

# Condition-based Maintenance: Determination of optimal deterioration levels to perform preventive activities by using a multi-objective evolutionary algorithm

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## ABSTRACT

Several authors insist on the fact that maintenance is a key activity in the manufacturing industry, because of its economic consequences. Within maintenance, Condition-Based Maintenance programs can provide significant advantages to industrial plants. This paper is focused on the problem of Condition-Based Maintenance optimization in an industrial environment, with the objective of determining both the critical age level to perform preventive maintenance activities and the amount of this type of activities to be executed before upgrading or substituting components. For this purpose, a mathematical model who jointly considers the evolution in quality and production speed along with condition based, corrective and preventive maintenance is presented. The cost and profit optimization process using a Multi-Objective Evolutionary Algorithm is applied to an industrial case.

## 1. INTRODUCTION

Modern industrial engineers are continually faced with the challenge of meeting increasing demands for high quality products while using a reduced amount of resources. Since systems used in the production of goods and delivery of services constitute the vast portion of capital in most industries, maintenance of such systems is crucial (Simmons Ivy & Black Nembhard, 2005). Several studies compiled in Ref. (Mjema 2002) show that maintenance costs represent from 3 to 40 % out of the total product cost (with an average value of a 28%).

Condition-Based Maintenance (CBM) techniques are an important part of maintenance. Nevertheless, and according

to Aven (1996) CBM models are usually, by its nature, rather sophisticated compared to the more traditional replacement models. Marhadi (2015) as well states that setting optimal alarm thresholds in vibration based CBM systems is inherently difficult. Within this maintenance strategy, Das & Sarkar (1999) distinguish two CBM subtypes, the On-Condition Maintenance (OCM) and the Condition Monitoring (CMT). OCM is based on periodic inspections, while CMT performs a continuous monitoring of the critical parameters through instrumentation to execute maintenance activities when the age of the component achieves a certain critical value. Literature review also can be divided using this two approaches. E.g. referred to the OCM models, Barbera et al. (1999) elaborate an OCM model with exponential failures and fixed inspection intervals for a two-unit system in series modeled using dynamic programming formulation that considers failure, repair and replacement costs. Grall et al. (2002) focus on the analytical modeling of a OCM inspection/replacement policy for a stochastically and continuously deteriorating single-unit system, considering both the replacement threshold and the inspection schedule as decision variables. The goal of the initiative is to minimize the long run expected corrective, preventive and inspection maintenance costs. Cassady et al. (2000) develop an OCM model using an age replacement Preventive Maintenance (PM) policy, with the application of a control chart to inspect the process and monitor the condition of the studied equipment; the model considers the cost of inspection, process downtime and poor quality. Castanier et al. (2005) develop an OCM policy model for a two-unit deteriorating system monitored by sequential non-periodic inspections to minimize set-up and replacement maintenance costs. Linderman et al. (2005) detail a model that uses Statistical Process Control (SPC) to monitor the deterioration of the process and conduct maintenance actions devoted to minimize the total costs

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associated to quality, maintenance, and inspection. Kuo (2006) also builds a model that uses a control chart to monitor the process with the goal of finding the optimal machine inspection, maintenance and product sampling policy that minimizes the expected total cost of inspection, repair and poor quality.

Related to CMT modeling, Scarf (1997) determined that no model was designed for modeling the deterioration level of a component beyond which preventive activities had to be applied. Since that affirmation, in the recent years and thanks to the explosion of Big Data, CMT has gained widespread acceptance (Mehairjan, Zhuang, Djairam, & Smit, 2015) as the possibility of embedding of CMT models into decision models open new possibilities for analyzing information coming from different data sources.

In this context, this paper focuses on the problem of CMT optimization in a manufacturing environment, with the objective of determining the optimal CMT deterioration levels beyond which PM activities should be applied under cost and profit criteria. The developed cost and profit model takes into account the interaction of productive speed, quality and maintenance aspects of a single machine.

Regarding the speed, quality and maintenance concepts modelled in this article, it is worth noting that only a few works developed by the authors (Oyarbide-Zubillaga, Sanchez, & Goti, 2007; Valčuha, Goti, Úradníček, & Navarro, 2011) show the productive speed loss associated to the deterioration of equipment (together with maintenance and quality costs). In difference to the current work Ref. (Oyarbide-Zubillaga et al., 2007) is oriented to the optimization of a time directed PM, whereas Ref. (Valčuha et al., 2011) models a CMT model under the same assumptions of this work but using a completely different aging or deterioration model for the components: specifically the Ref. (Valčuha et al., 2011) each maintenance activity is assumed to shift the origin of time from which the age of the component is evaluated, in this paper it shifts the origin of time from which the age of the component had its last maintenance intervention.

