A Gamma Process Based Degradation Model with Fractional Gaussian Noise

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Overview

Introduction

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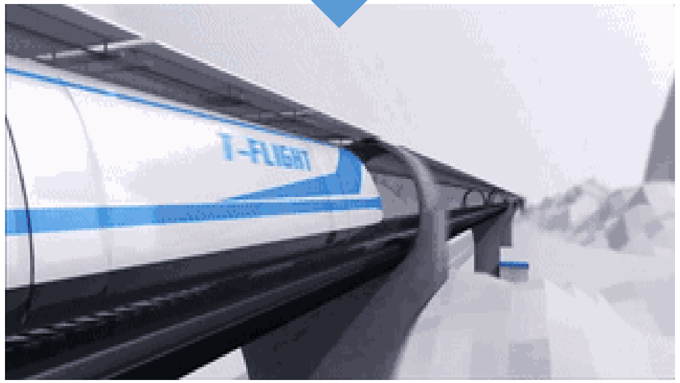
Aerospace

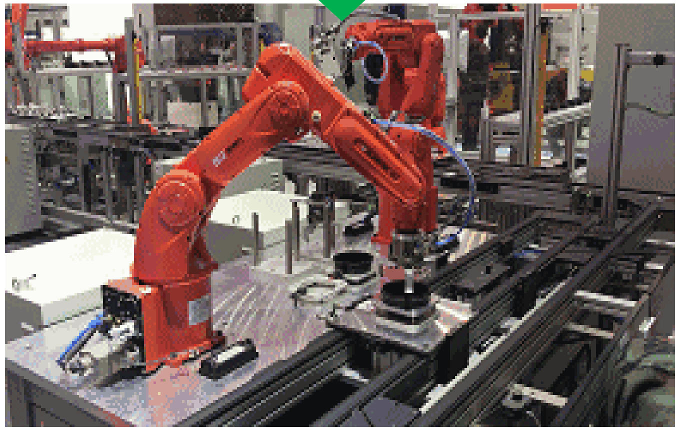
Energy

The **safety** and **reliability requirements** of modern equipment are **rising**, and the **cost** of management and maintenance is **increasing**.

Transportation



Industry



Introduction

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High speed rail accident

Power outage

Drilling platform explosion

Plane crash

Operational safety is crucial, and remaining useful life prediction is its key technology

