

Ballast Degradation Modeling for Turnouts based on Track Recording Car Data

SM. Asadzadeh¹, R. Galeazzi¹, E. Hovad², JF. Andersen², C. Thyregod², AFS. Rodrigues³

¹ *Department of Electrical Engineering, Technical University of Denmark, Kgs. Lyngby, Denmark*
semoasa@elektro.dtu.dk; rg@elektro.dtu.dk

² *Department of Applied Mathematics and Computer Science, Technical University of Denmark, Kgs. Lyngby, Denmark*
emilh@dtu.dk; jfan@dtu.dk; camt@dtu.dk

³ *Banedanmark. Strandvænget 46, 2100 København Ø*
afsr@bane.dk

ABSTRACT

Turnouts play a central role in the railway infrastructure since they enable increased network capacity and allow for minimal impact of train delays. Their performance is of paramount importance for infrastructure managers, who face large maintenance cost in order to secure proper turnouts operability. Railway turnouts are complex mechanical systems, whose dynamic performance depends on the health state of the different components of the superstructure and substructure. A key component is the ballast as it provides the elastic support to the track and the sleepers and it largely contributes to the safety and reliability of the infrastructure. Ballast degradation can be a root cause of excessive failures in other components. A track recording car is typically used to collect geometry data that is used to assess the quality of the railway tracks; however this type of data has not been widely used for ballast quality evaluation in turnouts. One reason is that maintenance decision for turnouts are dominantly made based on visual inspections and/or manual measurement of track geometry, as turnouts are significantly more complex than traditional railway track. This study presents the application of fractal dimensioning of track longitudinal level for the monitoring of ballast degradation in railway turnouts. In other words, the irregularities of the track vertical profile related to the ballast degradation are quantified as a ballast quality index. The ballast quality index is the basis for developing ballast degradation models in different sections of the turnout based on a segmentation scheme. Using track geometry data of 88 turnouts in the Danish railway network for the period 2012-2017, this study develops and compares ballast degradation models based on

regression analysis and stochastic processes (lognormal and Gamma processes). The models are estimated for different sections of the single turnout, for different turnouts at distinct geographical locations. The proposed method provides an efficient tool for the analysis of the effect of tamping on ballast degradation rate. Moreover, the effects on ballast degradation of track loading rate, train speeds and seasonal changes of weather conditions are quantified.

1. INTRODUCTION

Track geometry degradation is a major hindrance for safety, availability and ride quality. Modelling track degradation is an indispensable part of a track maintenance support system and is essential for prognostics function of predictive maintenance.

During the last three decades, a number of track geometry degradation models have been developed to predict the railway track geometry condition (Ferreira & Murray, 1997; Soleimanmeigouni, Ahmadi, & Kumar, 2016). With respect to the level of detail, Ferreira & Murray (1997) divide track models into three categories: microscopic models (involving detailed engineering analysis), deterioration models (involving condition forecasting based on engineering judgement) and macroscopic models (involving network analysis, investment and maintenance support system).

The approaches used for modelling track geometry degradation are classified into mechanistic/physical and statistical models (Soleimanmeigouni et al., 2016). Mechanistic models are based on modelling of the mechanical interactions of track components and usually make no use of geometry data (Zhang, Murray, & Ferreira, 2000). A major barrier towards the practical application of mechanistic models for track degradation forecasting is the

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high level of required details. In the face of uncertainty of the track behavior, missing only one influencing factor can give rise to a model generating invalid results.

On the other hand, statistical models by employing concepts from probability theory and statistical estimation can address uncertainty of track degradation more efficiently. Moreover developments of technology and methods for track geometry measurement as well as integrated data collection systems have favored the statistical modelling approach.

The successful application of the statistical approach requires having a proper measure to quantify track quality or track degraded level. The majority of studies use standard deviation of track longitudinal level as the track quality measure to be used for triggering preventive tamping actions (UIC, 2008; Vale & Lurdes, 2013; Andrade & Teixeira, 2015; Soleimanmeigouni et al, 2018). Other quality numbers based on a combination of geometry parameters with speed and lack of super elevation (Veit and Wogowitsch, 2002; Lyngby, 2009) however can be found in literature.

Identifying the factors that can influence the track geometry degradation is an essential part of degradation modelling. The significant factors studied in the literature are train speed, axle load, wheel quality (flats and shells), number of past tamping operations (maintenance actions aiming at restoring the quality of the ballast layer), weather condition (temperature, precipitation), type of rail, pads, sleepers and ballast, imperfections in the rail surface (corrugations, joints, welds, defects), and subgrade stiffness (Sato, 1995; Ferreira & Murray, 1997; Iwnicki, Grassie, & Kik, 2000; Lyngby, 2009; Guler, 2014). According to Andrade & Teixeira (2011), the initial quality of track in switches can also affect its degradation rate. Specific to turnouts, the discontinuity of rail in the switching or in the crossing sections can generate high impact forces that, in turn, can lead to accelerated ballast degradation. Based on unstressed measurement of track geometry in 13 turnouts, Jönsson et al. (2016) showed the geometry of turnouts on the straight main track to have a vertical elevation tendency towards the mid-section.

Currently, the monitoring of track geometry is performed by measuring geometry parameters 3-4 times per year with an average of approximately 100-day time intervals. Therefore, the problem of forecasting ballast degradation rate in the next 100 days is of relevance for railway infrastructure managers. The aim of this study is to propose a model for forecasting ballast degradation in the next coming 100 days.

