

# Particle Filtering-Based System Degradation Prediction Applied to Jet Engines

Peng Wang and Robert X. Gao

*Department of Mechanical Engineering, University of Connecticut, Storrs, CT, 06269, USA*

*pew12002@engr.uconn.edu*

*rgao@engr.uconn.edu*

## ABSTRACT

This paper investigates a real-time fault detection and degradation prediction scheme for dynamical systems such as jet engines, based on Regularized Particle Filtering (RPF). Particle Filtering is a prognosis method for the prediction of state degradation and remaining useful life (RUL) due to its demonstrated performance in handling non-linear and non-Gaussian situations. RPF overcomes the problem of sample impoverishment among particles over the resampling process. Based on measured data from hybrid sensing and nonlinear models, which link system parameters and degradation state to the measurement, RPF has been applied to establishing a framework for both state and parameter estimation, to achieve prognosis at the component level. In addition, a modified system evolution model is proposed to track both exponential and transient types of system performance degradation. The developed method is evaluated using simulated data created with C-MAPSS, which contains measured parameters associated with engine degradation under nominal and varied fault types (fan, compressor and turbine) during a series of flights. The developed system-parameter estimation method is found effective in state estimation and degradation prediction in jet engines.

## 1. INTRODUCTION

In most cases real world data contain failure signatures but little to no information about the failure evolution or state degradation, thus driving the need for health monitoring, diagnosis of faults, system performance degradations and trend prediction for dynamic systems, such as jet engines. Several prevalent sensing and diagnosis techniques have been proposed in past decades for health management in jet engines, such as gas path analysis (Volponi, 2003), exhaust composition and gas path debris (Simon, Garg, Hunter, Guo

& Semega, 2004). Gas path analysis (GPA) is one of the most popular techniques to quantify the thermodynamic performance of engines based on the hybrid sensing of temperature, pressure and other measurements. The approaches to establish the relationship between measurement and system state can be classified into two categories: data-driven and model-based. A data-driven approach requires a large amount of historical data for training and lacks generality (Peng, Dong & Zuo, 2010), while a model based approach takes advantage of merits of both physical knowledge and historical data information.

Depending on system types and noise assumptions, different methods including the Kalman filter (for linear system and Gaussian noise) (Kalman, 1960), the extended Kalman filter (for weak nonlinear system and Gaussian noise) (Julier & Uhlmann, 1997), and the particle filter (for nonlinear system and non-Gaussian noise) (Gordon, Salmond & Smith, 1993) can be applied to implement model based prognosis (Doucet & Johansen, 2009). Due to the stochastic and nonlinear nature of the engine system performance degradation, this paper presents a probabilistic degradation prediction method to achieve the diagnosis and prognosis at the component level by recursively updating the physical model with online measurement based on Regularized Particle Filtering (RPF), while RPF is proposed to overcome the sample impoverishment problem in the resampling stage of standard PF (Musso, Oudjane & Legland, 2001). Besides exponential degradation prediction, a modification of the state evolution model has been proposed to track transient changes in system state and parameters due to faults.

The rest of the paper is constructed as follows. Theoretical background of particle filtering and the modified system evolution model are introduced in Section 2, followed by the discussion of the system degradation model and thermodynamic measurement models of engines at the component level that are implemented in RPF based prognosis in Section 3. The effectiveness of the presented technique is demonstrated in Section 4, based on run-to-failure simulated data created with C-MAPSS. Finally, conclusions are drawn in Section 5.

## 2. FILTERING FRAMEWORK

In order to analyze and make inference about a dynamic system, the posterior probability density function (pdf) needs to be estimated and updated for the underlying system state, based on the availability of new measurements, in the Bayesian framework. The system model describing the evolution of the state (variables representing system performance degradation in this paper) with time and the measurement model relating observable noisy measurements to true state are not nonlinear in many dynamic systems. Particle Filtering, also referred as Sequential Monte Carlo (SMC) (Orchard, Cerda, Olivares & Silva, 2012), provides a numerical approximation for nonlinear system estimation, using a set of random samples (or particles) with associated weights to construct the pdf of a state (Gordon, 1993).

