

# A Qualitative Fault Isolation Approach for Parametric and Discrete Faults Using Structural Model Decomposition

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## ABSTRACT

With increasing complexity of engineering systems, fault diagnostics plays a significant role in ensuring that they operate safely. Such systems most often exhibit mixed discrete and continuous, i.e., hybrid, behavior, and may encounter both parametric faults (unexpected changes in system parameters) as well as discrete faults (unexpected changes in component modes). Diagnosis becomes computationally very complex due to the large number of possible system modes, and possible mode changes that occur near the point of fault occurrence. This paper presents a qualitative fault isolation framework for integrated diagnosis of both parametric and discrete faults in hybrid systems, based on structural model decomposition. Fault isolation is performed by analyzing the qualitative information of the residual deviations, and considering observation delay. The great advantage of structural model decomposition for this problem is that it essentially defines several smaller independent diagnosis problems that become more efficient to solve, and makes the overall diagnosis problem more scalable. To demonstrate and test the validity of our approach, we use a hydraulic multi-tank system as the case study in simulation. Results illustrate that the approach is both efficient and scalable.

## 1. INTRODUCTION

Fault diagnostics plays a critical role in ensuring that complex engineering systems operate in a safe manner. Most industrial systems are best represented as *hybrid systems* that have discrete behavioral modes, with each mode having its own continuous dynamics. Such systems, once fielded, typically en-

counter both *parametric faults* and *discrete faults*. Parametric faults are characterized by unexpected changes in the system parameters, e.g., a leak in a hydraulic tank; while discrete faults are characterized by unexpected changes in component modes, e.g., an electrical relay stuck open. The diagnosis of hybrid systems is computationally complex due to the large number of possible system modes and mode changes that occur before and during fault detection and isolation.

In previous work, approaches for hybrid systems diagnosis have focused on either hybrid system modeling (Rienmüller, Bayouh, Hofbaur, & Travé-Massuyès, 2009; Bayouh, Travé-Massuyès, & Olive, 2008), hybrid state estimation (Hofbaur & Williams, 2004), or a combination of on-line state tracking and residual (the difference between observed and predicted behavior) evaluation (Benazera & Travé-Massuyès, 2009). However, in all those approaches, the proposed solutions involve modeling and pre-enumeration of the set of *all* possible system-level modes, which grows exponentially with the number of switching components, and so do not scale well. Other works have instead built hybrid system models in a *compositional* way, where modes are defined at a local level (e.g., at the component level), and the system-level mode is defined implicitly by the local component-level modes thus avoiding the pre-enumeration of all the system-level (Narasimhan & Brownston, 2007; Trave-Massuyes & Pons, 1997).

Following the compositional modeling approach, in previous work (Daigle, Bregon, & Roychoudhury, 2015; Bregon, Daigle, & Roychoudhury, 2016), we proposed such a modeling framework for hybrid systems. A system is modeled as a set of interacting components, with each component defined by its own set of (local) modes, with a different set of mathematical constraints describing the continuous

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dynamics in each mode. For a given system mode, structural model decomposition (Roychoudhury, Daigle, Bregon, & Pulido, 2013) is used to construct minimal submodels for residual generation, and, based on efficient causality reassignment (Daigle, Bregon, & Roychoudhury, 2015), local submodels can be efficiently reconfigured upon mode changes. In (Bregon et al., 2016), we presented an approach for qualitative fault isolation of parametric faults in hybrid systems using submodel-based residuals.

In this paper, we extend this framework to also isolate discrete faults. We present how discrete faults can be described within the same qualitative fault isolation framework, thus enabling seamless integrated diagnosis of both types of faults. Allowing for observation delay, we also reason about the system mode in which the observations may come from, which complicates the diagnosis problem. Unlike previous approaches based on this methodology (P. J. Mosterman & Biswas, 1999; Daigle, Koutsoukos, & Biswas, 2009; Narasimhan & Biswas, 2007), we make use of structural model decomposition, and show how it improves the efficiency and scalability of the diagnosis task compared to global model-based approaches. Effectively, local independent diagnosis subproblems are defined that can be easily solved, because they have only a fraction of the faults and residuals, and each local submodel has only a small set of local modes that must be reasoned over, compared to a large set of system-level modes when using a global system model for residual generation. We use a hydraulic multi-tank system as a case study for demonstrating that our approach can correctly isolate single, persistent faults in hybrid systems in the presence of observation delay.

The paper is organized as follows. Section 2 summarizes the compositional modeling approach and introduces the multi-tank case study. Section 3 formulates the problem we solve in this paper. Section 4 describes the qualitative fault isolation approach for diagnosing both parametric and discrete faults in hybrid systems. Section 5 demonstrates the approach for the case study. Section 6 reviews the related work and current approaches for hybrid systems fault diagnosis and puts our work into context. Finally, Section 7 concludes the paper.

## 2. COMPOSITIONAL HYBRID SYSTEMS MODELING

In this section we review the compositional modeling approach for hybrid systems proposed in (Daigle, Bregon, & Roychoudhury, 2015). The basic idea in (Daigle, Bregon, & Roychoudhury, 2015) is that models are made up of a set of user-defined components, where each component is defined by a set of modes. Each mode is then described by a different set of constraints specifying the continuous dynamics. In the following, we summarize the main details of the modeling framework and the structural model decomposition approach. For additional details, we refer the reader to (Daigle, Bregon, & Roychoudhury, 2015).

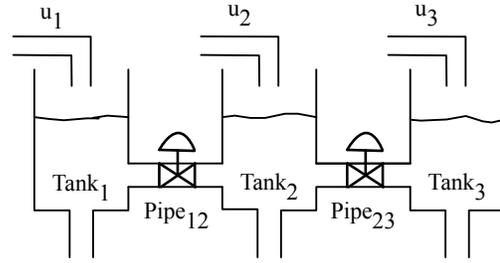


Figure 1. Three-tank system running example.

### 2.1. System Modeling

In each mode, the continuous dynamics of a component are modeled using a set of *variables* and a set of *constraints*. A constraint is defined as follows:

**Definition 1 (Constraint).** A constraint  $c$  is a tuple  $(\varepsilon_c, V_c)$ , where  $\varepsilon_c$  is an equation involving variables  $V_c$ .

A component is defined by a set of constraints over a set of variables. The constraints are partitioned into different sets, one for each component mode. A component is then defined as follows:

**Definition 2 (Component).** A *component*  $\kappa$  with  $n$  discrete modes is a tuple  $\kappa = (V_\kappa, \mathcal{C}_\kappa)$ , where  $V_\kappa$  is a set of variables and  $\mathcal{C}_\kappa$  is a set of constraints sets, where  $\mathcal{C}_\kappa$  is defined as  $\mathcal{C}_\kappa = \{C_\kappa^1, C_\kappa^2, \dots, C_\kappa^n\}$ , with a constraint set,  $C_\kappa^m$ , defined for each mode  $m = \{1, \dots, n\}$ .

To illustrate our proposal, throughout the paper we will use a three-tank system, shown in Fig. 1. The components of the three-tank system, defined in Table 1, are three tanks, Tank<sub>1</sub>, Tank<sub>2</sub> and Tank<sub>3</sub>; and two connecting pipes Pipe<sub>12</sub> and Pipe<sub>23</sub>. The connecting pipes have binary on/off valves, so each has two modes. Thus, there are four total modes in the system. For Tank <sub>$i$</sub> ,  $m_i$  is the fluid mass,  $u_i$  is the input flow,  $p_i$  is the tank pressure,  $K_i$  is the tank capacity,  $q_i$  is the flow out a drainage pipe, and  $R_i$  is the resistance of that pipe. For Pipe <sub>$ij$</sub> ,  $q_{ij}$  is the flow through the pipe, and  $R_{ij}$  is the resistance of the pipe.

**Example 1.** Consider the component Pipe<sub>12</sub>. It has two modes: *off* (represented as mode 1 in Table 1) and *on* (represented as mode 2). In the *off* mode, the flow through the pipe is set to 0, and in the *on* mode, it is dependent on the pressure difference between the two adjacent tanks.

