Preventive maintenance optimization using a Hybrid Multi-Objective Evolutionary Algorithm

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ABSTRACT

This paper is focused on the problem of preventive maintenance optimization in a manufacturing environment, to determine the optimal preventive maintenance frequencies for equipment under cost and profit criteria, considering production, quality and maintenance aspects. The paper is based on a previously developed maintenance model, to execute a benefit and cost optimization process using a Hybrid Multi-Objective Evolutionary Algorithm (Hybrid MOEA) that combines a global search method with a local one. The hybrid algorithm combines the capabilities of both worlds, using a global search technique to effectively explore wide parameter spaces, deal properly with function non-linearities and avoid falling into local optimal solutions, and combining it with the capacities of local search methods to efficiently converge into local optimal solutions. The hybridization is done according to two different schemes. Firstly, 'a posteriori' scheme has been implemented, where the MOEA runs for a number of generations obtaining an approximation of the Pareto front to apply then a local search from each non-dominated solution of the front. Secondly, an 'on-line' scheme has been developed, where in each generation (or after a reduced number of generations) of the evolutionary algorithm a local search is applied on each non-dominated solution to return then the improved solutions to the MOEA as the current population. Both hybrid schemes have been applied to an industrial manufacturing case where the benefit of implementing the hybrid optimization approach is shown, by comparing the hybrid schemes with the MOEA.

1. INTRODUCTION

Optimization of maintenance activities developed in the different systems of an industrial plant is of great importance in plant management and operation. In the past decades the maintenance optimization problem of production systems has been extensively studied in the literature and different methods and tools focused on the maintenance optimization have been proposed). In this context Sharma, Yadaba and Deshmukh (2011) present a review of the existing literature on maintenance optimization models. The authors classify the optimization models and they identify the possible gaps in the topic. Later de Almeida et al. (2015) elaborate a complete review of maintenance optimization problems from a multicriteria perspective.

For the optimization approach presented herein the cost and profit models developed by Sanchez and Goti (2006) are considered and applied to a production system of a Spanish company. The cost model includes corrective and preventive maintenance costs, along with quality costs and cost related to poor performance (reduction of the speed of equipment due to deterioration problems) and the profit model includes the benefits obtained to the sale of products. Thus, the objective of this paper is the preventive maintenance optimization based on costs and benefit criteria using a hybrid MOEA approach. This paper is organized as follows. Section 2 introduces a imperfect maintenance model. Section 3 presents the cost and benefit quantification models used in the optimization process, followed by Section 4, summarizing the multiobjective optimization problem. Section 5 presents the developed hybrid algorithms, whereas an application case focusing on maintenance intervals optimization for a simplified production system is shown in Section 6. Finally, Section 7 presents the conclusions.

2. IMPERFECT MAINTENANCE MODEL

Traditionally, the effect of the maintenance activities on the state of equipment is based on three situations: a) perfect maintenance activity which assumes that the state of the component after the maintenance is “As Good as New” (GAN), b) minimal maintenance which supposes that activity leaves the equipment in “As Bad as Old” (BAO) situation, and c) imperfect maintenance which assumes that the activity improves the state of the equipment by some degree depending on its effectiveness. Last situation is closer to many real situations.

Several approaches have been developed to simulate imperfect maintenance. In this paper, an age reduction preventive maintenance model, named Proportional Age-Set Back (PAS), proposed by Martorell et al. (1998) is used to model the effect of the maintenance activities on the equipment.

In the PAS approach, each maintenance activity is assumed to shift the origin of time from which the age of the component is evaluated. PAS considers that the maintenance activity reduces proportionally, in a factor of ε, the age that the component has immediately before it enters maintenance. ε ranges in the interval [0,1]. If $\varepsilon = 0$, the PAS model simply reduces to a BAO situation, while if $\varepsilon = 1$ it is reduced to a GAN situation. Thus, this model is a natural generalization of both GAN and BAO models in order to account for imperfect maintenance.

As stated in Sanchez and Goti (2006), in addition to the conditions established by the PAS model, if it is assumed that a) preventive maintenance is periodically realized on the component with constant effectiveness ε and periodicity M, and b) operation conditions remain steady, it is established that the relation between the age of the component $w_m(t)$ immediately before $w^-_{m\text{-}1}$ and after $w^+_{m\text{-}1}$ the m-1[th] maintenance activity is defined by Eqn. (1):

$$w^+_{m\text{-}1} = w^+_{m\text{-}1} \cdot (1-\varepsilon) \tag{1}$$

As maintenance activity reduces the deterioration level of the component applying the effectiveness parameter ε since the origin of time from which it was installed, the progression of the age of the component will follow an asymptotic behavior:

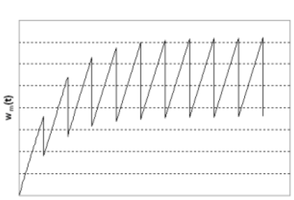

Figure 1. Evolution of the age of a component over the chronological time.