### 2.1. Regularized Particle Filtering

For the estimation of the underlying state in a nonlinear dynamic system, it is assumed the stochastic model of system evolution is known as:

$$x_k = f_k(x_{k-1}, w_{k-1}) \quad (1)$$

where  $f_k: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$  describes the state transition function from state  $x_{k-1}$  to  $x_k$  considering an order-one Markov process.  $w_{k-1}$  is the process noise representing uncertainty. The state is recursively estimated based on the measurements (Saha & Goebel, 2011):

$$z_k = h_k(x_k, v_k) \quad (2)$$

where  $h_k: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$  is the measurement function representing the relation between online measurements  $z_k$  and an unobservable degradation state  $x_k$ .  $v_k$  is the sequence of measurement noise.

In the Bayesian framework, estimation is fulfilled by recursively calculating the posterior pdf  $p(x_k | z_{1:k})$  of the state given the noisy measurements  $z_{1:k}$  (Wang, Wang & Gao, 2013). Taking into account the one-step Markov process, the pdf can be obtained using two stages: prediction and update, as shown in Eq. (3) and Eq. (4).

$$p(x_k | z_{k-1}) = \int p(x_k | x_{k-1}) p(x_{k-1} | z_{k-1}) dx_{k-1} \quad (3)$$

$$p(x_k | z_k) = \frac{p(x_k | z_{k-1}) p(z_k | x_k)}{p(z_k | z_{k-1})} \quad (4)$$

where  $p(z_k | z_{k-1})$  is the normalizing factor which can be calculated as:

$$p(z_k | z_{k-1}) = \int p(x_k | z_{k-1}) p(z_k | x_k) dx_k \quad (5)$$

In particle filters, the posterior pdf is represented and approximated by a set of random samples or particles  $\{x_{1:k}^i, i = 1, 2, \dots, N\}$  and associated importance weights  $w_k^i$ . The

weights are normalized with  $\sum_i w_k^i = 1$ . The integral operation in Eq. (3) is then approximated as the summarization of these random numbers as:

$$\begin{aligned} p(x_k | z_{k-1}) &= \int p(x_k | x_{k-1}) p(x_{k-1} | z_{k-1}) dx_{k-1} \\ &\approx \sum_{i=1}^N w_{k-1}^i \delta(x_{k-1} - x_{k-1}^i) p(x_k | x_{k-1}) = \sum_{i=1}^N w_{k-1}^i p(x_k | x_{k-1}^i) \end{aligned} \quad (6)$$

where the total number of particles  $N$  can affect the accuracy of the represented probability distribution, and computational efficiency. In the update step, the weight of each particle is updated based on the likelihood of the observation  $z_k$  at time  $k$  as:

$$w_k^i \propto w_{k-1}^i p(z_k | x_k^i) \quad (7)$$

Similarly, the posterior probability distribution  $p(x_{k+l} | z_k)$  in the  $l$ -step ahead prediction can be obtained as:

$$p(x_{k+l} | z_k) \approx \sum_{i=1}^N w_{k+l-1}^i p(x_{k+l} | x_{k+l-1}^i) \quad (8)$$

In constructing the particle filter, resampling is applied in every step to remove particles with small weights (justified by comparing the cumulative distribution function to a threshold within 0~1) and obtain equally weighted samples so as to avoid the degeneracy problem of the algorithm. After resampling, the weights of the new particle population are reset to  $w_k^i = 1/N$ . However, in the standard PF methods stated above, due to the fact that the samples are drawn from discrete distributions instead of continuous distributions, the problem of loss of diversity among the particles may arise. To overcome this problem, the Regularized Particle Filter (RPF) has been proposed. The fundamental idea is to change the discrete approximation to a continuous one of posterior pdf in the resampling stage with the rescaled kernel structure. The update process Eq. (4) becomes:

$$p(x_k | z_k) = \sum_{i=1}^N w_k^i K_h(x_k - x_k^i) \quad (9)$$

Where

$$K_h(x) = \frac{1}{h^{n_x}} K\left(\frac{x}{h}\right) \quad (10)$$

$K(\cdot)$  is the recalled kernel density and  $h$  is the kernel bandwidth, the selection of which is optimally related to the dimension of state  $n_x$  and the number of particles  $N$ .