1.1. Significance and Contribution

The current study performs a comprehensive investigation of the track geometry evolution in 88 turnouts of the Danish railway network. It proposes a statistical approach for

ballast degradation modelling in the turnouts. The paper contributes to the scientific field by proposing an integrated methodology for ballast degradation modelling in turnouts. The advantage of this methodology is two-fold. First, it is the first turnout-focused study that uses fractal dimensioning as a validated technique to monitor the health state of the ballast based only on vertical track geometry profile. Moreover, the effects of different influencing factors on the ballast degradation rate are examined by a regression model. Second, it proposes a Bayesian updating scheme to provide information on the type and the parameters of the probability distribution function best fit to the ballast degradation rate in turnouts. The Bayesian updating allows integrating the prior knowledge of ballast degradation in turnouts with the new information collected for a specific turnout. This is especially important for building predictive models for ballast degradation which helps railway infrastructure managers in the transition to a predictive maintenance strategy.

The rest of the paper is organized as follows. Section 2 presents the methodological steps for the development of the ballast degradation model in turnouts. Section 3 presents results of ballast degradation modelling and Bayesian updating. The paper ends in Section 4 with main findings and conclusions.

2. BALLAST DEGRADATION MODEL

This section presents the methodological steps for the development of the ballast degradation model in turnouts. Figure 1 provides a schematic overview of the overall proposed method by illustrating the path from raw data (turnout longitudinal level) to ballast degradation model. At the first step, the time-series data available for the turnouts are preprocessed to ensure their spatial alignment. Next, the fractal method is applied to calculate the time-series of *fractal2*, an index for ballast degradation over time. Then the degradation rate of the ballast is calculated as the slope of the best straight line fitted to the time-series of *fractal2* (referred to as *deltafractal2*). Having *deltafractal2* calculated for all the turnouts under study, a prior probability distribution function (pdf) is fitted. The candidate pdfs are Weibull, Gamma and lognormal distributions. Moreover, a multiple linear regression modelling is used to estimate the effects on *deltafractal2* of different contributing variables like the number of past tamping, maximum permissible train speed; passing million gross tons (MGT), the weather condition (seasonality, temperature and precipitation), the current degraded state of the track geometry, the current degraded shape of the turnout, the type of the turnout and also the section of the turnout. Finally, the information generated by the regression modelling is used to update the prior distribution to a posterior distribution of the *deltafractal2*, which can be utilized as a leading indicator for ballast degradation rate in turnouts.

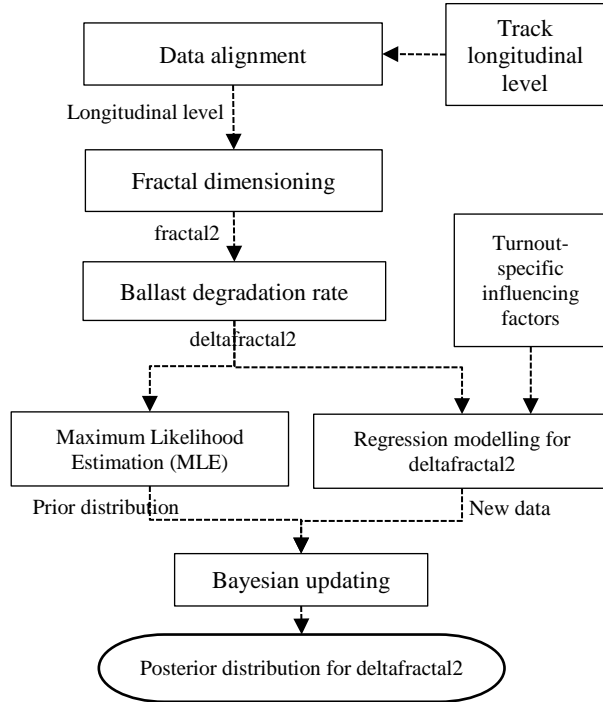


Figure 1: Ballast degradation model development

2.1. Data alignment

Each turnout has been measured several times over the years 2012-2017 (3-4 measuring campaigns each year). The track recording car can enter the turnout from two directions and the turnout’s divergent track can be on the left or right side. For the track recording car going into the straight track, this introduces 4 types of time-series data as shown in Figure 2. To simplify data analysis, all the time-series data are reconfigured to be type 1. In the type 3 configuration, the rail sides are switched. In the type 5, the direction of the data is reversed and rail sides are switched and in the type 7, the direction of the data is reversed. If a switch of the rails is performed, the sign of specific variables are changed based on the definition of the variables discussed in (Fongemie and Jensen, 2017) and (DIN Standards Committee Railway, 2008).

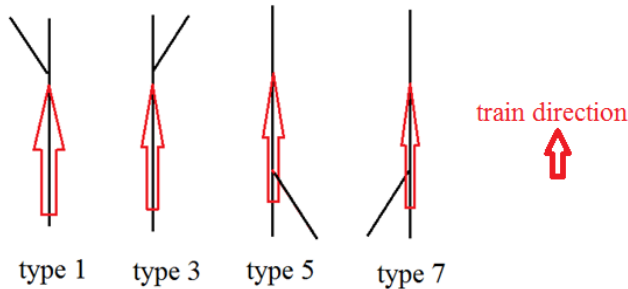


Figure 2: Definition of turnout type

The track recording car uses a GNS (Global Navigation Satellite) system for establishing the spatial position of the time-series data. Due to measurement uncertainty small spatial shifts occur in the time-series data from campaign to campaign within the same turnout. Therefore spatial alignment is performed as described in Hovad et al. (2018). A two steps procedure is performed to align the data: first a “within turnout alignment algorithm” followed by a “between turnout’s alignment algorithm”. The “within turnout alignment algorithm” uses the cross-correlation function (CCF) to perform a date to date alignment within the individual turnout’s removing the small spatial shifts. The “between turnout’s alignment algorithm” is based on a peak in the track gauge parameter monitored at the crossing nose position. An overall spatial position of this peak is determined as the average position of the peaks from all the turnout’s. This overall peak is used as a reference point to which each turnout is aligned (Figure 3). For some of the turnout’s the peak in the track gauge was not present. The cross-correlation function is used in a similar way to “the within turnout alignment algorithm” to align these remaining turnouts.