As stated in Sanchez and Goti (2006), this asymptotic behavior is limited by the age values just before and after a maintenance activity is performed, shown in Eqns. (2) and (3):

$$w^-_{m\text{-}1} = w^-_a = \frac{M}{\varepsilon} \tag{2}$$

$$w^+_{m\text{-}1} = w^+_a = M \cdot \left(\frac{1}{\varepsilon} - 1\right) \tag{3}$$

Due to this behavior it is possible to obtain an age-dependent reliability model in which the induced or conditional failure rate, in the period m, after the maintenance number m-1, is given by:

$$h_m(w) = h(w(t,\varepsilon)) + h_0 \qquad w \geq w^+_{m\text{-}1} \tag{4}$$

where h_0 represents the initial failure rate of the component, that is, when the equipment is installed and h(w(t,ε)) is the failure rate of the component. Adopting a Weibull model for the failure rate, the expression for the induced failure rate after the maintenance number m-1 can be written as:

$$h_m(w) = \left\{\lambda^\gamma \cdot \gamma \cdot [w_m(t,\varepsilon)]^{\gamma-1}\right\} + h_0 \quad w \geq w^+_{m\text{-}1} \tag{5}$$

where λ is the scale parameter, γ the shape parameter, and $w_m(t,\ \varepsilon)$ represents the age of the component:

$$w_m(t,\ \varepsilon) = w^+_{m-1} + t \qquad w \geq w^+_{m\text{-}1} \tag{6}$$

Influenced by the age, function $h_m(w)$ is also asymptotic, determined by Eqns. (7) and (8):

$$h^-_a = \lambda^\gamma \cdot \gamma \cdot \left(w^-_a\right)^{\gamma-1} + h_0 \tag{7}$$

$$h^+_a = \lambda^\gamma \cdot \gamma \cdot \left(w^+_a\right)^{\gamma-1} + h_0 \tag{8}$$

Finally, in order to introduce the effect of maintenance activities into the cost and profit models, to be presented in the following section, it is derived an averaged standby failure rate over the component’s life based on a double averaging process. First, it is formulated the average failure rate, h^*_m, over the period between two consecutive maintenance activities, m-1 and m. Next, it is formulated the average failure rate, h^*, over the analysis period, L. Thus, the value of h^* is given by:

$$h^* = (M)^{\gamma-1} \cdot \left(\frac{\lambda}{\varepsilon}\right)^\gamma \cdot \left[1 - (1-\varepsilon)^\gamma\right] + h_0 \tag{9}$$

Thus, as the influence of periodicities of preventive maintenance activities on the deterioration level of equipment are modelled based on the Eqns. (1) to (9), the following Section details how this deterioration effects on productive costs and benefits.

3. COST AND BENEFIT MODELS

This Section presents the models used in the optimization process. Cost related to preventive and corrective maintenance activities, cost related to the production speed lost, quality cost and profit model are presented. All these parameters are affected by the decision vector **x**, composed by m maintenance activities executed each of them on a single component with an M periodicity value.

3.1. Maintenance costs

The relevant maintenance costs of the equipment include the contributions due to preventive maintenance (PM) activities, along with the corrective maintenance (CM) events, consequence of idling, minor stoppage and failure/breakdowns. Thus, the cost associated with preventive maintenance can be evaluated in the analysis period, L, as:

$$C_{pm}(\mathbf{x}) = \frac{L}{M} d_{pm} \cdot c_{hpm} \tag{10}$$

where d_{pm} and ch_{pm} represent, respectively, the mean time and the average hourly cost of performing PM, and **x** represents the vector of decision variables.

The cost contribution due to corrective maintenance is given by:

$$C_{cm}(\mathbf{x}) = u_r(\mathbf{x}, M).\frac{L}{M}.c_{hcm} \cdot d_{cm} \tag{11}$$

being d_{cm} and $c_{hcm,}$ respectively, the mean time and the average hourly cost of performing corrective maintenance and $u_r(\mathbf{x},M)$ is time-dependent unreliability for discontinuous evaluated as:

$$u_r(x,M) = \rho + (1-\rho).\left(1 - e^{-h^*.M}\right) \tag{12}$$

where ρ is the probability of failure on demand, and h* is evaluated using Eqn. (9).

3.2. Cost related to the production speed lost due to aging

Traditionally, in the literature the production rate (speed) of the equipment is assumed to be predetermined and constant along the component life. Nevertheless, it is logical to think that the production speed as consequence of the aging of equipment decreases. Thus, Nakajima (1988) justifies the gap between the real equipment speed and the designed speed causes significant economic losses which are often neglected or underestimated.

