Reliability Growth Analysis of Satellite Systems

[1]Dr. John W. Evans, [2]Dr. Mark P. Kaminskiy, and [3]Mr. Luis D. Gallo Jr.

[1,2,3]*NASA Goddard Space Flight Center, Greenbelt, MD, 20771, U.S.A*

John.W.Evans@NASA.gov
Mark.P.Kaminskiy@NASA.gov
Luis.D.Gallo@NASA.gov

ABSTRACT

A reliability trend/growth analysis methodology for satellite systems is suggested. A satellite system usually consists of many satellites successively launched over many years, and its satellites typically belong to different satellite generations. This paper suggests an approach to reliability trend/growth data analysis for the satellite systems based on grouped data and the Power Law (Crow-AMSAA) Non-Homogeneous Poisson process model, for both one (time) and two (time and generation) variables. Based on the data specifics, the maximum likelihood estimates for the Power Law model parameters are obtained. In addition, the Cumulative Intensity Function (CIF) of a family of satellite systems was analyzed to assess its similarity to that of a repairable system. The suggested approaches are illustrated by a case study based on Tracking and Data Relay Satellite System (TDRSS) and Geostationary Operational Environmental Satellite (GOES) data.

1. INTRODUCTION

The objective of this study is to develop a reliability growth analysis methodology applicable to *satellite systems*. A satellite system usually consists of many satellites successively launched during many years, and its satellites can belong to different satellite generations. For example, the United States National Environmental Satellite, Data, and Information Service (NESDIS) is now developing its fourth generation (gen.) of the GOES satellites. The GOES first satellite, GOES 1, was launched in 1975 and the latest, GOES 15, was launched in 2010 (see Table 2).

During the system life, its satellites can be in different states, like *active*, *in-orbit testing*, *failed*, *standby*, *retired*, etc. The satellite system reliability improvements are based on the analysis of anomalies (failures) observed on the in-orbit satellites, and the respective corrective actions can usually be implemented only in the next and the following satellites to be launched. In other words, the traditional reliability growth "Test-Analyze-Fix" concept is not applicable to the on-orbit satellite systems, which makes the data model and data analysis of the satellite systems rather different.

2. DATA AND RELIABILITY GROWTH MODEL

A satellite system (SS) is considered. Let's assume that the SS currently consists of k satellites S_1, S_2, …S_k , where S_1 is the first (oldest) successfully launched satellite, S_2 is the second satellite, . . . , and S_k is the latest successfully launched launched satellite. Let T_1, T_2 , . . . T_k denote, respectively, the cumulative times during which the S_1, S_2, …S_k anomalies were recorded, and let N_1, N_2 , . . . N_k denote the random numbers of corresponding failures (anomalies). These data can be represented using Table 1.

The Crow-AMSAA model is suggested to apply for the SS reliability trend analysis. This model is the most popular reliability growth model. The model is used in Military Handbook 189 (MIL-HDBK -189 C, 2011). The model was applied in the following traditional form:

$$\lambda(t) = \lambda_0 \beta t^{\beta - 1} \quad (1)$$

where $\lambda(t)$ is the ROCOF for a given satellite, λ_0 and β are positive parameters, and t is the satellite order number, so that the variable t takes on the following values: 1, 2, 3, 4, 5, ... Other choices of the independent variable t can be budget or other monetary or manpower resources spent to improve the satellite reliability. It should be noted that in the case of reliability growth, the parameter β should satisfy the following inequality: $0 < \beta < 1$. The model (1) is sometimes referred to as the Weibull process, because it coincides with the failure rate of the Weibull distribution.

Satellite	Time Interval	Number of Anomalies
S_1	T_1	N_1
S_2	T_2	N_2
…	…	…
S_k	T_k	N_k

Table 1. Satellite System Anomaly Data

3. Data Analysis

For each satellite of the system considered, the ROCOF estimate is calculated as

$$\hat{\lambda}(i) = \frac{N_i}{T_i} \quad i = 1, 2 \ldots, k \tag{2}$$

where the estimator (2) is known as the *natural* estimator of ROCOF (Basu, A.P. & Rigdon S.E., 2000; Crowder, M. J., Kimber A. C., Smith, R. L., & Sweeting, T. J., 1991) .