We define a system model as a set of components:

**Definition 3 (Model).** A *model*  $\mathcal{M} = \{\kappa_1, \kappa_2, \dots, \kappa_k\}$  is a finite set of  $k$  components for  $k \in \mathbb{N}$ .

The set of variables for a model,  $V_{\mathcal{M}}$ , is the union of all

Table 1. Components of the three-tank system.

Component	Mode	Constraints
Tank <sub>1</sub>	1	$\dot{m}_1 = u_1 - q_1 - q_{12}$
		$p_1 = m_1 / K_1$
		$q_1 = p_1 / R_1$
		$q_1^* = q_1$ $m_1 = \int_{t_0}^t \dot{m}_1$
Pipe <sub>12</sub>	1	$q_{12} = 0$
	2	$q_{12} = (p_1 - p_2) / R_{12}$
Tank <sub>2</sub>	1	$\dot{m}_2 = u_2 + q_{12} - q_2 - q_{23}$
		$p_2 = m_2 / K_2$
		$q_2 = p_2 / R_2$
		$q_2^* = q_2$ $m_2 = \int_{t_0}^t \dot{m}_2$
Pipe <sub>23</sub>	1	$q_{23} = 0$
	2	$q_{23} = (p_2 - p_3) / R_{23}$
Tank <sub>3</sub>	1	$\dot{m}_3 = u_3 + q_{23} - q_3$
		$p_3 = m_3 / K_3$
		$q_3 = p_3 / R_3$
		$q_3^* = q_3$ $m_3 = \int_{t_0}^t \dot{m}_3$

the component variable sets, i.e., for  $d$  components,  $V_{\mathcal{M}} = V_{\kappa_1} \cup V_{\kappa_2} \cup \dots \cup V_{\kappa_d}$ .  $V_{\mathcal{M}}$  consists of five disjoint sets, namely, the set of state variables,  $X_{\mathcal{M}}$ ; the set of parameters,  $\Theta_{\mathcal{M}}$ ; the set of inputs (variables not computed by any constraint),  $U_{\mathcal{M}}$ ; the set of outputs (variables not used to compute any other variables),  $Y_{\mathcal{M}}$ ; and the set of auxiliary variables,  $A_{\mathcal{M}}$ . Parameters,  $\Theta_{\mathcal{M}}$ , include explicit model parameters that are used in the model constraints (e.g., fault parameters). Auxiliary variables,  $A_{\mathcal{M}}$ , are additional variables that are used to simplify the structure of the equations.

**Example 2.** In the three-tank system model, we have  $X_{\mathcal{M}} = \{m_1, m_2, m_3\}$ ,  $\Theta_{\mathcal{M}} = \{R_1, R_2, R_3, R_{12}, R_{23}, K_1, K_2, K_3\}$ ,  $U_{\mathcal{M}} = \{u_1, u_2, u_3\}$ , and  $Y_{\mathcal{M}} = \{q_1^*, q_2^*, q_3^*\}$ . Remaining variables belong to  $A_{\mathcal{M}}$ . Here, the \* superscript is used to denote a measured value of a physical variable, e.g.,  $q_3$  is the flow and  $q_3^*$  is the measured flow.

The interconnection structure of the model is captured using shared variables between components, i.e., components  $\kappa_i$  and  $\kappa_j$  are connected if  $V_{\kappa_i} \cap V_{\kappa_j} \neq \emptyset$ .

**Example 3.** In the three-tank system model, component  $\kappa_2$  (Pipe<sub>12</sub>) is connected to  $\kappa_1$  (Tank<sub>1</sub>) through  $q_{12}$  and  $p_1$ , and to  $\kappa_3$  (Tank<sub>2</sub>) through  $q_{12}$  and  $p_2$ .

In our work, a fault is the cause of an unexpected, persistent deviation of the system behavior from the acceptable nominal behavior. Differently to our previous work in (Bregon et al., 2016), we consider both parametric and discrete faults and link faults to the set of parameters  $\Theta_{\mathcal{M}}$  and to the set of switching components, respectively. More formally, faults are defined as follows.

**Definition 4 (Parametric Fault).** A parametric fault  $f$  is a persistent constant deviation of exactly one parameter  $\theta \in \Theta_{\mathcal{M}}$  of the system model  $\mathcal{M}$  from its nominal value.

**Definition 5 (Discrete Fault).** A discrete fault  $f$  is persistent change in the mode of exactly one component  $\kappa \in \mathcal{M}$  from its nominal value.

The model constraints,  $C_{\mathcal{M}}$ , are the union of the component constraints over all modes, i.e.,  $C_{\mathcal{M}} = C_{\kappa_1} \cup C_{\kappa_2} \cup \dots \cup C_{\kappa_d}$ . Constraints are exclusive to components, that is, a constraint  $c \in C_{\mathcal{M}}$  belongs to exactly one  $C_{\kappa}$  for  $\kappa \in \mathcal{M}$ .

To refer to a particular mode of a model we use the concept of *mode vector*. A mode vector  $\mathbf{m}$  specifies the current mode of each of the components of a model. So, the constraints for a mode  $\mathbf{m}$  are denoted as  $C_{\mathcal{M}}^{\mathbf{m}}$ .

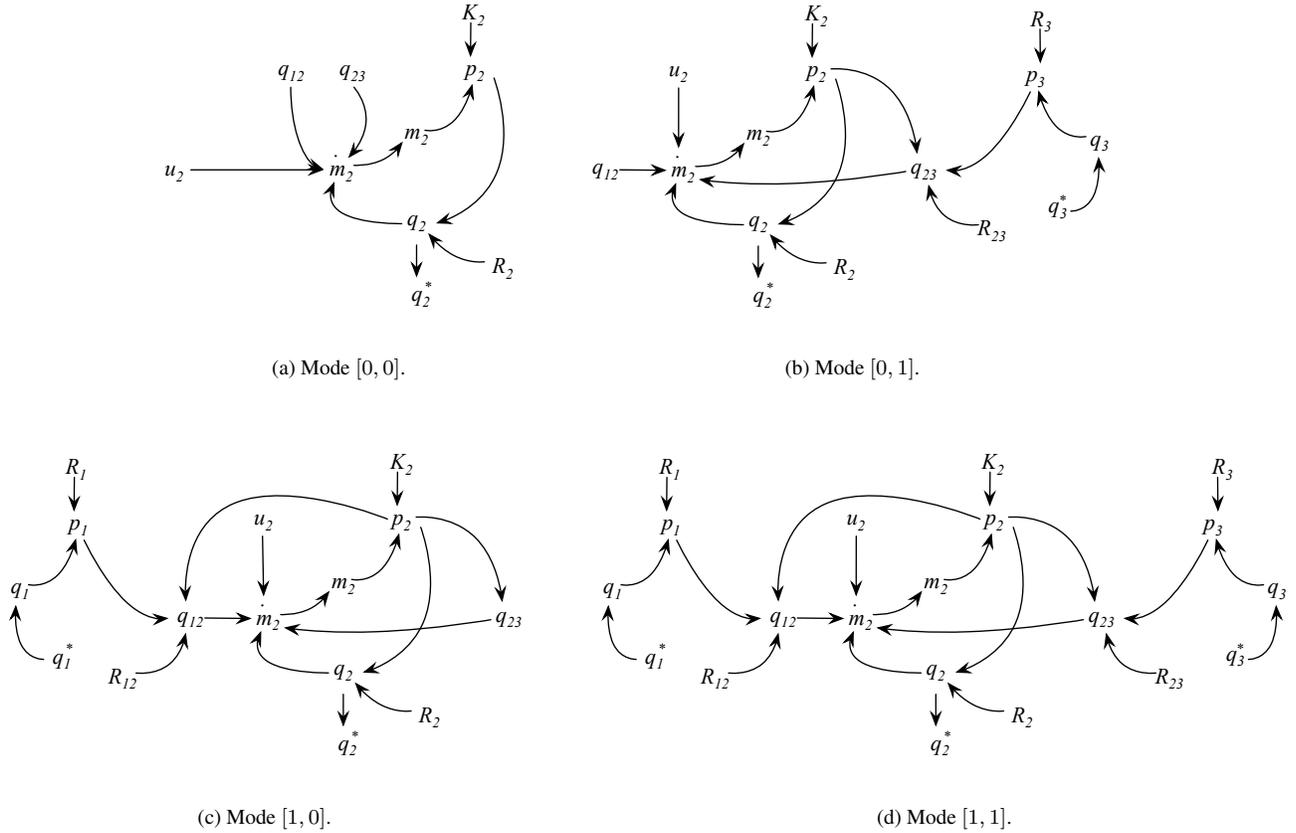
For shorthand, we will refer to the modes only of the components with multiple modes. For the three-tank system, since connecting pipes can be either on or off, we use a 0 to denote on and a 1 to denote off. So, for example, mode  $[1, 0]$  means that Pipe<sub>12</sub> is off (mode 1), and Pipe<sub>23</sub> is on (mode 2).