In this paper it is assumed that the production rate depends on the equipment age. According to this assumption, the equipment can be working at different production rates depending on its age. Herein, for sake of simplicity of the formulation it is assumed a linear relationship between equipment age and production speed. Based on it the production speed after the m-maintenance activity can be evaluated as:

$$S_m(w) = S_o - \tau.w_m(t,\varepsilon) \tag{13}$$

where S_0 is the initial (e.g. as per design) production speed, τ represents the speed reduction coefficient and $w_m(t,\varepsilon)$ is the age of the component after the maintenance m-1, which adopting a PAS model is given by Eqn. (6).

The behavior of the function $S_m(w)$ is also asymptotic, as it was observed for the age of the component when the PAS model is adopted which are given by:

$$S_a^- = S_0 - \tau.\frac{M}{\varepsilon} \tag{14}$$

$$S_a^+ = S_0 - \tau \cdot M \cdot \left(\frac{1}{\varepsilon} - 1\right) \tag{15}$$

The production speed loss effect on the equipment can be introduced in the cost model, considering an averaged production speed over the analysis period L. So, using a similar process as the one described for the induced failure rate in Section 2, it is possible to derive the following expression for the production speed under the PAS approach:

$$S^* = S_0 - \left[\tau \cdot M \cdot \left(\frac{2-\varepsilon}{2\varepsilon}\right)\right] \tag{16}$$

Adopting, the value of the speed production average S* given by Eqn. (23) is possible to determine the “production time lost” related to a reduced speed (t_{sl}) and considering only the fraction of the production system is available as follows:

$$t_{sl}(\mathbf{x}) = \left(1 - A_s(x) \cdot \frac{S^*}{S_0}\right).L \tag{17}$$

where $A_s(\mathbf{x})$ is the availability system which is obtained as

$$A_s(\mathbf{x}) = 1 - U_s(\mathbf{x}) \tag{18}$$

being $U_s(\mathbf{x})$ the unavailability system is to be evaluated using the system fault tree and the single component unavailability contributions. These contributions are $u_r(\mathbf{x})$ which is evaluated using Eq. (12), $u_{cm}(\mathbf{x})$ which is the unavailability due to corrective maintenance given by:

$$u_{cm}(\mathbf{x}) = \frac{1}{M} \cdot u_r(\mathbf{x}, M) \cdot d_{cm} \tag{19}$$

and $u_{pm}(\mathbf{x})$ that represents the unavailability associated with preventive maintenance in the L period given by:

$$u_{pm}(\mathbf{x}) = \frac{1}{M} \cdot d_{pm} \tag{20}$$

Finally, the cost related to the production speed loss of the equipment (C_{sl}) in the period L can be evaluated proportionally to the production time lost as:

$$c_{sl}(\mathbf{x}) = c_{hsl} \cdot t_{sl}(\mathbf{x}) = C_{hsl} \cdot L \cdot \left(1 - A_s(\mathbf{x}) \cdot \frac{S^*}{S_0}\right) \cdot L \tag{21}$$

where c_{hsl} is the average hourly cost due to non-produce items.

3.3. Quality costs

Aging of equipment is one of the factors that cause defective product outputs. However, although a strong relationship between aging and quality exist, traditionally both factors are modeled as separate problems (Ben-Daya & Rahim, 2000). Aging of equipment depend on maintenance activities developed on it, so equipment can be maintained in optimal operational conditions through adequate maintenance preventive. Therefore, to optimize the maintenance schedule it is necessary to develop a model that links the quality and maintenance. PAS model, presented in Section 2, assumes that each preventive maintenance activity reduces the age of the equipment depending on an effectiveness parameter. The change in the age of the equipment introduced by the PAS model affects the time distribution to the system swaps to the out-of-control and consequently the expected amount of nonconforming items.

In this Section it is derived a quality cost model which considers the preventive maintenance effect on the component age based on the PAS model. The model is developed under the following assumptions: 1) The equipment only produces non-conforming items with constant rate, α, while the process is out-of-control 2) The time to the system swaps out-of-control follows a Weibull distribution which depends on the age of the equipment, 3) The preventive maintenance and the process inspection are performed simultaneously, 4) Inspections are error free, and 5) the process is restored to the in-control state when the preventive maintenance activity is realized. To model the quality costs it is necessary to determine the fraction of time during which the process is in-control state named $\kappa_m(w)$.

$$\kappa_m(w) = \int_{w_{m-1}^{+}}^{w} t \cdot f_m(w(t,\varepsilon))dw \tag{22}$$

where $f(w_m(t,\varepsilon))$ is the density function, which can be obtained using the conditional hazard function as:

$$f_m(w(t,\varepsilon)) = h^* \cdot e^{-\int_{w_{m-1}^{+}}^{w_m^{-}} h^* dw} \approx h^* \cdot e^{-h^* \cdot M} \tag{23}$$

as a consequence of the asymptotic behavior of the hazard function when a PAS model and a low failure rate are considered.

$$\kappa^* = \frac{1}{M}\int_0^M t \cdot f_m(w(t,\varepsilon)) \cdot dw \approx \frac{1}{2} \cdot h^* \cdot e^{-h^* \cdot M} \tag{24}$$

Thus, quality costs are:

$$c_q = S^* \cdot \left[A_s(x)(1-\kappa^*)\right] \cdot C_\alpha \cdot \alpha \cdot L \tag{25}$$

3.4. Profit

In order to quantify the consequences of a given preventive maintenance schedule in economic terms it is necessary to consider the benefits obtained to its implementation. So, a net profit function, T, that denotes the benefits obtained to the sale of products, is introduced as:

$$T = n \cdot \psi \tag{26}$$

where n is the number of non-defective items produced in the period analysis, L, and ψ is the estimated margin of a single product.