Assuming that ROCOF is constant (but different) for each satellite, it is clear that N_i is distributed according to the Poisson distribution with the mean equal to $\lambda(i)T_i$, where $\lambda(i)$ is the unknown true value of ROCOF for *i*th satellite. If the number of the observed failures N_i is great enough, the distribution can be approximated by the Normal Distribution, having the same mean and the variance equal to this mean.

Based on the above considerations, the following regression model (3) is suggested for estimating the parameter of the Crow-AMSAA model (1)

$$\lambda(t_i) = \lambda_0 \beta t_i^{\beta-1} \quad + \delta_i \tag{3}$$

where δ_i is a normally distributed error with zero mean and the variance is inversely proportional to the number of the observed failures N_i; t_i is the satellite order number, taking on the following values: 1, 2, 3, ... The model (3) is non-linear in the parameters regression model, where parameters λ_0 and β should be estimated under the following restrictions: $\lambda_0 > 0$ and $1 > \beta > 0$.

Another way to estimate the parameters of the reliability growth model (1) is to apply the Maximum Likelihood (ML) approach. For the data discussed above, the likelihood function $L(\lambda_0, \beta)$ can be written as

$$L(\lambda_0, \beta) = \cdots \tag{4}$$

$$= \prod_{i=1}^{k} \frac{\left(\int_0^{T_i} \lambda_0 \beta t_i^{\beta-1} d\tau\right)^{N_i} exp\left(-\int_0^{T_i} \lambda_0 \beta t_i^{\beta-1} d\tau\right)}{N_i!} \tag{4}$$

$$= \prod_{i=1}^{k} \frac{\lambda_0^{N_i} \beta^{N_i} t_i^{N_i(\beta-1)} T_i^{N_i} \exp\left(-\lambda_0 \beta t_i^{\beta-1} T_i\right)}{N_i!} \tag{4}$$

and its logarithm as

$$ln\big(L(\lambda_0, \beta)\big) = \sum_{i=1}^{k} N_i ln(\lambda_0) + N_i\, ln(\beta) + \cdots \tag{5}$$

$$N_i(\beta - 1)\, ln(t_i) + N_i\, ln(T_i) - \lambda_0 \beta t_i^{\beta-1} T_i - ln(N_i!) \tag{5}$$

Writing the first derivatives of (5) with respect to λ_0 and β and equating them to zero, we arrive at the following system of non-linear equations for λ_0 and β:

$$\frac{\partial}{\partial \lambda_0}\big(L(\lambda_0, \beta)\big) = \frac{1}{\lambda_0} \sum_{i=1}^{k} N_i - \beta \sum_{i=1}^{k} t_i^{\beta-1} T_i = 0 \tag{6}$$

$$\frac{\partial}{\partial \beta}\big(L(\lambda_0, \beta)\big) = \frac{1}{\beta} \sum_{i=1}^{k} N_i - \ldots \tag{7}$$

$$\lambda_0 \sum_{i=1}^{k} T_i \left[t_i^{\beta-1} + \beta(\beta - 1) t_i^{\beta-2}\right] = 0 \tag{7}$$

which must be solved under the restrictions: $\lambda_0 > 0$ and $1 > \beta > 0$.

4. Case Study: Tracking and Data Relay Satellite System

The Tracking and Data Relay Satellite System (TDRSS) is a network of satellites (each called a *Tracking and Data Relay Satellite* or TDRS) and ground stations used for space communications. The TDRSS space segment currently consists of nine on-orbit TDRSs located in geosynchronous orbit, distributed to provide global coverage.

The available data on the TDRSs are 327 NASA Spacecraft Orbital Anomaly Report System (SOARS) records related to the satellites of the first TDRS generation (A, C, D, E, F and G) and the second TDRS generation (H, I, J), listed in Table 2. It should be noted that there is much less data on TDRS H, I and J (only about 25 cumulative mission years) compared to the first generation, i.e., TDRS A, C, D, E, F and G (about 101.4 cumulative mission years).