The CMT model is applied to a plastic injection machine of a manufacturing plant in a Multiple-objective Optimization Problem (MOP), and optimized by using a Multi-Objective Evolutionary Algorithm (MOEA).

This paper is organized as follows. Section 2 introduces the model of imperfect maintenance model proposed, which is based on a Proportional Age Reduction (PAR) model: initially the aging models and its assumptions are presented, to then develop the reliability and availability models of a CMT maintenance strategy. Section 3 shows the cost and benefit quantification models used in the optimization process. Section 4 describes the optimization process. Section 5 briefly details the optimization algorithm used, whereas Section 6 presents an application case focusing on

the optimization critical age for intervention and minimum time between two consecutive interventions under cost and profit criteria for a simplified production system. Finally, Section 7 presents the conclusions.

## 2. IMPERFECT MAINTENANCE MODEL

### 2.1 Aging model

Traditionally, the effect of the maintenance activities on the state of equipment is based on three situations: a) perfect maintenance activity which assumes that the state of the component after the maintenance is “As Good as New” (GAN), b) minimal maintenance which supposes that activity leaves the equipment in “As Bad as Old” (BAO) situation, and c) imperfect maintenance which assumes that the activity improves the state of the equipment by some degree depending on its effectiveness. Last situation is closer to many real situations.

There exist several models developed to simulate imperfect maintenance, e.g. Proportional Age Set-back (PAS) of Martorell et. al (1998). In this paper, the PAR imperfect maintenance model proposed by Malik (1979) is used to model the effect of the PM activities on equipment.

In the PAR approach, each maintenance activity is assumed to shift the origin of time from which the age of the component had its last maintenance intervention. PAR model in Ref. (Malik 1979) considers that the maintenance activity reduces proportionally, in a factor of  $\varepsilon$ , the age gained from previous maintenance, where  $\varepsilon$  ranges in the interval  $[0,1]$ . If  $\varepsilon = 0$ , the PAR model simply reduces to a BAO situation, while if  $\varepsilon = 1$  it is reduced to a GAN situation. Thus, this model is a natural generalization of both GAN and BAO models in order to account for imperfect maintenance.

The following assumptions are considered to obtain the expressions for the PAR model: a) working conditions (environmental and operational) are assumed to keep constant and normal during the operational time, b) the component is continuously monitored, so that no preventive action is taken until it arrives to a critical age  $w_c$ , c) corrective maintenance is performed with minimal repair, that is, repairing failures do not improve the age of components (BAO model is adopted), d) preventive maintenance effect is modelled considering a PAR model and a effectiveness  $\varepsilon$ , e) as in a PAR model only an  $\varepsilon$  proportion of the age gained since the last PM activity is shifted, the time between two PM activities  $M$  gets shorter and shorter. Thus,  $M$  tends to zero, and it becomes necessary to fix a limit value for the time between two PM activities,  $M_{\min}$ , from which the component will be upgraded. The model developed herein considers an upgrading of  $\varepsilon = 1$  (GAN model).

Thus, the system behaves as it is shown in Figure 1.

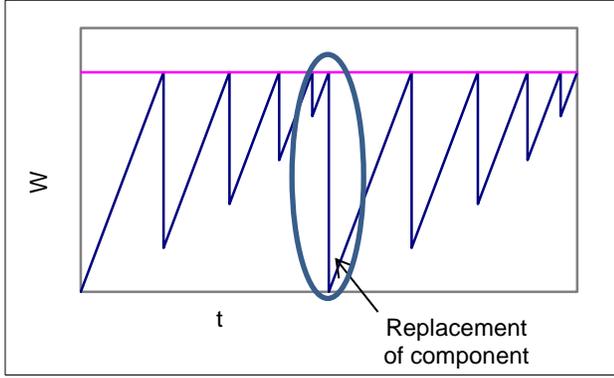


Figure 1. Age vs chronological time in a PAR model under a CMT maintenance strategy.