### 2.2. System Model for Transient Degradation

System estimation includes state estimation and parameter estimation. In most cases, the parameters are included in the state transition function  $f_k: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ , and then it becomes the joint state and parameter estimation. For most dynamical models like the performance degradation, the parameters are assumed to be constant within in a small range and the artificial evolution law is adopted (Liu & West, 2001), then the state will decay in an exponential

way. However these state models do not consider the case of transient degradation due to faults, which would cause a transient change in both parameters and states (Daroogheh, Meskin & Khorasani, 2013). The idea to handle this problem proposed in this paper is to include the output prediction error or measurement innovation into the state evolution model.

If fault occurs between sampling time  $k$  and  $k+1$ , the parameters used to predict the state  $x_{k+1}$  and output  $z_{k+1}$  are assumed to be consistent with values in previous sampling times  $1:k$ . Thus there will be transient change of the output prediction error between time  $k$  and  $k+1$ . The solution is to compare the cost function

$$J = E\left[\frac{1}{2}(z_{k+1} - \hat{z}_{k+1})(z_{k+1} - \hat{z}_{k+1})^T\right] \quad (11)$$

to a predefined threshold. Where  $\hat{z}_{k+1}$  is the predicted output at time  $k+1$ . If the cost function exceeds the threshold, the state evolution model Eq. (1) becomes:

$$x_{k+1} = x_k + \gamma_k u + w_k \quad (12)$$

where  $u$  is the unit step function and  $\gamma_k$  is the time varying gain related to the cost function  $J$ . The additional item  $\gamma_k u$  is to track the state change due to failures.

### 3. MODEL FORMULATION

Gas path analysis relies on discernable changes in observable parameters to detect physical faults. The fundamental tenet underlying this approach is that physical faults occurring in components (fan, low/high pressure compressor and high/low pressure turbine) of engines induce a change in component performance (modeled as efficiency, flow capacity, etc.), which in turn produce observable changes in measureable parameters (temperature, pressure, speeds, etc.). This inverse relationship offers the approach for engine performance estimation (Volponi, 2003). In the implementation of fault detection and degradation trend prediction of engines at the component level, using the proposed estimation method, the efficiency of each component is considered as the state needing to be estimated from observable measurements.

The exponential behavior of the fault evolution or system performance degradation is common for all degradation models (Saxena, Goebel, Simon & Eklund, 2008). Thus, a generalized state evolution model in this paper is assumed as:

$$x_k = x_{k-1} - \exp(A_{k-1} \tau^{B_{k-1}}) + w_{k-1} \quad (13)$$

where  $A_{k-1}$  is the scaling factor and  $B_{k-1}$  is the time-varying factor determining the degradation rate at sampling  $k-1$ .  $\tau$  is the sampling interval and  $w$  is the associated process noise. In the training stage, parameters  $A$  and  $B$  are estimated using RPF iteratively. In the prediction stage, the latest updated parameters assigned with each particle joint with

state evolution model would provide the predicted states. Namely, the parameters stay constant in the prediction stage (Zhu, Yoon, He, Qu & Bechhoefer, 2009).

The nonlinear measurement equations that relate state (efficiency) and measurements for compressor and turbine (Moran & Howard, 2004) are listed as follows

$$CPR = \frac{P_{Cout}}{P_{Cin}} \quad (14)$$

$$T_{Cout} - T_{Cin} = \frac{T_{Cin}}{\eta_C} (CPR^{\gamma_C} - 1)$$

$$T_{Tout} - T_{Tin} = \frac{T_{Tin}}{\eta_T} \left( \left( \frac{P_{Tout}}{P_{Tin}} \right)^{\gamma_T} - 1 \right) \quad (15)$$

where,  $T_{Cin}$ ,  $T_{Cout}$ ,  $T_{Tin}$  and  $T_{Tout}$  denote the temperature of the inlet and outlet of the compressor (low/high pressure) and turbine (low/high pressure), respectively, and  $P_{Cin}$ ,  $T_{Cout}$ ,  $T_{Tin}$  and  $T_{Tout}$  denote the temperature of the inlet and outlet of the compressor and turbine, respectively. CPR is the abbreviation of compressor pressure ratio.  $\gamma_C$  and  $\gamma_T$  denote the specific heat ratio of the compressor and turbine, which are assumed to be constant.  $\eta_C$  and  $\eta_T$  denote the efficiency of the compressor and turbine, which are also assigned as the state parameter to represent engine status.

