of f_{RMC1} is directly observed in the output ω_{H1} because the evolution of ω_{H1} depends on q_{HC1}. Residual r_{1001} is not directly influenced because q_{CO1} is not used to estimate ω_{H1}. The fault f_{RMC1} will first impact the state q_{GO1} which then will influence the estimation of q_{GH1} and affects ω_{H1}. During those steps, the fault magnitude f_{RMC1} is divided by a constant bigger than one, attenuating the effect of the fault in the output. This makes the residual r_{1321} more suitable to detect f_{RMC1}. The same analysis can be extended to faults f_{qCO1} and $f_{\omega_O 1}$, where the magnitude of the faults is attenuated before affecting the output of residual r_{1001}.

6.3. Simulation results

To test the performance of the residuals (17) in simulation, an Unscented Kalman Filter (UKF) was calculated for each residual. This state estimator can deal with any type of nonlinearities and gives accurate estimations up to the third order of Taylor expansion (Wan & Van Der Merwe, 2000). The unscented transformation parameters were set at the default values, which gives an optimal solution for Gaussian distributions, with $\alpha = 0.001$, $\kappa = 0$, and $\mathcal{B} = 2$. The multi-engine cluster model was implemented using Simulink, and measurement noise was added. It is a white noise with zero mean, and the standard deviation varies according to the sensor specifications. For the rotational frequency of the turbopumps ω_{Oi}, ω_{Hi}, the Standard Deviation (SD) is 0.1% the rotational frequency when the engine at its nominal operating point. For the low-pressure values (P_{si}, P_H, P_{ull}), the SD is 0.1%, the nominal pressure value. For the high pressures (P_{Gi}, P_{Ci}) the SD is 0.2% the nominal value. The mixture ratios RMC_i and RMG_i have a standard deviation of 0.3%, the nominal value. The measurement noise covariance matrix $\mathcal{R}$ of the UKF was defined according to the standard deviations. The model parameters are considered perfectly modeled, so the process noise covariance matrix $\mathcal{Q}$ is proportionally defined ten times smaller than $\mathcal{R}$. To simulate the behavior of the system in closed-loop when a fault is injected, three PIDs were designed for each engine using the classical configuration (Pérez Roca, 2020). The PIDs use the valves to control the outputs $y_{PID} = [RMC_i, RMG_i, P_{Ci}]$. The closed loop system has a settling time to the step response of two seconds without overshooting.

Five faults are simulated in rocket engine 1, and each fault stays active for two seconds. The fault injection time and parameters are presented in Tab. 3.

The residuals generated by the UKFs are the difference between the output estimated by the state observer and the measured output, they are illustrated in Fig. 2. The UKF calculated from residual generator r_{166} is denoted UKF$_{166}$, etc.

From Fig. 2, the theoretical fault signature matrix of the residuals defined in Table 2 is observed in simulation. The exception is residual r_{1001} where the faults f_{qCO}, f_{wO} and f_{RMC} are attenuated by the residual's equations, and the impact of those faults cannot be seen when measurement noise is added. From simulation results, it is confirmed that the subset R_2 from (17) is more suitable for fault detection and isolation due to the higher sensitivity of r_{1321} when compared with r_{1001}.

Table 3. Fault parameters

Fault	Time	Effect
f_{VGH1}	12s-14s	Blockage 50% V_{GH1}
f_{VCH1}	16s-18s	Blockage 10% V_{CH1}
f_{qC1}	20s-22s	LOX Leak. of 0.8% nominal q_{CO1}
f_{wO1}	24s-26s	Bias of 1% nominal ω_{O1} value
f_{RMC1}	28s-30s	Bias of 5% nominal RMC_1 value

