

Anomaly Detection in Gas Turbine Compressor of a Power Generation Plant using Similarity-based Modeling and Multivariate Analysis

Tomás Carricajo¹, Felipe Kripper², Marcos E. Orchard³, Luis Yacher⁴, Rodrigo Paredes⁵

^{1,2,4}*CONTAC Ingenieros Ltda., Avda. Américo Vespucio Sur 315, Ñuñoa, Santiago 7760005, Chile*
(tcarricajo, fkripper, lyacher)*@contac.cl*

³*Department of Electrical Engineering, Universidad de Chile, Santiago 8370451, Chile*
morchard*@ing.uchile.cl*

⁵*Endesa Chile, Av. Santa Rosa 76, Santiago 8330099, Chile*
rapb*@endesa.cl*

ABSTRACT

This paper introduces advances on the implementation of anomaly detection modules based on a combination of nonparametric models and multivariate analysis of residuals. The proposed anomaly detector utilizes similarity-based modeling (SBM) techniques to represent the process behavior and principal component analysis (PCA) for the study of model residuals; while partial least squares (PLS) is used to select an optimal subset of process variables to be included in the design of the detection module. In addition, the method considers a structured algorithm for the optimal inclusion of representative samples from the data set that is used to define the normal operation of the system. The method is validated using data that characterizes the operation of a compressor in a power generation plant.

1. INTRODUCTION

An anomaly detector (Orchard & Vachtsevanos, 2007) is basically a module that intends to recognize abnormal conditions within the operation of a monitored system. In this regard, the implementation of anomaly detectors is one of the first and most important steps needed to ensure operational continuity of the process, plant safety, as well as high quality standards. Conventional anomaly detection and fault diagnosis algorithms (Isermann, 1997) are typically designed to provide a solution for the supervision of a finite number of fault modes that are believed to be severe, frequent, and “testable”; fault modes that are selected on the basis of a Failure Modes, Effects, and Criticality Analysis (FMECA). This task needs to be performed while

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simultaneously minimizing the probability of false alarms and the detection time (time between the initiation of a fault and its detection), given a fixed threshold that represents the maximum risk (associated to the fault condition) that is allowed in the system.

Classical fault detection and identification (FDI) methods rely on an accurate model of the system under consideration and the utility of an innovation or “discrepancy” between the actual plant output and the model output, for all possible operating conditions, to detect an unanticipated fault (Isermann, 1997; Isermann & Balle, 1997). The innovation (or residual) method captures the fault signature, and suggests which residuals are normal or which ones result from fault conditions. A variety of techniques have been proposed based on estimation theory, failure sensitive filters, multiple hypothesis filter detection, generalized likelihood ratio tests, model-based approach, statistical analysis, and information theory (Ayhan *et al.*, 2008; Khan & Rahman, 2009; Lebaroud & Cleac, 2008).

If process/system dynamics are not well understood, then verification, calibration, and validation of parametric models may represent a difficult challenge. In contrast, nonparametric models offer a direct representation of nonlinear systems that requires the availability of historical data and a minimal comprehension of the relationships that exist between process variables. The definition of “normal” operation is done only by selecting an appropriate number of data samples that could illustrate moments where the process behaved accordingly to a particular set of requirements or standards; the need of a particular structure or linear/Gaussian assumptions is thus avoided.

In this regard, this article shows the implementation of a monitoring scheme that identifies abnormal operating conditions in a compressor of a power generator plant,

utilizing a nonparametric modeling approach known as *Similarity-based Modeling* (SBM). Provided that the plant represents a multivariate nonlinear system, the use of SBM allows generating estimates of the system output that can be used to compute residuals, when compared with actual measurements. Partial least squares (PLS) is used to select an optimal subset of process variables to be included in the design of the detection module, considering to this end the impact that those variables may have in terms of the mean-squared error of model residuals for data associated to “normal” operation.

The method also considers a structured algorithm for the optimal inclusion of representative samples from the data set that is used to define the normal operation of the system. This feature is critical since it is possible that the process model may exhibit problems simply because the database that is being considered for training purposes does not represent all possible operation conditions. Furthermore, in case of implementing a fault detection scheme, the addition of new samples to the database must be done with special attention of not incorporating samples corresponding to these abnormal conditions, since if this is done, the SBM algorithm will consider faulty conditions as known, and hence, normal.

