Lamb Wave-based Damage Indicator for Plate-Like Structures

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ABSTRACT

Structural health monitoring based on ultrasonics typically involves complex data analysis. Ultrasound monitoring based on Lamb waves techniques are extensively used nowadays due to their efficiency in exploring large areas with relatively small attenuation. In recent years, baseline based methods have been developed to identify structural damage based on the mismatch between the measured signal and the baseline one. To this end, complex time-frequency transformations are required to obtain signal features such as the time of arrival or the energy content, as indicators of damage onset and growth. Notwithstanding this, on-board applications require highly efficient processing techniques due to information storage and exchange limitations. This paper proposes a very high efficiency signal processing methodology to obtain a novel cumulative damage factor using Lamb wave raw data. The new methodology has been tested using ultrasonic and damage data from a fatigue test in carbon-epoxy composite laminates. The data is taken from NASA Prognostics data repository. In view of the results, the method is able to efficiently detect the onset and extent of damage from early stages of degradation. Moreover, the results demonstrate a remarkable agreement between the growth of delamination area and the predicted cumulative damage factor.

1. INTRODUCTION

The assessment of the structural integrity is a key problem in the aerospace industry with important implications in safety, serviceability, and cost. A significant number of key structures are placed in poorly accessible locations. In this context, advanced technologies for structural health monitoring (SHM) that are able to work remotely and autonomously, in addition with advanced data processing techniques emerge as key enabling technology (Staszewski, Boller, & Tomlinson, 2004). More specifically, aircraft structures are prone to damage during service that propagates over time such as cracks (in metallic structures) and barely visible impact and fatigue damages (in composite materials). Monitoring the onset and growth of these damage features is crucial in terms of safety and maintenance efficiency.

Amongst the SHM techniques available, ultrasonics based on Lamb waves have been extensively used in plate-like structures due to its ability to explore large areas with relatively small attenuation (Su, Ye, & Lu, 2006). Generally, signal features such as energy content and time-of-flight are obtained straightforwardly from the time-amplitude Lamb wave representation. Alternatively, model-based inverse problems methodologies can be applied to extract a higher level of information from the signal (Chiachío, Bochud, Chiachío, Cantero, & Rus, 2017), but it is at the cost of a significant increase of computational time. To avoid such computational complexity while providing a high level of information, some authors have looked at signal features like the obtained from time frequency representations. These signal features are commonly monitored to obtain a better understanding of the Lamb wave behaviour when it interacts with damage. Then, the correlation of those signal features with physical damage features such as the length of fatigue crack or the area of delamination has been explored by many authors. In particular, the correlation between energy-based indices and crack growth in aluminium plates by using time-frequency transforms has been explored in (Ihn & Chang, 2004; Park, Yun, Roh, & Lee, 2006). On the other hand, the use of time-amplitude signal features has been reported in the literature to be a useful feature to monitor damage in aluminium plates (Leong, Staszewski, Lee, & Scarpa, 2005; Lee & Staszewski, 2007). Other damage indicators have been also reported in the literature. Amongst them, both the energy density and the time-of-flight have been related to the diameter of delamination in composite laminates in (Si & Wang, 2016). Likewise, energy-based probability damage index has been explored and empirically correlated with crack growth in a full-scale aircraft structure in (Qiu, Yuan, & Boller, 2017). Lastly, a mixed model based on phase and amplitude changes was developed to monitor crack length, and it was experimentally validated in alluminium plates in (Yang et al., 2016).

On a side note, such structural damage features are normally used to classify structures into several health states. Thus, the general understanding of the damage importance is facilitated to the end user. To this end, fuzzy logic provides a powerful tool that accounts for: (1) the structural damage classification in predefined importance levels and (2) the classification uncertainty, i.e. providing with the degree of membership of a damage into some predefined importance levels (Pawar & Ganguli, 2011). The damage monitoring of structures by using fuzzy techniques has been explored by many authors in the literature. Particularly, fuzzy sets have been used as the basis of the inference system for bridge damage diagnosis (Zhao & Chen, 2002). Bayesian updating framework has also been used to define health patterns in (Taha & Lucero, 2005). The dynamic behaviour of a structure was established by a wavelet norm index, which was then used to classify the health state according to the patterns.