The switching behavior of each component can be defined in many different ways, such as using a finite state machine or a similar type of control specification. For the purposes of this paper, we view the switching behavior as a black box where the mode change event is given, and refer the reader to many of the approaches already proposed in the literature for modeling the switching behavior (Henzing, 2000; P. Mosterman & Biswas, 2000).

## 2.2. Structural Model Decomposition for Hybrid Systems

Our framework is based on structural model decomposition to divide the system model into submodels for the purpose of computing residuals, i.e., the difference between observed and predicted system behavior, which are then used for diagnosis as we describe later in Section 4. The main advantage of structural model decomposition, in contrast to using a global model for residual generation, is that each residual only responds to a subset of the faults, thus decreasing the complexity of the diagnosis process. Further, it allows the diagnosis task to be distributed, improving scalability, as proven in (Bregon et al., 2014). Later, in Section 4, we will show the specific advantages that structural model decomposition provides for diagnosis of hybrid systems.

In order to derive submodels, we need to assign causality to the system. Given a constraint  $c$ , belonging to a specific mode of a specific component, the notion of a *causal assignment* is used to specify a possible computational direction, or *causality*, for the constraint  $c$ . This is done by defining which  $v \in V_c$  is the dependent variable in equation  $\varepsilon_c$ . For a given mode, we have the set of causal assignments over the entire model in that mode, and with that we can compute the

Figure 2.  $q_2^*$  submodel graphs for the four modes of the system.

minimal submodels, using the `GenerateSubmodel` algorithm described in our previous work (Roychoudhury et al., 2013). The algorithm finds a submodel, which computes a set of local outputs given a set of local inputs, by searching over the causal model. It starts at the local inputs, and propagates backwards through the causal constraints, finding which constraints and variables must be included in the submodel. When possible, causal constraints are inverted in order to take advantage of local inputs. Additional information and the pseudocode are provided in (Roychoudhury et al., 2013).

In the context of residual generation, we set the local output set to a single measured value, and the local inputs to all other measured values and the (known) system inputs. That is, we exploit the analytical redundancy provided by the sensors in order to find minimal submodels to estimate values of sensor outputs. In this framework, we consider one submodel per sensor, each producing estimated values for that sensor. Assuming that the set of sensors does not change from mode to mode, we will always have one submodel per sensor. Since the set of constraints changes from mode to mode, the set of submodels will change as well, however, by taking advantage

of causality information, reconfiguring the submodels can be done efficiently (Daigle, Bregon, & Roychoudhury, 2015).

**Example 4.** Submodels can be represented visually using a graph notation, where vertices correspond to variables, and edges correspond to constraints with causal assignments, i.e., a directed edge from  $v_i$  to  $v_j$  means that  $v_j$  is computed using  $v_i$ . The submodel graphs for  $q_2^*$  for all possible modes are shown in Fig. 2. Note that in all modes, the submodel for  $q_2^*$  is decoupled from the dynamics of the states of its neighboring tanks, by taking the flow measurements in the neighboring tanks as inputs. So, in an  $n$ -tank system, the submodel for tank  $i$  would have at most four local modes, because any switches beyond the tank's neighboring connecting pipes will be decoupled by the decomposition. Further, any faults in tanks other than  $i$ ,  $i + 1$ , and  $i - 1$  would have no effect on the residual for tank  $i$ . Note also that changes in  $K_{i+1}$  and  $K_{i-1}$  are decoupled from the residual for tank  $i$ .

### 3. PROBLEM FORMULATION

In (Daigle, Roychoudhury, & Bregon, 2015) we presented our qualitative fault diagnosis approach that works by reasoning

over measurement deviations from their expected nominal behavior. These observations are formed from a qualitative abstraction of residual signal deviations. Residuals are computed as the difference between predicted nominal,  $\hat{y}(t)$ , and measured,  $y(t)$ , system variables, i.e. a residual  $r(t)$  is computed as  $r(t) = y(t) - \hat{y}(t)$ . In our proposed framework, predicted system variables  $\hat{y}(t)$  are computed using the minimal submodels as mentioned in the previous section. Once residuals are computed, fault detection is performed by determining statistically significant nonzero deviations in the residuals, which are then abstracted into a symbolic representation to form *fault signatures*. These symbols are computed from the residuals using symbol generation, as described in (Daigle, Roychoudhury, & Bregon, 2015). Finally, fault isolation is carried out by comparing predicted signatures against observed signatures.

In the context of hybrid systems, the structure of the residual generators changes from mode to mode, causing the set of fault signatures to also change. Observing mode change events can help to match the observations to both the fault and the mode in which they occurred. Further, if there is a delay in the observation of fault signatures, then the mode in which the deviation actually occurred may not be the current mode in the system in which it was observed, and consequently, the fault signature for the estimated fault could mismatch the fault signature for the current mode. A hybrid system diagnosis algorithm must handle each of these challenges.

We restrict the problem to single, persistent faults.

**Assumption 1.** Only single faults occur in the system.

**Assumption 2.** Faults are persistent.

Thus, we define a diagnosis as follows.

**Definition 6** (Diagnosis). For a system with fault set  $F$ , a *diagnosis* is a fault  $f \in F$  that is consistent with a given finite sequence of observations. A set of diagnoses is denoted as  $D$ .

The diagnosis problem can then be formally defined as follows.

**Problem 1.** For a system with fault set  $F$ , given a finite sequence of observations  $O$ , find the set of diagnoses  $D \subseteq F$  that is consistent with  $O$ .

#### 4. QUALITATIVE FAULT ISOLATION FOR HYBRID SYSTEMS

Generally speaking, for the purposes of diagnosis, we consider an observation to be an event observed at a particular time.

**Definition 7** (Observation). An *observation* is a tuple  $(e, t)$ , where  $e$  is an observed event and  $t$  is the time of observation.

We consider two types of observable events: (i) fault signature events and (ii) mode change events. Section 4.1 reviews the event-based fault modeling framework based on the concepts of fault signatures (Daigle et al., 2009), and extends it to hybrid systems, adding on to the framework developed in (Bregon et al., 2016). Following that, Section 4.2 describes how diagnostic reasoning can be performed under this new framework in the presence of mode changes, and for both parametric and discrete faults.

##### 4.1. Event-based Fault Modeling

The basis of the qualitative fault isolation approach is the concept of a fault signature.

**Definition 8** (Fault Signature). A *fault signature* for a fault  $f$  and residual  $r$  in mode  $m$ , denoted by  $\sigma_{f,r,m}$  is a set of symbols representing changes in  $r$  caused by  $f$  at the point of the occurrence of  $f$  in mode  $m$ . The set of all fault signatures for a fault  $f$  over residuals  $R$  in mode  $m$  is denoted as  $\Sigma_{f,R,m}$ .

In this work, we adopt a set of two symbols to define a fault signature: the qualitative change in residual magnitude, and the qualitative change in residual slope, each of which take on the values + (increase), - (decrease), and 0 (no change). These symbols are based on the transient that is produced when a fault occurs (P. J. Mosterman & Biswas, 1999). We write always the magnitude symbol followed by the slope symbol, e.g., a signature  $+ -$  represents an increase in magnitude and a decrease in slope.