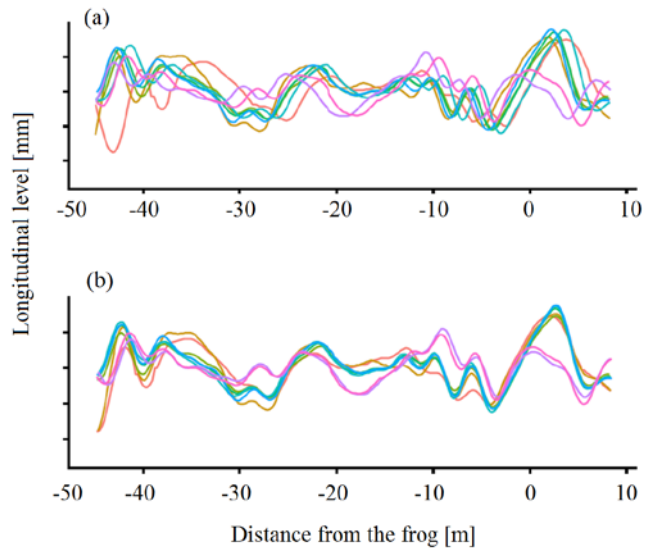


Figure 3: The alignment of the track recording car time-series data, a) un-aligned sample data, b) aligned data

2.2. Fractal analysis of track vertical profile

Fractal analysis is used to characterize irregular geometry patterns and to quantify patterns that are seemingly chaotic and random (Mandelbrot, 1983). Fractal analysis has proven potential to obtain useful information about the substructure condition of the track by meaningfully quantifying the vertical-profile geometry patterns (Hyslip, 2002; Landgraf, Hansmann and Marschnig, 2014; Vidovic, Landgraf & Marschnig, 2017). For details of fractal dimensioning and its calculation procedure the interested readers are referred to the above references.

Applying the fractal method to railway track geometry data includes calculating the roughness of the geometry signal for different wavelength regions. According to Hyslip (2002) the track vertical geometry profile has two orders of roughness, first-order and second order fractal dimensions. Irregularities associated with the first order fractal dimension are related to superstructure components like the rail and the sleepers while second order fractal dimension relates to track irregularities caused by the substructure components. Landgraf et al. (2014) also evidenced that three orders of roughness can be drawn from open track longitudinal level, from which the dimension for wavelengths between 3 and 30 meters can reflect the condition of the ballast. However, in the experiments performed in the current study, the fractal dimensioning for track longitudinal level in turnouts results in two orders of roughness, one for wavelength below 3 meters and another for wavelengths between 3 and 20 meters, due to the limited area of the turnouts. The length of the type of turnouts under study does not exceed 60 meters and this puts a limit on the maximum wavelength possible for fractal dimensioning. Therefore, the irregularities of the track vertical profile related to the ballast degradation are quantified as the second order fractal dimension, which we address as *fractal2*.

To illustrate the applicability and usefulness of the fractal dimensioning of track longitudinal level for the monitoring of ballast degradation in railway turnouts, Figure 4 exemplifies the effect of a tamping operation in September 2016. This is clearly observable in the proposed ballast quality index (*fractal2*). Moreover, the gradual deterioration of the ballast (increasing irregularities of the vertical profile) from November 2016 to November 2017 is traceable from the ballast quality index. It is noted that fractal dimensioning always generates a negative output for ballast quality index: smaller values relate to higher degradation of the ballast layer.

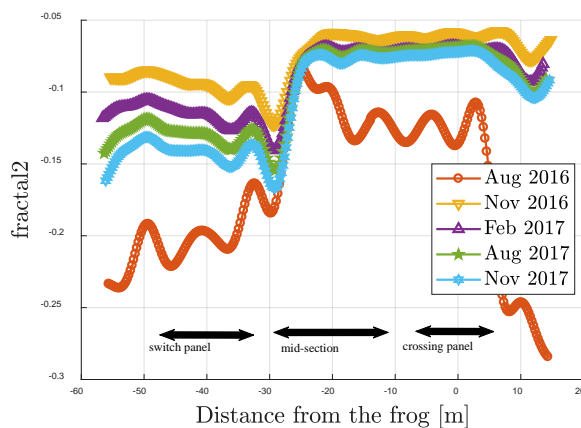


Figure 4: Ballast quality changes in a turnout system.

Figure 5 shows the time series of *fractal2* in two other turnouts. In these plots, the evolution of *fractal2* is shown for three different sections of the turnout system; the switch panel, the mid-section and the crossing panel. As seen, the change in *fractal2* is different for the sections of the same turnout, which indicates that the degradation does not occur homogeneously across the turnout. Moreover, tamping operations which are characterized by a sudden jump in *fractal2* do not have the same effect across all the sections. For example, in Fig (5a), a tamping had occurred around 1000 days before 1/1/2018, and this tamping has restored the ballast condition in the crossing section from about -0.5 to -0.17 (reduction in the geometry roughness) but the change is minimal in the mid-section and not noticeable in the switch panel.