Introduction

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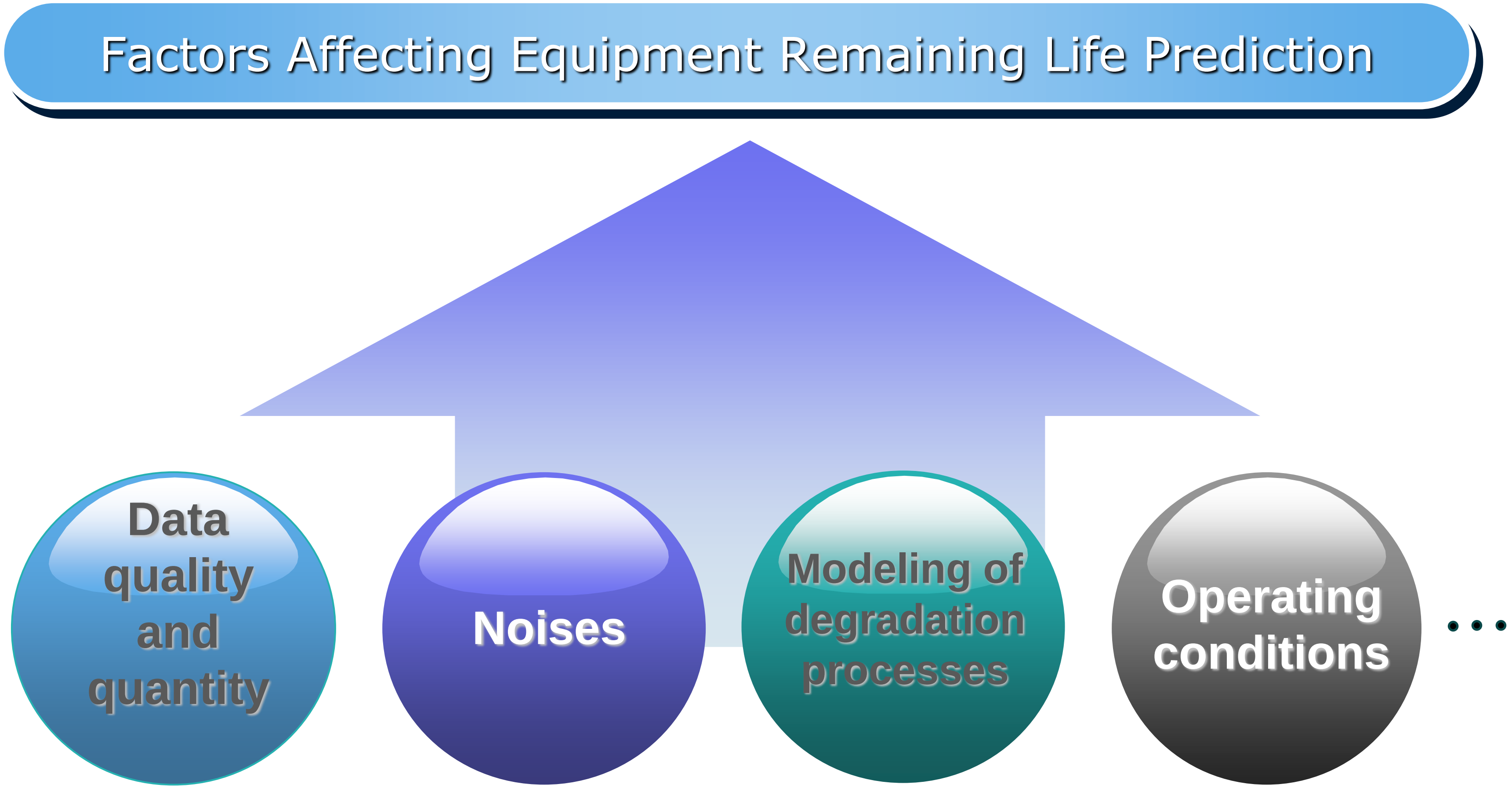
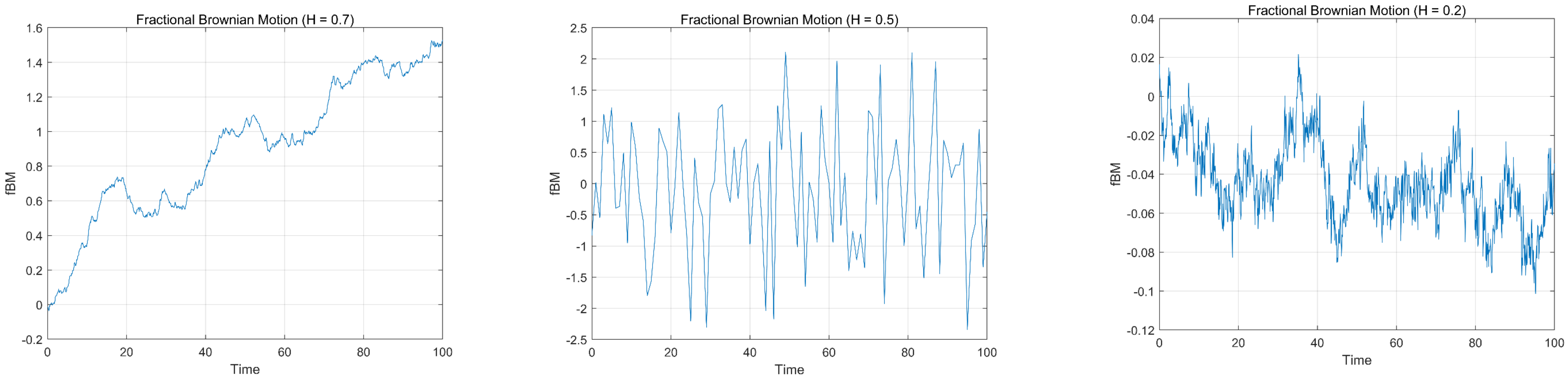


Eliminating **noises** interference and extracting the potential state of equipment are key challenges in RUL prediction.