The Crow-AMSAA model (1) and the data from Table 2 were used for the reliability trend analysis. The parameters of the reliability growth model were estimated as: λ_0 = 3.156917 1/day and β = 0.006. The ROCOF estimates and the fitted Crow-AMSAA model are shown in Figure 1 below. The model provides a good fit to the data: the squared correlation coefficient $R^2 = 0.963$. Using the fitted model, the ROCOF for the future TDRS M was predicted as 0.00151/day. The predicted value indicates a 30% – 40% reliability growth for TDRS 13 (TDRS M) compared to TDRS 10 (TDRS J) in terms of ROCOF.

Generation (Gen.)	Satellite Name	Other Satellite Name	Launch Date	Last Record Date	Time Interval, days	Number of records	ROCOF (λ_{est}) 1/day
1	TDRS A	TDRS 1	4-Apr-83	5/04/2006	8431	192	0.0228
1	TDRS C	TDRS 3	29-Sep-88	9/28/2004	5843	28	0.0048
1	TDRS D	TDRS 4	13-Mar-89	11/02/2010	7904	35	0.0044
1	TDRS E	TDRS 5	2-Aug-91	11/06/2004	4845	21	0.0043
1	TDRS F	TDRS 6	13-Jan-93	7/16/2006	4932	20	0.0041
1	TDRS G	TDRS 7	13-Jul-95	6/4/2009	5075	8	0.0016
2	TDRS H	TDRS 8	30-Jun-00	9/02/2010	3716	5	0.0013
2	TDRS I	TDRS 9	8-Mar-02	9/26/2010	3124	12	0.0038
2	TDRS J	TDRS 10	4-Dec-02	7/21/2009	2421	6	0.0025

Table 2. Data and Estimated Rate of Occurrence of Failures

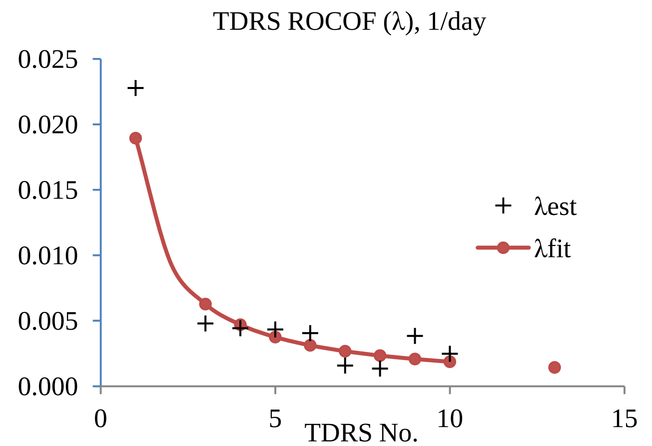

Figure 1. Estimated ROCOF and fitted reliability growth model. The extreme right point is the projected ROCOF for TDRS M (13).

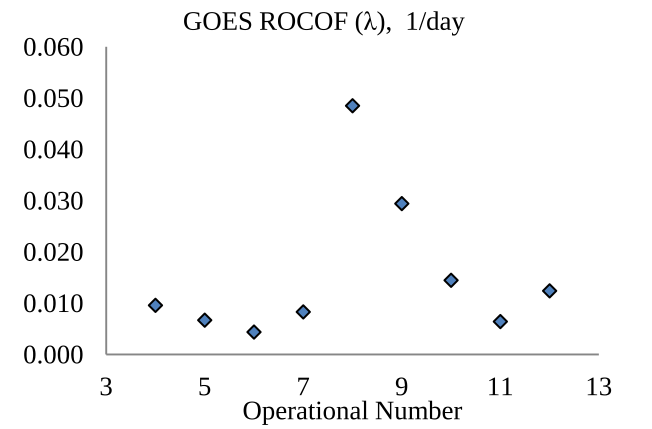

Figure 2. GOES ROCOF dependence on satellite operational number. The interval [4, 7] is the first generation of satellites; the interval [8, 12] is the second generation of GOES satellites.

5. Reliability Growth Model with Two Variables

Earlier, we applied the power law (Crow-AMSAA) relationship to model satellite ROCOF dependence on the satellite order (operational) number. The relationship we are going to introduce below can be used to take into account a possible jump of ROCOF attributed to a new satellite generation, which is illustrated by the GOES ROCOF (see Figure 2 and Table 3).

