A Gamma Process Based Degradation Model with Fractional Gaussian Noise

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ABSTRACT

In modern industrial and engineering systems, stochastic degradation models are widely used for reliability analysis and maintenance decision-making. However, due to imperfect sensors and environmental influences, it is difficult to directly observe the latent degradation states. Traditional degradation models typically assume that measurement errors have simple statistical properties, but this assumption often does not hold in practical applications. To address this issue, this paper constructs a degradation model based on the Gamma process (GP) and assumes that measurement noise can be characterized by the fractional Gaussian noise (FGN). Furthermore, this paper proposes a method combining Gibbs sampling with the stochastic expectation-maximization (SEM) algorithm to achieve efficient estimation of the model parameters and accurate inference of the latent degradation states. Simulation results demonstrate that the proposed model, validated solely through numerical simulations, exhibits improved generalizability compared to the GP model with Gaussian noise.

1. INTRODUCTION

Prognostics and health management (PHM) aims to utilize data analysis and monitoring techniques to predict the degradation of system components or equipment, and to implement appropriate maintenance measures to ensure their reliability and safety (Gebraeel et al., 2023; Xi et al., 2018). In recent years, this technology has gained widespread attention because it not only reduces maintenance costs but also improves system reliability (Liu et al., 2023). The collected data for predicting the future degradation behavior is typically classified into two categories: (1) event data and (2) condition monitoring (CM) data. However, collecting sufficient event data can be costly, and in addition, some systems rarely experience failure events (Hong et al., 2022). With the continuous improvement of data preprocessing techniques, CM data can help analyze the health status and performance degradation of systems (Xi et al., 2020). Simultaneously, the degradation state is typically represented by CM data.

In practical applications, systems often involve various uncertain factors both in their internal characteristics and external environment. In the presence of such uncertainty, stochastic processes demonstrate significant advantages. In the selection of degradation models, the GP is commonly used to model systems with monotonic increments of degradation. For example, the GP has been applied to model the degradation of rolling element bearings, assessing the approximate failure time distribution when crack size exceeds a certain threshold (Wang et al., 2021). Additionally, accelerated degradation tests have been proposed to efficiently obtain reliable degradation information for light-emitting diodes (Ling et al., 2014). To address different failure mechanisms, a GP-based method has been developed for extrapolating failure times in high-reliability products (Li et al., 2022). The aforementioned studies obtained the failure time distribution at a given threshold through approximate methods, and interested readers can refer to review articles for more details (Li et al., 2024).

It is noteworthy that degradation measurements are often influenced by sensor accuracy and external environmental interference, resulting in the true degradation state being obscured by the measurement noise. Complicated measurement noises can hide the monotonic nature of the GP-based degradation processes, posing challenges for data analysis and model prediction. Therefore, modeling the observation results is necessary. For instance, a GP model with measurement noise interference has been used to estimate the hidden degradation state, employing Gibbs sampling for the estimation process (Le Son et al., 2016). Additionally, methods like particle filtering have been utilized to estimate latent degradation states in stochastic systems with uncertain degradation paths (Si et al., 2019). Furthermore, the significance of sensor degradation has been emphasized in the context of predicting system health status and making informed decisions (Mukhopadhyay et al., 2023). However, these studies assumed that the measurement noise follows a simple Gaussian distribution, which may limit the model applicability in practice. **??** depicts the temperature changes caused by the reduction in furnace wall thickness, based on temperature increment data from the 6th layer of a certain blast furnace. Based on the general increasing trend presented by the blast furnace wall temperature in **??**, GP can be considered as a suitable degradation process. However, due to the existence of non-monotonic fluctuation phenomena, further processing is required to comply with the GP model. In such cases, it is expected that a more generalised noise model would be more appropriate to describe the complex measurement noise.

In this paper, we introduce a novel degradation modeling method. Unlike traditional models that assume Gaussian noise, we incorporate FGN to represent measurement noise. Although this model introduces more parameters, it offers better estimation performance and robustness. Additionally, we combine the SEM approach with the Gibbs sampler and the Metropolis-Hastings (MH) algorithm in the Markov Chain Monte Carlo (MCMC) method to achieve higher efficiency in parameter estimation and model fitting for complex models.

The remainder of this paper is organized as follows. Section 2 introduces the degradation model based on GP and FGN. Section 3 details the degradation state estimation methods and the parameter identification algorithm. Section 4 provides and discusses the numerical simulation results to demonstrate the effectiveness of the proposed methods. Conclusions and future work are presented in section 5.