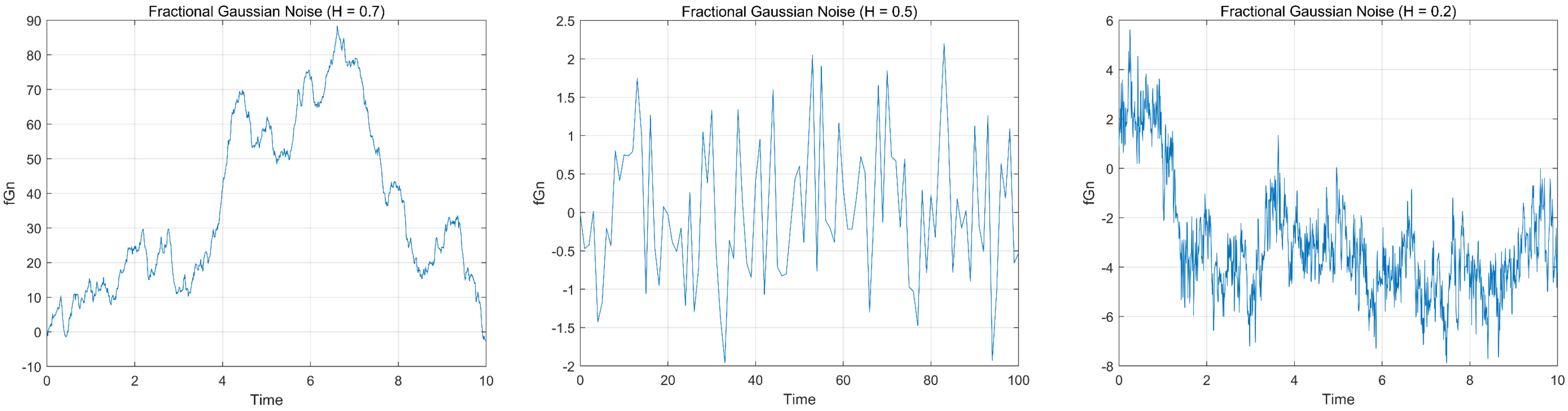
Overview

Long-range dependence



- **Long-range dependence** refers to a statistical property of a time series or stochastic process where the correlations between observations decay more slowly than the exponential rate typical in short-range dependent processes.
- The **Hurst exponent (H)** is a statistical measure used to characterize the long-term memory or dependence of time series data.

Fractional Gaussian Noise (FGN)



- **Gaussian Noise**: **H** is typically around 0.5, indicating a random walk (no memory).
- **Fractional Gaussian Noise (FGN)**: **H** can vary between 0 and 1. Values less than 0.5 indicate mean-reverting behavior, while values greater than 0.5 indicate persistent trends.
- FGN is more general than Gaussian noise.

Purpose of the work

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- By incorporating FGN into the Gamma process degradation model, we can better account for **persistent trends** and **self-similarity** in degradation processes.
- The use of FGN aims to improve the model's **predictive accuracy** and reliability, particularly in complex systems where Gaussian or white noise assumptions may lead to errors.
- This innovation provides a more **robust framework** for analyzing and forecasting degradation behavior, benefiting fields such as reliability engineering and maintenance management.
- The ultimate goal is to offer improved tools for understanding and managing system degradation in **various practical applications**.