Figure 2 shows a significant jump in ROCOF between the first generation and the second generation of GOES. This increase in ROCOF of the second generation can be explained by more complex satellite design and functions. The figure also reveals a minor ROCOF increase for each last satellite of the first generation and the second generation. The GOES 7 increase in ROCOF compared to its predecessor GOES 6 can be attributed to the GOES 7 new feature – it was the first GOES satellite capable of detecting 406 MHz distress signals from emergency beacons carried aboard aircraft and vessels and sending them to ground stations. In its turn, the GOES 12 increase in

ROCOF (compared to GOES 11) can be attributed to the new instrument -- GOES 12 was the first satellite to carry a Solar X-Ray Imager (SXI) type instrument.

In order to take into account a ROCOF dependence on the satellite generation, the following model is suggested:

$$\lambda(t_i, T) = \lambda_0 t_i^{\beta_1} T_j^{\beta_2} \qquad (8)$$

The model has the following two independent variables- operational number (t_i = 4, 5, . . . , 12) and a dummy variable, T_j, (j = 1, 2,..., J), which is the satellite generation order number. The dummy variable T value is e for the first generation, i.e., $T_1 = e$, and T takes on the value $e^o \equiv 1$ for the second satellite generation, i.e., $T_2 = 1$. The choice of these values becomes obvious if we take the natural logarithm of (8) in order to make the model linear:

$$ln\big(\lambda(t_i, T)\big) = ln(\lambda_0) + \beta_1\, ln(t_i) + \beta_2\, ln(T_j) \qquad (8.1)$$

It is clear that the transition from the first generation to the second generation changes the intercept of the above linear dependence by β_2 because of a unit change in $\ln(T_j)$, i.e., $\ln(T_1) - \ln(T_2) = 1$. The variable T_j can be called the *generation code*. The available GOES ROCOF data needed to fit the above model are given in Table 3.

GOES Gen.	Gen. Code (T)	ln(T)	GOES Oper. Number (t)	ROCOF (λ_{est},) 1/day
1	EXP(1)	1	4	0.00956
1	EXP(1)	1	5	0.00667
1	EXP(1)	1	6	0.00435
1	EXP(1)	1	7	0.00828
2	EXP(0)	0	8	0.04848
2	EXP(0)	0	9	0.02940
2	EXP(0)	0	10	0.01445
2	EXP(0)	0	11	0.00638
2	EXP(0)	0	12	0.01239

Table 3. GOES History and Estimated Rate of Occurrence of Failures (ROCOF)

Using the above data, the parameter estimates of model (8.1) are given in Table 4.

	Parameter Estimate	Std. Err.	t(6)	p-level
$ln(\lambda_0)$	0.4017	2.3920	0.1679	0.8722
β_2	-2.1074	0.7308	-2.8839	0.0279
β_1	-1.9407	1.0380	-1.8697	0.1107

Table 4. Regression analysis summary of model (8.1)

As it follows from Table 4, the parameter $\ln(\lambda_0)$ is statistically insignificant, so that our model (8) can be written as:

$$\lambda(t_i, T) = t_i^{\beta_1} T_j^{\beta_2} \qquad (8.2)$$

The fitted model is shown in Figure 3.

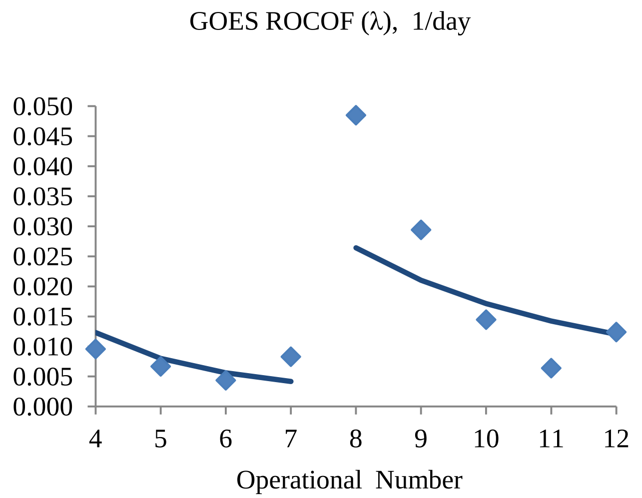

Figure 3. The GOES ROCOF and fitted model (8.2)

Based on the model and data, the jump in values of ROCOF $\frac{\lambda(8,T_2)}{\lambda(7,T_1)}$ is about 8.