This paper proposes a novel cumulative damage factor (CDF) based on time-of-flight differences between consecutive measurements in the same structure. The index relies on fuzzy logic principles which makes it robust against noise. The efficiency of this approach in terms of signal post-processing makes it suitable for real-time on-board applications. Additionally, a remarkable correlation between the CDF and the growth of delamination area in a carbon-fibre composite plate has been empirically observed.

The paper is organised as follows: Section 1 contains the introduction and motivation. Section 2 comprises the methodology used to obtain the CDF. Section 3 shows the effectiveness of the proposed method in a case study. Section 4 contains the discussion of the results obtained, and Section 5 concludes the paper and points out the future work.

2. METHODOLOGY

This Section provides the insight to the proposed methodology based on fuzzy logic principles. Firstly, the basic hypotheses and fundamentals are presented, secondly, the round-robin strategy is developed, and finally, the CDF is explained.

2.1. Fuzzy Fundamentals

The proposed methodology is based on three main hypotheses: (1) the structure under consideration is assumed to degrade and not self-healing is expected, (2) internal changes in the structure can be evidenced by time-of-flight differences between consecutively measured maximum and minimum peaks (at the same frequency) of the signal, and (3) noise is expected to occur from the acquisition system which will cause time-of-flight and amplitude mismatch for several measurements carried out in the same structural health conditions. From this standpoint, an algorithm based on fuzzy sets (Zadeh, 1965) is proposed to detect structural damage. These fuzzy sets are built from consecutive measurements of the structure in the same health state. Since a non-perfect signal is expected due to noise and other factors, different time-of-flights will be obtained for the same structure. The *range* (time from the maximum to the minimum) of time-of-flights for one particular peak in the signal is the basis for building the fuzzy sets. To this end, a band-pass filter (Smith et al., 1997) is applied in order to reduce noise and obtain a signal with the frequency of interest, i.e. the frequency of excitation. The time window representation of the band-pass filter in addition to the magnitude representation in frequency domain can be observed in Fig. 1. Then, an algorithm specifically designed to get maximum positive and minimum negative points of the cycles of the signal is applied. As a result, an arbitrary number of peaks N_{pk}, i.e. defined by the user, is obtained to built the fuzzy sets.

The test is then repeated an arbitrary number of times (N_t) in order to obtain a more robust information. This is carried out in the same structural health state and the time-of-flight of the peaks in each test is stored. Given the noisy nature of the measurements, instead of having the same time-of-flight for all the peaks, it results in a *range* with a certain spread, which is a representative measurement of the system uncertainty. The fuzzy sets are defined so that certain membership functions evaluate whether or not there has been a time mismatch. Therefore, when evaluating these functions with a new peak, the degree of membership of that peak in a predefined fuzzy set is obtained. Several potential scenarios arise when the function evaluation is carried out: (1) the new peak fits within the limits of the *range* and the membership function assigns it a value of 1, (2) the new peak is in a zone close

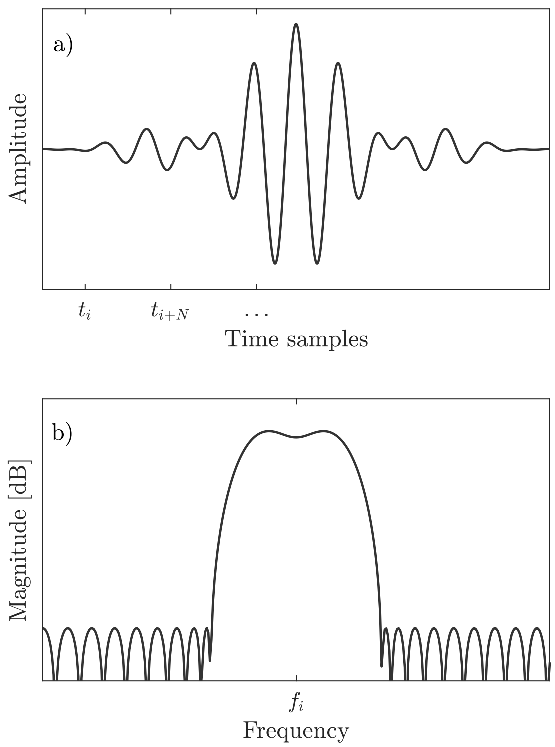

Figure 1. a) Time-amplitude representation of band-pass filter. b) Magnitude response of the filter. The same is designed so that the filter is centred in the frequency of interest (f_i).

to the *range*, in which is neither 0 nor 1, but an arbitrary membership function assesses the degree of membership, and (3) the new peak is out of the *range* and the fuzzy zone, then a zero value (0) is assigned. Note that the value of the function lies within the interval $[0, 1]$.