2. GP-BASED DEGRADATION MODEL

Suppose that the actual degradation of a system can be represented by the non-homogeneous gamma process $\{X(t), t \geq 0\}$ with a scale parameter ς and a shape parameter $\nu(t)$. Here, $\nu(t)$ is a non-decreasing, right-continuous, real-valued function for $t \geq 0$ with $\nu(0) \equiv 0$. Empirical studies provide evidence that the shape parameter at a given time point, denoted as t, frequently exhibits a proportional relationship with a power law form, expressed as $\nu(t) = at^b$ (Le Son et al., 2016). Here, $a > 0$ represents a constant coefficient, while b denotes a constant exponent that is obtained through data analysis. According to the properties of the gamma process, the degradation model exhibits two fundamental characteristics

1. For any time instants $0 < t_1 < \ldots < t_n$, the health state increments, *i.e.*, $\Delta x_1, \Delta x_2, \ldots, \Delta x_n$, are mutually independent random variables, where $\Delta x_j = x(t_j) - x(t_{j-1})$, for all $t_j > t_{j-1} \geq 0$.
2. Given any time intervals $[t_{j-1}, t_j]$, the corresponding increment Δx_j follows a gamma distribution with the mean $\Delta \nu_j \varsigma$ and the variance $\Delta \nu_j \varsigma^2$, where $\Delta \nu_j = \nu(t_j) - \nu(t_{j-1})$.

The degradation state $X(t) \sim Ga(x; \nu(t), \varsigma)$ with probability density function is given by

$$f_X(x; \nu, \varsigma) = \frac{x^{\nu(t)-1}}{\varsigma^{\nu(t)} \Gamma(\nu(t))} \exp\left(-\frac{x}{\varsigma}\right) \mathbb{I}_{[0,+\infty]}(x) \tag{1}$$

where $\Gamma(\tau) = \int_0^\infty z^{\tau-1} e^{-z} dz$ is the Euler gamma function, $\mathbb{I}_{[0,\infty]}(x) = 1$ for $x \in [0, \infty]$, and $\mathbb{I}_{[0,\infty]}(x) = 0$ otherwise. It is assumed that the degradation level is obscured within noisy observations, rendering it unobservable directly. Let $Y_j = Y(t_j)$ denote the observation at monitoring time t_j, the measurement model can be expressed as

$$Y_j = X_j + \omega_j \tag{2}$$

where ω_j represents the measurement error, independent of X_j. Assuming that ω_j follows FGN(Coeurjolly, 2000), which can be defined as

$$\omega_j = \sigma_H (B_H(t_j) - B_H(t_{j-1})) \tag{3}$$

where $B_H(\cdot)$ represents standard fractional Brownian motion (FBM), H is the Hurst exponent, with $0 < H < 1$, and σ_H is the diffusion coefficient. According to the past literature (Sottinen, 2001), $B_H(t)$ can be expressed in the following kernel integral form

$$B_H(t) = \int_0^t Z_H(t,s) \, \mathrm{dB}(\mathrm{s}) \tag{4}$$

where $B(\cdot)$ represents standard Brownian motion (BM), $Z_H(t,s)$ is an explicitly square-integrable kernel given by

$$Z_H(t,s) = c(H-0.5) s^{(0.5-H)} \int_0^t (u-s)^{(H-1.5)} u^{(H-0.5)} \, \mathrm{du} \tag{5}$$

with a normalization coefficient c expressed as

$$c = \sqrt{\frac{H(2H-1)}{\int_0^1 (1-\varepsilon)^{(1-2H)} \varepsilon^{(H-1.5)} \, \mathrm{d}\varepsilon}} \tag{6}$$

According to the previous research (Zhang et al., 2017), as the time interval $\tau \to 0$, and $H > 0.5$, FBM can weakly converge to the following form.

$$B_H(t)=\lim_{\tau\to 0}\sum_{i=1}^{\lfloor \frac{t}{\tau}\rfloor}\frac{1}{\sqrt{\tau}}\left[\int_{\lfloor \frac{t}{\tau}-1\rfloor\tau}^{\lfloor \frac{t}{\tau}\rfloor\tau} Z_H\left(\left\lfloor\frac{t}{\tau}\right\rfloor\tau,s\right)\mathrm{ds}\right]\xi_i \tag{7}$$

where $\lfloor \frac{t}{\tau}\rfloor$ denotes the greatest integer less than or equal to $\frac{t}{\tau}$, ξ_i are independent identically distributed random variables with $\mathbb{E}(\xi_i)=0$ and $\mathrm{Var}(\xi_i)=1$.