The assessment of the system behavior cannot be performed purely considering each variable residual, since the process is inherently multiple-input multiple-output; consequently, multivariate analysis techniques such as *Principal Component Analysis* (PCA) (Jackson, 1991; Fuente *et al.*, 2009) are employed in order to reduce the space dimension, while ensuring an adequate representation of the residual vector. Additionally, hypothesis testing procedures such as the Hotelling’s test (Beale & Kim 2002) are also considered to ensure that the modeling errors remain in a statistically acceptable region.

This paper presents some extensions and results obtained after the implementation of the scheme that was presented in (Tobar, 2010) at facilities of a Chilean power generation company: Endesa-Chile. For confidentiality issues, process labels and time stamps have been discarded in all figures.

This article is organized as follows. Section II presents the necessary theoretical resources to understand the implementation of the proposed system monitoring scheme; i.e., the fundamentals of SBM, partial least squares, principal component analysis, and the Hotelling’s test. Section III explains the considerations regarding the data preprocessing procedures, a justification for the implementation of the proposed schemes, and the results obtained for the anomaly detector when using two different sets of process variables as inputs/outputs of the SBM model for the compressor of a power generation plant. Finally, Section IV states the concluding remarks and suggests guidelines for future research work in this field.

2. THEORETICAL BACKGROUND

2.1 Similarity-based Modeling for System Monitoring

One advantage of the nonparametric modeling techniques is that they do not require an a priori knowledge of the system, since its implementation is based on the identification of similarities and relationships between a given data set and online observations, instead of the construction of algebraic structures based upon these observed data. A particular case of such structures is the Similarity-based Model (SBM), which estimates the system output by comparing online measurements and a historical database which represents the system under study. SBM has proven to be a successful estimator when used in high dimension systems using considerably low number of training samples (Gong *et al.*, 2009).

In order to understand the SBM basic concept for systems modeling, consider the static system defined by (1):

$$y = f(x), x \in R^m, y \in R^p, \quad (1)$$

where x and y are the system input and output respectively, and $f(\bullet)$ is an unknown function.

When input and output measurements are available for the system described in (1), it is possible to define the following matrices to be used for model identification purposes (D_i and D_o stand for input and output matrices, respectively):

$$D_i = [x_1, x_2, \dots, x_n] \in R^{m \times n}, \quad (2.a)$$

$$D_o = [y_1, y_2, \dots, y_n] \in R^{p \times n}, \quad (2.b)$$

where $y_i = f(x_i)$, $\forall i = 1, \dots, n$, and the pairs $[x_i, y_i]_{i=1..n}$ accurately represent the system behavior; i.e., they span the regions containing the system operations points.

Hence, SBM assumes that for a given an input x^* , it is possible to estimate $y^* = f(x^*)$ by a linear combination of the columns of D_o denoted by \hat{y}^* . Consequently, the problem of estimating $y^* = f(x^*)$ can be regarded as the determination of a vector w such that $\hat{y} = D_o w$.

This vector can be computed as in (3):

$$w = \frac{\hat{w}}{\mathbf{1}^T \cdot \hat{w}}, \quad (3.a)$$

$$\hat{w} = (D_i^T \Delta D_i)^{-1} (D_i^T \Delta x_i), \quad (3.b)$$

where Δ is a similarity operator (Gong *et al.*, 2009; Pivoso *et al.*, 1994). SBM is not restricted to any particular similarity operator; however, according to the literature, the selected similarity operator must hold certain properties. For two elements $A, B \in \mathfrak{R}^u$, $A \Delta B \in \mathfrak{R}^+$ must be symmetric,

reaching its maximum in $A=B$ and monotonically decreasing with $\|A - B\|$.

Literature does not provide a framework for choosing a suitable similarity operator based on the available measurements. In this work, all similarity operators were based on saturated linear kernels.