Under these assumptions, it is possible to model the time between two consecutive preventive maintenance activities. Eqn. (1) shows the time between the installation of the component and the execution of the first activity,  $M_1$ :

$$M_1 = t_1 = w_c \quad (1)$$

The interval for the second activity  $M_2$  and the chronologic time  $t_2$  when the second activity is executed are evaluated as follows:

$$\begin{aligned} w_2^- &= w_1^+ + (t_2 - t_1) \\ w_1^+ &= (1 - \varepsilon) \cdot t_1 = (1 - \varepsilon) \cdot w_c \\ M_2 &= w_c \cdot \varepsilon = t_2 - t_1 \\ t_2 &= w_c + w_c \cdot \varepsilon = w_c \cdot (1 + \varepsilon) \end{aligned} \quad (2)$$

where the superscript “ - ” is included in order to denote the age of the component immediately before entering maintenance, while the superscript “ + ” is included to denote the age of the component immediately after the maintenance activity. As generalization for the  $m$  preventive maintenance activity the time between  $m-1$ <sup>th</sup> and  $m$ <sup>th</sup> maintenance activities,  $M_m$ , is given by:

$$M_m = w_c \cdot \varepsilon^{m-1} \quad (3)$$

An overall expression of  $w_m^+$  can be obtained by Eqn. (4):

$$w_m^+ = w_c (1 - \varepsilon) \sum_{k=0}^{m-1} \varepsilon^k \quad (4)$$

As it indicated previously, it will be necessary to substitute the component or upgrade it into a GAN situation when  $M_{m+1} < M_{\min}$ . As consequence, for prefixed  $w_c$  and  $M_{\min}$  values, it is possible to obtain the number of PM activities executed before a component is replaced,  $e$ :

$$\begin{aligned} w_c \cdot \varepsilon^{e-1} &\leq M_{\min} \\ (e-1) &\leq \ln \frac{M_{\min}}{w_c} \\ e-1 &\leq \frac{\ln \frac{M_{\min}}{w_c}}{\ln \varepsilon} \\ e &\leq 1 + \ln \left( \frac{M_{\min}}{w_c} \right)^{1/\varepsilon} \\ e &\leq 1 + \frac{1}{\varepsilon} \ln \left( \frac{M_{\min}}{w_c} \right) \end{aligned} \quad (5)$$

It is worth stating that at the  $e$ <sup>th</sup> PM activity an upgrade of the component will be executed. So that a component will be installed during  $L_c$  time to them be upgraded into a GAN situation:

$$L_c = \sum_{k=1}^e M_k \quad (6)$$

And as a consequence, there will be  $n_c$  components substituted during  $L$ , the analysis period:

$$n_c \approx \frac{L}{L_c} \quad (7)$$

Finally, it is worth noting that after the installation of a new component the  $(e+1)$ <sup>th</sup> PM activity will have the same effect as the first one, the  $(e+2)$ <sup>th</sup> as the second one, etc.

## 2.2 Reliability and availability models

Considering the conditions and the aging model presented in the previous section, it is possible to obtain an age-dependent reliability model in which the induced or conditional failure rate, in the period  $m$ , after the PM activity  $m-1$ , is given by:

$$h_m(w) = h(w(t, \varepsilon)) + h_0 \quad w \geq w_{m-1}^+ \quad (8)$$

where  $h_0$  represents the initial failure rate of the component, that is, when the equipment is installed.

Considering the age of the component after maintenance  $m$  is given by Eqn. (4), and adopting a Weibull model for the failure rate, the expression for the induced failure rate after the maintenance number  $m$  can be written as:

$$h_m(w) = \left\{ \lambda^\gamma \cdot \gamma \cdot [w_m(t, \varepsilon)]^{\gamma-1} \right\} + h_0 \quad w \geq w_{m-1}^+ \quad (9)$$

Where  $\lambda$  is the scale parameter,  $\gamma$  is known as the shape parameter. For Weibull distribution, the accumulated failure rate  $H_m(w)$  is defined by:

$$H_m(w) = H(w_m) = [\lambda \cdot w_m(t, \varepsilon)]^\gamma + h_0 \cdot w_m(t, \varepsilon) \quad (10)$$

As all PM activities will be executed when the age or deterioration level reaches a same critical age  $w_c$ , it is possible to obtain the accumulated failure rate for  $w_m^-$  with the expression shown in Eqn. (11):

$$\begin{aligned} H_m^- &= H_m(w_m^-) = [\lambda \cdot w_m^-]^\gamma + h_0 \cdot w_m^- \\ &= [\lambda \cdot w_c]^\gamma + h_0 \cdot w_c \end{aligned} \quad (11)$$

A similar process can be executed for  $w_{m-1}^+$  based on Eqn. (4) and Eqn. (10):

$$\begin{aligned} H_{m-1}^+ &= H_{m-1}(w_{m-1}^+) = [\lambda \cdot w_{m-1}^+]^\gamma + h_0 \cdot w_{m-1}^+ \\ &= \left[ \lambda \cdot w_c (1-\varepsilon) \sum_{k=0}^{m-2} \varepsilon^k \right]^\gamma + h_0 \cdot w_c (1-\varepsilon) \sum_{k=0}^{m-2} \varepsilon^k \end{aligned} \quad (12)$$