The number of non-defective items produced in the period L can be evaluated considering the time that the process has been in-control and out-of-control state. Thus, if the process is in an out-of-control state is produced (1-α)% of non-defective product while if the process is in an in-control state elaborates a 100% non-defective product. Therefore, the profit can be evaluated as:

$$T = S^* \cdot A_s(\mathbf{x}) \cdot \left[\kappa^* + \left(1-\kappa^*\right) \cdot \left(1-\alpha\right)\right] \cdot L \cdot \psi \tag{27}$$

4. COST AND PROFIT OPTIMIZATION PROBLEM

Maintenance intervals optimization based on cost and profit criteria can be formulated as a multi-objective optimization problem (MOP). A general MOP includes a set of parameters (decision variables), a set of objective functions, and a set of constraints. Objective functions and constraints are defined in terms of the decision variables using the models presented in the previous section. The optimization goal can be formulated to optimize a vector of functions of the form (Martorell, Sanchez, Carlos, & Serradell, 2004):

$$\mathrm{y}(\mathbf{x}) = \mathrm{f}(\mathbf{x}) = (\mathrm{f}_1(\mathbf{x}), \mathrm{f}_2(\mathbf{x}), \ldots, \mathrm{f}_n(\mathbf{x})) \tag{28}$$

subject to the vector of constraints:

$$\mathrm{g}(\mathbf{x}) = (\mathrm{g}_1(\mathbf{x}), \mathrm{g}_2(\mathbf{x}), \ldots, \mathrm{g}_n(\mathbf{x})) \tag{29}$$

where:

$$\mathbf{x} = \{\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{x}_n\} \in \mathrm{X} \tag{30}$$

$$\mathbf{y} = \{\mathrm{y}_1, \mathrm{y}_2, \ldots, \mathrm{y}_n\} \in \mathrm{Y} \tag{31}$$

and **x** is the decision vector (vector of decision variables), **y** the objective vector, **X** the decision space and **Y** is called the objective space, that is to say **Y**=f(**X**).

In the optimization proposed in this paper the cost and profit criteria are formulated using the expressions obtained in Section 3. Both models depend on maintenance intervals which act as decision variable and are encoded in the decision vector, **x**. So, the vector of bi-objective function, f(**x**), is defined as:

$$\mathrm{f}(\mathbf{x}) = \{\mathrm{C}(\mathbf{x}), \mathrm{T}(\mathbf{x})\} \tag{32}$$

and the objective is to minimize the function C(**x**) and maximize a profit function T(**x**). C(**x**) is the cost system which is evaluated as sum of the maintenance, production speed lost and quality costs for each component of the system which are evaluated using Eqns. (10), (11), (21) and (25), respectively. T(**x**) is the profit function which is evaluated using Eqn. (27).

In addition, the vector of restriction, g(**x**) is formulated as:

$$\mathrm{g}(\mathbf{x}) = \{\mathrm{C}(\mathbf{x}) \le \mathrm{C}_i, \mathrm{U}(\mathbf{x}) \le \mathrm{U}_i\} \tag{33}$$

where C_i and U_i represent the cost, profit and unavailability associated with the initial values for the decision vector, respectively.

5. HYBRID MULTI-OBJECTIVE EVOLUTIONARY ALGORITHMS

The optimization of maintenance models under multiple criteria can be solved by using MOEAs. The Multi-Objective maintenance optimization problem of production systems using MOEAs has been extensively studied in the literature. Although MOEAs are extremely efficient optimization techniques they may not be the most efficient ones in all the phases of the search process. Their performances may therefore be improved by performing a hybridization of the algorithms with other techniques (such as Simulated Annealing, Tabu Search, etc.) which perform better in a given search phase. The hybridization of a MOEA with a Local Search (LS) method may be an interesting approach as it combines the main advantages of each technique: thus, MOEAs confer hybrid algorithms a high exploratory capability, and decreases at the same time the number of generations needed to reach convergence, whereas the low precision of classical MOEAs is compensated by the local method which can locate accurately the closest solution associated to each chromosome.