According to Eqs. (3) and (7), ω_j can be reconstructed as follows.

$$\omega_j=\sigma_H\lim_{\tau\to 0}\sum_{i=1}^{\lfloor \frac{t_j-t_{j-1}}{\tau}\rfloor}\frac{1}{\sqrt{\tau}}\left[\int_{(\lfloor \frac{t_j-t_{j-1}}{\tau}\rfloor-1)\tau}^{\lfloor \frac{t_j-t_{j-1}}{\tau}\rfloor\tau} Z_H\left(\left\lfloor\frac{t}{\tau}\right\rfloor\tau,s\right)\mathrm{ds}\right]\xi_i \tag{8}$$

According to Eq. (8), the measurement noise ω_j can be approximately represented by BM. The next section will introduce the methods for parameter identification and degradation state estimation based on the SEM algorithm and the Gibbs sampler.

3. DEGRADATION MODEL IDENTIFICATION

To effectively identify degradation model parameters, it is necessary to estimate the latent degradation states. Let $\boldsymbol{\theta}=(a,b,\varsigma,H,\sigma_H)$, the conditional density function of the latent states $\boldsymbol{X}=(X_1,X_2,\ldots,X_n)$ given the observations $\boldsymbol{Y}=(Y_1,Y_2,\ldots,Y_n)$ and the parameters $\boldsymbol{\theta}$ is defined as

$$P(x_1,...,x_n|\boldsymbol{Y},\boldsymbol{\theta})=\frac{P(x_1,...,x_n,y_1,...,y_n)}{\int\ldots\int P(x_1,...,x_n,y_1,...,y_n)\mathrm{dx_1...dx_n}} \tag{9}$$

where $(x_1,...,x_n)$ and $(y_1,...,y_n)$ represent the realizations of $\boldsymbol{X}$ and $\boldsymbol{Y}$. According to Eqs. (1), (2), and (8), the derivation of the joint probability density $P(x_1,...,x_n,y_1,...,y_n)$ is as follows.

$$\begin{aligned}P(x_1,\ldots,x_n,y_1,\ldots,y_n)&=\mathscr{K}\exp(-\frac{x_n}{\varsigma})\\&\times\prod_{j=1}^{n}(x_j-x_{j-1})^{[a(t_j^b-t_{j-1}^b)-1]}\exp\left[-\frac{(y_j-x_j)^2}{2\sigma_j^2}\right]\end{aligned} \tag{10}$$

where $\mathscr{K}$ is a normalization constant that enables $P(x_1,\ldots,x_n,y_1,\ldots,y_n)$ to be a density function, and σ_j, $j=1,\ldots,n$ is defined as follows

$$\begin{aligned}\sigma_j=\sigma_H\lim_{\tau\to 0}\sum_{i=1}^{\lfloor \frac{t_j-t_{j-1}}{\tau}\rfloor}\frac{1}{\sqrt{\tau}}\Bigg[\int_{(\lfloor \frac{t_j-t_{j-1}}{\tau}\rfloor-1)\tau}^{\lfloor \frac{t_j-t_{j-1}}{\tau}\rfloor\tau} c(H-0.5)s^{(0.5-H)}\\ \times\int_{(i-1)\tau}^{i\tau}(u-s)^{(H-1.5)}u^{(H-0.5)}\,\mathrm{d}u\,\mathrm{d}s\Bigg]\end{aligned} \tag{11}$$

However, due to the involvement of a large number of integrals, calculating Eq. (9) becomes quite challenging, and even difficult to implement. To avoid computing complex distributions in the high-dimensional space, the Gibbs sampler algorithm is proposed to estimate $P(x_1,...,x_n|\boldsymbol{Y},\boldsymbol{\theta})$.

Therefore, given the other $\bar{\boldsymbol{x}}_j$ of $\boldsymbol{X}$, where $\bar{\boldsymbol{x}}_j=(x_1,\ldots,x_{j-1},x_{j+1},\ldots,x_n)$, the marginal conditional density function for each x_j can be obtained based on Eq. (10). Specifically, for $1\le j\le n-1$, we have

$$\begin{aligned}&P(x_j|x_1,...,x_{j-1},x_{j+1},...,x_n,\boldsymbol{Y})=\Upsilon_{1,j}\\&\times(x_j-x_{j-1})^{[a(t_j^b-t_{j-1}^b)-1]}(x_{j+1}-x_j)^{[a(t_{j+1}^b-t_j^b)-1]}\\&\times\exp\left[-\frac{(y_j-x_j)^2}{2\sigma_j^2}\right]\end{aligned} \tag{12}$$

where $\Upsilon_{1,j}$ is a normalization constant, and $x_0=0$.