Even if no fault occurs, the engine performance still decays in an exponential way, causing an accumulative efficiency loss of each component, which in turn is represented by discernable changes of observable measurements. Fig (1) gives an example of accumulative efficiency loss and corresponding measurement change of the high pressure compressor (HPC). More details on implementation of degradation trend prediction and transient decay detection using proposed diagnosis and prognosis method are discussed in the next section.

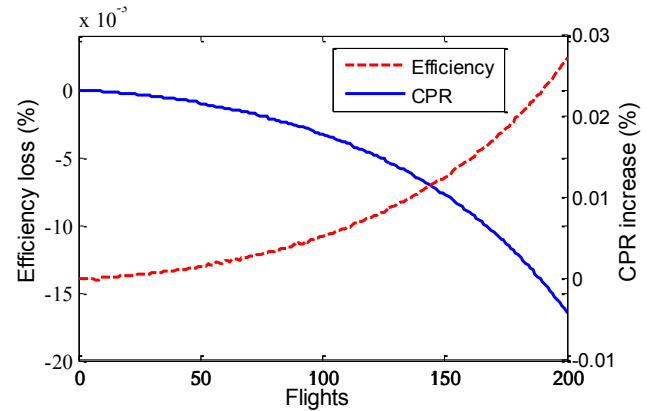


Figure 1. Accumulative efficiency loss and corresponding CPR increase of HPC

4. PERFORMANCE EVALUATION

To evaluate the performance of the proposed RPF based engine degradation prediction method, a set of high fidelity system level engine simulation data is used (Saxena, 2008). The data is created with a Matlab Simulink tool called C-MAPSS, designed to simulate normal and fault engine degradation over a series of flights. Each flight is a combination of a series of flight conditions with a reasonable transition period to allow the engine to change from one flight condition to the next. For the normal condition case, the engine is given an exponentially degrading fuel flow and efficiency profile, which denote the degradation of system performance. For fault condition cases, the engine is assigned one of five possible faults (fan, LPC, HPC, HPT and LPT) at a random flight. The fault is manifested by increasing the efficiency parameters degradation from the fault time point until the end of the simulation for the remaining flights. After a flight is simulated, a snapshot of all engine parameters is taken in the middle of cruise and applied to estimate engine state and predict the degradation trend.

In the learning stage, based on the state equations (denoted by Eq. (12) and Eq. (13)) and measurement equations (denoted by Eq. (14) and Eq. (15)), the state transition probability  $p(x_k|x_{k-1})$  and measurement probability  $p(z_k|x_k)$  can be obtained *a priori*, then the posterior distribution function of efficiency state  $p(x_{k+1}|z_k)$  can be predicted using the RPF. In the system equation, the model parameters  $A$  and  $B$  in Eq. (13) are modeled as probability distributions following the uniform distribution, to incorporate the stochastic property of the engine component degradation. The latest update of these two parameters helps construct the state transition probability  $p(x_k|x_{k-1})$  and subsequently the degradation prediction. Fig. 2 shows an example of HPC efficiency degradation prediction based on the developed methods under a normal case (natural decay, no fault occurrence), using the information of the first 160 flights as the prior knowledge to predict the efficiency trend.