A fault signature is to be interpreted as a prediction as to what observations will be made, given that we are in a particular mode and some fault happens. For a parametric fault, this is a straightforward concept. For discrete faults, the interpretation remains the same even though a discrete fault will change the mode. That is, if we are in mode  $m$  and a discrete fault occurs from that mode (thus changing the mode), the signatures in  $\Sigma_{f,R,m}$  for the discrete fault  $f$  will be those observations predicted for the fault occurring in mode  $m$ , and not the mode in which the fault drives the system into. So, if we know the system is in mode  $m$  and fault signatures are observed, we always look in  $\Sigma_{f,R,m}$  for every  $f \in F$  to reason about which fault has occurred.

Changes may be observed in each residual that responds to the fault. Thus, when a fault occurs, we observe a *sequence* of fault signatures. *Relative residual orderings* define a partial order of signatures for a given fault, and thus define all the possible fault signature sequences that can be produced by a fault.

**Definition 9** (Relative Residual Ordering). A *relative residual ordering* for a fault  $f$  and residuals  $r_i$  and  $r_j$  in mode  $m$  is a tuple  $(r_i, r_j)$ , denoted by  $r_i \prec_{f,m} r_j$ , representing that  $f$  always manifests in  $r_i$  before  $r_j$  in mode  $m$ . The set of all

Table 2. Fault signatures and orderings for global model for mode  $[0, 0]$ .

Fault	$q_1^*$	$q_2^*$	$q_3^*$	Residual Orderings
$K_1^-$	+ -	0 0	0 0	$q_1^* \prec q_2^*, q_1^* \prec q_3^*$
$K_2^-$	0 0	+ -	0 0	$q_2^* \prec q_3^*, q_2^* \prec q_1^*$
$K_3^-$	0 0	0 0	+ -	$q_3^* \prec q_2^*, q_3^* \prec q_1^*$
$R_1^+$	- +	0 0	0 0	$q_1^* \prec q_2^*, q_1^* \prec q_3^*$
$R_{12}^+$	0 0	0 0	0 0	$\emptyset$
$R_2^+$	0 0	- +	0 0	$q_2^* \prec q_3^*, q_2^* \prec q_1^*$
$R_{23}^+$	0 0	0 0	0 0	$\emptyset$
$R_3^+$	0 0	0 0	- +	$q_3^* \prec q_2^*, q_3^* \prec q_1^*$
$Pipe_{12}^{on}$	0 -	0 +	0 0	$q_1^* \prec q_3^*, q_2^* \prec q_3^*$
$Pipe_{23}^{on}$	0 0	0 -	0 +	$q_2^* \prec q_1^*, q_3^* \prec q_1^*$
$Pipe_{12}^{off}$	0 0	0 0	0 0	$\emptyset$
$Pipe_{23}^{off}$	0 0	0 0	0 0	$\emptyset$

orderings for a fault  $f$  over residuals  $R$  in mode  $m$  is denoted as  $\Omega_{f,R,m}$ .

 Table 3. Fault signatures and orderings for global model for mode  $[0, 1]$ .

Fault	$q_1^*$	$q_2^*$	$q_3^*$	Residual Orderings
$K_1^-$	+ -	0 0	0 0	$q_1^* \prec q_2^*, q_1^* \prec q_3^*$
$K_2^-$	0 0	+ -	0 +	$q_2^* \prec q_3^*, q_2^* \prec q_1^*, q_3^* \prec q_1^*$
$K_3^-$	0 0	0 +	+ -	$q_2^* \prec q_1^*, q_3^* \prec q_2^*, q_3^* \prec q_1^*$
$R_1^+$	- +	0 0	0 0	$q_1^* \prec q_2^*, q_1^* \prec q_3^*$
$R_{12}^+$	0 0	0 0	0 0	$\emptyset$
$R_2^+$	0 0	- +	0 +	$q_2^* \prec q_3^*, q_2^* \prec q_1^*, q_3^* \prec q_1^*$
$R_{23}^+$	0 0	0 +	0 -	$q_2^* \prec q_1^*, q_3^* \prec q_1^*$
$R_3^+$	0 0	0 +	- +	$q_2^* \prec q_1^*, q_3^* \prec q_2^*, q_3^* \prec q_1^*$
$Pipe_{12}^{on}$	0 -	0 +	0 +	$q_2^* \prec q_3^*$
$Pipe_{23}^{off}$	0 0	0 +	0 -	$q_2^* \prec q_1^*, q_3^* \prec q_1^*$
$Pipe_{12}^{off}$	0 0	0 0	0 0	$\emptyset$
$Pipe_{23}^{on}$	0 0	0 0	0 0	$\emptyset$

**Example 5.** Tables 2–5 show the fault signatures and orderings for the four modes of the tank system for the global model residuals. For example, in mode  $[0, 1]$ ,  $R_3^+$  will cause a -+ in  $r_{q_3^*}$ , i.e., a decrease in magnitude and increase in slope. Then on  $r_{q_2^*}$  it will cause 0+, i.e., no change in magnitude and an increase in slope. In this mode, the first tank is decoupled since the connecting pipe is turned off, so no effect on  $r_{q_1^*}$  will be observed. In the same mode, the fault  $Pipe_{12}^{on}$  will connect the first and second tanks, and so we will see 0- on  $r_{q_1^*}$  (since now flow is also exiting through the connecting pipe), 0+ on  $r_{q_2^*}$ , and 0+ on  $r_{q_3^*}$  (since flow is entering through the connecting pipe into the second tank).

 Table 4. Fault signatures and orderings for global model for mode  $[1, 0]$ .

Fault	$q_1^*$	$q_2^*$	$q_3^*$	Residual Orderings
$K_1^-$	+ -	0 +	0 0	$q_2^* \prec q_3^*, q_1^* \prec q_2^*, q_1^* \prec q_3^*$
$K_2^-$	0 +	+ -	0 0	$q_2^* \prec q_3^*, q_2^* \prec q_1^*, q_1^* \prec q_3^*$
$K_3^-$	0 0	0 0	+ -	$q_3^* \prec q_2^*, q_3^* \prec q_1^*$
$R_1^+$	- +	0 +	0 0	$q_2^* \prec q_3^*, q_1^* \prec q_2^*, q_1^* \prec q_3^*$
$R_{12}^+$	0 +	0 -	0 0	$q_2^* \prec q_3^*, q_1^* \prec q_3^*$
$R_2^+$	0 +	- +	0 0	$q_2^* \prec q_3^*, q_2^* \prec q_1^*, q_1^* \prec q_3^*$
$R_{23}^+$	0 0	0 0	0 0	$\emptyset$
$R_3^+$	0 0	0 0	- +	$q_3^* \prec q_2^*, q_3^* \prec q_1^*$
$Pipe_{12}^{off}$	0 +	0 -	0 0	$q_2^* \prec q_3^*, q_1^* \prec q_3^*$
$Pipe_{23}^{on}$	0 -	0 -	0 +	$q_2^* \prec q_1^*$
$Pipe_{12}^{on}$	0 0	0 0	0 0	$\emptyset$
$Pipe_{23}^{off}$	0 0	0 0	0 0	$\emptyset$

 Table 5. Fault signatures and orderings for global model for mode  $[1, 1]$ .