for $j=n$,

$$\begin{aligned}&P(x_n|x_1,...,x_{n-1},\boldsymbol{Y})=\Upsilon_{1,n}\exp(-\frac{x_n}{\varsigma})\\&\times(x_n-x_{n-1})^{[a(t_n^b-t_{n-1}^b)-1]}\exp\left[-\frac{(y_n-x_n)^2}{2\sigma_n^2}\right]\end{aligned} \tag{13}$$

Based on Eqs. (12) and (13), the degradation state estimation can be performed by Gibbs sampler, which is implemented as algorithm 1.

Algorithm 1 Gibbs sampler for the GP-based degradation model

Initialization: Give the initial parameters $\boldsymbol{\theta}_0$ and the degradation states $\boldsymbol{X}^{(0)}=(\boldsymbol{x}_1^{(0)},\ldots,\boldsymbol{x}_n^{(0)})$.

for $s\in 1,\ldots,\mathrm{S}$ **do**
  if $j=1$ **then**
    Generate the value $x_1^{(s)}$ of $\boldsymbol{x}_1^{(s)}$ following the marginal distribution of x_1:

$$P(x_1|x_2^{(s-1)},...,x_n^{(s-1)})$$

  else if $2\leqslant j\leqslant n-1$ **then**
    Generate the value $x_j^{(s)}$ of $\boldsymbol{x}_j^{(s)}$ following the marginal distribution of x_j:

$$P(x_j|x_1^{(s)},...,x_{j-1}^{(s)},x_{j+1}^{(s-1)},...,x_n^{(s-1)})$$

  else
    Generate the value $x_n^{(s)}$ of $\boldsymbol{x}_n^{(s)}$ following the marginal distribution of x_n:

$$P(x_n|x_1^{(s)},...,x_{n-1}^{(s)})$$

  end if
end for

The number of iterations S for the Gibbs sampler should be sufficiently large to obtain the well-approximated value of $(x_1,\ldots,x_n)$.

However, the marginal distribution Eqs. (12) and (13) are not known distribution functions, and generating random variables based on these distribution functions is not a simple task. In view of this, the MH algorithm is used to generate random variables (Pang et al., 2021). By combining the MH algorithm, it is possible to enhance the robustness of the sampling method through the design of appropriate proposal distributions and acceptance criteria, making it adaptable to different types of models and data distributions.

Due to the presence of latent variables and random effects in the model, this study employs the SEM algorithm to obtain the estimates of unknown model parameters. Assuming there are I devices, given the latent degradation states $\boldsymbol{X}_{i,1:n_i}$, and the observations $\boldsymbol{Y}_{i,1:n_i}$, $i = 1,\dots,\mathrm{I}$. Since the degradation process and the measurement noise are mutually independent, the expectation of the log-likelihood function can be decomposed into two parts as

$$\begin{aligned}
&\mathbb{E}[\log(L(\boldsymbol{\theta}))] \simeq \mathbb{E}[\log(\prod_{i=1}^{\mathrm{I}}\{f(\boldsymbol{Y}_{i,1:n_i},\boldsymbol{X}_{i,1:n_i}|\boldsymbol{\theta})\})]\\
&= \mathbb{E}[\log(\prod_{i=1}^{\mathrm{I}}\{f(\boldsymbol{Y}_{i,1:n_i}|\boldsymbol{X}_{i,1:n_i},\boldsymbol{\theta_2})\cdot f(\boldsymbol{X}_{i,1:n_i}|\boldsymbol{\theta_1})\})]\\
&= \mathbb{E}[\log(\prod_{i=1}^{\mathrm{I}}\{f(\boldsymbol{Y}_{i,1:n_i}|\boldsymbol{Y}_{i,1:n_i},\boldsymbol{\theta_2})\})]\\
&+ \mathbb{E}[\log(\prod_{i=1}^{\mathrm{I}}\{f(\boldsymbol{X}_{i,1:n_i}|\boldsymbol{\theta_1})\})]
\end{aligned} \tag{14}$$

where $\boldsymbol{\theta}_1 = \{a,b,\varsigma\}$, and $\boldsymbol{\theta}_2 = \{\sigma_H,H\}$.