The averaged failure rate between  $t_m - t_{m-1}$  ( $h_m^*$ ), necessary in the calculation process of unavailability rates is obtained as shown in Eqn (13):

$$\begin{aligned} h_m^* &= \frac{1}{w_m^- - w_{m-1}^+} (H_m^- - H_{m-1}^+) = \\ &= \frac{1}{w_c - \left( w_c (1-\varepsilon) \sum_{k=1}^{m-1} \varepsilon^{k-1} \right)} = \\ &= \frac{\left\{ [\lambda \cdot w_c]^\gamma + h_0 \cdot w_c - \left( [\lambda \cdot w_c (1-\varepsilon) \sum_{k=1}^{m-1} \varepsilon^{k-1}]^\gamma + h_0 \cdot w_c (1-\varepsilon) \sum_{k=1}^{m-1} \varepsilon^{k-1} \right) \right\}}{w_c \left( 1 - (1-\varepsilon) \sum_{k=1}^{m-1} \varepsilon^{k-1} \right)} \\ &= \frac{\lambda^\gamma \cdot w_c^{\gamma-1} \cdot \left( 1 - \left[ (1-\varepsilon) \sum_{k=1}^{m-1} \varepsilon^{k-1} \right]^\gamma \right) + h_0}{1 - (1-\varepsilon) \sum_{k=1}^{m-1} \varepsilon^{k-1}} \end{aligned} \quad (13)$$

And finally  $h^*$ , the averaged failure rate of the component over its lifetime is calculated as shown:

$$h^* = \frac{\sum_{m=1}^e h_m^*}{e} \quad (14)$$

The availability of the system to be studied  $A_s(\mathbf{x})$ , which depends on the decision vector  $\mathbf{x}$  (vector of decision variables), is obtained as follows:

$$A_s(\mathbf{x}) = 1 - U_s(\mathbf{x}) \quad (15)$$

being  $U_s(\mathbf{x})$  the system unavailability, to be evaluated using the system fault tree and the single component unavailability contributions. These contributions are a) the unavailability related to the execution of PM activities  $u_{pm}(\mathbf{x})$  and b) the unavailability due to corrective maintenance  $u_{cm}(\mathbf{x})$ . Assuming a sufficiently large analysis period, where several components are replaced,  $u_{pm}(\mathbf{x})$  is given by the following Eqn.:

$$u_{pm}(\mathbf{x}) = \frac{(e-1) \cdot d_{pm} + d_u}{L_c} \quad (16)$$

where  $d_{pm}$  and  $d_u$  respectively represent the mean time for executing preventive maintenance and upgrading activities.  $u_{cm}(\mathbf{x})$  can be obtained using the time-dependent unavailability due to random failures for discontinuous equipment,  $u_r^*(\mathbf{x})$ :

$$u_{r_m}^*(\mathbf{x}) = \rho + (1-\rho) \cdot (1 - e^{-h_m^* \cdot M_m}) \quad (17)$$

Being  $\rho$  the probability of failure on demand (and  $e$  the 'e' number). Thus  $u_{cm}(\mathbf{x})$  is calculated by the following expression:

$$u_{cm}(\mathbf{x}) = \frac{\sum_{i=1}^e u_{r_m}^*(\mathbf{x}) \cdot d_{cm}}{L_c} \quad (18)$$

### 3. COST AND BENEFIT MODELS

#### 3.1 Maintenance costs

The relevant maintenance costs of the equipment include the contributions due to condition monitoring (CMT), preventive maintenance (PM), corrective maintenance (CM), consequence of idling, minor stop and failure/breakdowns), and upgrading or substitution of components ( $C_u$ ). Thus, the cost associated to condition monitoring ( $C_{cmt}$ ) is:

$$C_{cmt} = L \cdot c_{hct} \quad (19)$$

Where  $L$  represents the analysis period and  $C_{hct}$  the hourly cost of monitoring. Preventive maintenance costs can be evaluated as:

$$C_{pm}(\mathbf{x}) = n_c \cdot (e-1) \cdot d_{pm} \cdot c_{hpm} \quad (20)$$

where  $c_{hpm}$  is the average hourly cost of performing PM. The cost contribution due to corrective maintenance is given by:

$$C_{cm}(\mathbf{x}) = u_{cm}(\mathbf{x}) \cdot L_c \cdot c_{hcm} \quad (21)$$

Being  $c_{hcm}$  the average hourly cost of performing corrective maintenance.