In addition, in order to get more information about the signal mismatch, i.e. whether there has been a lag or a lead in the time-of-flight, two different membership functions are simultaneously evaluated (Yen & Langari, 1999): (i) S-shaped membership function (SMF), which evaluates the mismatch in the left-part of the *range* and (ii) Z-shaped membership function (ZMF), which evaluates the right-part of the *range*. Figure 2 depicts the process of evaluating one peak over the two membership functions explained above. Observe that the new peak has a value of 1 in the ZMF, whereas the value in the SMF is lower than 1. Membership values for maximum or minimum peak (denoted with superscripts j and i respectively) of SMF (μ_ℓ^i) and ZMF (μ_r^i) are then combined into a unique value, by multiplying them as follows:

$$\mu^i_{\{min\}} = \mu^i_\ell \cdot \mu^i_r \tag{1}$$

$$\mu^j_{\{max\}} = \mu^j_\ell \cdot \mu^j_r \tag{2}$$

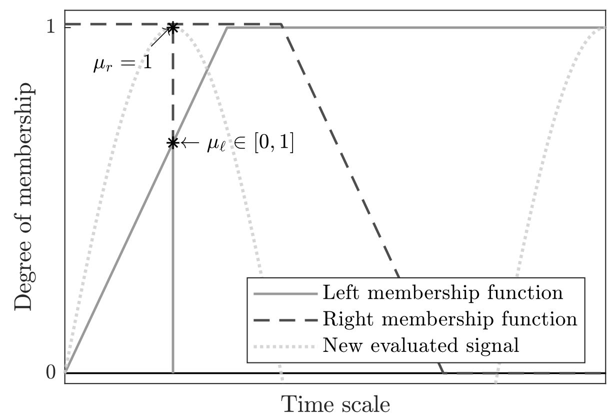

Figure 2. Fuzzy interval and membership function evaluation for both left and right-hand functions. The new signal is evaluated by selecting the maximum point of the cycle and simultaneously evaluating both membership functions. The membership value $\mu^i_{\{max\}}$ for the peak i would be the result of applying Eq. 2.

where $\mu^i_{\{min\}}$ stands for the membership value of a minimum peak and $\mu^j_{\{max\}}$ for a maximum peak. Finally, the minimum membership value of the total number of peaks (N_{pk}) is selected as follows:

$$\mu_m = \min\left(\mu^i_{\{min\}}, \mu^j_{\{max\}}\right) \tag{3}$$

where μ_m is the representative value of the signal m in terms of health assessing. This unique value is the indicator of the degree of health of the structure. In this regard, $\mu_m = 1$ means that no change in the structure is observed, $\mu_m = 0$ means that the structure has significantly changed, and a value $\mu_m \in [0, 1]$ indicates a certain degree of modification. Figure 3 summarises the proposed methodology in a flowchart. Two stages are depicted, the first one to define the fuzzy sets and membership functions, and the second one to evaluate a new signal and obtain the *structural health information*.

2.2. Round-Robin Configuration

Once obtained the representative membership value μ_m of the signal under evaluation, the interpretation of the data in terms of deciding whether or not there is damage in the structure is not trivial. Experimental failures, false positives or sensor faults can be source of potential misidentification. Hence, a more robust method to be able to diagnose a significant change in the structure is proposed based on a round-robin procedure. It consists of emitting from one actuator (A_n) and receiving with all the sensors (S), and then, repeating the pro-