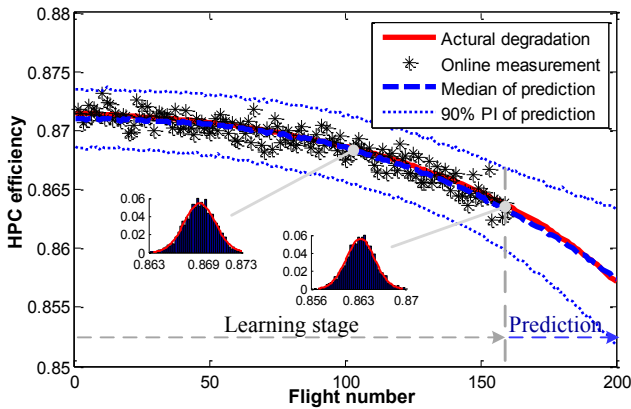


Figure 2. Predicted HPC efficiency degraded in an exponential way without external fault

Fig. 3 shows the HPC efficiency prediction under fault case, where the transient decay occurs at the 23<sup>rd</sup> flight by a 0.25% loss. Also, the information about the first 80 flights is taken as the prior knowledge for the proposed method to predict the efficiency evolution of the last 20 flights. The simulation result indicates that the proposed method can track the both exponential and transient types of system performance degradation. Because the cost function of estimated output at previous sampling time, as denoted by Eq. (11), is checked each step, there is a time delay for the estimation to track the transient change. Fig. 4 is the evolution of distribution of parameters  $A$  and  $B$  in Eq. (13). It is noted that the value of both parameters are consistent before and after the transient change. In addition, because there is no new information to update the parameters, they stay the same in the prediction stage.

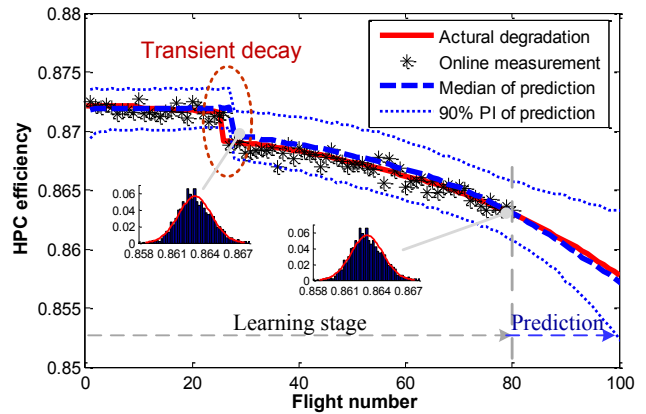


Figure 3. Predicted HPC efficiency with a transient decay under the effect of external fault

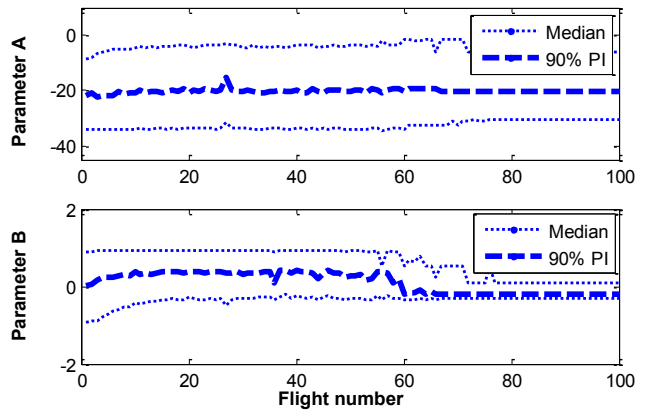


Figure 4. Evolution of distribution of parameters  $A$  and  $B$  for HPC efficiency estimation

Fig. 5 shows another example of LPT efficiency prediction in the fault case, where the fault occurs at 33<sup>rd</sup> flight. Fig. 6 is the corresponding evolution of parameters distribution.

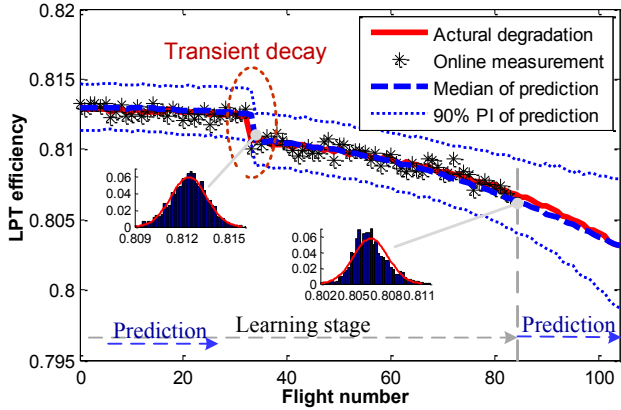


Figure 5. Predicted LPT efficiency with a transient decay under the effect of external fault