Since *fractal2* is an index of ballast quality, the change of this index over time gives an indicator of degradation rate. The main maintenance operations that can change the quality of the ballast in turnouts, in the order of expected service life, are ballast profiling and stabilization, tamping and ballast cleaning/replacement. Between every two consecutive tamping/cleaning operations, the decrease in *fractal2* is due to traffic-induced gradual ballast degradation. But looking at the trend of *fractal2* between two consecutive tamping, there are also some minor ups and downs which are treated as the noise in the *fractal2*. This uncertainty in the time series of *fractal2* can be addressed by fitting a linear degradation trendline (bold dotted lines in Figure 5). This is a linear regression model where the dependent variable is *fractal2* and the only independent variable is time (number of days). This single-variable regression line is estimated by the ordinary least squares method. According to Dahlberg (2001), the settlement of ballasted track occurs in two phases: the first phase directly after tamping in which track settlement best modeled by a logarithmic function of track loading cycles. The second phase in which the settlement occurs linearly with cumulative load. As track measurement with the loaded car is normally performed a few days after tamping, it is assumed that the measurements of track geometry are collected during the second phase of track settlement and this justifies the linearity assumption. The quality of this linear fit is quantified by the regression coefficient of determination (R^2). $\Delta fractal2$ is the slope of the fitted lines multiplied by 100. In other words, $\Delta fractal2$ is the change in fractal in a time period of 100 days.

One observation with $\Delta fractal2$ in the turnouts under study is the high level of variation from one turnout to another and even for a single turnout in two different time-periods. This observation can be both seen in Fig (5a,b) where $\Delta fractal2$ -0.19 and -0.04 are calculated for the crossing section for the turnout in Fig (5a). The variation of $\Delta fractal2$ in the other turnout (Fig 5b) is even higher with the calculated values of -0.62, -0.07, and -0.03 for the switch panel.

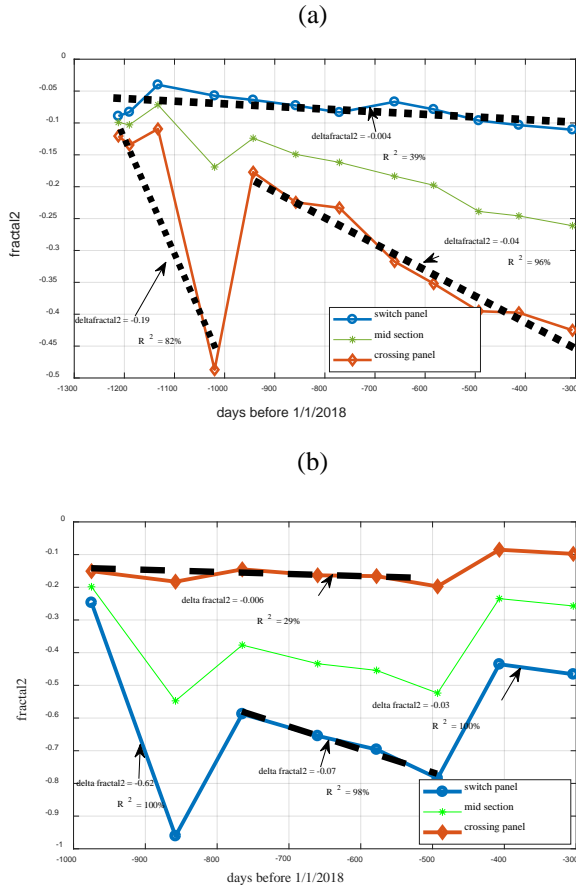


Figure 5: Delta-fractal2 calculation in two turnouts.

Another observation with deltafractal2 is the linearity of ballast degradation between consecutive tamping which justifies the assumption of linear ballast degradation in turnouts. Figure 6 depicts the regression coefficient of determination (R^2) versus the estimated deltafractal2. It is noted that in the cases where $R^2=1$, there are only two measurements. Generally, as the estimated deltafractal2 decreases, the degradation path shows better fit to linear degradation, due to higher R^2 value. Especially, when degradation rate is below -0.01 , R^2 is mostly above 60%. That is to say that when a considerable degradation occurs, fractal2 follows a linear descending trajectory.

2.3. Prior distribution

The method described for deltafractal2 is used to calculate degradation rate for all the turnouts under study in the time period 2012-2017. The calculation has been performed for the longitudinal level for both the right and the left rail in 88 turnouts. Table 1 presents the descriptive statistics of deltafractal2. The counting for deltafractal2 shows the total number of the degradation paths drawn between consecutive tamping operations in the 3 sections of all the 88 turnouts under study.

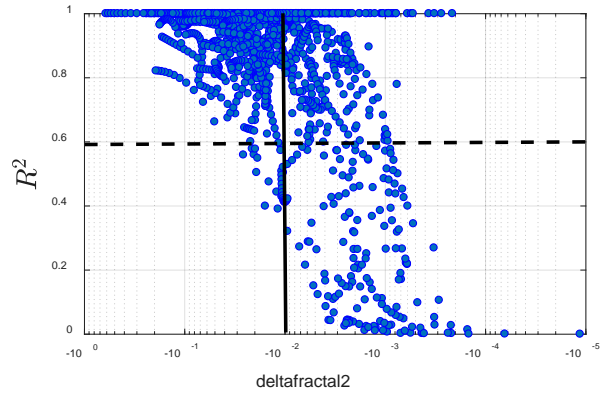


Figure 6: Linearity of fractal-based ballast degradation.

Table 1. Descriptive statistics of deltafractal2.