For $\{\boldsymbol{X}\}$ in Eq. (14), it can be obtained by using the Gibbs sampler. In detail,

$$\begin{aligned}
&\mathbb{E}[\log(\prod_{i=1}^{\mathrm{I}}\{f(\boldsymbol{X}_{i,1:n_i}|\boldsymbol{\theta_1})\})] = \sum_{i=1}^{\mathrm{I}}\mathbb{E}[\log(\{f(\boldsymbol{X}_{i,1:n_i}|\boldsymbol{\theta_1})\})]\\
&= \sum_{i=1}^{\mathrm{I}}\sum_{j=1}^{n_i}\{(-a(t_{i,j}^b - t_{i,j-1}^b)\ln\varsigma) - \ln(\Gamma(a(t_{i,j}^b - t_{i,j-1}^b)))\\
&-\mathbb{E}(x_{i,j}-x_{i,j-1})/\varsigma + (a(t_{i,j}^b - t_{i,j-1}^b)-1)\ln(\mathbb{E}(x_{i,j}-x_{i,j-1}))\}
\end{aligned} \tag{15}$$

According to Eqs. (2) and (8), the other part can be formulated as follows

$$\begin{aligned}
&\mathbb{E}[\log(\prod_{i=1}^{\mathrm{I}}\{f(\boldsymbol{Y}_{i,1:n_i}|\boldsymbol{X}_{i,1:n_i},\boldsymbol{\theta_2})\})]\\
&= \sum_{i=1}^{\mathrm{I}}\mathbb{E}[\log(\{f(\boldsymbol{Y}_{i,1:n_i}|\boldsymbol{X}_{i,1:n_i},\boldsymbol{\theta_2})\})]\\
&= \sum_{i=1}^{\mathrm{I}}\sum_{j=1}^{n_i}\{-\frac{1}{2}(\ln(2\pi)+2\ln(\sigma_j)) - \frac{[(y_{i,j}-\mathbb{E}(x_{i,j}))]^2}{2\sigma_j^2}\}
\end{aligned} \tag{16}$$

The Nelder-Mead algorithm is then applied to maximize the log-likelihood functions. The model parameters are estimated and updated with each iteration of the Gibbs sampler until convergence is achieved.

4. SIMULATION

In the simulation study, a non-homogeneous GP model with a shape parameter following a power-law is established to simulate the hidden degradation states. Measurement noise is modeled using FGN, and Coeurjolly (Coeurjolly, 2000) introduced methods for generating FGN random variables. The parameter settings for the above model are shown in Table 1. Figure 1 displays a set of simulated paths. In order to evaluate the model performance, one curve was randomly selected as a test set and the remaining four curves were used as a training set. For comparison purposes, this paper employs two models for parameter estimation and degradation state estimation: the model proposed in this article (referred to as M_1) and the GP model with Gaussian noise, as used in the previous study (Le Son et al., 2016) (referred to as M_2). In all experiments, the first 200 iterations are considered as the burn-in phase, and the samples from the subsequent 600 iterations are retained for analysis. Note that the burn-in phase allows the Markov chain to move from the initial state to the high-probability regions of the target distribution, reducing the influence of the initial state on the final samples. Table 2 presents the parameter estimation results for M_1 and M_2. Table 2 summarizes the average and root mean square error (RMSE) of parameter estimation in the degradation model.

Table 1. Model parameters for the simulation study

Parameter	a	b	ς	H	σ_H
Value	0.5	1.1	1	0.7	20

The sampling interval is set to 1, and the total time is 100.

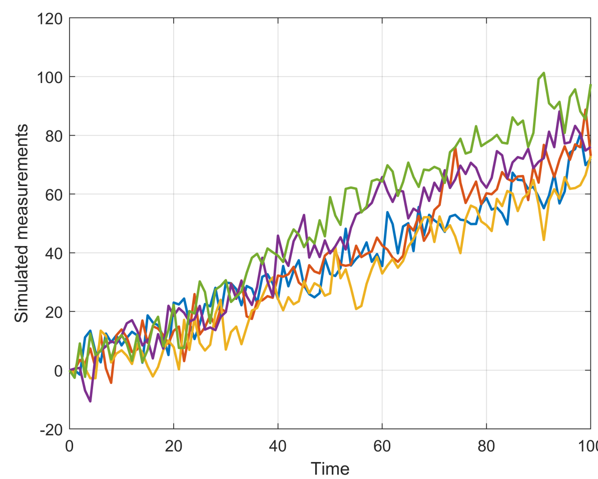

Figure 1. The simulation data

Table 2. The estimation results of model parameters

	M_1						M_2				
	a	b	ς	σ_H	H	AIC	a	b	ς	σ	AIC
Mean	0.4911	1.0909	0.9129	21.6436	0.7326	22.7986	0.6327	1.0544	0.9706	28.5285	61.5423
RMSE	0.0312	0.0232	0.1133	2.5437	0.4163	–	0.1161	0.0869	0.4664	4.3141	–