In this case, the Nondominated Sorting Genetic Algorithm (NSGA-II) (Deb, Pratap, Agarwal, & Meyarivan, 2002) is combined with the Nelder and Mead Simplex Search method or NMS (Nelder & Mead, 1965), a local search method, to develop a hybrid MOEA.

5.1. Nondominated Sorting Genetic Algorithm II

Although the NSGA-II has been improved for more than two objective optimizations, it is still a very efficient MOEA for bi-objective optimizations, it incorporates: a) a faster non-dominated sorting approach, b) an elitist strategy i.e. the best non-dominated individuals are preserved from one generation to another, and c) no niching parameter.

Diversity is preserved using a crowding comparison criterion in the tournament selection and in the phase of population reduction. The crowding comparison operator assumes that each individual, **x**, in the population has two attributes: nondomination rank ($\mathbf{x}_{rank}$) and crowding distance ($\mathbf{x}_{distance}$), which is a measure of density of solutions in the neighbourhood. The crowding distance value of a solution **x** is obtained by the perimeter of the cuboid formed by using the nearest neighbours as vertices. Solutions of the last accepted front are ranked according to the crowding comparison distance. Considering both attributes a partial order relation is used as follows:

$$\begin{aligned} &\mathbf{x} \le \mathbf{y} \text{ if } (\mathbf{x}_{rank} < \mathbf{y}_{rank}) \\ &\text{or } ((\mathbf{x}_{rank} = \mathbf{y}_{rank}) \text{ and } (\mathbf{x}_{distance} > \mathrm{y}_{distance})) \end{aligned} \tag{34}$$

Thus, the solutions with the lower rank are preferred and, if two solutions have the same rank the solution with larger crowding distance is preferred.

An important feature to be considered is how the algorithm guides searches towards feasible region in presence of constraints considering domination concepts. According to the procedures of NSGA-II, the definition of domination between two solutions is the stated below: a solution is *i* said to constrained-dominate a solution *j*, if any of the following conditions is true.

1) Solution *i* is feasible and solution *j* is not.

2) Solutions *i* and *j* are both infeasible, but solution *i* has a smaller overall constraint violation.

3) Solutions *i* and *j* are feasible and solution *i* dominates solution *j*.

The effect of using this constrained-domination principle is that any feasible solution has a better non-domination rank than any infeasible solution. All feasible solutions are ranked according to their non-domination level based on the objective function values. However, among two infeasible solutions, the solution with a smaller constraint violation has a better rank.

The NSGA-II follows the working procedure defined next:

Step 1. Fix N (population size), i=1 (number of generation), t = N (number of f(**x**) evaluations) and t_{maxGA}.(maximum number of f(**x**) evaluations using the genetic algorithm)

Step 2. Create a random parent population P_i of size N.

Step 3. If $t>t_{maxGA}$ return P_i else:

Step 4. Form a combined population of size 2N as $T_i = P_i \cup Q_i$.

- Q_i (i>1)= offspring population
- T_i size N and equal to P_i in the first interaction

Step 5. Penalization (according to restriction violations).

Step 6. Identify non dominated fronts F_1, F_2, …., F_k. Thus each solution is assigned a fitness equal to its non-domination level.

Step 7. For each non-dominated front calculate crowding distance of the solutions F_j.

Step 8. Create P_{i+1} as the N best individuals from P_i.

Step 9. Select randomly N couples from P_{i+1} using a binary tournament selection.

Step 10. Create offspring population Q_{i+1} applying crossover and mutation.

Step 11. Calculate the M genes which have not been evaluated in previous generations, and evaluate the N-M unevaluated genes

Step 12. Do i=i+1 and t=t+N-M.

Step 13. Go to step 4.

5.2. Nelder–Mead Simplex method

The NMS is a popular direct search minimization method. Using the NMS, if the objective function has n decision variables, the algorithm begins from the n+1 initial function values of these decision variables which define a simplex. A simplex is a geometrical figure consistent in n dimensions of n+1 points, V_0, V_1, V_2, … , V_n. For the initial generation, if a point of a simplex is taken as the origin, the n other points define vector directions that span the n-dimension vector space. The method finds a minimum for the objective function by performing concurrent searches following multiple directions, determined by the points that constitute the limits of a simplex, until a termination criterion is verified. So, the objective function is evaluated at every point that constitutes the simplex. In a minimization problem, the point with the highest numerical value is replaced by a better point which is obtained through reflection, expansion and contraction operations.

The reflection consists on the worst point is reflected through the simplex centroid. The reflection can be followed with an expansion if the reflected point is the best out of all points. The expansion step extends the reflection point along with the reflection point. The contraction step is performed when the worst point is at least as good as reflected point.