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Proposed methodology

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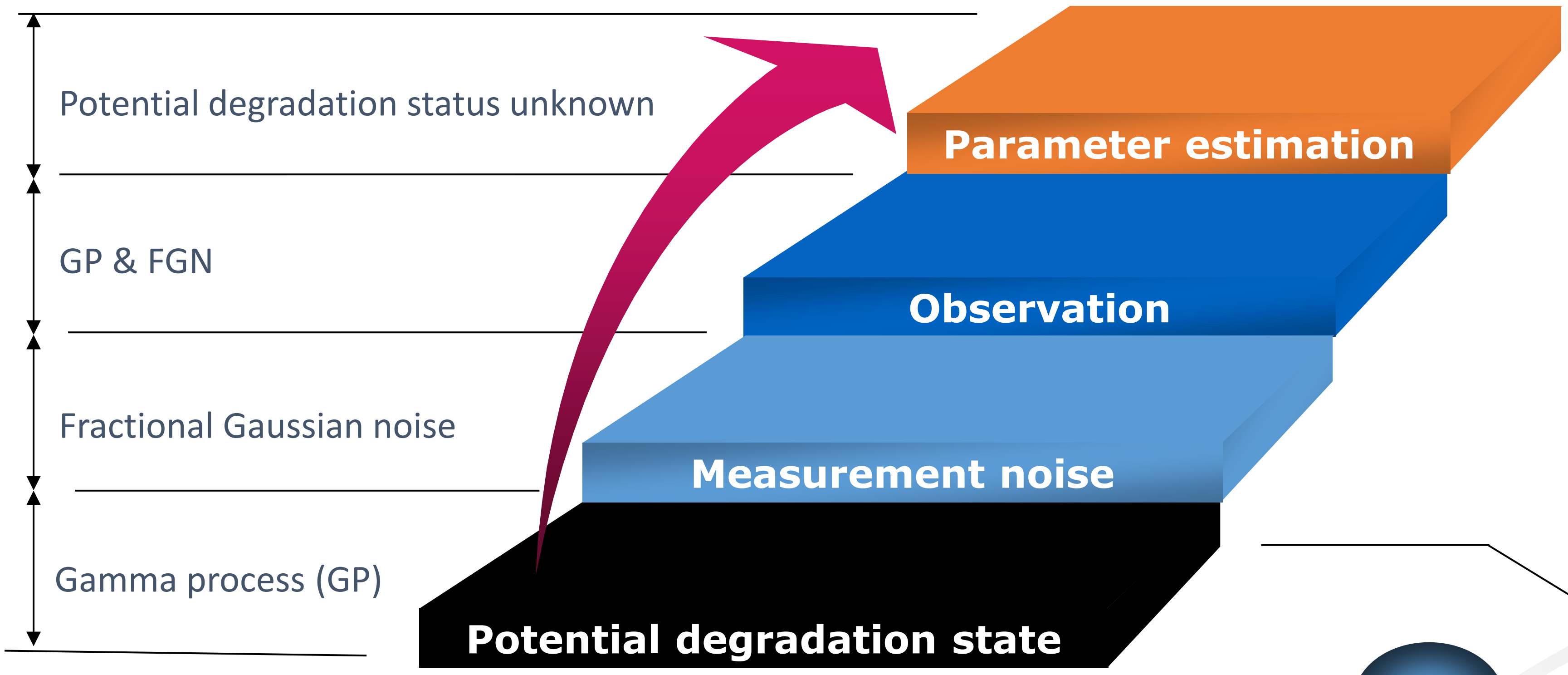
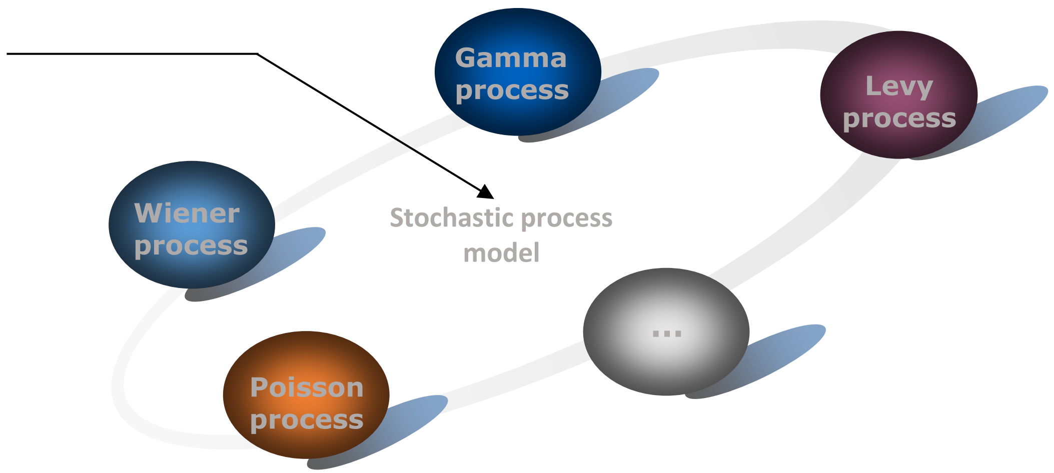


Advantages:

1. Coping with uncertainty;
2. Handling noise;
3. Updating status in real time;
4. Adapting to different degradation behaviors.