Finally, for calculating the costs of upgrading components ( $C_u$ ) the amount of components changed  $n_c$  with the cost of upgrading or substituting a component  $C_c$  must be combined:

$$C_u = C_c \cdot n_c \quad (22)$$

Where  $C_c$  includes the costs of both the upgrade of the component and the one related to its installation

### 3.2 Cost related to the production speed lost due to aging

Traditionally, in the literature the production rate (speed) of the equipment is assumed to be predetermined and constant along the component life. Nevertheless, it is logical to think that production speed as consequence of the aging of equipment decreases. As in Ref. (Oyarbide-Zubillaga, Goti, & Sanchez, 2008), in this paper it is assumed that the production rate falls linearly as equipment ages. Based on this hypothesis the production speed after the  $m$ -maintenance activity can be evaluated as:

$$V_m(w) = V_0 - \tau \cdot w_m(t, \varepsilon) \quad (23)$$

where  $V_0$  is the initial (e.g. as per design) production speed,  $\tau$  represents the speed reduction coefficient and  $w_m(t, \varepsilon)$  is the age of the component after the maintenance  $m$ , which adopting a PAR model is given by Eqn. (4). Thus, the behavior of the productive speed over the time ( $V_m(t)$ ) shown by Figure 2:

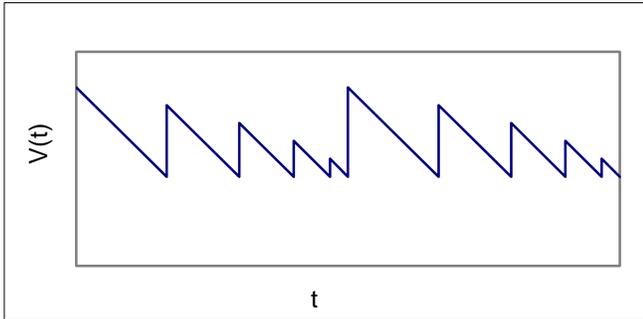


Figure 2. Production speed vs chronological time in a PAR model under a CMT maintenance strategy.

For obtaining the cost of the lost production speed it is necessary to calculate an average value of the production speed between activities  $m-1$  ( $V_{m-1}^+$ ) and  $m$  ( $V_m^-$ ),  $V_m^*$ .

This process is shown in Eqs (24), (25) and (26), where it is assumed that the unavailability of the component does not affect the linear fall of the speed over the time. As it happens with the accumulated failure rate, the productive speed will be limited by the critical age when the component is maintained preventively and the effectiveness of the activities executed on it:

$$V_m^- = V_0 - \tau \cdot w_c \quad (24)$$

$$V_{m-1}^+ = V_0 - \tau \cdot w_c (1 - \varepsilon) \cdot \sum_{k=0}^{m-2} \varepsilon^k \quad (25)$$

$$V_m^* = \frac{1}{M_m} \int_{w_{m-1}^+}^{w_m^-} v(t) \cdot dt = V_0 - \tau \cdot w_c \left[ \frac{1 + (1 - \varepsilon) \cdot \sum_{k=1}^{n-1} \varepsilon^{k-1}}{2} \right] \quad (26)$$

As with the failure rate, it is possible to get an averaged production speed for the lifetime of a component,  $V^*$ :

$$V^* = \frac{\sum_{m=1}^c V_m^* \cdot M_m}{L_c} \quad (27)$$

Adopting the value of  $V^*$  the averaged production speed given by Eqn. (27) it is possible to determine the "production time lost" related to a reduced speed ( $t_{sl}$ ). Considering only the fraction of the production system is available  $t_{sl}$  is calculated as follows:

$$t_{sl}(\mathbf{x}) = \left( 1 - A_s(\mathbf{x}) \cdot \frac{V^*}{V_0} \right) \cdot L \quad (28)$$

where  $A_s(\mathbf{x})$  is the availability system which is obtained as described in Eqn. (15). Finally, the cost related to the production speed loss of the equipment ( $C_{sl}$ ) in the period  $L$  can be evaluated proportionally to the production time lost as:

$$c_{sl}(\mathbf{x}) = c_{hsl} \cdot t_{sl}(\mathbf{x}) = C_{hsl} \cdot L \cdot \left( 1 - A_s(\mathbf{x}) \cdot \frac{V^*}{V_0} \right) \cdot L \quad (29)$$

where  $c_{hsl}$  is the average hourly cost due to non-production of items.