SBM is a nonparametric modeling technique that is mainly used to identify static systems (or at least, systems where the dominant time constant is negligible with respect to the data sampling period). In this regard, and especially when the process exhibits noticeable dynamics, the model structure requires some adjustments before its implementation. For example, past observations (both inputs and outputs) may be incorporated as regressors to estimate the system response in time. For this particular case study, though, system dynamics were neglected (thus the process was regarded as a static one). As it was mentioned above, this concept can only be applied when the data is acquired at a very low frequency with respect to the system dominant constant.

SBM residuals can be computed simply using the difference between the model outputs (SBM estimates) and online measurements as in (4). If the estimates differ considerably from the actual measurements in the training data (w.r.t. a given criteria such as mean-squared error), it could be inferred that the associated operating point has not been incorporated yet into the SBM structure, and consequently the optimal database that ultimately defines the SBM model must be extended with samples representing the unknown condition. After the process of incorporating samples to the database is complete, i.e. once for every input x^* the estimation error given by

$$\begin{aligned} e &= y^* - \hat{y}^* \\ &= f(x^*) - D_0 \frac{(D_i^T \Delta D_i)^{-1} (D_i^T \Delta x^*)}{\bar{1}^T \cdot (D_i^T \Delta D_i)^{-1} (D_i^T \Delta x^*)} \in \mathfrak{R}^p \end{aligned} \quad (4)$$

is acceptable under a specified criteria, the relationships between the measured variables should be assessed to ensure consistency with the operation conditions represented in the database. Due to the large number of variables that are present in industrial systems, multivariate-processing algorithms should be implemented to verify these relationships.

2.2 Partial Least Squares

Partial least squares, also referred to as “projection to latent structures”, is a parametric modeling technique. This technique allows system modeling through a reduction of the problem dimensionality and the maximization of the covariance between projections of the input data matrix X and the output data matrix Y (Chiang *et al.*, 2001). It uses a matrix $X \in \mathfrak{R}^{n \times m}$ and a matrix $Y \in \mathfrak{R}^{n \times p}$, where m is the

number of variables predictors, n is the total number of observations of data and p is the number of observed variables in Y .

First, the matrices X and Y must be centered on the mean and normalized by their variances. Then, the matrix X can be decomposed into an array called scores $T \in \mathfrak{R}^{n \times a}$ and a loading matrix $P \in \mathfrak{R}^{m \times a}$, where a is the reduced order of the data, the residue matrix $E \in \mathfrak{R}^{n \times m}$.

$$X = TP^T + E \quad (5)$$

The matrix $T \cdot P^T$ can be expressed as the sum of products of vectors scores t_j and load vectors p_j .

$$X = \sum_{j=1}^a t_j p_j^T + E \quad (6)$$

Similarly, the matrix Y is decomposed into matrices:

$$Y = UQ^T + F = Y = \sum_{j=1}^a u_j q_j^T + F \quad (7)$$

If “ a ” is equal to $\min(m, n)$, then E and F are zero and this technique is reduced to the ordinary least squares. Choosing “ a ” smaller than $\min(m, n)$ noise is reduced. The objective is to determine the loading and scores vectors which maximize the correlation between X and Y .

PLS estimates the scores vectors u_j with scores vector t_j as:

$$u_j = t_j b_j^T, \text{ or equivalently } U = TB \quad (8)$$

Finally,

$$Y = TBQ^T + F \quad (9)$$

where F is the prediction error matrix. The matrix B is selected to minimize the norm of F . T and U scores matrices are calculated as to maximize the covariance between X and Y for each component “ a ”.

Although PLS is typically used to generate a linear parametric Multiple-Input Multiple-Output (MIMO) model for the process, as a result of an appropriate selection for the number of projection components, there are other important complementing aspects that can also be studied. Particularly, this article uses an analysis of the coefficients in matrices B , T and U to assess the impact of each of the inputs variables X on each of the output variables Y .

2.3 Principal Component Analysis

Principal Component Analysis (PCA) is a dimensionality reduction technique for correlated variables; i.e. for a given a set of correlated variables, it aims at finding a set of uncorrelated indicators that can help to characterize the variability of the process in a smaller dimension. PCA performs a linear transformation of the data, which is optimal in terms of capturing its variability, and determines a new data set ordered by the level of representation of the entire process variance.