Proposed methods

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Phase 1: Degradation modeling

The degradation state $X(t) \sim Ga(x; \nu(t), \varsigma)$ *with probability density function is given by*

$$f_X(x;\nu,\varsigma) = \frac{x^{\nu(t)-1}}{\varsigma^{\nu(t)}\Gamma(\nu(t))}\exp\left(-\frac{x}{\varsigma}\right)\mathbb{I}_{[0,+\infty]}(x)$$

where $\Gamma(\tau) = \int_0^\infty z^{\tau-1}e^{-z}\,dz$ *is the Euler gamma function,* $\mathbb{I}_{[0,\infty]}(x) = 1$ *for* $x \in [0,\infty]$, and $\mathbb{I}_{[0,\infty]}(x) = 0$ *otherwise.*

Here, $\nu(t)$ *is a non-decreasing, right-continuous, real-valued function for* $t \geq 0$, *with* $\nu(0) \equiv 0$. *Empirical studies provide evidence that the shape parameter at a given time point, denoted as* t, *frequently exhibits a proportional relationship with a power law form, expressed as* $\nu(t) = at^b$.

Proposed methods

Let $Y_j = Y(t_j)$ denote the observation at monitoring time t_j, the measurement model can be expressed as

$$Y_j = X_j + \omega_j$$

where ω_j represents the measurement error, independent of X_j. Assuming that ω_j follows FGN, which can be defined as

$$\omega_j = \sigma_H (B_H(t_j) - B_H(t_{j-1}))$$

where $B_H(\cdot)$ represents standard fractional Brownian motion (FBM), H is the Hurst exponent, with $0 < H < 1$, and σ_H is the diffusion coefficient.

According to weak convergence theory, ω_j can be reconstructed as

$$\omega_j = \sigma_H \lim_{\tau \to 0} \sum_{i=1}^{\left\lfloor \frac{t_j - t_{j-1}}{\tau} \right\rfloor} \frac{1}{\sqrt{\tau}} \left[\int_{\left(\left\lfloor \frac{t_j - t_{j-1}}{\tau} \right\rfloor - 1\right)\tau}^{\left\lfloor \frac{t_j - t_{j-1}}{\tau} \right\rfloor \tau} Z_H \left(\left\lfloor \frac{t}{\tau} \right\rfloor \tau, s \right) \mathrm{ds} \right] \xi_i$$

where $\left\lfloor \frac{t}{\tau} \right\rfloor$ denotes the greatest integer less than or equal to $\frac{t}{\tau}$, ξ_i are independent identically distributed random variables with $\mathbb{E}(\xi_i) = 0$ and $\mathrm{Var}(\xi_i) = 1$, $\tau \to 0$.

Proposed methods

Phase 2 : DEGRADATION MODEL IDENTIFICATION

Gibbs sampling with the stochastic expectation-maximization (SEM) algorithm

Initialization:

Give the initial parameters $\boldsymbol{\theta}_0$ *and the degradation states* $\mathbf{X}^{(0)} = (\mathbf{x}_1^{(0)}, \dots, \mathbf{x}_n^{(0)})$.

for $s \in 1, \dots, S$ **do**

if $j = 1$ **then**

Generate the value $x_1^{(s)}$ *of* $\mathbf{x}_1^{(s)}$ *following the marginal distribution of* x_1:

$$P(x_1 \mid x_2^{(s-1)}, \dots, x_n^{(s-1)})$$

else if $2 \leqslant j \leqslant n-1$ **then**

Generate the value $x_j^{(s)}$ *of* $\mathbf{x}_j^{(s)}$ *following the marginal distribution of* x_j:

$$P(x_j \mid x_1^{(s)}, \dots, x_{j-1}^{(s)}, x_{j+1}^{(s-1)}, \dots, x_n^{(s-1)})$$

Proposed methodology

else

Generate the value $x_n^{(s)}$ *of* $\mathbf{x}_n^{(s)}$ *following the marginal distributionof* x_n :

$$P(x_n \mid x_1^{(s)},\ldots,x_{n-1}^{(s)})$$

end if

end for

The number of iterations S *for the Gibbs sampler should be sufficiently large.*

Assuming there are I *devices, given the latent degradation states* $\mathbf{X}_{i,1:n_i}$, *and the observations* $\mathbf{Y}_{i,1:n_i}$, $i = 1,\ldots,\mathrm{I}$.