### 3.3 Quality costs

PAR model presented in section 2 assumes that each preventive maintenance activity reduces suddenly the age of

the equipment depending on an effectiveness parameter. The change in the age of the equipment introduced by the PAR model affects the time distribution when the system swaps to the out-of-control and consequently the expected amount of nonconforming items.

In this section it is derived a quality cost model which considers the effects on PM and upgrading activities on the component age based on the PAR model. The model is developed under the following assumptions: 1) The equipment only produces non-conforming items with constant rate,  $\alpha$ , while the process is out-of-control 2) The time to the system swaps out-of-control follows a Weibull distribution which depends on the age of the equipment, 3) The preventive maintenance and the process inspection are performed simultaneously, 4) Inspections are error free, and 5) the process is restored to the in-control state when the preventive maintenance activity is realized. To model the quality costs it is necessary to determine the fraction of time during which the process is in UC state named  $\kappa_m(w)$ .

$$\kappa_m(w) = \int_{w_{m-1}^+}^{w_m^-} w_m \cdot f(w_m) \cdot dw_m \quad (30)$$

where  $f(w_m)$  is the density function, attainable using the conditional hazard function as:

$$f(w_m) = \lambda \cdot \gamma \cdot [\lambda \cdot w_m(t, \varepsilon)]^{\gamma-1} \cdot \exp[-(\lambda \cdot w_m(t, \varepsilon))^\gamma] \quad (31)$$

Obtaining that:

$$\begin{aligned} \kappa_m(w) &= \int_{w_{m-1}^+}^{w_m^-} w_m \cdot f(w_m) \cdot dw_m = \\ &= \int_{w_c}^{w_c} \lambda \cdot \gamma \cdot [\lambda \cdot w_m(t, \varepsilon)]^{\gamma-1} \cdot \exp[-(\lambda \cdot w_m(t, \varepsilon))^\gamma] \cdot dw_m \\ &= \sum_{k=1}^{m-1} c^{k-1} \end{aligned} \quad (32)$$

Finally, once obtained the time with the process under control between two maintenance activities ( $M_m - \kappa_m$ ), it is possible to obtain quality costs,  $C_q$  as:

$$C_q(\mathbf{x}) = n_c \cdot C_\alpha \cdot \sum_{m=1}^e V_m^* \cdot (M_m - \kappa_m) \cdot A_s(\mathbf{x}) \cdot \alpha \quad (33)$$

Where  $C_\alpha$  is the cost of the non-conforming unit.

### 3.4 Profit

In order to quantify the consequences of a given preventive maintenance schedule in economic terms it is necessary to consider not only the costs but the benefits obtained to its

implementation. So, a net profit function,  $P$ , that denotes the benefits obtained to the sale of products, is introduced as:

$$P = n \cdot \psi \quad (34)$$

where  $n$  is the number of non-defective items produced in the period analysis,  $L$ , and  $\psi$  is the estimated margin of a single product.

The number of non-defective items produced in the period  $L$  can be evaluated considering the time that the process has been in-control and out-of-control state. Thus, if the process is in an out-of-control state it produces  $(1-\alpha)\%$  of non-defective product while if the process is in an in-control state it elaborates  $0\%$  of defective product. Therefore, the profit can be evaluated as:

$$P(\mathbf{x}) = n_c \cdot A_s(\mathbf{x}) \cdot \sum_{m=1}^e [(M_m - \kappa_m)(1-\alpha) + \kappa_m] \cdot V_m^* \cdot \psi \quad (35)$$

## 4. COST PROFIT PROBLEM FORMULATION AND OPTIMIZATION PROCEDURE

### 4.1 Problem formulation

Critical age ( $w_c$ ) and minimum interval between two preventive activities ( $M_{\min}$ ) optimization based on cost and benefit criteria can be formulated as a multi-objective optimization problem (MOP), aiming at optimizing a vector of functions of the form (Martorell et al. 2004):

$$\mathbf{y}(\mathbf{x}) = f(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_n(\mathbf{x})) \quad (36)$$

Being subject to the vector of constraints

$$\mathbf{g}(\mathbf{x}) = (g_1(\mathbf{x}), g_2(\mathbf{x}), \dots, g_n(\mathbf{x})) \quad (37)$$

where:

$$\mathbf{x} = \{x_1, x_2, \dots, x_n\} \in \mathbf{X} \quad (38)$$

$$\mathbf{y} = \{y_1, y_2, \dots, y_n\} \in \mathbf{Y} \quad (39)$$

and  $\mathbf{x}$  is the decision vector (vector of decision variables, as indicated in section 2.2),  $\mathbf{y}$  the objective vector,  $\mathbf{X}$  the decision space and  $\mathbf{Y}$  the objective space,  $\mathbf{Y}=f(\mathbf{X})$ .