Fault	$q_1^*$	$q_2^*$	$q_3^*$	Residual Orderings
$K_1^-$	+ -	0 +	0 +	$q_2^* \prec q_3^*, q_1^* \prec q_2^*, q_1^* \prec q_3^*$
$K_2^-$	0 +	+ -	0 +	$q_2^* \prec q_3^*, q_2^* \prec q_1^*$
$K_3^-$	0 +	0 +	+ -	$q_2^* \prec q_1^*, q_3^* \prec q_2^*, q_3^* \prec q_1^*$
$R_1^+$	- +	0 +	0 +	$q_2^* \prec q_3^*, q_1^* \prec q_2^*, q_1^* \prec q_3^*$
$R_{12}^+$	0 +	0 -	0 -	$q_2^* \prec q_3^*$
$R_2^+$	0 +	- +	0 +	$q_2^* \prec q_3^*, q_2^* \prec q_1^*$
$R_{23}^+$	0 +	0 +	0 -	$q_2^* \prec q_1^*$
$R_3^+$	0 +	0 +	- +	$q_2^* \prec q_1^*, q_3^* \prec q_2^*, q_3^* \prec q_1^*$
$Pipe_{12}^{off}$	0 +	0 -	0 -	$q_2^* \prec q_3^*$
$Pipe_{23}^{off}$	0 +	0 +	0 -	$q_2^* \prec q_1^*$
$Pipe_{12}^{on}$	0 0	0 0	0 0	$\emptyset$
$Pipe_{23}^{on}$	0 0	0 0	0 0	$\emptyset$

**Example 6.** Tables 6–9 show the fault signatures and orderings for the four modes of the tank system for the local submodel residuals. Consider the fault  $K_2^-$ . In  $[0, 1]$ , it will cause + - on  $r_{q_2^*}$ . No other residuals will be affected, since the fault is decoupled from them due to the decomposition. In the global model residuals, however, an additional residual (that for  $q_3^*$ ) will deviate. In the same mode, the fault  $Pipe_{12}^{on}$  will connect the first and second tanks, and so we will see 0- on  $r_{q_1^*}$  and 0+ on  $r_{q_2^*}$ . We will not see any change in  $r_{q_3^*}$ , since the submodel generating that residual is decoupled from that mode change.

A single sequence of fault signatures is termed a *fault trace*.

**Definition 10** (Fault Trace). A *fault trace* for a fault  $f$  over a set of residuals  $R$  in mode  $m$ , denoted by  $\lambda_{f,R,m}$ , is a se-

Table 6. Fault signatures and orderings for local submodels for mode  $[0, 0]$ .

Fault	$q_1^*$	$q_2^*$	$q_3^*$	Residual Orderings
$K_1^-$	+-	00	00	$q_1^* \prec q_2^*, q_1^* \prec q_3^*$
$K_2^-$	00	+-	00	$q_2^* \prec q_3^*, q_2^* \prec q_1^*$
$K_3^-$	00	00	+-	$q_3^* \prec q_2^*, q_3^* \prec q_1^*$
$R_1^+$	-+	00	00	$q_1^* \prec q_2^*, q_1^* \prec q_3^*$
$R_{12}^+$	00	00	00	$\emptyset$
$R_2^+$	00	-+	00	$q_2^* \prec q_3^*, q_2^* \prec q_1^*$
$R_{23}^+$	00	00	00	$\emptyset$
$R_3^+$	00	00	-+	$q_3^* \prec q_2^*, q_3^* \prec q_1^*$
$Pipe_{12}^{on}$	0-	0+	00	$q_1^* \prec q_3^*, q_2^* \prec q_3^*$
$Pipe_{23}^{on}$	00	0-	0+	$q_2^* \prec q_1^*, q_3^* \prec q_1^*$
$Pipe_{12}^{off}$	00	00	00	$\emptyset$
$Pipe_{23}^{off}$	00	00	00	$\emptyset$

Table 7. Fault signatures and orderings for local submodels for mode  $[0, 1]$ .

Fault	$q_1^*$	$q_2^*$	$q_3^*$	Residual Orderings
$K_1^-$	+-	00	00	$q_1^* \prec q_2^*, q_1^* \prec q_3^*$
$K_2^-$	00	+-	00	$q_2^* \prec q_3^*, q_2^* \prec q_1^*$
$K_3^-$	00	00	+-	$q_3^* \prec q_2^*, q_3^* \prec q_1^*$
$R_1^+$	-+	00	00	$q_1^* \prec q_2^*, q_1^* \prec q_3^*$
$R_{12}^+$	00	00	00	$\emptyset$
$R_2^+$	00	-+	0+	$q_2^* \prec q_1^*, q_3^* \prec q_1^*$
$R_{23}^+$	00	0+	0-	$q_2^* \prec q_1^*, q_3^* \prec q_1^*$
$R_3^+$	00	0+	-+	$q_2^* \prec q_1^*, q_3^* \prec q_1^*$
$Pipe_{12}^{on}$	0-	0+	00	$q_1^* \prec q_3^*, q_2^* \prec q_3^*$
$Pipe_{23}^{off}$	00	0+	0-	$q_2^* \prec q_1^*, q_3^* \prec q_1^*$
$Pipe_{12}^{off}$	00	00	00	$\emptyset$
$Pipe_{23}^{on}$	00	00	00	$\emptyset$

quence of fault signatures that can be observed given the occurrence of  $f$  in mode  $m$ .

Fault traces are grouped into *fault languages*.<sup>1</sup>

**Definition 11** (Fault Language). The *fault language* for a fault  $f$  and residual set  $R$  in mode  $m$ , denoted by  $L_{f,R,m}$ , is the set of all fault traces for  $f$  over  $R$  in  $m$ .

For the purposes of this paper, we assume that signatures and orderings are correctly observed.<sup>2</sup>

**Assumption 3** (Correct Observation). If a fault  $f$  occurs in

<sup>1</sup>Fault languages can be automatically derived for certain classes of system models (Daigle, 2008), obtained via simulation, or obtained experimentally. In this work, we assume that the fault languages are given as input.

<sup>2</sup>Relaxation of this assumption has been explored for continuous systems in (Daigle, Roychoudhury, & Bregon, 2014).

Table 8. Fault signatures and orderings for local submodels for mode  $[1, 0]$ .

Fault	$q_1^*$	$q_2^*$	$q_3^*$	Residual Orderings
$K_1^-$	+-	00	00	$q_1^* \prec q_2^*, q_1^* \prec q_3^*$
$K_2^-$	00	+-	00	$q_2^* \prec q_3^*, q_2^* \prec q_1^*$
$K_3^-$	00	00	+-	$q_3^* \prec q_2^*, q_3^* \prec q_1^*$
$R_1^+$	-+	0+	00	$q_2^* \prec q_3^*, q_1^* \prec q_3^*$
$R_{12}^+$	0+	0-	00	$q_2^* \prec q_3^*, q_1^* \prec q_3^*$
$R_2^+$	0+	-+	00	$q_2^* \prec q_3^*, q_1^* \prec q_3^*$
$R_{23}^+$	00	00	00	$\emptyset$
$R_3^+$	00	00	-+	$q_3^* \prec q_2^*, q_3^* \prec q_1^*$
$Pipe_{12}^{off}$	0+	0-	00	$q_1^* \prec q_3^*, q_2^* \prec q_3^*$
$Pipe_{23}^{on}$	00	0-	0+	$q_2^* \prec q_1^*, q_3^* \prec q_1^*$
$Pipe_{12}^{on}$	00	00	00	$\emptyset$
$Pipe_{23}^{off}$	00	00	00	$\emptyset$

Table 9. Fault signatures and orderings for local submodels for mode  $[1, 1]$ .

Fault	$q_1^*$	$q_2^*$	$q_3^*$	Residual Orderings
$K_1^-$	+-	00	00	$q_1^* \prec q_2^*, q_1^* \prec q_3^*$
$K_2^-$	00	+-	00	$q_2^* \prec q_3^*, q_2^* \prec q_1^*$
$K_3^-$	00	00	+-	$q_3^* \prec q_2^*, q_3^* \prec q_1^*$
$R_1^+$	-+	0+	00	$q_2^* \prec q_3^*, q_1^* \prec q_3^*$
$R_{12}^+$	0+	0-	00	$q_2^* \prec q_3^*, q_1^* \prec q_3^*$
$R_2^+$	0+	-+	0+	$\emptyset$
$R_{23}^+$	00	0+	0-	$q_2^* \prec q_1^*, q_3^* \prec q_1^*$
$R_3^+$	00	0+	-+	$q_2^* \prec q_1^*, q_3^* \prec q_1^*$
$Pipe_{12}^{off}$	0+	0-	00	$q_1^* \prec q_3^*, q_2^* \prec q_3^*$
$Pipe_{23}^{off}$	00	0+	0-	$q_2^* \prec q_1^*, q_3^* \prec q_1^*$
$Pipe_{12}^{on}$	00	00	00	$\emptyset$
$Pipe_{23}^{on}$	00	00	00	$\emptyset$

mode  $m$ , then if the system does not change mode after the occurrence of the fault, the observed fault trace will belong to  $L_{f,R,m}$ .