Proposed methods

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E-Step:

The expectation of the log-likelihood function can be decomposed into two parts as

$$\mathbb{E}[\log(L(\boldsymbol{\theta}))] \simeq \mathbb{E}[\log(\prod_{i=1}^{I}\{f(\mathbf{Y}_{i,1:n_i}, \mathbf{X}_{i,1:n_i} \mid \boldsymbol{\theta})\})]$$

$$= \mathbb{E}[\log(\prod_{i=1}^{I}\{f(\mathbf{Y}_{i,1:n_i} \mid \mathbf{X}_{i,1:n_i}, \boldsymbol{\theta}_2) \cdot f(\mathbf{X}_{i,1:n_i} \mid \boldsymbol{\theta}_1)\})]$$

$$= \mathbb{E}[\log(\prod_{i=1}^{I}\{f(\mathbf{Y}_{i,1:n_i} \mid \mathbf{Y}_{i,1:n_i}, \boldsymbol{\theta}_2)\})]$$

$$+\mathbb{E}[\log(\prod_{i=1}^{I}\{f(\mathbf{X}_{i,1:n_i} \mid \boldsymbol{\theta}_1)\})] \qquad \text{where } \boldsymbol{\theta}_1 = \{a, b, \varsigma\}, \text{ and } \boldsymbol{\theta}_2 = \{\sigma_H, H\}.$$

M-Step:

The Nelder-Mead algorithm is then applied to maximize the log-likelihood functions.

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Case Study

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In the simulation study, a **non-homogeneous GP model** is established to simulate the hidden degradation states. Measurement noise is modeled using FGN, and the parameter settings for the above model are shown in Table 1.

Figure 1 displays a set of simulated paths.

Table 1. Model parameters for the simulation study

Parameter	a	b	ς	H	σ_H
Value	0.5	1.1	1	0.7	20

The sampling interval is set to 1, and the total time is 100.