In order to compare the reliability growth rate for GOES generations 1 and 2, the following ROCOF model was fitted for each generation:

$$\lambda(t_i) = \lambda_0 t_i^{\beta} \qquad (8.3)$$

The fitted models are shown in Figures 4 and 5.

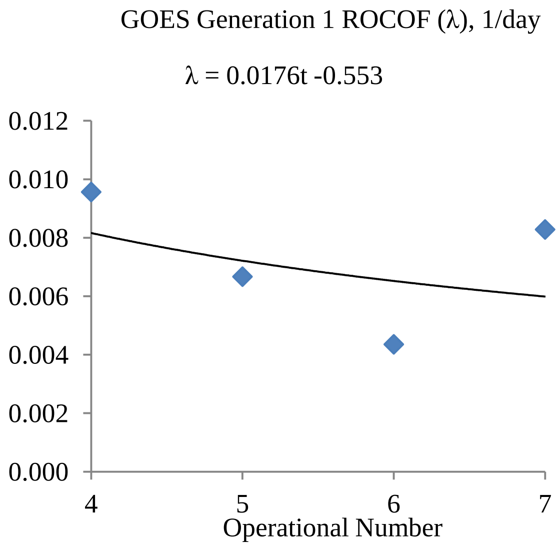

Figure 4

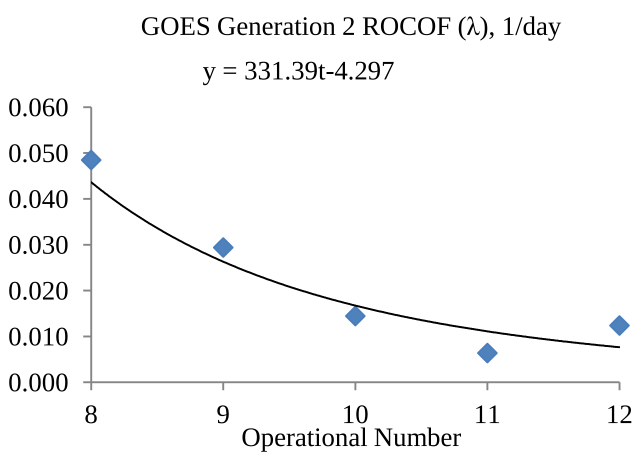

Figure 5

6. Failure Time Occurrences during Each Mission

Based on its cumulative intensity function (CIF), each satellite in a system of satellites, such as GOES, can be considered as a repairable system. The cumulative intensity function of an idealized repairable system is depicted in Figure 6. At the beginning of mission, the CIF is concave down (has a decreasing derivative (ROCOF)). This part of system mission lifetime corresponds to the reliability growth. Then CIF becomes approximately linear, which corresponds to constant in time ROCOF and normal (from reliability standpoint) system operation. At the end of system life, the CIF becomes concave up, corresponding to increasing ROCOF, and this part of the system mission lifetime corresponds to the reliability deterioration (aging).

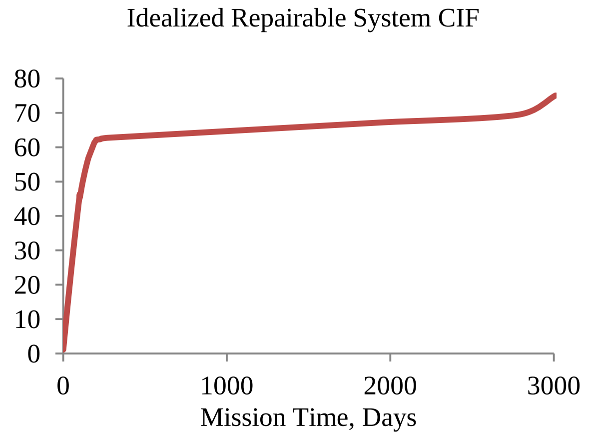

Figure 6. Cumulative Intensity Function of Idealized Repairable System

Figures 7a through 7i display the real CIF for a variety of GOES missions. These cumulative intensity functions have shapes similar to the idealized CIF.