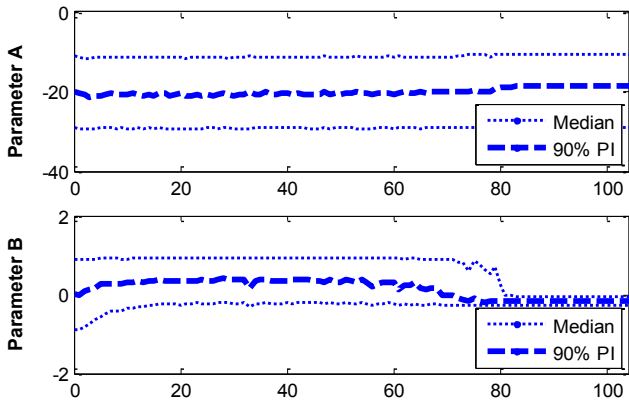


Figure 6. Evolution of distribution of parameters A and B for LPT efficiency estimation

To evaluate the effectiveness and robustness of proposed method on degradation prediction, Monte Carlo simulation is applied to derive the comprehensive simulation results. Each scenario has been run for 100 times. Mean and root mean square (RMS) of root mean square error (RMSE) of median prediction are listed in Table 1.

Table 1 Monte Carlo simulation result of proposed method

	Normal HPC	Fault HPC	Fault LPT
Mean	0.086%	0.1%	0.13%
RMS	0.097%	0.12%	0.14%

Extended Kalman filter (EKF) is selected here as the alternative method to compare with PF, while the results are shown in Fig. 7. Maximum likelihood (ML) integrated with EKF is adopted to estimate the unknown parameters in the state evolution model, based on which prediction is performed. It is found that prediction accuracy of PF is over EKF+ML, and the prediction accuracy of natural degradation over the mixed degradation.

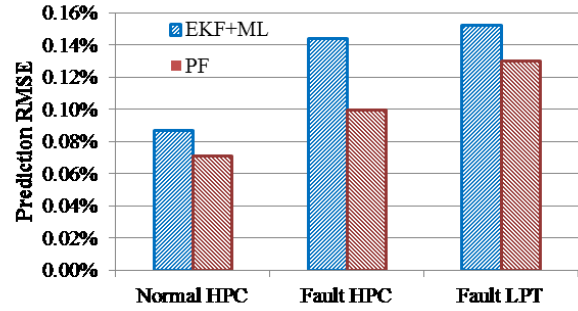


Figure 7. Performance comparison between PF and EKF

5. CONCLUSION

Particle Filtering has been investigated as a prognostic method for both state and parameter estimations in determining the efficiency degradation of jet engines as an example of dynamical system prognosis, at the component level. State estimator is modified by a cost function that compares the predicted measurements to updated measurements, and enables the tracking of transient decays in addition to exponential type of degradations. Simulated data sets including normal and fault cases generated by the C-MAPSS program have been used to evaluate the effectiveness of the developed algorithm for engine degradation state prediction, with quantified confidence intervals to manage uncertainty. In the three examples considered, the results indicate that the method can track transient changes within two steps, and the prediction error is less than 1%. Future research will investigate the robustness of the developed algorithm for different applications under different operational conditions, using experimental data.

ACKNOWLEDGEMENT

This research has been partially supported by National Science Foundation under grant CMMI-1300999 and CNS-1239030.

REFERENCES

Doucet A., & Johansen A. (2009). A Tutorial on Particle Filtering and Smoothing: Fifteen Years Later. In Crisan D., & Rozovsky B., *The Oxford Handbook of Nonlinear Filtering*, (656-704). Oxford: Oxford University Press.

Darogheh, N., Meskin, N., & Khorasani, K. (2013). Particle filtering for state and parameter estimation in gas turbine engine fault diagnostics. *Proceedings of 2013 American Control Conference*. June 17-19, Washington, DC, USA.