Rail	Turnout Count	Deltafractal2 calculation count	Mean	Std
Right	88	1801	-0.30	0.046
Left	88	1776	-0.28	0.038

Figure 7 shows the histogram of deltafractal2 estimated for all the turnouts under study.

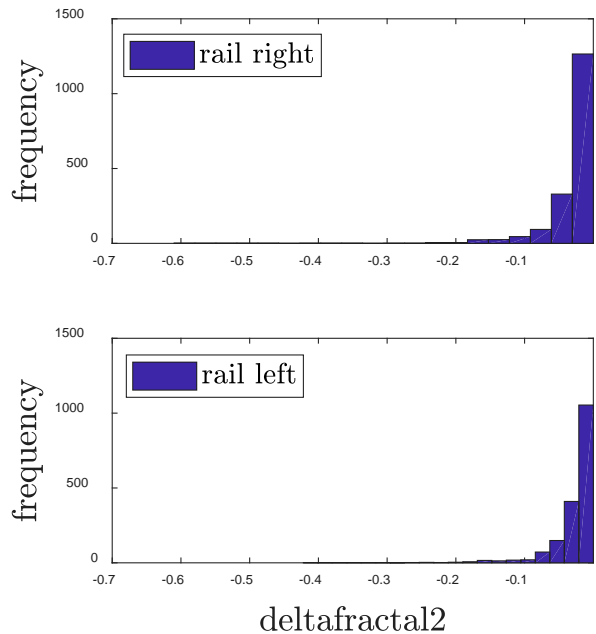


Figure 7: Histogram of deltafractal2.

The proposed deltafractal2 is compared with another common feature extracted from the track longitudinal level, a measure of the geometric quality of railroad tracks which

is calculated as the standard deviation of the band-pass filtered longitudinal measurements taken 25 mm apart within a 50 m track section of the turnout area. The engineering idea to monitor the track substructure is to filter the measurements in waveband 3-25 m (Berggren 2010). It should be noted that according to the study by Spooner, Thyregod, Stockmarr & Ersbøll (2015) a section of 120 m of track in the turnout area would give more stable and reliable results for standard deviation calculation, however there are practical issues with applying this length of the track in the turnout area as it may include another turnout in the case of two very close turnouts or it may include a great portion of open track in the analysis which is undesirable for a turnout-focused study.

In this study, the method described in Figure 5 to calculate the rate of degradation is also applied for the standard deviation of the longitudinal level and the calculated statistics is called delta standard deviation (*DSD*). In order to compare deltafractal2 with DSD, deltafractal2 is multiplied by (-1) to have positive values.

The common distributions for geometry degradation modelling are Gamma, lognormal and Weibull distributions. For prior distribution of deltafractal2, these distributions are considered and among them lognormal and Gamma distributions show better fit to the data. Figure 8 shows the probability plot of deltafractal2 and DSD displayed on a lognormal plot. As it is seen, deltafractal2 calculated both for longitudinal level for the right and the left rail has a good fit to the lognormal line. However, DSD show discrepancy from lognormal and better fits to Gamma distribution.

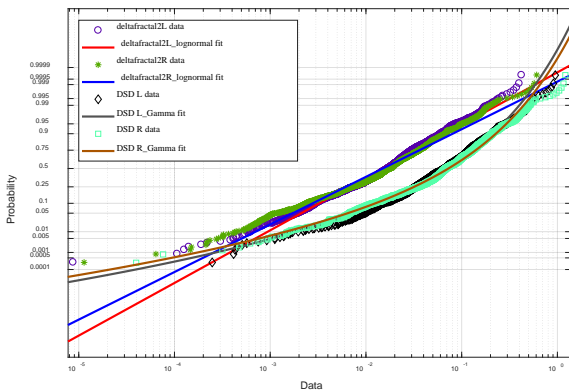


Figure 8: Distribution fit for deltafractal2 and DSD.

Table 1 compares the log-likelihood of the lognormal and Gamma distributions fits to the deltafractal2 and DSD data. Due to higher likelihood, deltafractal2 has better fit to the lognormal distribution whereas DSD best fits to the Gamma distribution.

Table 2. Log-likelihood of the fitted distribution.

Data \ Distribution	Lognormal	Gamma
deltafractal2 – rail right	4535	4520
deltafractal2 – rail left	4648	4606
DSD – rail right	2268	2370
DSD – rail left	2400	2439

As fractal2 and deltafractal2 are dimensionless quantities, it is worth to find a relationship between deltafractal2 and DSD, as DSD, being the change in the standard deviation of track longitudinal level, is more recognized among practitioners. Figure 9 shows the relationship between deltafractal2 and DSD values calculated in the turnouts under study. As seen, the average line fitted to this relationship shows that one unit change in deltafractal2 implies 2.6 mm change in DSD.

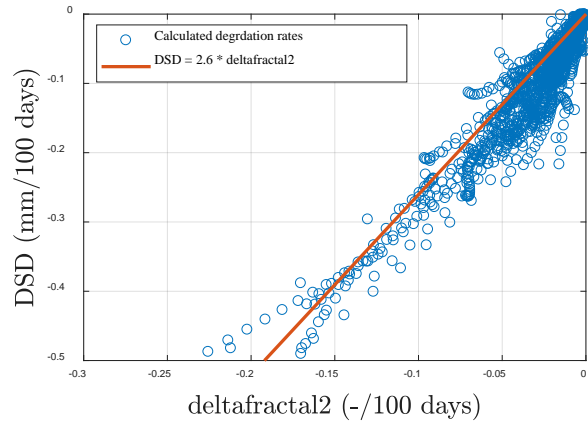


Figure 9: Relationship between deltafractal2 and DSD.