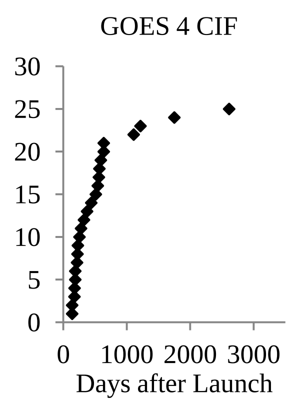

Figure 7a

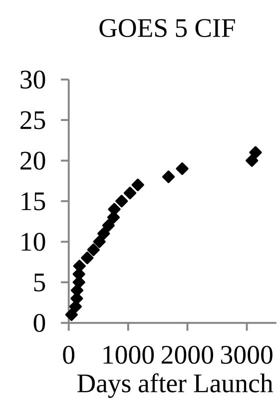

Figure 7b

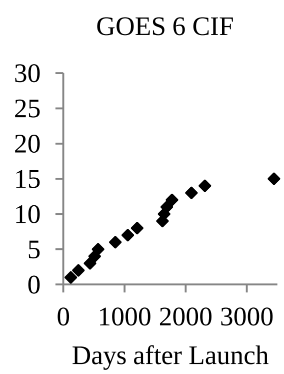

Figure 7c

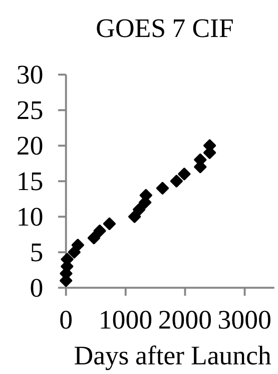

Figure7d

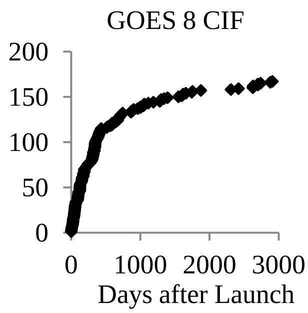

Figure7e

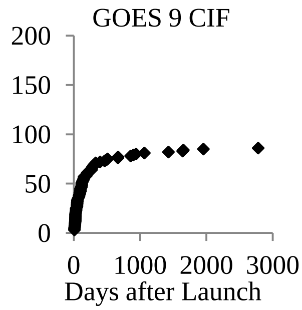

Figure 7f

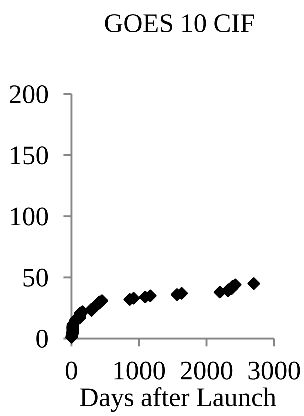

Figure 7g

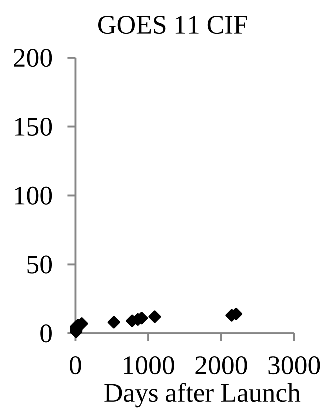

Figure 7h

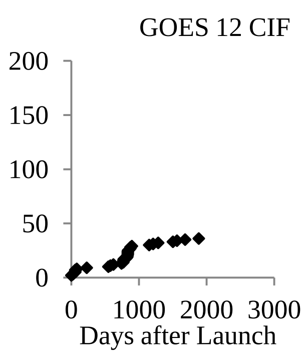

Figure 7i

7. Conclusion

In this study we set out to suggest a reliability trend/growth analysis methodology for satellite systems. We used the number of recorded anomalies for a given satellite mission over a given time period of operation as the data to measure this growth. Using this data, we modeled reliability growth as both a function of time and as a function of both time and satellite generation. Finally, we observed trends in ROCOF over a single satellite's operational lifetime and discussed the implications of all the observed trends on the evolution of satellite systems. In order to do this, we assumed that an anomaly entry in the SOARS database corresponded to a failure of the given satellite, since anomalies are positively correlated with failures. Such an assumption is reasonable as long as the model used to fit the data is not expected to predict/measure the number of actual failures of a given satellite system.