Gordon, N. J., Salmond, D., & Smith, A. (1993). Novel approach to nonlinear/non-Gaussian Bayesian state estimation. *IEE Proceedings-F Radar and Signal Processing*, vol. 140(2), pp. 107-113. doi: 10.1049/ip-f-2.1993.0015

- Julier, S., & Uhlmann, J.,(1997). A new extension of Kalman Filter to nonlinear systems. *Proceedings of 11th International Symposium on Aerospace/Defense sensing, Simulation and Controls, Multi Sensor Fusion, Tracking and Resource Management*. July 28, Orlando, FL, USA. doi: 10.1117/12.280797
- Kalman, R. E. (1960). A new approach to linear filtering and prediction problems. *Transactions of the ASME-Journal of Basic Engineering*, vol. 82, pp. 35-45. doi: 10.1115/1.3662552
- Liu, J., & West, M. (2001). Combined parameter and state estimation in simulation based filtering. In Doucet, A., Freitas, N., & Gordon, N., *Sequential Monte Carlo Methods in Practice*, (197-223). NY: Springer New York.
- Moran, M., & Howard, N. S. (2004). *Fundamentals of Engineering Thermodynamics*. NJ: Wiley.
- Musso, C., & Oudjane, N. (2001). Improving regularized particle filters. In Doucet, A., Freitas, N., & Gordon, N., *Sequential Monte Carlo Methods in Practice*, (197-223). NY: Springer New York.
- Orchard, M.E., Cerda, M., Olivares, B., & Silva, J. (2012). Sequential Monte Carlo methods for Discharge Time Prognosis in Lithium-Ion Batteries. *International Journal of Prognostics and Health Management*, vol. 3(2) 010, pp. 1-12.
- Peng, Y., Dong, M., & Zuo, M. J. (2010). Current status of machine prognostics in condition-based maintenance: a review. *International Journal of Advanced Manufacturing Technology*. vol. 50, pp. 297-313. doi: 10.1007/s00170-009-2482-0
- Saha, B., & Goebel, K. (2011). Model Adaptation for Prognostics in a Particle Filtering Framework. *International Journal of Prognostics and Health Management*, vol. 2(1) 006, pp. 1-10
- Simon, D. L., Garg, S., Hunter, G. W., Guo, T. H., & Semega, K. J. (2004). Sensor needs for control and health management of intelligent aircraft engines. Technical memo. NASA/TM-2004-213202
- Saxena, A., Goebel, K., Simon, D., & Eklund, Neil. (2008). Damage propagation modeling for aircraft engine run-to-failure simulation. *Proceedings of International Conference on Prognostics and Health Management*. October 6-9, Denver, CO. doi: 10.1109/PHM.2008.4711414
- Volponi, A. J. (2003). Foundation of gas path analysis. In Mathludakls, K., & Sieverding, C. H., *GasTurbine Condition Monitoring and Fault Diagnosis*, (1-16). Belgium: von Karman Institute.
- Wang, J., Wang, P., & Gao, R. (2013). Tool life prediction for sustainable manufacturing. *Proceedings of 11<sup>th</sup> Global Conference on Sustainable Manufacturing*, September 23-25, Berlin, Germany.
- Zhu, J., Yoon, J., He, D., Qu, Y., & Bechhoefer, E. (2013). Lubrication Oil Condition Monitoring and Remaining Useful Life Prediction With Particle Filtering.

*International Journal of Prognostics and Health Management*: vol. 4 020, pp. 1-15.

#### BIOGRAPHIES



**Peng Wang** is a PhD Candidate at the University of Connecticut. He received his B.S. and M.S. from Beijing University of Chemical Engineering, China in 2010 and 2013 respectively. His research interests include system estimation, health monitoring, fault diagnosis and prognosis.



**Robert X. Gao** received his Ph.D. from the Technical University of Berlin, Germany in 1991. He is a professor and the Pratt and Whitney Chair of Mechanical Engineering at the University of Connecticut. His research interests include signal transduction, smart structures and materials, and signal processing for diagnosis and prognosis of dynamic systems. He is a recipient of multiple awards, and a Fellow of IEEE, SME, and ASME.