In the optimization proposed in this paper the cost and profit criteria are formulated using the expressions obtained in section 3. Both models depend on  $w_c$  and  $M_{\min}$  which act as decision variable and are encoded in the decision vector,  $\mathbf{x}$ . So, the vector of bi-objective function,  $f(\mathbf{x})$ , is defined as:

$$f(\mathbf{x}) = \{C(\mathbf{x}), P(\mathbf{x})\} \quad (40)$$

and the objective is to minimize the function  $C(\mathbf{x})$  and maximize a profit function  $P(\mathbf{x})$ .  $C(\mathbf{x})$  is the cost system which is evaluated as sum of the maintenance, production

speed lost and quality costs for each component of the system which are evaluated using Eqns. (19), (20), (21), (22), (29) and (33).  $P(\mathbf{x})$  is the profit function which is evaluated using Eqn. (35).

**5. OPTIMIZATION ALGORITHM**

In this case, a pre-packed Matlab version of the the Nondominated Sorting Genetic Algorithm (NSGA-II) (Deb, Pratap, Agarwal, & Meyarivan, 2002) has been applied to this process. The NSGA-II is still a very efficient MOEA for bi-objective optimizations (although it has been improved by the NSGA-III for the joint optimization of more objectives). Convergence in results in MOEAs during an amount of generations is considered a standard finalization criterion. Thus, as termination criterion it has been established the convergence in results of 5 generations.

**6. APPLICATION CASE**

The cost and profit models described in section 3 and the hybrid algorithms are applied herein to the optimization problem of continuously monitored 3 components of a simplified injection system. The simplified system (shown in Figure 3) is installed in a Spanish manufacturing company of the ‘Mondragon Cooperative Corporation’ and it consists of three groups of components (C1-Electric-Electronic components, C2-Hydraulic components and C3- Others) in serial configuration.

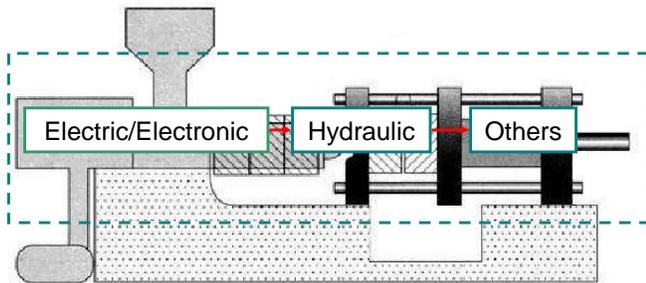


Figure 3. Graphical description of the simplified plastic injection system.

Three maintenance activities (M1, M2 and M3) are applied on the components in order to reduce the deterioration level of the studied equipment: M1 is applied on C1, M2 on C2 and M3 on C3. Currently these activities are time directed PM activities, and this study is oriented to offer an alternative of these PM approach with a CMT one. Thus, monitoring of temperature of electric-electronic devices along with oil and vibration analysis are suggested to respectively monitor C1, C2 and C3.

**6.1 Simulation values for the studied equipment**

Tables 1, 2, 3, 4 and 5 show the relevant component reliability, preventive maintenance, corrective maintenance and cost data for this case of application, respectively.

Table 1. Reliability data.

Component	$\lambda(10^{-4}/\eta)$	$\gamma$
C1	5	2
C2	2	2.9
C3	4	2

Table 2. Parameters related to preventive maintenance.

Activity	$\varepsilon$	$d_{pm}$ (h)	$d_u$ (h)
M1	0,9	0,5	0,5
M2	0,9	0,5	0,5
M3	0,9	1	1

Table 3. Parameters related to corrective maintenance.

Component	Duration (h)
C1	0,5
C2	0,5
C3	1

Table 4. Parameters related to quality, speed loss and unavailability.

$h_0$	$\tau$	$S_0$	$\rho$	$\alpha$
	( $u^*/h^2$ )	(u/h)	( $10^{-3}$ )	
0	0.0017	180	1	0.03

\* u: product unit

Table 5. Parameters related to cost.