2.4. Bayesian update

In this section, the posterior distribution for deltafractal2 is drawn based on a Bayesian update scheme. The prior distribution of deltafractal2 (y) is assumed to follow a lognormal distribution with parameters μ_0 and σ_0 , that is

$$y \sim \text{Lognormal}(\mu_0, \sigma_0^2)$$

$$f(y) = \frac{1}{\sqrt{2\pi} y \sigma_0} e^{-\frac{1}{2} \left(\frac{\log(y) - \mu_0}{\sigma_0} \right)^2}, y > 0, \quad (1)$$

$$-\infty < \mu_0 < \infty, \sigma_0 > 0$$

Regression model implies

$$\log(y) = \bar{y}_R + \varepsilon; \quad \varepsilon \sim N(0, \sigma^2) \quad (2)$$

In which \bar{y}_R is the mean value estimate for the logarithm of y . Here, the observations on the explanatory (independent) variables of the regression model are used to update the prior distribution. Therefore, assuming that \bar{y}_R is a random

variable, we are interested in the conditional pdf: $f(\bar{y}_R|y)$. From the regression model (2), we have

$$\bar{y}_R = \log(y) - \varepsilon \quad (3)$$

Conditional on the value of y , from Equation 2, we have:

$$f(\bar{y}_R|y) = N(\log(y), \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{\bar{y}_R - \log(y)}{\sigma}\right)^2} \quad (4)$$

Bayes' rule reads

$$f(y|\bar{y}_R) = \frac{f(\bar{y}_R|y)f(y)}{f(\bar{y}_R)}$$

Plugging (1) and (4) into Bayes' rule will give

$$f(y|\bar{y}_R) = \frac{1}{2\pi y \sigma_0 \sigma} e^{-\frac{1}{2}\left[\left(\frac{\log(y) - \mu_0}{\sigma_0}\right)^2 + \left(\frac{\bar{y}_R - \log(y)}{\sigma}\right)^2\right]} \quad (5)$$

After doing some algebra, from (5) we will have

$$f(y|\bar{y}_R) = \frac{1}{\sqrt{2\pi} y \sigma_1} e^{-\frac{1}{2}\left(\frac{\log(y) - \mu_1}{\sigma_1}\right)^2}, \quad (6)$$

$$y > 0, -\infty < \mu_1 < \infty, \sigma_1 > 0$$

in which

$$\mu_1 = \frac{\sigma^2}{\sigma^2 + \sigma_0^2} \mu_0 + \frac{\sigma_0^2}{\sigma^2 + \sigma_0^2} \bar{y}_R \quad (7)$$

$$\sigma_1^2 = \frac{\sigma^2}{\sigma^2 + \sigma_0^2} \sigma_0^2 \quad (8)$$

The distribution in Eq. (6) shows that the posterior distribution of deltafractal2 is a lognormal random variable with parameters μ_1 and σ_1 . The posterior mean parameter μ_1 is a linear combination of the prior mean parameter μ_0 and the estimated mean value \bar{y}_R , each weighted proportional with the inverse of their variance.

The posterior standard deviation parameter σ_1 is a fraction of the prior standard deviation parameter σ_0 . The fraction is determined by the ratio of the error variance of the regression model to the total variance of regression error and the prior lognormal distribution.

Based on these results, an average and two-sided 95% confidence interval (CI) can be set for deltafractal2 as:

$$E(y|\bar{y}_R) = e^{(\mu_1 + \sigma_1^2/2)} \quad (9)$$

$$95\% \text{ CI for } y|\bar{y}_R = [F_{0.025}^{-1}(\mu_1, \sigma_1^2), F_{0.975}^{-1}(\mu_1, \sigma_1^2)] \quad (10)$$

where F_α^{-1} is the inverse of lognormal cumulative distribution with $\Pr(y < F_\alpha^{-1}) = \alpha$.

3. RESULTS

3.1. Regression modeling for ballast degradation rate

Regression analysis estimates the relationship between a dependent variable (response) and a set of independent variables (explanatory variables). According to the related literature (Sato, 1995; Ferreira & Murray, 1997; Iwnicki, Grassie, & Kik, 2000; Lyngby, 2009; Guler, 2014; Jönsson et al., 2016), the variables that can affect geometry degradation in open tracks are the number of past tamping operations, maximum permissible train speed; passing tonnage (MGT), the weather condition (seasonality, temperature and precipitation) and the current degraded state of the track geometry. Specific to the turnouts, the current degraded shape of the turnout and the section of the turnout can also influence degradation rate. In this study, the effects of all these factors on the ballast degradation rate are examined by using a linear regression model.

The degraded shape of the turnout is depicted in Figure 10 where the level of degradation in different sections of the turnout is different. This factor contributes in the regression model by introducing a variable named *front2mid* which is the ratio of degradation level in the front section (switch panel) to the level in the middle section.

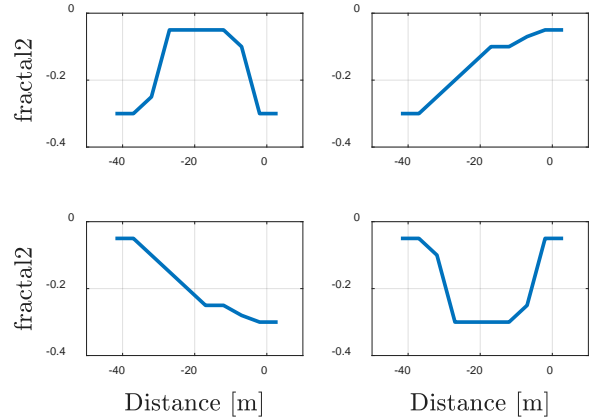


Figure 10: Degraded shape of turnout.