## 4.2. Hybrid Systems Diagnosis

For hybrid systems, fault signatures, residual orderings, fault traces, and fault languages are a function of the system mode. If the mode does not change between the point of fault occurrence and the diagnosis of the fault, then the problem reduces to the continuous systems case. Otherwise, we will observe some new trace that may not belong to any mode-specific fault language, i.e., it may be a trace that is composed of partial traces for a fault from the different modes encountered during diagnosis.

**Example 7.** For example, consider the global model residu-

als. Assume that the system starts in  $[0, 1]$  and  $R_1^+$  occurs. Then we would observe  $r_{q_1^+}^{-+}$ . So far, this partial trace can be found as a prefix to a trace in  $L_{R_1^+, R, [0, 1]}$ . Now, assume that the system moves to mode  $[1, 1]$ , now we would observe  $r_{q_2^+}^{0+}$  followed by  $r_{q_3^+}^{0+}$ .

Thus, the first challenge is that now observed fault traces may contain some subtraces corresponding to one mode, and other subtraces corresponding to other modes. Thus, the fault isolation reasoning must span over several potential mode changes. If we know the system mode, then we know which fault language corresponds to the predicted observations for each fault. If there are unobservable mode changes, this adds another layer of complexity, because we must not only diagnose which fault has occurred but also what mode the system is currently in. This is also complicated by discrete faults, which are themselves unobservable mode changes. If we do not know whether a parametric or a discrete fault has occurred, then we do not know the true mode of the system.

We define mode change events specific to components.

**Definition 12** (Mode Change Event). An event  $(\kappa, m)$  represents component  $\kappa$  changing to its mode  $m$ .

For the purposes of this paper, we assume that all commanded mode change events are observable. Since we assume that discrete faults are permanent, commanded mode change events for the faulty component will be ignored, and so this must be taken into account within the diagnostic reasoning.

**Assumption 4** (Mode Change Observability). All commanded mode change events are observable.

However, even if we know the current mode of the system, there is another layer of complexity to consider: *observation delay*. Specifically, in our framework, this corresponds to the observations of fault signatures being delayed. The difficulty is that the system may be in one mode, but when the observation arrives we have moved to a different system mode, and thus we do not know in which mode the observation was actually made.

**Example 8.** Consider the global model residuals, with the system in mode  $[0, 1]$ , and  $R_2^+$  occurs. We observe  $r_{q_2^+}^{-+}$ , and then change to mode  $[0, 0]$ . Say that  $r_{q_3^+}^{0+}$  occurred in the previous mode, but we only get the observation now. This observation is not consistent with  $R_2^+$  in mode  $[0, 0]$ .

Observation delay can manifest in different ways. For example, fault detection is usually performed by checking whether a residual crosses some threshold. To make this approach robust to noise, usually we check that the mean of the residual, computed over some small time window, has crossed that threshold. This means that the signal could actually cross the

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**Algorithm 1** FaultIsolation
 