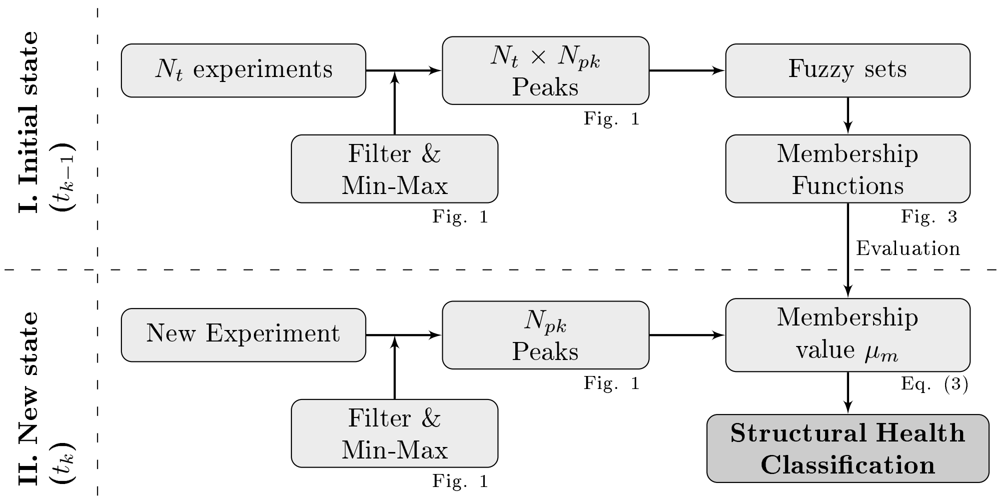

Figure 3. Flowchart of the proposed methodology for getting the health information of a structure by using Lamb wave raw data.

cedure with a different actuator, but receiving with the same sensors. Thus, a greater area can be swept and more information about the structure's health can be obtained. The method identifies not only a single membership value μ_m that stands for the degree of health (DoH) of the structure, but a matrix ($\mathbf{H_k}$) of $N_a \times N_s$ μ_m values, where N_a are the number of actuators and N_s the number of sensors at a time t_k.

DoH matrices $\mathbf{H_k}$ obtained at time t_k need to be assessed in depth. The information that carries is crucial in terms of health assessment. Several different scenarios are likely to appear which correspond with either damage status or errors' detection. Some examples are shown in Eqs. (4-6).

$$\mathbf{H_k} = \begin{array}{c c} \begin{matrix} A_1 & A_2 & A_3 & \dots & A_{N_a} \end{matrix} & \\ \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix} & \begin{matrix} S_1 \\ S_2 \\ S_3 \\ \\ S_{N_s} \end{matrix} \end{array} \tag{4}$$

$$\mathbf{H_k} = \begin{array}{c c} \begin{matrix} A_1 & A_2 & A_3 & \dots & A_{N_a} \end{matrix} & \\ \begin{bmatrix} 0.7 & 0.2 & 0.4 & \cdots & 0.4 \\ 0.3 & 0.5 & 0.5 & \cdots & 0.2 \\ 0.4 & 0.3 & 0.2 & \cdots & 0.3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0.2 & 0.3 & 0.6 & \cdots & 0.9 \end{bmatrix} & \begin{matrix} S_1 \\ S_2 \\ S_3 \\ \\ S_{N_s} \end{matrix} \end{array} \tag{5}$$

$$\mathbf{H_k} = \begin{array}{c c} \begin{matrix} A_1 & A_2 & A_3 & \dots & A_{N_a} \end{matrix} & \\ \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} & \begin{matrix} S_1 \\ S_2 \\ S_3 \\ \\ S_{N_s} \end{matrix} \end{array} \tag{6}$$

Equation (4) shows the example of a round robin test with no structural change detected. Equation (5) shows a certain level of mismatch in the signal which stands for a certain degree of damage. Lastly, Eq. (6) shows the case of a round-robin test when the structure has changed significantly with respect to the baseline state.

Note that decisions on whether the structure has changed or not can be made based on DoH matrices. Therefore, the identification of damage in plate-like structures is covered. Notwithstanding this, the proposed methodology is designed to detect structural modifications between two instants of time (t_k and t_o). Thus, an extension of this methodology applied to measurements taken throughout the life of the material, may provide information about the degree of degradation of the material.

2.3. Cumulative Damage Factor

In the field of SHM and PHM, a damage factor that is able to efficiently correlate with the level of degradation of a particular structure with damage features is desired. To this end, by subtracting 1 to the DoH matrix obtained above, the degree-of-damage (DoD) matrix can be obtained. DoD matrix provides a spacial damage indication, since the Round-Robin configuration allows to explore different paths in the plate-like structure. However, a representative value of the structure as a whole is assumed to be the mean of the values of the DoD matrix. This value is denoted as the DoD factor and is representative of the structural health state when compared with a baseline.