The regression model is shown in Eq. (11)

$$\log(\text{deltafractal2}) = f(\text{speed}, \text{ntamp}, \text{section}, \text{MGT}, Y_0, \text{season}, \text{front2mid}, \text{speed} * \text{MGT}, \text{speed} * Y_0, \text{MGT} * Y_0, \text{section} * \text{front2mid}) + \varepsilon \quad (11)$$

in which $f(\cdot)$ is a linear function and $\varepsilon \sim N(0, \sigma^2)$ is the error term. The normality of the error term is justified because deltafractal2 has shown best fit to lognormal distribution. Hence it is reasonably assumed that the dependent variable in this regression model i.e. $\log(\text{deltafractal2})$ is normally distributed. The variable *speed* is the maximum permissible train speed in the turnout and can be 120 or 250 km/h. The variable *ntamp* is the number of past tamping operations for

the turnout and can be 0, 1 or 2. The variable *section* indicates if *deltafractal2* is calculated for the switch panel, mid-section or the crossing panel. *MGT* is the average annual passing tonnage over the turnout. Y_0 is the degradation level or the *fractal2* at the time $t=0$ where *deltafractal2* is calculated for its next 100 days interval i.e. $[t, t+100]$. The variable *season* represents an index of cold/warm seasons in the time period $[t, t+100]$. For calculation of the *season*, the warmth of the seasons is accounted for by the values of 0,1,2,3 for the winter, autumn, spring and summer, respectively. The asterisk (*) in Eq. (11) shows the interaction of two variables. Four second order interactions of *speed*MGT*, *speed*Y₀*, *MGT*Y₀*, and *section*front2mid* are included in the model. It should be noted that in the regression model (11), the first three variables *speed*, *ntamp* and *section* are categorical variables since each can get discrete values.

A preliminary analysis of differences between the two rails right and left shows that the degradation rate of standard deviation of the longitudinal level is similar for both rails. This is also verified by Vale and Lurdes (2013), therefore, in the rest of the paper the analysis is presented for the rail right.

Table 3. Estimation of linear regression

Coefficient	Estimate	SE	tStat	Sig
(Intercept)	-6.41	0.20	-32.74	0.000
speed_250	0.61	0.18	3.43	0.001
ntamp_1	0.30	0.06	4.77	0.000
ntamp_2	0.63	0.09	6.90	0.000
section_mid (2)	0.86	0.20	4.33	0.000
section_crossing (3)	1.23	0.22	5.55	0.000
MGT	0.07	0.01	6.26	0.000
Initial deg. (Y_0)	-2.26	0.43	-5.20	0.000
season	0.22	0.05	4.07	0.000
front2mid	0.54	0.13	4.21	0.000
speed_250*MGT	-0.07	0.01	-6.57	0.000
speed_250* Y_0	-8.25	1.02	-8.06	0.000
MGT* Y_0	0.21	0.05	4.46	0.000
section_2*front2mid	-0.76	0.16	-4.59	0.000
section_3*front2mid	-1.02	0.19	-5.52	0.000
Estimated Dispersion: 1.32; $R^2 = 25\%$				
F-statistic vs. constant model: 40.6, p-value = 0.000				

With the data of all the turnouts under study, the regression model in the Eq. (11) is estimated (Table 3). The total number of data observation, as presented in Table 1 is 1801. The model is estimated with the ordinary least squares method. As seen, all the variables in the model are significant in 99.9% confidence level since the P-values of

t-tests all are below 0.1%. The regression model is significant and the estimated variance for the error term is $\hat{\sigma}^2 = 1.32$.

The results of estimated coefficients show that increasing speed limit, tamping, being in the crossing section, *MGT*, initial degradation level (Y_0), warm season, higher ratio of the front to the mid-section, all have increasing effect on ballast degradation rate in turnouts.

3.2. Bayesian updating for posterior distribution

The new information generated based on linear regression for the logarithm of degradation rate is used to update prior distribution of *deltafractal2* to posterior distribution. The prior lognormal distribution has parameters $\mu_0 = -4.21$ and $\sigma_0^2 = 1.314$ with mean 0.029 and 95th percentile 0.13. The error term in the regression model has the variance $\sigma^2 = 1.32$. Figure 10 shows the relation between the degradation rates from the regression i.e. $\exp(\bar{y}_R)$ and from the posterior distribution, where the mean value and 95% CI for the posterior degradation rate are presented.

As it can clearly be seen, the mean value of posterior degradation rate remains lower than the rate estimated by the regression model. From the prior distribution the 95th percentile for degradation rate is 0.13. Therefore, rates higher than 0.13 should be considered as severe degradation. For these high degradation rates, Figure 11 suggests a relatively wide confidence interval which implies a high level of uncertainty in the modelling environment.