The NMS method requires a single objective function. A weighted sum of multiple objectives is often used for local searches in hybrid MOEAs. This weighted sum (defined as r(**x**)) can be implemented using the following scalar fitness function:

$$r(\mathbf{x}) = \sum_{i=1}^{n} q_i \cdot r_i(\mathbf{x}) \tag{35}$$

where q_i is a weight for the i-objective function satisfying the following conditions:

$$\sum_{i=1}^{n} q_i = 1 \qquad q_i \geq 0 \tag{36}$$

Additionally, if the minimization of objectives functions is assumed, $r_i(\mathbf{x})$ represents the i^{th} normalized function, which can be obtained as:

$$r_i(x) = \frac{f_i^{max} - f_i(x)}{f_i^{max} - f_i^{min}} \tag{37}$$

where f_i^{max} and f_i^{min} are the minimum and the maximum values of the objective function f_i from the obtained set solutions from the MOEA. The steps followed by the NMS are described next:

Step 1. Fix simplex parameters (τ, ω, γ, β, δ), maximum number evaluations (t_{max}) and current number evaluations (t = 0). Generate n vertex from the starting point and evaluate the objective function at the n+1 vertex of the simplex and. Do t=n+1.

Step 2. If $f(V_n)-f(V_0))<\tau$ or $t>t_{max}$ return the i element which has the minimum $f(V_i)$ else:

Step 3. Order the n+1 vertex of the simplex according to the objectives function values, thus is, $f(V_0) \leq f(V_1) \leq .. \leq f(V_n)$.

Step 4. Reflection. Calculate $V_r = \overline{V} + \omega \cdot (\overline{V} - V_n)$, and compute $f(V_r)$. Do t=t+1. (Note: $\overline{V} = \sum_{i=0}^{n-1} V_i / n$)

Step 5. If $f(V_0) \leq f(V_r) < f(V_n)$ replace V_n with V_r go to step 2.

Step 6. Expansion. If $f(V_r) < f(V_0)$ calculate $V_e = \overline{V} + \gamma \cdot (V_r - \overline{V})$ and compute $f(V_e)$. Do t=t+1.

If $f(V_e) < f(V_r)$ replace V_n with V_e and go to step 2.

Else replace V_n with V_r and go to step 2.

Step 7. Outside Contraction. If $f(V_{n-1}) \leq f(V_r) < f(Vn)$ Calculate $V_{CO} = \overline{V} + \beta \cdot (V_r - \overline{V})$. Do t=t+1.

If $f(V_{CO}) \leq f(V_r)$ replace V_{n+1} by V_{CO} and go to step 2.

else go to step 8.

Step 8. Inside Contraction. If $f(Vn) \leq f(V_r)$ calculate $V_{CI} = \overline{V} - \beta \cdot (\overline{V} - V_{n+1})$ and compute $f(V_{CI})$. Do t=t+1.

If $f(V_{CI}) \leq f(V_n)$ replace V_n with V_{CI} and go to step 2.

Else go to step 8.

Step 9. Calculate $V_i' = (V_i + V_1).\delta$ for i= 2,3, .., n and evaluate to replace all V_i by V_i . Do t=t+(n-1). Go to step 2.

It is worth adding that the standard NMS has certain standard parameter values for ω, γ, β and δ, which are shown in Table 1. This table also includes the conditions that these values must acquire.

Table 1. Standard NMS values.

Parameter	Standard value	Conditions
ω	1	ω>0
γ	2	γ>1 and γ>ω
β	0.5	0<β<1
δ	0.5	0<δ<1

5.3. Hybrid MOEAs

In the literature it is possible to analyze mainly two hybrid schemes: the 'a posteriori' approach where the MOEA runs for a fixed number of generations obtaining an approximation of the Pareto front to apply then a local search from each non-dominated solution of the front, and the 'on-line' approach, where in each generation (or after a reduced number of generations) of the evolutionary algorithm it is applied a local search on each non-dominated solution to return then the improved solutions to the MOEA as the current population.

These approaches are shown in the next subsections, but previously it is shown how the hybridization of both methods is performed.

5.4. Hybridization

The two hybrid algorithms developed employ the hybridization scheme shown in Figure 2:

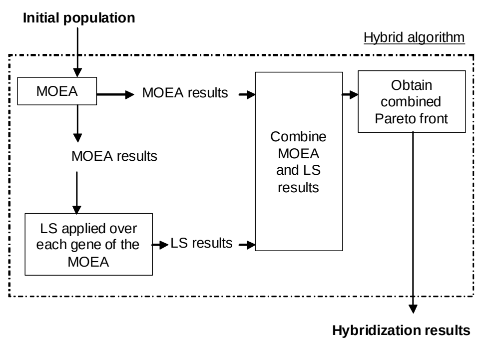

Figure 2. Hybridization scheme of the implemented algorithm.

In this scheme the MOEA runs for a fixed number of generations obtaining a set of solutions. After evolving the population using the algorithm a local search is applied from each non-dominated solution of the front. Local search boundaries related to each input gene are established determining a symmetric upper and lower limit of the input over each genome. Local search applied to each gene generates another solution so it is created another population. In a third step the populations obtained using both MOEA and LS are combined to re-generate the solution front. In case the front size exceeds the established maximum Pareto size worst genes are removed according to the criteria defined in Eq (34).