In order to use the DoD as a indicator of degradation in the structure, several consecutive measurements are to be taken from the structure. However, if a measurement taken at time t_k is evaluated in the fuzzy sets created with the baseline signal (i.e. measured in t_0) and the membership value results in $\mu_m = 0$, e.g. because of a severe damage scenario, it is expected that further measurements in $t_{k+1}, t_{k+2}, \ldots$ the mismatch between the measured signal with respect to the baseline is even greater, hence obtaining $\mu_m = 0$ for the new scenarios. Thus, only the damage scenarios between t_0 and t_k could be identified, and no further information about the degradation process could be extracted. When the structure modify its behaviour as much as the DoD factor is around 1, consecutive measurements will always result around the maximum DoD factor. A procedure that is able to detect and quantify further damage is required to use this damage factor as an estimator of degradation with high sensitivity. Hence, the proposed methodology consists of establishing a new baseline for each new measurement, so that the measurement at time t_k is compared against the baseline taken at time t_{k-1}.

The novel index is represented in time-DoD axes and linearly interpolated between consecutive measurement points t_{k-1} and t_k. Then, it is integrated along the time to obtain the *cumulative damage factor* (CDF, see Eq. (7)) that provides information about the degree of degradation of the structure under consideration.

$$CDF(t_k) = \alpha \left(\int_0^{t_k} DoD(t)\, dt \right)^{\beta} \tag{7}$$

Where α and β denote the power and scaling factors of the proposed degradation model. The model is selected in a power law form based on the general expression of a fatigue propagation process (Elber, 1971).

3. CASE STUDY

This section describes the case study applied to test the proposed methodology over a carbon fibre composite subjected to fatigue damage degradation. The proposed algorithm was applied to the ultrasonic measurements available through the SHM system based on PZT sensors (SMART Layer ® from Acellent Technologies Inc) which were placed on top and bottom of the specimen. The results were then assessed and compared with other SHM techniques that were carried out during the experiment, as shown below.

3.1. Experimental Set-up Considered

The fatigue test data has been taken from NASA Ames repository (`https://ti.arc.nasa.gov/tech/dash/groups/pcoe/prognostic-data-repository/`) (Saxena, Goebel, Larrosa, & Chang, 2008). These data were obtained from a set of run-to-failure fatigue experiments in cross-ply graphite-epoxy laminates. Torayca T700G unidirectional pre-impregnated (commonly known as *prepreg*) material was used for 15.24cm $\times$ 25.4cm coupons with dogbone geometry and $[0_2, 90_4]_s$ stacking sequence. A notch was created in these coupons to induce damage modes other than matrix-cracks, such as delamination. The tests were conducted under load-controlled tension-tension cycling loading with a maximum applied load of 31.13 (KN), a frequency $f = 5$ (Hz), and a stress ratio $R = 0.14$ (relation between the minimum and maximum stress for each cycle) (Saxena et al., 2011). Monitoring data were collected from a network of 12 piezoelectric (PZT) sensors using Lamb wave signals. In addition, X-rays images of the samples were taken from where the delamination area was estimated.

3.2. Relation with Damage Features

The proposed damage indicator, shown in Fig. 4, has a cumulative trend. The scaling and power parameters of Eq. (7) are obtained in this case study so that they fit the cumulative trend of the delamination area curve and the values are $\alpha = 1.5$ and $\beta = 0.5$, respectively. The DoD calculated by comparing consecutive measurements is shown in Fig. 5. It can be observed that at $10\,000$ cycles it has the severest damage episode with a DoD of 0.88. In addition, in the second measurement, at 2 cycles, another severe damage with DoD of 0.76 was also observed. A correlation between the proposed damage indicator and delamination area has been carried out. Remarkable agreement between the new factor and the delamination area can be observed in Fig. 4. It can be observed that the proposed CDF fits fairly well with the delamination area in the cross-ply laminate. Then, a direct comparison between the delamination area and the novel CDF has been explored.

Figure 6 a) shows the direct relation between the CDF and the delamination area. Linear, quadratic and cubic regression models are applied to assess the best polynomial fitting. The level of agreement for linear regression is notable in the range from 120 to 180 units of CDF. However, in the range from 0 to 100 units of CDF there are fewer data of delamination area, which makes difficult the assessment of the regression models' performance. In addition, Fig. 6 b) shows the residuals of the different polynomial curve fitting. The norm of the residuals for the different fittings are: (1) $1.84 \cdot 10^{-4}$ for linear fitting, (2) $1.81 \cdot 10^{-4}$ for quadratic fitting, and (3) $1.53 \cdot 10^{-4}$ for cubic fitting.