Theoretically, for the data matrix

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{bmatrix}, \quad (10)$$

which comprises n observations for each one of the m variables, PCA finds a loading matrix $P \in \mathfrak{R}^{m \times a}$, $a \leq m$ which relates X to the first principal components being contained in the score matrix:

$$T = XP. \quad (11)$$

Denoting the i^{th} column of T by t_i , the transformation performed by PCA holds (Chiang *et al.*, 2001) the following properties:

1. $\text{Var}(t_1) \geq \text{Var}(t_2) \geq \dots \geq \text{Var}(t_a)$.
2. $\text{Mean}(t_i) = 0, \forall i$.
3. $t_i^T t_k = 0, \forall i \neq k$.
4. There is no other transformation of “ a ” components that captures more variations in the data.

Additionally, the projection back on an a -dimensional space is given by (Wise *et al.*, 1990):

$$\hat{X} = TP^T, \quad (12)$$

and hence, the difference between the original data stored at X and its projection is the residual matrix E :

$$E = X - \hat{X}, \quad (13)$$

which captures the variations of space generated by the remaining $(m - a)$ components, and has typically low signal-to-noise ratio. It has been formally justified (Golub *et al.*, 1983) that, when “ a ” is properly chosen, these remaining components represent the random noise of the measurements, whereas the first “ a ” components describe dynamic variations.

The application of PCA in our system monitoring framework is to reduce the dimension of the error vector “ e ”, simplifying in that manner the anomaly detection procedure (in terms of the associated computational cost). Indeed, once the PCA linear transformation has been applied to the error vector, one can easily recognize if the system is behaving in an anomalous manner through the application of a hypothesis test formulated in terms of the main principal components.

2.4 Hotelling’s Test

The one sample Hotelling’s T^2 index is typically used to test $H_0: \mu = \mu_0$ vs. $H_A: \mu \neq \mu_0$ in a 2-class classification

problem. However, when applied to multivariate Gaussian residual vectors, it also provides the means to compute a scalar threshold that characterizes the maximum acceptable deviation of the model residual, for a given level of significance (Gonzalez *et al.*, 2003). To properly introduce the Hotelling’s T^2 test, consider the sample covariance of the data matrix X given by

$$S = \frac{1}{n-1} X^T X. \quad (14)$$

The Hotelling’s T^2 test states that a particular observation $x \in \mathfrak{R}^m$ belongs to the same class as the data in X if the statistic

$$T^2 = x^T S^{-1} x, \quad (15)$$

is below the threshold

$$T_\alpha^2 = \frac{m(n-1)(n+1)}{n(n-m)} F_\alpha(m, n-m), \quad (16)$$

where $\Pr(Z \leq F_\alpha(g, k)) = \alpha$ if $Z \sim F_\alpha(g, k)$, an F -distribution with degrees of freedom g and k . When the data matrix X characterizes the model residuals obtained when the process is healthy, then an anomaly may be detected by analyzing the time instants when the alternative hypothesis is accepted.

3. ANOMALY DETECTION IN GAS TURBINE COMPRESSOR OF POWER GENERATION PLANT USING SIMILARITY-BASED MODELING, PLS AND PCA

A monitoring scheme for the detection of anomalies in the operation of the compressor of a Chilean natural-gas power generation plant was implemented using SBM to model the operation of the compressor at many different operating points (even including operation after the execution of maintenance procedures), and PCA for residual analysis. Selection of input/output variables within the structure of the SBM model considered the analysis of the coefficients in matrices associated to PLS models for the aforementioned plant. In this regard, all process variables that exhibited comparatively small weights in the PLS loading-plot (Chiang *et al.*, 2001) were discarded.