We can model reliability growth across multiple satellite generations in a satellite system with a Crow-AMSAA model. This model is a good fit, having a squared correlation coefficient that is close to one (R2 = 0.963). The fitted model indicates that there is 30% – 40% reliability growth for the TDRS 13 (TDRS M) satellite compared to TDRS 10 (TDRS J), in terms of ROCOF. The overall trend of ROCOF decrease with time implies an improving level of reliability over time and thus reliability growth in the TDRS family of spacecraft. These results are intuitive, since each satellite generation is relatively similar in design to the previous one, allowing for consecutive generation designs to be more refined.

We can model reliability growth across multiple satellite generations in a satellite system with greater accuracy after a slight modification to the Crow-AMSAA model. This modification involves introducing a dummy variable, Tj, (j = 1, 2,..,J), which represents the satellite generation order number. This modification allows the model to capture any major generational changes in satellite system ROCOF data due to new technologies. The reliability growth of the model fitted to GOES satellite data is greater/more pronounced among the newer generation of GOES satellites and is able to capture and explain the radical change in ROCOF data corresponding to a significant change in technology introduced by the second generation of GOES satellites, beginning with GOES 8. This model provides a better fit than would have been possible with the single variable Crow-AMSAA model due to its ability to capture the inflection introduced by GOES 8.

We considered the plausibility of considering satellite systems, such as GOES, as repairable systems. Such systems experience a rapid increase in the reported number of failures over an initial period of operation, and maintain a fixed, less sharply increasing rate of failures for an extended period of operation, until finally the rate of failures increases again towards the end of system life (i.e. the bathtub curve effect). This turned-out to be a plausible consideration, since the observed CIF of each of the GOES family systems displayed some, if not all, of these repairable system characteristics.

We can improve the current models by introducing a Bayesian prior distribution over its parameters (i.e. λ_0, β), considering them as random variables, and employing Bayesian inference, as opposed to classical Maximum Likelihood Estimation. All of these considerations should be made in future studies of these data.

REFERENCES

Rigdon, S. E., & Basu, A. P. (2000). *Statistical methods for the reliability of repairable systems*. New York: Wiley.

Castet, J., & Saleh, J. (2009). Geosynchronous Communication Satellite. *27th IET/AIAA international communications satellite systems conference 2009: (ICSSC 2009), Edinburgh, United Kingdom, 1-4 June 2009.* (p. 431). Stevenage: IET.

Crowder, M. J., Kimber A. C., Smith, R. L., & Sweeting, T. J. (1991). *Statistical analysis of reliability data*. London: Chapman & Hall.

Military Specification (MIL)-HDBK-189C, Military Handbook: Reliability growth management (DOD, 2011).

Saleh, J. H., & Castet, J. (2011). *Spacecraft reliability and multi-state failures a statistical approach*. Chichester, West Sussex, UK: John Wiley.

Seber, G. A., & Wild, C. J. (1989). *Nonlinear regression*. New York: Wiley.

Dr. John Evans is a Senior Aerospace Engineer in the Reliability and Risk Assessment Branch of the NASA Goddard Space Flight Center. He is an internationally recognized expert in reliability growth and its application to complex systems. John is the author or coauthor of several books and numerous conference and journal articles on electronic system reliability, failure analysis and manufacturing technology.

Dr. Mark Kaminskiy is a Principal Reliability Engineer at Ares, providing Reliability Analysis support to the NASA Goddard Space Flight Center. Dr. Kaminskiy has served as the chief statistician at the Center of Technology and Systems Management, University of Maryland (College Park). He possesses extensive experience as a researcher in reliability engineering, life data analysis, and risk analysis of engineering systems and has conducted numerous research and consulting projects funded by the government and industrial companies. He has taught several graduate courses on reliability engineering at the University of Maryland and is the author or coauthor of over 50 publications in journals, conference proceedings, and reports. Dr. Kaminskiy possesses an extensive background in parametric and nonparametric distribution estimation, multivariate regression modeling for reliability applications, accelerated life testing, and probabilistic damage modeling.

Mr. Luis D. Gallo Jr. is an Aerospace Engineer in the Reliability and Risk Analysis branch of the NASA Goddard Space Flight Center. He holds a Bachelors of Science Degree in Electrical Engineering from Florida International University and his research interests include Bayesian inference in graphical models of spacecraft systems and machine learning as applied to reliability and risk analysis.