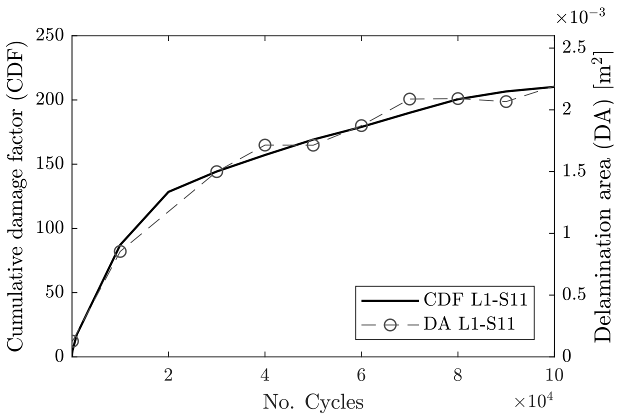

Figure 4. Cumulative damage factor (CDF) compared with delamination area (DA) in the laminate.

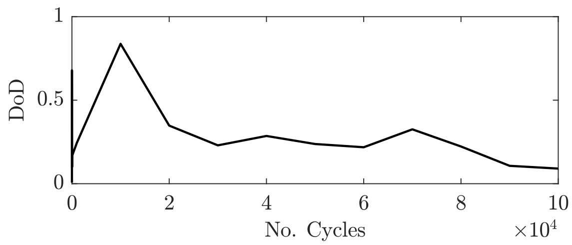

Figure 5. DoD computed over the fatigue test consecutive measurements.

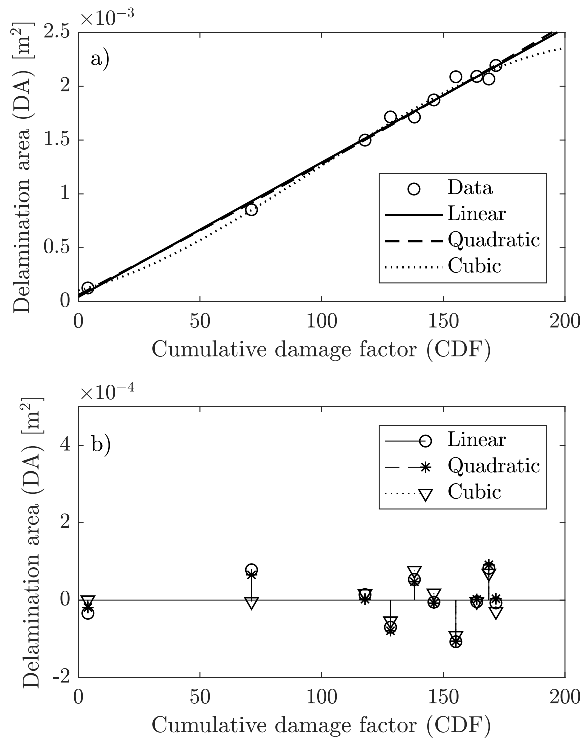

Figure 6. a) Delamination area vs CDF for the laminate. Furthermore, polynomial curve fitting of several degrees is shown. b) Residuals between the data and the polynomial curves is depicted.

4. DISCUSSION

The proposed CDF is shown to be highly correlated with the delamination area with a certain relation in cross-ply laminates. The structural modification that the delamination induces in the plate, e.g. a decrease in the Young's modulus, is affecting the time-of-flight of the ultrasonic measurements by a slowing-down of the wave propagation velocity. This is captured by the DoD and then integrated over the time to obtain the CDF. Thus, it has been shown that the CDF is able to capture a significant change in the structural behaviour of the laminate. However, in the dataset, low and high values of CDF lack of delamination area information. Therefore, a desirable further work would be the increase of measurements at the beginning of the loading process, as well as for high number of cycles in the fatigue test, thus providing more information in order to decide the more appropriate regression type. Even though, the agreement between the three models observed in Fig. 6 is remarkable, but the problem of model selection arises. The cubic polynomial curve provides the best fitting in terms of norm of residuals, however, the linear and quadratic ones also represents the trend reasonably well. In this regard, a Bayesian model-class discussion would solve the question more rigorously, giving the trade-off between information gained and data fitting of each model.

Once a robust model that relates the delamination area with the CDF is established, one can obtain a powerful SHM tool that provide indirect and fast estimations of the delamination area. Amongst the potential benefits expected from this methodology, the following can be highlighted: (1) the strength of using an efficient model with potential robust applications, e.g. in noisy environments and (2) the possibility to avoid expensive techniques as X-rays to infer the delamination area due to the remarkable agreement between the CDF and the growth of delamination area.