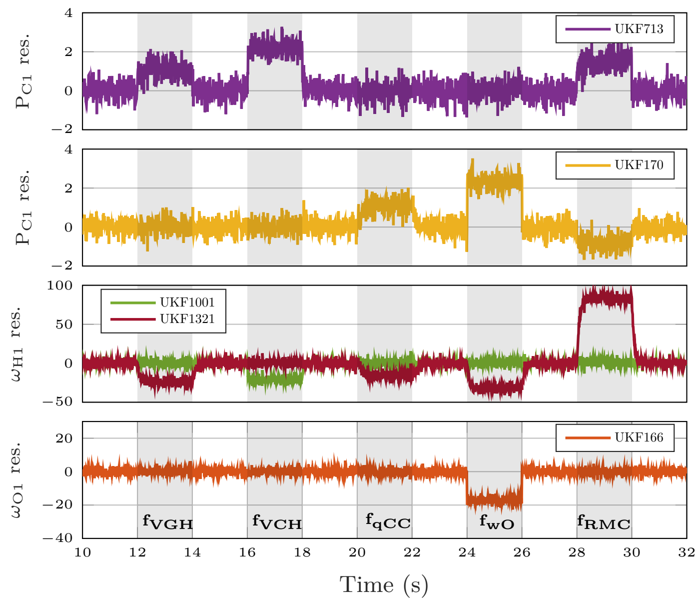

Figure 2. Residuals

7. Conclusion

A novel algorithm to find all possible subsets of residual generator candidates capable of detecting and isolating all faults with minimal cardinality has been presented. The minimal cardinality is achieved using a less restrictive isolability constraint based on the fault signature. Since the algorithm cannot be applied to a large number of residual generator candidates due to combinatorial explosion, two criteria to decrease the number of residual candidates were established. Those criteria take into account the residual generator method based on state observers, i.e. the reduced sensitivity to uncertainty when the number of state equations is minimal per residual. A procedure was presented to evaluate the selected residuals and compare the subsets with minimal cardinality returned by the algorithm. The proposed methods were applied in a model of a multi-engine propulsion cluster where five different faults were considered. From 24433 residual generator candidates, the algorithm found 40 subsets, each one containing four different residual generators, that were capable of detecting and isolating the five faults. Two of those subsets of residual generators were implemented using Unscented Kalman Filter. Simulation results showed that the subsets can be used to detect and isolate all faults, and as a result the effectiveness of the proposed selection algorithm and quantitative sensitivity evaluation.

ACKNOWLEDGMENTS

This paper is a result of a study supervised by CNES and ONERA, involving experts in propulsion and FDI domains, to propose solutions for the diagnosis of reusable launchers.

REFERENCES

Campbell, S. L., Linh, V. H., & Petzold, L. R. (2008). Differential-algebraic equations. *Scholarpedia*, *3*(8), 2849.

Escobet, T., Bregon, A., Pulido, B., & Puig, V. (2019). *Fault diagnosis of dynamic systems*. Springer.

Frisk, E., Krysander, M., & Jung, D. (2017). A toolbox for analysis and design of model based diagnosis systems for large scale models. *IFAC-PapersOnLine*, *50*(1), 3287–3293.

Isermann, R. (2005). Model-based fault-detection and diagnosis–status and applications. *Annual Reviews in control*, *29*(1), 71–85.

Jung, D., & Frisk, E. (2018). Residual selection for fault detection and isolation using convex optimization. *Automatica*, *97*, 143–149.

Jung, D., & Sundström, C. (2017). A combined data-driven and model-based residual selection algorithm for fault detection and isolation. *IEEE Transactions on Control Systems Technology*, *27*(2), 616–630.

Kalman, R. E. (1960). A new approach to linear filtering and prediction problems.

Krysander, M., Åslund, J., & Nyberg, M. (2007). An efficient algorithm for finding minimal overconstrained subsystems for model-based diagnosis. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, *38*(1), 197–206.

Luenberger, D. G. (1964). Observing the state of a linear system. *IEEE transactions on military electronics*, *8*(2), 74–80.

Majumdar, A., & Steadman, T. (2001). Numerical modeling of pressurization of a propellant tank. *Journal of Propulsion and Power*, *17*(2), 385–390.

Miquel, V. (2020). *Propellant feeding system of a liquid rocket with multiple engines.* KTH Royal Institute of Technology.

Pérez Roca, S. (2020). *Model-based robust transient control of reusable liquid-propellant rocket engines* (Theses). Université Paris-Saclay.

Simani, S., Fantuzzi, C., Patton, R. J., Simani, S., Fantuzzi, C., & Patton, R. J. (2003). *Model-based fault diagnosis techniques*. Springer.

Svärd, C., Nyberg, M., & Frisk, E. (2013). Realizability constrained selection of residual generators for fault diagnosis with an automotive engine application. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, *43*(6), 1354–1369.

Wan, E. A., & Van Der Merwe, R. (2000). The unscented kalman filter for nonlinear estimation. In *Proceedings of the ieee 2000 adaptive systems for signal processing, communications, and control symposium* (pp. 153–158).

Zhang, J., & Rizzoni, G. (2017). Selection of residual generators in structural analysis for fault diagnosis using a diagnosability index. In *2017 ieee conference on control technology and applications (ccta)* (pp. 1438–1445).