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1: Inputs:  $D_i, \lambda_i, \sigma_{i+1}, M_{r, \Delta}$ 
2: Outputs:  $D_{i+1}$ 
3:  $D_{i+1} \leftarrow \emptyset$ 
4: for all  $q \in M_{r, \Delta}$  do
5:   for all  $f \in D_i \cap F_{r, q}$  do
6:     if  $\sigma_{i+1} \in \Sigma_{f, r_{\sigma_{i+1}}, m}$  and  $\neg \exists r' \in (R - R_{\lambda_i})$  s.t.  $r' \prec r_{\sigma_{i+1}} \in \Omega_{f, R - R_{\lambda_i}, m}$  then
7:        $D_{i+1} \leftarrow \{f\}$ 
  
```

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threshold in one mode, but the mean of the signal could cross only in the next mode. Thus, the observation of this signature is delayed. In practice, we can assume that observation delay is finite and bounded.

**Assumption 5** (Bounded Observation Delay). The delay of any observation is no greater than  $\Delta$ .

Given our assumptions, the algorithm for a single step of fault isolation for hybrid systems is shown as Algorithm 1.<sup>3</sup> Because we reason about discrete faults through fault signatures, the same as for parametric faults, this algorithm is the same as that presented for only parametric faults in (Bregon et al., 2016). As inputs, it takes the current diagnosis,  $D_i$ , the previous sequence of fault signatures,  $\lambda_i$ , the new fault signature,  $\sigma_{i+1}$ , and the set of recent modes that falls within  $[t - \Delta, t]$ ,  $M_{r, \Delta}$ , for the model/submodel that generates residual  $r$ . The change from the continuous systems case is that we need to check signatures and orderings for each of the recent modes.

The set of recent modes is dependent on the model used for isolation, and so is a function of the residual associated with the signature. When using the global model to generate residuals, the residual generator contains all system modes, but for a local submodel generating  $r$ , it contains only the local modes of that submodel, which is less than the number of system modes. Thus, fewer modes must be searched and efficiency is improved.

If the signature it is consistent in any of the modes, it must be added to  $D_{i+1}$ . Here, for a given mode  $m$ , we need to check only the subset of faults that are included in the current diagnosis and can actually affect this residual in this mode, denoted as  $F_{r, m}$ . To check consistency, we check that the predicted signature for the residual associated with  $\sigma_{i+1}$ , denoted as  $r_{\sigma_{i+1}}$ , can be found in the signature set for that fault and residual, and that the orderings, with respect to residuals that have not yet deviated (those in  $R - R_{\lambda_i}$ , where  $R_{\lambda_i}$  denotes the residuals associated with the trace  $\lambda_i$ ), are not violated.

Algorithm 1 executes a single reasoning step, given a newly

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<sup>3</sup>Because fault languages can become prohibitively large, we implement the fault isolation step directly using the signatures and orderings, which is more efficient (Daigle et al., 2009).

observed fault signature. This would be placed within a progressive monitoring algorithm, that keeps track of the current diagnosis, and computes the set of recent modes based on the times events are observed.

### 4.3. Scalability

The complexity of the fault isolation algorithm is dependent on the number of faults,  $|F|$ , the number of residuals,  $|R|$ , and the number of modes,  $|M|$ . For the global model case, all faults, residuals, and modes in  $M_{r,\Delta}$  must be searched. Because  $r$  is computed using the global model, it is a function of the system-level mode. For an  $n$ -tank system, there are  $n-1$  switching components and so  $2^{n-1}$  system-level modes. Clearly, diagnosis in this case will not scale.

For the local submodel case, each residual is generated by a minimal submodel. Each minimal submodel has only a single residual that it produces, contains only a subset of the faults, and has only a few modes. Thus, on average it will scale much better. The more decomposition can be achieved, the better it will scale. For an  $n$ -tank system, each residual  $r_{q_i^*}$  for tank  $i$  will have at most 4 modes, because it depends only on the switching behavior of the two adjacent connecting pipes. So there will be at most 4 modes to search through for each residual deviation, compared to  $2^{n-1}$  for the global model case. Here, then, this scales linearly with the number of tanks, not exponentially, and thus will have significant efficiency gains as the system size grows.

## 5. DEMONSTRATION OF APPROACH

In this section, we demonstrate the approach through some example scenarios using the three-tank system. In each experiment, the system always starts in mode  $[0, 0]$ , goes through some mode changes, and a fault is injected. The complete set of faults considered is that listed in Tables 3–6: 8 parametric faults and 4 discrete faults. In each case, we compare the performance of the global model approach and the local submodel approach.

The symbol generation approach described in (Daigle, Roychoudhury, et al., 2010) is used, which uses the Z-test for statistical fault detection and symbol generation. A window of samples is used to compute the mean, and thus can produce a delay that increases with window size. For the particular fault detector settings, we consider the bounded observation delay to be  $\Delta = 5$  s.

In the first scenario, we consider the parametric fault  $K_1^-$ . Initially, the system is in mode  $[0, 0]$ , and moves to mode  $[1, 0]$  at 10.0 s. At 15.0 s,  $K_1^-$  occurs, with the value reducing by 50%. For the global model residuals, we detect first  $+ -$  in  $r_{q_1^*}$  at this time, along with  $0+$  in  $r_{q_2^*}$ . The signatures may have come from either of the past two modes, since the fault was detected within 5.0 s of the first mode change. The only

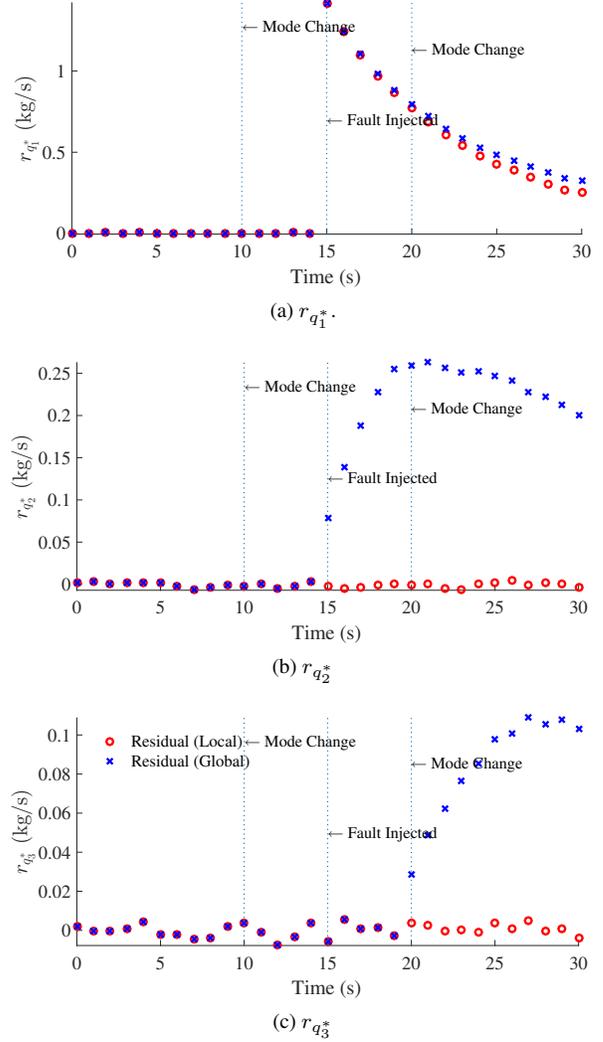


Figure 3. Residual values with a decrease in  $K_1$  at  $t = 15$  s.

consistent fault is  $K_1^-$ , for both of these modes. Then, the system moves to mode  $[1, 1]$  at 20.0 s. With this second mode change,  $r_{q_3^*}$  becomes connected to the fault and so at 21.0 s,  $0+$  is detected, which is still consistent with the diagnosis of  $K_1^-$ . For the local submodel residuals, we observe only  $+ -$  in  $r_{q_1^*}$ . This residual's submodel is different for the previous two modes, so both must be considered. Again, only  $K_1^-$  is consistent, and is the diagnosis.

In the second example, we consider the discrete fault  $Pipe_{12}^{off}$ . Initially, the system is in mode  $[0, 0]$ , and moves to mode  $[1, 0]$  at 10.0 s. At 15.0 s,  $Pipe_{12}^{off}$  occurs. In the global model residuals, we see first  $0+$  in  $r_{q_1^*}$  and  $0-$  in  $r_{q_2^*}$  at 16.0 s. In this case only mode  $[1, 0]$  needs to be considered as the mode in which the fault occurred, and these signatures are consistent only with  $Pipe_{12}^{off}$ . For the local submodel residuals, we observe the same signatures and reach the same conclusion.