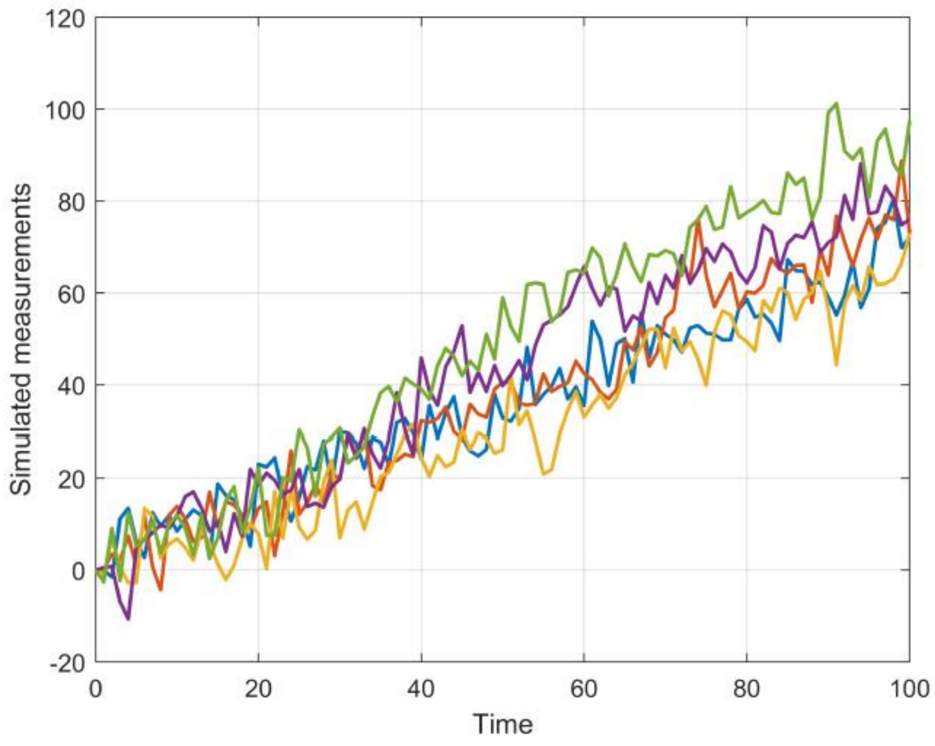

Figure 1. The simulation data

Overview

Experiments and results

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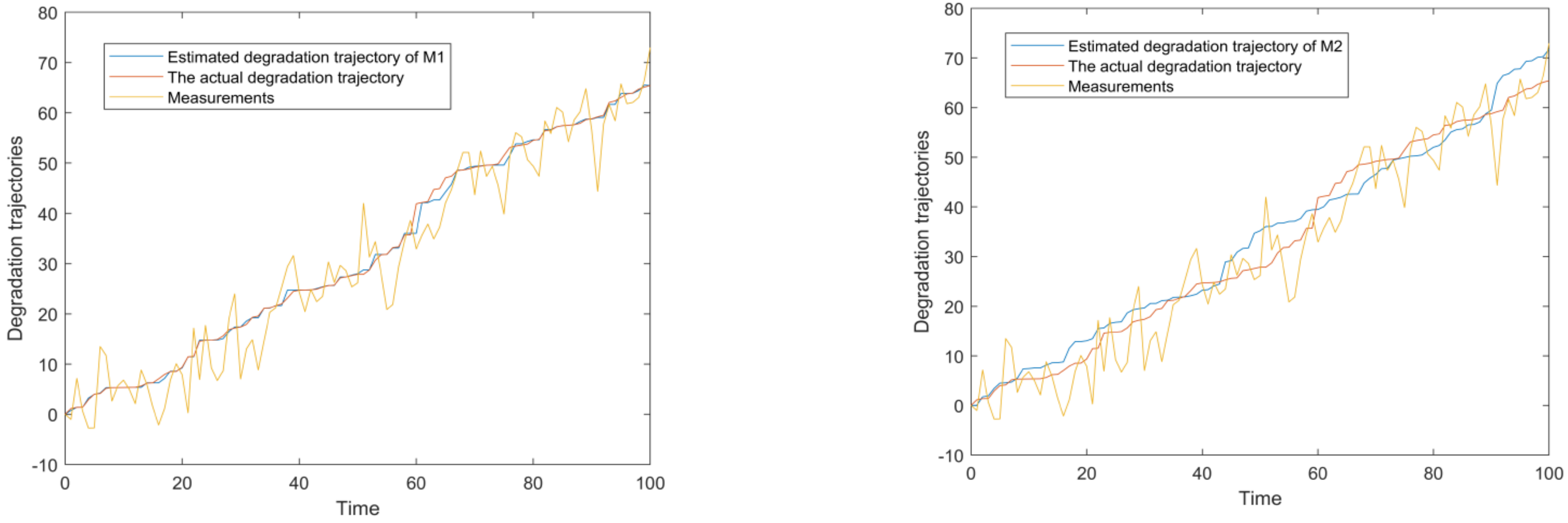
The model proposed in this article (referred to as M_1) and the GP model with Gaussian noise(referred to as M_2).



(a) (b)

Figure 2. The estimation results of degradation states. (a) M_1. (b) M_2.

Experiments and results

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In Figure 2(a), the estimated degradation state of M_1 *aligns closely with the actual degradation trajectory.*

Table 2 summarizes the average and root mean square error $(RMSE)$ *of parameter estimation in the degradation model. From the table,* M_1 *demonstrates a certain advantage in parameter identification accuracy.*

Table 2. The estimation results of model parameters

	M_1						M_2				
	a	b	ς	σ_H	H	AIC	a	b	ς	σ	AIC
Mean	0.4911	1.0909	0.9129	21.6436	0.7326	22.7986	0.6327	1.0544	0.9706	28.5285	61.5423
RMSE	0.0312	0.0232	0.1133	2.5437	0.4163	–	0.1161	0.0869	0.4664	4.3141	–

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Conclusions

- Unlike existing Gaussian-noise-based models, the proposed model uses the Hurst exponent (H) to characterize non-Markovian forms of noise, which enhances the modeling flexibility.

- Numerical studies show a superior estimation accuracy of the parameters and the latent degradation states.

- In future research, it would be valuable to apply this model to specific real-world systems, such as blast furnaces, power grid, and high-speed trains, and carry out example validations.

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