$C_\alpha$	$C_{hsl}$	$c_{hcm}$	$c_{hpm}$	$c_{c1}$	$c_{c2}$	$c_{c3}$	$c_{het}$
(€/u)	(€/h)	(€/h)	(€/h)	(€/u)	(€/u)	(€/u)	(€/h)
6	25	45	30	30	30	60	1

Optimization criteria considered herein correspond to the ones formulated for the cost and profit problem in Eqn. (39) for a  $L = 10$  years working period. The models for  $C(\mathbf{x})$  and  $P(\mathbf{x})$  depend on  $w_c$  and  $M_{min}$ , so for each PM activity it is necessary to define two parameters  $w_c$  and  $M_{min}$ , so each of the genes will be composed by both of them. Variable  $w_c$

will be an integer value with range between 1 and 260 days (average amount of working days at the company), whereas  $M_{\min}$  will be represented with an integer value between 1 and  $w_c$ .

**6.2 Simulation values of the algorithms**

All but selection, crossover and mutation parameters of the NSGA-II were parametrized using standard parametrization options. For the selection, crossover and mutation the simulation values were chosen by performing a design of experiments to then choose the ones that performed best in terms of computational effort. Specifically, the values used in the NSGA-II (shown in Table 6.) are obtained by performing a 3 level 3 variable experiment on the selection, crossover and mutation rates (with values of 0.25, 0.5 and 0.75 for each of the three parameters).

Table 6. Values used in the NSGA-II.

Parameter	Value
Population Size	100
Selection rate	0.25
Crossover rate	0.5
Mutation rate	0.75

**6.3 Results**

Figure 4 shows the Pareto Front obtained after the simulation stopped (74 generations):

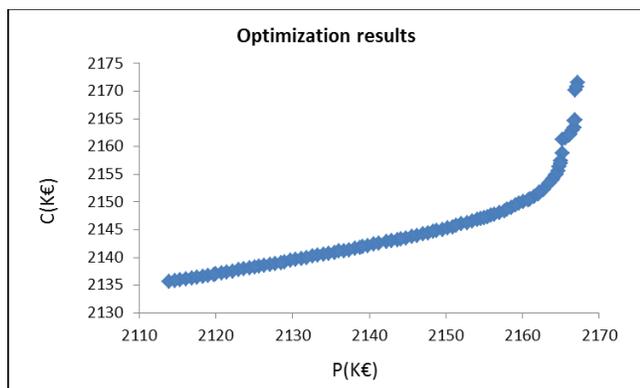


Figure 4. Optimization results.

As indicated previously, the set of solutions generated satisfies the constraints imposed to the problem. Thus, the decision maker can select a solution in accordance with his preferences. Nevertheless, in appearance it can be detected that part of the Pareto front has not been fully generated, meaning that another convergence criterion (or a convergence in results of more than 5 generations) could fit better to this case.

**7. CONCLUSIONS**

Maintenance economic optimization is a challenging topic. This paper presents a model which can be useful to calculate the profitability of a condition monitoring strategy applied to equipment. The model considers jointly unavailability, equipment’s productive speed loss, quality costs, maintenance (corrective, preventive, condition monitoring and component replacement) costs and the profit (as the margin of the sold products) related to a maintenance strategy, modeled using an imperfect maintenance model. In difference to other works modelling these same concepts, this work is innovative as it applies a different aging model. Specifically, past research bases on the PAS deterioration model whereas this paper applies the PAR one.

Referring to the case study, Genetic Algorithms are very likely the most widely known type of Evolutionary Algorithms. In the last years, there has been a growing effort to apply GAs to general constrained optimization problems, as most of engineering optimization problems often see their solution constrained by a number of restrictions imposed on the decision variables. In this paper, a Multi-Objective Nondominated-Sorting GA has been implemented and successfully applied to perform the constrained optimization of condition monitored maintenance activities. Nevertheless, it is worth noting that the partly stochastic nature of MOEA and the finalization criterion imposed to the case can be affecting the convergence of the simulation especially at the final stages of the simulations. Thus, further research will be oriented to the combination of a local search method to improve the effectiveness of the optimization process.

Regarding the application case, it is worth commenting that the results of this modeling and optimization process were discussed with the production and maintenance managers of the production line studied herein. As indicated in Section 6, the current maintenance policy for the studied components is time directed PM. In a comparison between the CMT approaches proposed and the time based PM, the managers found that monitoring of oil viscosity and control of temperature could be an interesting approach considering its cost and the reliability of its diagnoses, whereas they yet have some doubts with vibrations for the following two reasons: first they consider they have their time based PM activity (visual inspection and tightening of mechanical clearances) quite optimized, and second they do not rely too much on an average vibration value to monitor equipment as it can lead to false positive results. Thus, following the indications of the plant managers, further research regarding this area will be oriented to the incorporation of the effect of having false positives.

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