Training and validation data included 19,530 observations for each one of the main process variables. All measurements were acquired using OSIsoft PI software (OSIsoft 2013); including signals associated to pressures, temperatures, valves positions, voltages, speed of rotating parts, and other Boolean states that indicated if certain control loops were active. Data from all measured process variables were grouped in an “input” data matrix $X \in \mathfrak{R}^{19530 \times 42}$ and an “output” data matrix $Y \in \mathfrak{R}^{19530 \times 53}$, being 42 and 53 the number of input and output variables in the process, respectively. The i^{th} rows of the matrices X and Y was respectively denoted by $x_i \in \mathfrak{R}^{42}$ and $y_i \in \mathfrak{R}^{53}$, and the

matrix containing all the acquired measurements was denoted as $M=[X,Y] \in \mathcal{R}^{19530 \times 95}$. For monitoring purposes, these data points were processed sequentially in order to emulate online observations, although an intermediate normalization step was used in order to avoid biased results due to the different variables magnitude. All the numerical implementations were performed in MATLAB® software.

The implementation of a nonparametric monitoring scheme requires data bases with a comprehensive representation of different process operation conditions. Thus training data considered different operating points for healthy operation, as well as post-maintenance data and abnormal system operation.

3.1 Data Pre-processing

The use of nonparametric models and SBM can only be justified if the system exhibits nonlinearities and the existence of several operating points. PCA was used to quickly identify the existence of these operating points; using only four principal components of training data for this purpose. Figure 1 shows the results of the aforementioned analysis, which captures the 87% of the data variability, where it is evidenced that there are clustered regions for the operation of the compressor. Failure to characterize all these operating points using simply a collection of linear-in-the-parameters models (Gonzalez *et al.*, 2003) inspired the use of a monitoring technique based on SBM. It must be noted that, for confidentiality reasons, data labels cannot be clearly indicated on this article. In addition it is important to mention that, for all practical purposes, the models only incorporated a static characterization of the system. The latter is based on the fact that all thermo-dynamical and mechanical subsystems were always controlled in closed loops that ensured dominant time constants smaller than the data sampling period. Although it is always possible to increase the sampling frequency to a point where the dynamics of the control loops are in evidence, the company explicitly decided to incorporate those features as part of future research activities.

Being stated that the data admits the use of SBM techniques, and assuming that the system dynamics can be neglected, a suitable similarity operator should be defined with respect to the statistical properties of the measurements. After a preliminary study, the similarity operator that best captured the data variability was the saturated triangular operator defined in (17).

$$A\Delta B = \begin{cases} d - \|A - B\|, & \|A - B\| \leq d + \varepsilon \\ \varepsilon, & \|A - B\| > d + \varepsilon \end{cases} \quad (17)$$

where $\varepsilon > 0$ is a small number that ensures $A\Delta B > 0$, and $d > 0$ is a threshold depending on the observations variance. The definition of these parameters heavily depends on the

distribution of clusters and the distance between samples in the training data set.

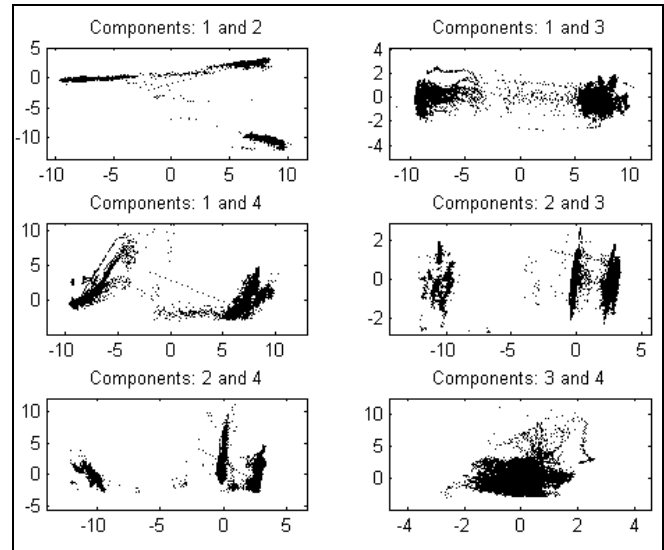


Figure 1. Principal component analysis (PCA) of data from power generation plant. Clusters are the first indication of the existence of several operating points within the data set.