5. CONCLUSIONS AND FUTURE WORK

A novel methodology to obtain a new damage factor for plate-like structures has been proposed. In addition, a case study with a complex structure, a CFRP composite cross-ply laminate, has been shown with a remarkable level of agreement between the factor and the delamination area. The proposed procedure has proven effective in efficiently modelling the behaviour of the growth of delamination area in cross-ply laminates.

A more in-depth study with more specimens is under development. In addition, a probabilistic study to select the most plausible model between the novel CDF and the delamination area is under study by the authors. The base for a new damage factor for both probabilistic SHM and PHM will be explored as well.

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REFERENCES

Chiachío, J., Bochud, N., Chiachío, M., Cantero, S., & Rus, G. (2017). A multilevel Bayesian method for ultrasound-based damage identification in composite laminates. *Mechanical Systems and Signal Processing*, *88*, 462–477.

Elber, W. (1971). The significance of fatigue crack closure. In *Damage tolerance in aircraft structures.* ASTM International.

Ihn, J.-B., & Chang, F.-K. (2004). Detection and monitoring of hidden fatigue crack growth using a built-in piezoelectric sensor/actuator network: I. Diagnostics. *Smart materials and structures*, *13*(3), 609.

Lee, B., & Staszewski, W. (2007). Lamb wave propagation modelling for damage detection: II. Damage monitoring strategy. *Smart materials and structures*, *16*(2), 260.

Leong, W., Staszewski, W., Lee, B., & Scarpa, F. (2005). Structural health monitoring using scanning laser vibrometry: III. Lamb waves for fatigue crack detection. *Smart Materials and Structures*, *14*(6), 1387.

Park, S., Yun, C.-B., Roh, Y., & Lee, J.-J. (2006). PZT-based active damage detection techniques for steel bridge components. *Smart Materials and Structures*, *15*(4), 957.

Pawar, P. M., & Ganguli, R. (2011). *Structural health monitoring using genetic fuzzy systems*. Springer Science & Business Media.

Qiu, L., Yuan, S., & Boller, C. (2017). An adaptive guided wave-Gaussian mixture model for damage monitoring under time-varying conditions: Validation in a full-scale aircraft fatigue test. *Structural Health Monitoring*, *16*(5), 501–517.

Saxena, A., Goebel, K., Larrosa, C. C., & Chang, F.-K. (2008). *CFRP Composites Data Set.* NASA Ames Research Center, Moffett Field, CA.

Saxena, A., Goebel, K., Larrosa, C. C., Janapati, V., Roy, S., & Chang, F.-K. (2011). *Accelerated aging experiments for prognostics of damage growth in composite materials* (Tech. Rep.). National Aeronautics and Space Administration Moffett Field CA Ames Research Center.

Si, L., & Wang, Q. (2016). Rapid multi-damage identification for health monitoring of laminated composites using piezoelectric wafer sensor arrays. *Sensors*, *16*(5), 638.

Smith, S. W., et al. (1997). *The scientist and engineer's guide to digital signal processing*. California Technical Pub. San Diego.

Staszewski, W., Boller, C., & Tomlinson, G. R. (2004). *Health monitoring of aerospace structures: smart sensor technologies and signal processing*. John Wiley & Sons.

Su, Z., Ye, L., & Lu, Y. (2006). Guided Lamb waves for identification of damage in composite structures: A review. *Journal of sound and vibration*, *295*(3-5), 753–780.

Taha, M. R., & Lucero, J. (2005). Damage identification for structural health monitoring using fuzzy pattern recognition. *Engineering Structures*, *27*(12), 1774–1783.

Yang, J., He, J., Guan, X., Wang, D., Chen, H., Zhang, W., & Liu, Y. (2016). A probabilistic crack size quantification method using in-situ Lamb wave test and Bayesian updating. *Mechanical Systems and Signal Processing*, *78*, 118–133.

Yen, J., & Langari, R. (1999). *Fuzzy logic: intelligence, control, and information* (Vol. 1). Prentice Hall Upper Saddle River, NJ.

Zadeh, L. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338 - 353.

Zhao, Z., & Chen, C. (2002). A fuzzy system for concrete bridge damage diagnosis. *Computers & structures*, *80*(7-8), 629–641.