In the third example, we consider the parametric fault  $R_{2,3}^+$ .

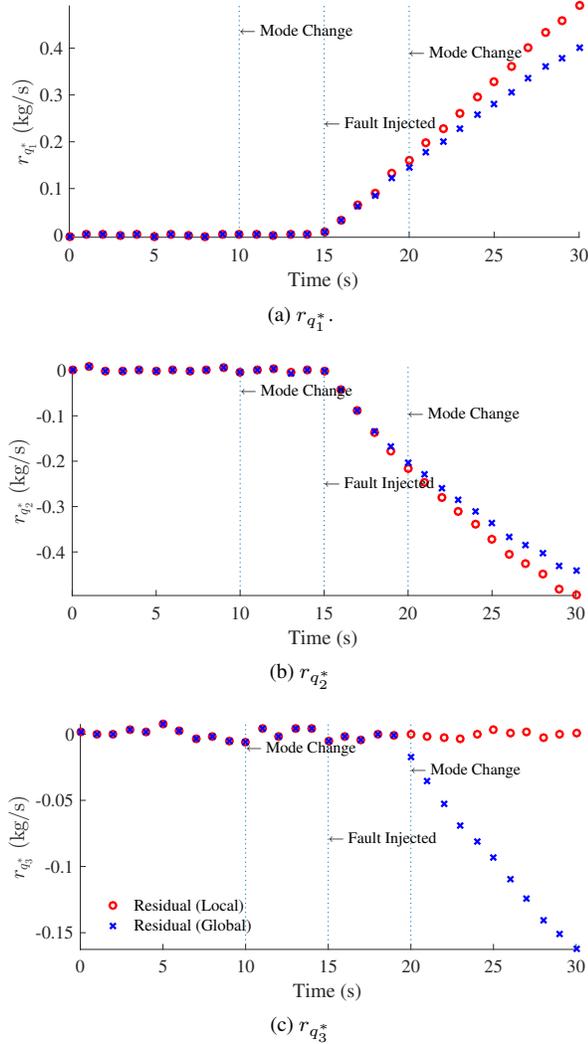


Figure 4. Residual values with  $Pipe_{12}^{off}$  at  $t = 15$  s.

Initially, the system is in mode  $[0, 0]$ , and moves to mode  $[0, 1]$  at 10.0 s, and  $[1, 1]$  at 12.0 s. At 15.0 s,  $R_{2,3}^+$  occurs, doubling in value. Here, we consider  $\Delta = 6$  s. At 16.0 s,  $0+$  in  $r_{q_2}^*$  and  $0-$  in  $r_{q_3}^*$  are detected in the global model residuals, and both modes  $[0, 1]$  and  $[1, 1]$  must be considered. In both cases, both  $R_{2,3}^+$  and  $Pipe_{12}^{off}$  are consistent, and cannot be distinguished further. For the local submodel residuals,  $0+$  in  $r_{q_2}^*$  and  $0-$  in  $r_{q_3}^*$  are detected at 16.0 s. Because the local submodel for  $r_{q_3}^*$  only changes modes to  $Pipe_{23}$ , which has not changed in the last  $\Delta = 6$  s, only the last known system mode needs to be considered. The diagnosis is the same as in the global model case, but it is arrived at with less computation (fewer searches over past modes).

In the fourth example, we consider the discrete fault  $Pipe_{12}^{on}$ . Initially, the system is in mode  $[0, 0]$ , and moves to mode  $[1, 0]$  at 10.0 s, and then to  $[1, 1]$  at 15.0 s. At 17.0 s,  $Pipe_{12}^{on}$  occurs. In this mode, however, it is not observable because

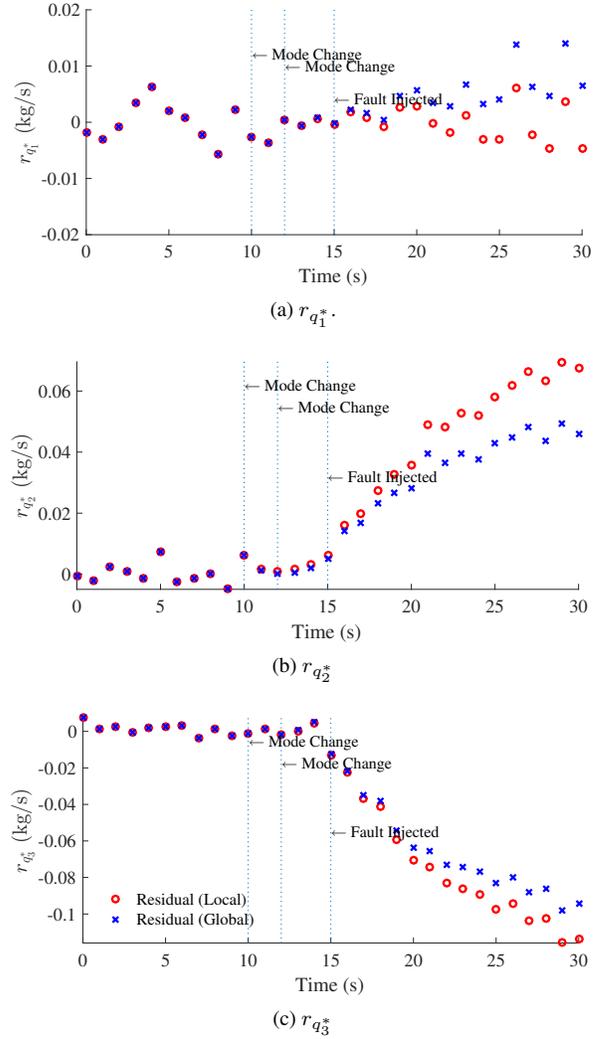


Figure 5. Residual values with an increase in  $R_{2,3}$  at  $t = 15$  s.

$Pipe_{12}$  is already *on*. At 20.0 s, the system changes to mode  $[0, 1]$ . At this point, the fault becomes observable. In both the global model and local submodel residuals we observe  $0-$  in  $r_{q_1}^*$  and  $0+$  in  $r_{q_2}^*$ , consistent only with  $Pipe_{12}^{on}$ . At 23.0 s,  $0+$  is observed in  $r_{q_3}^*$  for the global model residuals, confirming the diagnosis.

## 6. RELATED WORK

During the last decade or so, modeling and diagnosis for hybrid systems have been an important topic of researchers from both the FDI and DX communities. In the FDI community, several hybrid system diagnosis approaches have been developed. In (Cocquemot, El Meznyani, & Staroswiecki, 2004), parameterized ARRs are used. However, the approach is not suitable for systems with high nonlinearities or a large set of modes. In the DX community, some approaches have used different kind of automata to model the complete set of modes and transitions between them. In those cases, the

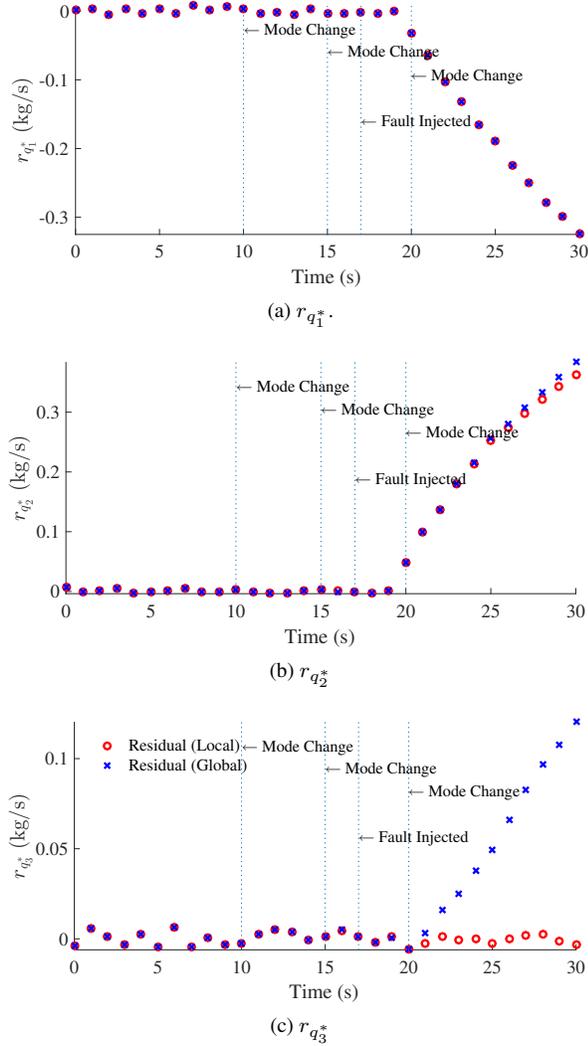


Figure 6. Residual values with  $Pipe_{12}^{on}$  at  $t = 17$  s.

main research topic has been hybrid system state estimation, which has been done using probabilistic (e.g., some kind of filter (Koutsoukos, Kurien, & Zhao, 2003) or hybrid automata (Hofbauer & Williams, 2004)) or set-theoretic approaches (Benazera & Travé-Massuyès, 2009).

Another solution has been to use an automaton to track the system mode, and then use a different technique to diagnose the continuous behavior (for example, using a set of ARR for each mode (Bayouhd et al., 2008), or parameterized ARR for the complete set of modes (Bayouhd, Travé-Massuyès, & Olive, 2009)). Nevertheless, one of the main difficulties regarding state estimation using these techniques is the need to pre-enumerate the set of possible system-level modes and mode transitions, which is difficult for complex systems. We avoid this problem by using a compositional approach.

In (Alonso, Bregon, Alonso-González, & Pulido, 2013), the authors present a qualitative fault isolation approach for hy-

brid systems that is based on structural model decomposition. This approach, however, unlike ours, does not take into account observation delays. Moreover, the approach presented in (Alonso et al., 2013) is applicable only to systems that are modeled using hybrid bond graphs.

In (Bregon, Narasimhan, Roychoudhury, Daigle, & Pulido, 2013), the authors had developed an efficient model-based methodology for diagnosis that integrated structural model decomposition within the Hybrid Diagnosis Engine (HyDE) architecture (Narasimhan & Brownston, 2007). The HyDE architecture offers flexibility to choose the modeling paradigm and reasoning algorithms for diagnosis of hybrid systems. The authors show how the integration of the structural model decomposition reduces the computational complexity associated with the fault diagnosis of hybrid systems. In our paper, similar reduction in computational complexity of fault diagnosis is observed, further bolstering the support for using structural model decomposition for hybrid systems diagnosis.

The approach presented in this paper is similar to that in (Daigle, 2008; Daigle, Koutsoukos, & Biswas, 2010), but differs in two major ways. First, the former work was based on modeling using hybrid bond graphs (HBGs). The modeling framework used here is less restrictive and, in our opinion, easier to work with. In fact, HBGs are a special case of our modeling framework, as elements of HBGs can be modeled directly as components in our framework. Second, that work was based on a global system model. In this work, we have demonstrated why using structural model decomposition is a better approach.

## 7. CONCLUSIONS

In this work, we developed a qualitative fault isolation approach for diagnosing both parametric and discrete faults in hybrid systems. We used structural model decomposition to improve the computational complexity of this diagnosis approach. The fault isolation is performed by analyzing the qualitative information of the residual signals. The approach was demonstrated using a three-tank system. In this work, we presented four experimental scenarios, in which the diagnosis based on structural model decomposition performed as well as the diagnosis based on the global model in terms of isolation accuracy, but improved in terms of efficiency and scalability. As part of future work, we will further develop the hybrid diagnosis approach for the diagnosis of multiple faults. We would also like to relax the assumption about all mode changes being observed using the ideas developed in (Narasimhan & Biswas, 2007). Finally, we would like to implement this hybrid diagnosis approach on large-scale real systems to evaluate its efficiency and scalability.

## ACKNOWLEDGMENTS

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## NOMENCLATURE

$c$	constraint
$C$	constraint set
$\epsilon$	equation
$v$	variable
$V$	variable set
$\kappa$	component
$m$	mode
$\mathcal{M}$	(sub)model
$\theta$	parameter
$f$	fault
$F$	fault set
$d$	diagnosis
$D$	diagnosis set
$r$	residual
$\sigma$	fault signature
$\lambda$	fault trace
$L$	fault language
$\Delta$	delay

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