3.2 Database description and a first implementation of the proposed anomaly detection scheme

A subset of data samples was selected from the acquired input/output data for purposes of SBM training and weight characterization. Training data was chosen to incorporate different modes of operation through a novel iterative method that focused on a two-objective optimization problem that minimized of the number of data samples to be included in the training set, while also minimizing the mean-squared error of the resulting SBM-based model residual. This is a critical procedure since, as Figure 2 shows, many operating points are presents within the data that was acquired to characterize the operation of the turbine power and its compressor. In fact, some of the data depicted in Figures 2a, 2b, and 2c contain two different instances of faulty operation, as well as healthy turbine operation, one maintenance procedure, and operation after maintenance. After each fault, the plant always stopped its operation and, during maintenance, the plant was shut down for extended periods of time.

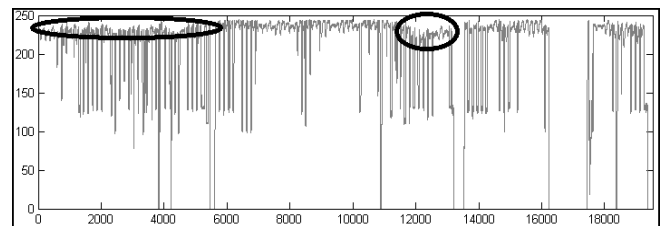


Figure 2a. Illustration of a compressor fault from the standpoint of the power turbine operation.

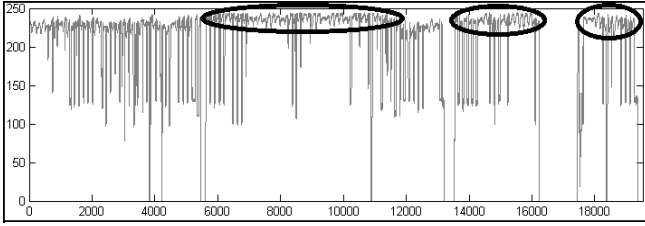


Figure 2b. Normal operation in turbine. Last operating point corresponds to post- maintenance.

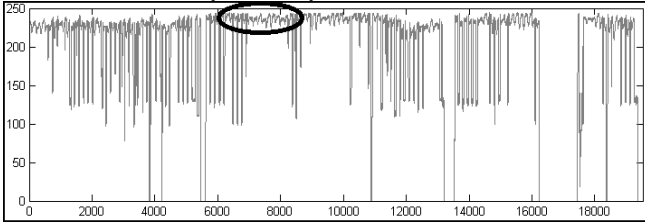


Figure 2c. Data subset used to train the SBM-based anomaly detection algorithm.

A first implementation of the proposed SBM-based anomaly detection scheme was performed for all process variables (i.e., the model had 42 input and 53 output variables), without the dimensionality reduction that some multivariate techniques such as PLS could suggest. Particularly in this case, the obtained mean-squared error (MSE) of the SBM-based normalized power output estimate is presented in Figure 3. It can be seen that using the specified database, the MSE that is related to SBM estimates remains considerably low for the region that contain the training data, other normal operating regions, and even for the post-maintenance data. As expected, the T^2 index is greater than the threshold for data associated to faulty operation. It must be noted that the principal components of the error matrix $E = Y - \hat{Y}$ (which statistically characterizes the training set, where \hat{Y} represents the SBM-based estimate) were used to compute the T^2 index threshold. Additionally, Hotelling's test has been applied to find the 95% confidence ellipse; using for these purposes the software SCAN developed by the Chilean company CONTAC Engineers Ltda (SCAN 2013).

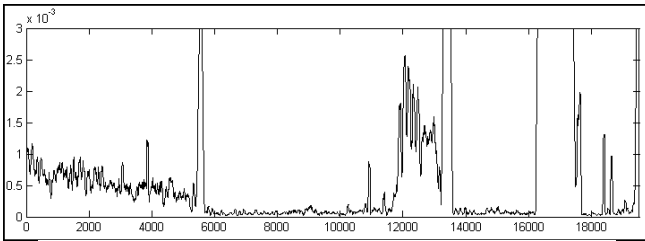


Figure 3. MSE associated to SBM-based estimates for output variables in power generation plant (MSE = 0.000594).

As Figure 4 shows, the T^2 index for the SBM-based residual of the process output variables is adequate to detect a fault in the compressor of the gas turbine.