From the table, it is evident that although there are discrepancies between the estimated values and the true values, M_1 demonstrates a certain advantage in parameter identification accuracy. Regarding RMSE, the FGN assumption notably outperforms the Gaussian noise assumption. Therefore, M_1 exhibits greater versatility in handling complex noise scenarios.

Figure 2 illustrates the estimated degradation states of M_1 and M_2. In Figure 2 (a), the estimated degradation state of M_1 aligns closely with the actual degradation trajectory. In contrast, Figure 2 (b) shows a significant discrepancy between the estimated degradation state of M_2 and the actual degradation trajectory, despite the parameter estimation having converged. Since Gaussian noise is a special case of FGN when H = 0.5, FGN demonstrates greater robustness in constructing state-space models.

To further illustrate the superiority of M_1, this study compares the mean squared error (MSE) of M_1 and M_2. The MSE for M_1 is 0.1206, whereas for M_2 is 0.8152. This comparison of MSE values further corroborates the conclusions drawn from Figure 2.

5. CONCLUSION

This paper presents a novel degradation model based on GP influenced by FGN. Unlike existing Gaussian noise based models, the proposed model uses the Hurst exponent to characterise non-Markovian forms of noise, which enhances the modeling flexibility. For model identification, the proposed method employs the Gibbs sampler in combination with the SEM algorithm for parameter estimation and updating, and the MH algorithm for degradation state estimation and updating. Numerical studies show a superior estimation accuracy of the parameters and the latent degradation states. The proposed degradation model is more robust in dealing with non-Markovian measurement noises.

In future research, it would be valuable to apply this model to specific real-world systems, such as blast furnaces, power grid, and high-speed trains, and carry out example validations. Other degradation processes, such as the Wiener process, can also be improved by integrating FGN. In addition, the Gaussian noise environments can be effectively extended to non-Gaussian noise scenarios for estimating the remaining useful life of complex equipment.

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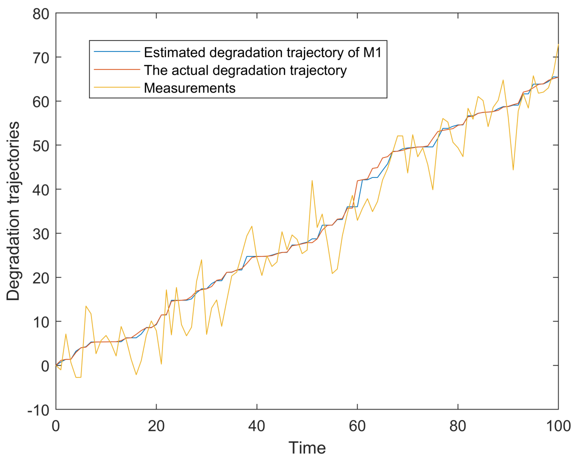
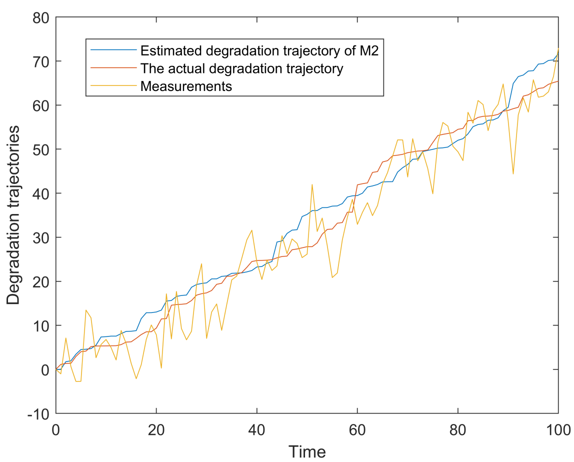


(a)



(b)

Figure 2. The estimation results of degradation states. (a) M_1. (b) M_2.

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ACKNOWLEDGMENT

Marcos Orchard would like to thank FONDECYT Chile Grant Nr. 1210031 and the Advanced Center for Electrical and Electronic Engineering, AC3E, Basal Project FB0008, ANID.