APPENDIX

The Liquid-Propellant Rocket Engine models are all derived from (Pérez Roca, 2020). All variables used in the equations that are not states (12), inputs and outputs (13) or faults (15) are known constants. Only the equations of the oxygen side are presented. The equations of the hydrogen side have the same structure. The only difference is the index $._O$ is replaced by $._H$ on the hydrogen side. The effect of the faults in the dynamic equations of the cluster is highlighted in red.

The pressure at the output of the oxygen turbopump P_{pOi} is given using manufacturer data:

$$P_{pOi} = \left(\frac{ap_O}{\rho_O} + R_{OGC}\right)(q_{COi} + q_{GOi})^2 + bp_O(q_{COi} + q_{GOi})\omega_{Oi} + cp_O\rho_O\omega_{Oi}^2. \tag{21}$$

The evolution of the oxygen mass flow that enters the combustion chamber q_{COi} and the gas generator q_{GOi} are derived from conservation of the momentum equation:

$$\dot{q}_{GOi} = \frac{1}{I_{GO}}\left[P_{si} + P_{pOi} - P_{Gi} - \left(\frac{1}{2\rho_O(V_{GOi} + f_{VGOi})^2} + R_{OG} + R_{OGC}\right)q_{GOi}^2\right] \tag{22}$$

q_{COi} has the same structure, where the subscript $._G$ is replaced by $._C$.

The hot gases mass flows are given by

$$\dot{q}_{THi} = \frac{1}{I_{TH}}\left(P_{Gi} - P_{THi}\right) - ky_{TH}R_{outG}\frac{T_G}{P_{Gi}}q_{THi}^2 \quad (23)$$

$$\dot{q}_{TOi} = \frac{1}{I_{TO}}(P_{Gi} - P_{TOi}) - Zr_iR_{outGG}\frac{T_G}{P_{Gi}}q_{TOi}^2 \quad (24)$$

where Zr_i is the equivalent resistive coefficient of the valve. The combustion chamber pressure P_{Ci} evolution can be approximated by first order Taylor expansion

$$\dot{P}_{Ci} = k1_C(q_{CHi} + q_{COi} - f_{qCOi}) - k2_C\sqrt{T_C}P_{Ci}. \quad (25)$$

The oxygen turbine pressure P_{TOi} is defined as

$$\dot{P}_{TOi} = k1_{TO}T_Gq_{TOi} - k2_{TO}\sqrt{T_G}P_{TOi}. \quad (26)$$

Finally, the rotational speed's evolution is given by manufacturer data

$$\dot{\omega}_{Oi} = \frac{1}{J_O}[T_{TOi} - \frac{ac_O}{\rho_O}(q_{COi} + q_{GOi})^2 - bc_O(q_{COi} + q_{GOi})\omega_{Oi} - cc_O\rho_O\omega_{Oi}^2] \quad (27)$$

where the motor torque T_{TOi} is given by $T_{TOi} = ST.W_i$ with ST the specific torque and W_i the work provided by the turbine pressure P_{TOi}.

For the feeding lines model, the evolution of the mass flow q and outlet pressure P in one rigid pipe, considering the effects of the fluid inertia, dynamic compressibility and neglecting the fluid thermal expansion, can be described by the momentum and mass balance equations:

$$\begin{aligned} \dot{q} &= \frac{S}{L}\left(P_{in} - P - \frac{f_rL}{2\rho S^2D}q^2\right) \\ \dot{P} &= \frac{\alpha^2}{V}(q - q_o) \end{aligned} \quad (28)$$

this pair of equations must be repeated for each pipe to model the feeding lines illustrated in Fig. 1.

The governing equations of pressurization of a propellant tank are obtained from (Majumdar & Steadman, 2001). The output pressure of the tank is defined as

$$P_{sT} = P_{ull} + \rho_O[a_L + g\cos(b)]H_d \quad (29)$$

Considering that a cylinder can approximate the shape of the tank, the gravitational head H_d is defined as

$$H_d = \frac{V_{LOX}}{\pi r^2}, \quad V_{LOX} = V_{LOX_0} - \frac{\int_0^t qm\,dt}{\rho_0} \quad (30)$$

The rocket's acceleration a_L can be approximated by a bivariate quadratic function total thrust T generated by the engines and the mass of the rocket m_R:

$$a_L = k_{1a} + k_{2a}T + k_{3a}m_R + k_{4a}Tm_R + k_{5a}m_R^2 \quad (31)$$

Finally, the mass of the rocket m_R can be calculated as:

$$m_R = m_{R_0} - \int qm\,dt - \int q_H\,dt \quad (32)$$

where $q_H = q_{CH} + q_{GH}$ is the total hydrogen mass flow used by the three engines, and m_{R_0} is the initial mass of the rocket.