5.5. 'A posteriori' approach

As it was defined previously, in the 'a posteriori' approach the MOEA runs for a fixed number of generations obtaining an approximation of the Pareto front. After evolving the population using the algorithm a local search is applied from each non-dominated solution of the front. Then the populations obtained using both the MOEA and the LS are combined to re-generate the solution front, reducing it in case the front size exceeds the established maximum Pareto size. The working scheme of this approach is shown in Figure 3:

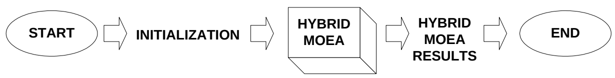

Figure 3. Scheme of the 'a posteriori' approach.

5.6. 'On-line' approach

In the 'on-line' initiative, after a reduced number of generations of the MOEA its results are taken as in-puts in the LS method. Then a local search is applied from each solution the same way it is performed in the 'a posteriori' approach. The results of both methods are combined to obtain the solution front, reducing it in case the maximum population size is exceeded. Then this solution front is returned to the MOEA as the current population. These operations are executed until a finalization criterion is achieved. The scheme of the 'on-line' approach is shown in Figure 4:

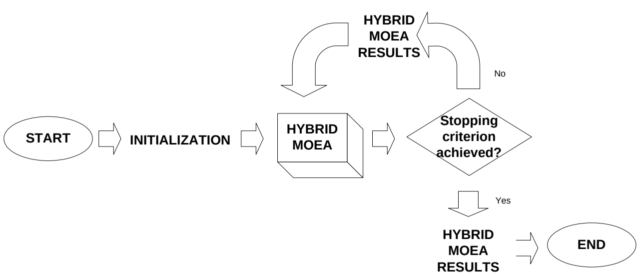

Figure 4. Scheme of the 'on-line' approach.

5.7. Termination criterion

The convergence in results in the MOEA during an amount of generations of global search method is considered as finalization criterion for the three cases; in other words, it is necessary to have a convergence in results in a predefined amount of generations in the NSGA-II for all of the approaches, thus is, a) NSGA-II in 'stand-alone' mode, b) 'a posteriori' and c) 'on-line' to end up the simulation. In the specific case of the 'a posteriori' scheme, a local search on each of the optimal solutions found is be executed before retrieving the set of optimal solutions.

For the NSGA-II in stand-alone mode, this convergence is established in 10 generations, whereas for the hybrid algorithms the limit is set on 5 generations. The reason for being more demanding in the convergence of the NSGA-II running in 'stand-alone' mode is that, if the same amount of generations are set up as convergence criterion, the NSGA-II will always be faster than the 'a posteriori'.

6. APPLICATION CASE

The cost and profit models described in Section 3 and the hybrid algorithms are applied herein to the problem of optimization the preventive maintenance activities of a simplified injection system. The developed hybrid algorithms were also compared with the NSGA-II considering simulation time and number of evaluations of the objective function as performance ratios. The simplified system is installed in a installed in a Spanish manufacturing company of the 'Mondragon Cooperative Corporation' and it consists of three groups of components (C1-Electric-Electronic components, C2-Hydraulic components and C3-Others) in serial configuration. Firstly, C1's deterioration influences only unavailability. Secondly, C2's deterioration affects unavailability and productive speed loss. Thirdly, C3's deterioration has an effect on unavailability and quality. Three maintenance activities (M1, M2 and M3) are applied on the components in order to reduce the deterioration level of the studied equipment: M1 is applied on C1, M2 on C2 and M3 on C3.

6.1. Simulation values

6.1.1. Simulation values for the studied equipment

Tables 2, 3, 4, 5 and 6 show the relevant component reliability, preventive maintenance, corrective maintenance, quality, availability, speed loss and cost data for this case of application:

Table 2. Reliability data.

Component	$\lambda(10^{-4}/\eta)$	γ
C1	5	2
C2	2	2.9
C3	4	2

Table 3. Parameters related to preventive maintenance.

Activity	ε	Duration d_{pm} (h)
M1	0,9	0,5
M2	0,9	0,5
M3	0,9	1

Table 4. Parameters related to corrective maintenance.

Component	d_{cm} (h)
C1	0,5
C2	0,5
C3	1

Table 5. Parameters related to quality, speed loss and unavailability.

h_0	τ	S_0	ρ	α
	(u^*/h^2)	(u/h)	(10^{-3})	
0	0.0017	180	1	0.03

*u: product unit

Table 6. Parameters related to quality, speed loss and unavailability.

C_α	C_{hsl}	c_{hcm}	c_{hpm}
(€/u)	(€/h)	(€/h)	(€/h)
6	25	45	30

Optimization criteria considered herein correspond to the ones formulated for the cost and profit problem in Eqn. (32) for a L = 10 years working period. The models for C(**x**) and T(**x**) depend on preventive maintenance activity frequency (M1, M2, and M3) which act as decision variables.