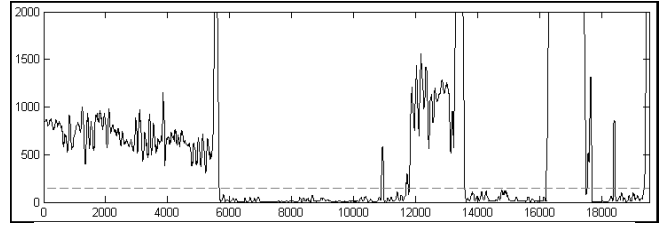


Figure 4. Anomaly detection using a detection threshold based on the Hotelling's T^2 index for training data. Hotelling's T^2 threshold is set in 150.

3.3 Selection of variables and Second Implementation

Utilizing PLS property to maximize the covariance between the input matrix X and the output matrix Y , a method of dimensionality reduction is proposed based on the analysis of correlations. A reduced set of variables is chosen in order to keep Hotelling's test detecting system faults, while maximizing the correlation and variability between inputs and outputs.

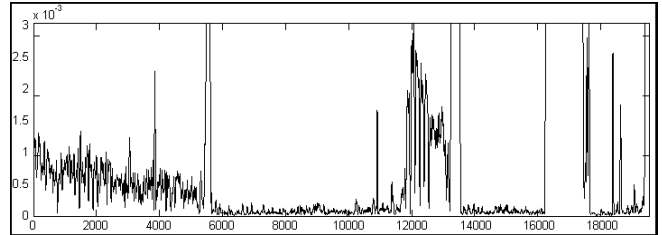


Figure 5. MSE associated to SBM-based estimates for output variables in power generation plant (MSE = 0.001586).

In this case study, and using the proposed methodology, it is discovered that only 5 input and 3 output variables are sufficient for anomaly detection purposes, thus helping to define new matrices $X \in \mathcal{R}^{19530 \times 5}$ and $Y \in \mathcal{R}^{19530 \times 3}$. Figure 5 shows the square error for the normalized power output from a new SBM-based structure. It is appreciated that for this new set of variables the estimate exhibits a larger MSE in general, although the dimensionality reduction associated to it allows to perform all the necessary computations in real-time. It must be noted, though, that the model still maintains its capability of discriminating normal from abnormal behavior in the plant.

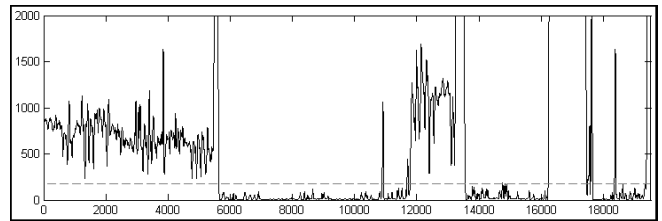


Figure 6. Anomaly detection using a detection threshold based on the Hotelling's T^2 index for training data. Hotelling's T^2 threshold is set in 150.

Using the same methodology as above, a Hotelling's T^2 index is constructed using projections on the space determined by the PCA of the model residuals, using the software SCAN. The results are depicted in Figure 6. As Figure 6 illustrates, and comparing with the results shown in Figure 4, the methodology allowed generating equivalent results for the anomaly detection module although the total number of variables included in the SBM model was reduced from 95 to 8; ensuring appropriate detection of faults in the compressor of the gas turbine. The computational cost was significantly lessened in the second implementation of the detector.

4. CONCLUSION

This article presents and validates a scheme to detect anomalies in the compressor of a gas turbine in a Chilean power generation plant, by comparing the process outputs with SBM-based estimates. The proposed scheme also provides the means to select the data samples that will be included in the training data set by an optimal procedure that minimizes the number of samples while also minimizing the MSE of the model residuals. The use of PCA and PLS techniques helped to dramatically reduce the dimension of the detection problem to a point where it was possible to build the SBM-based detector using only 8 process variables as sources of information. Once a representative training set is constructed, the proposed scheme estimate the system output, exhibiting a reduced MSE and also capturing the relationships between input and output variables even after maintenance procedures. Finally, it is important to know that the detector presented in this paper is monitoring the compressor (the same compressor with which the model was trained) online for more than a year ago. During this time, all anomalies have been confirmed by operators as true faulty conditions.

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