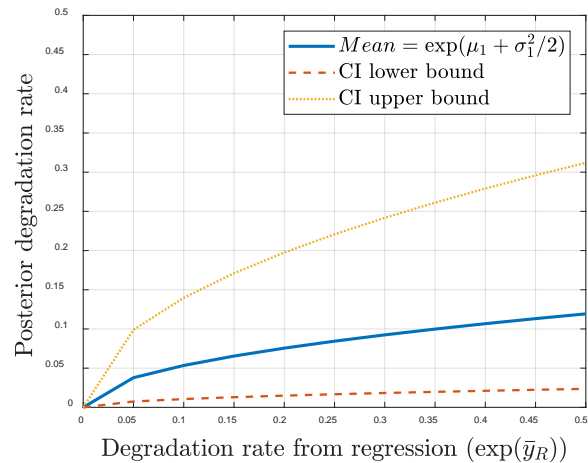


Figure 11. Posterior mean and 95% confidence interval for ballast degradation rate

From the results in Figure 11, it is also possible to compare the results of regression modelling and Bayesian updating for ballast degradation rate. For example, when the degradation rate from the regression model is 0.2, the 95% CI from the posterior distribution is [0.02, 0.2] which indicates regression model is over-estimating the

degradation rate compared to Bayesian approach and this comparison remains valid for rates above 0.13.

4. CONCLUSION

This article proposes a methodology for blending information of a generic ballast degradation rate model with the turnout-specific information to generate new information about ballast degradation rate in turnouts. The methodology uses the aligned measurements of track longitudinal level to calculate an index of ballast degradation. The proposed index is based on fractal dimensioning of vertical track profile. A regression modeling is introduced to estimate the effects of turnout-specific contributing factors on the ballast degradation rate. The paper also suggests a Bayesian updating scheme to estimate posterior distribution of ballast degradation rate. A demonstration of the methodology has been done on a large set of data coming from Danish railway turnouts. Lognormal distribution was the best fit distribution to the ballast degradation rate in turnouts. By building a posterior distribution, it is shown that the property of lognormal distribution is still valid for posterior distribution and the updated variance of posterior lognormal becomes considerably smaller when the variance of the error term in the regression model decreases. This paper produces distributions of the magnitude of ballast degradation in a 100-day time period. This degradation distribution is therefore of relevance to the turnouts' maintenance managers, avoiding the need to use generic and possibly inappropriate data for asset management.

The paper contributes to the scientific field by proposing an integrated methodology for ballast degradation modelling in turnouts. The advantage of this methodology is two-fold. First, it is the first turnout-focused study that uses fractal dimensioning as a validated technique to monitor the health state of the ballast based only on vertical track geometry profile. Second, its Bayesian updating allows integrating the prior knowledge of ballast degradation in turnouts with the new information collected for a specific turnout. This is especially important for building predictive models for ballast degradation which helps railway infrastructure managers in the transition to a predictive maintenance strategy.

Results also showed that the level of uncertainty in ballast degradation rate in turnouts is high, thus constructing degradation model based on the historical loaded track geometry data entails a high level of uncertainty in the estimated degradation rates. This conclusion calls for new monitoring strategies which are based on more frequent measurements or even continuous monitoring of ballast degradation. One possible solution is to use track-side measurement systems that make it possible to base ballast monitoring on track dynamic response when loaded by passing trains.

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BIOGRAPHIES

S. M. Asadzadeh, born in northern Khorasan, Iran 1979, obtained his M.Sc. and PhD degrees in Industrial Engineering from University of Tehran, Iran in 2013. His research interests are maintenance optimization, ergonomics and human performance in maintenance organizations, fault diagnosis and prognosis, data analytics for predictive maintenance, and statistical and machine learning. He is currently working as a postdoc at the Automation and Control group, Department of Electrical Engineering at the Technical University of Denmark.

Roberto Galeazzi was born in Ancona, Italy, on August 14th 1979. He received his M.Sc. degree in Electronic Engineering from the Technical University of Marche, Ancona, Italy, in 2005 and the Ph.D. degree in Automation and Control from the Technical University of Denmark, Kgs. Lyngby, Denmark, in 2010. He is an associate professor with the Department of Electrical Engineering at the Technical University of Denmark. His research interests include fault diagnosis and prognosis, fault-tolerant and reconfigurable control, nonlinear and adaptive control, state estimation. His focus areas are safety critical systems, intelligent autonomous systems and wind turbines. He is the co-author of 7 ISI journal papers, 25 peer-reviewed conference papers and 2 book chapters. Further he is the coeditor of one IFAC conference proceedings. He is member of IEEE and IFAC; he seats on the IEEE Technical

Committee on Aerospace Control and on the IFAC Technical Committee on Marine Systems, where he also serves as Co-Chair of the Technical Committee.

E. Hovad, born in Fredericia, Denmark 1982, obtained his M.Sc. in Mathematical Modelling and Computing (MMC) from the Technical University of Denmark (DTU) in 2011 and his industrial PhD at the Department of Mechanical Engineering at DTU together with the company DISA Industries A/S at 2017. He is currently working as a data scientist at the Department of Applied Mathematics and Computer Science at DTU COMPUTE.

JF. Andersen, born in Fredericia, Denmark 1991, obtained his M.Sc. in Mathematical Modelling and Computation from the Technical University of Denmark (DTU) in 2017. He is currently pursuing a PhD degree at the Department of Applied Mathematics and Computer Science at DTU. His research interests include scheduling in predictive maintenance, prognostics and discrete-event simulation.

Camilla Thyregod is special consultant in statistics and data analysis at DTU Compute, DTU. She received the M.Sc. in applied statistics (1998) and industrial PhD in applied statistics (2002) from IMM, DTU. She has extensive experience within industrial applications of statistics as well as within a broad range of other areas of application.

AFS. Rodrigues obtained his PhD in Civil Engineering from the New University of Lisbon (FCTUNL) in 2017, on the topic "*Viability and applicability of simplified models for the dynamic analysis of ballasted railway tracks*". He currently works as a Railway Engineer at the Track Systems and Development department at Banedanmark (Rail Net Denmark), focusing on rail maintenance through data analysis, including prognosis and prevention of rail defects.