The constraints imposed apply over: 1) the objective functions and 2) the values the decision variables can take. In the first case, we apply constraints over the total maintenance costs and the unavailability:

$$U(\mathbf{x}) \leq 1.2E\text{-}3 \qquad C(\mathbf{x}) \leq 2024916 \text{ €}$$

values that correspond to the preventive maintenance scheduling implemented at present. In the second case, constraints are directly imposed over the decision variable values (M1, M2 and M3), which must take typified values, representing each one a day, two days, etc.

6.1.2. Simulation values of the algorithms

The simulation values shown in this Section were chosen by performing designs of experiments choosing the ones that performed best in terms of computational effort.

Values used in the NSGA-II are shown in Table 7. For the determination of selection, crossover and mutation rates a design of experiments with values of 0.25, 0.5 and 0.75 for the three parameters was conducted.

Table 7. Values used in the NSGA-II.

Parameter	Value
Population Size	100
Selection rate	0.25
Crossover rate	0.5
Mutation rate	0.75

Values for the hybrid approaches are shown in Table 8:

Table 8. Values used in the hybrid algorithms.

Parameter	a posteriori	on-line
local search tolerance (± days)	6	6
t_{max}	10	10
NSGA-II generations per turn in the loop	-	5

6.2. Results

The Pareto fronts obtained by the compared algorithms are shown in Figure 5:

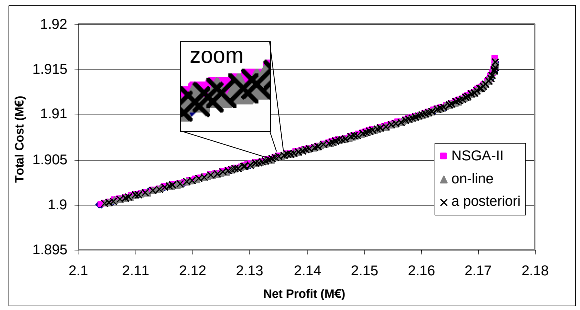

Figure 5. Pareto fronts of the developed algorithms

As it can be seen in Figure 5, the front obtained using the tested algorithms are practically equal. In order for performance comparisons to be meaningful across the four implementations, normalization is done. In this paper number of function evaluations and time values are used so as to express computation efforts referred to each algorithm. These results are shown in Table 9:

Table 9. Performance results of the developed algorithms.

	NSGA-II	A posteriori	On-line
Total simulation time (s)	36.03	30.84	33.88
NSGA-II execution time during simulation (s)	36.03	22.35	10.56
NMS execution time during simulation (s)	-	8.49	23.33
f(x) evaluations performed by simulation	4733	4339	4249
f(x) evaluations performed by the NSGA-II	4733	3244	1491
f(x) evaluations performed by the NMS	-	1095	2758

As it can be seen in Table 9, results obtained using the NSGA-II algorithm in this optimization problem are improved by hybrid algorithms considering both function evaluations and time terms.

As analysis of the results, it can be stated that the final convergence phase is influencing the results of the NSGA-II negatively, due to two reasons: on the one side (as it is stated in Section 6.1.2) it is worth noting that the mutation value obtaining best results after the design of experiments is 0.75, a high value that may transforms the search process into a partially-random one. Additionally, the convergence in results in the MOEA during 5 more generations (compared to the hybrid approaches) forces the NSGA-II to perform almost 500 function evaluations more.

7. CONCLUSIONS

This paper presents a useful model to calculate the profitability of a preventive maintenance strategy applied in equipment. The model considers jointly maintenance, equipment's productive speed loss and quality costs along with the profit related to a maintenance strategy.

Genetic Algorithms are very likely the most widely known type of Evolutionary Algorithms. In this paper, two variants of a hybrid MOEA have been implemented and successfully applied, to perform the constrained optimization of PM activities. In the studied case, the capabilities of efficiency and fast convergence of the hybrid approaches have been compared with the performance of the NSGA-II algorithm, obtaining positive results.

Further research regarding the hybrid algorithm would be oriented to the comparison of these hybrid algorithms against others evaluating optimizations performed using standardized functions for the comparative. Concerning the maintenance optimization problem the inclusion of other concepts in the model (e.g. short-term maintenance planning, discounted cost, age at the end of planning horizon), the modeling of other aging models, he comparison among different maintenance policies and the study of which class of system can be actually applied to each aging model constitute basis for the future work lines.

Regarding this last comment, it is worth commenting that the results of this modeling and optimization process were discussed with the production and maintenance managers of the production line studied herein. The feedback received from them indicates that the results of the model align with the information and impressions they manage when talking about group C1-Electric-Electronic components, whereas they distance when talking about C2-Hydraulic components and C3-Others.

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BIOGRAPHIES

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