Condition-Based Maintenance with both Perfect and Imperfect Maintenance Actions

Phuc Do Van, Alexandre Voisin, Eric Levrat, and Benoit Iung

Lorraine University, CRAN, CNRS UMR 7039
Campus Sciences BP 70239, 54506 Vandoeuvre, France
van-phuc.do@univ-lorraine.fr, alexandre.voisin@univ-lorraine.fr, eric.levrat@univ-lorraine.fr, benoit.iung@univ-lorraine.fr

ABSTRACT

This paper deals with a condition-based maintenance (CBM) model considering both perfect and imperfect maintenance actions for a deteriorating system whose condition is aperiodically monitored according to a remaining useful life (RUL) based-inspection policy. Perfect maintenance actions restore completely the system to the 'as good as new' state. Their related cost are however often high. Imperfect preventive maintenance restores partially the system with reduced maintenance cost. Nevertheless, it may however make the system more susceptible to future deterioration. The aim of the paper is to propose a CBM model which can help to construct optimal maintenance policies when both perfect and imperfect maintenance actions are possible. To illustrate the use of the proposed CBM model, a numerical example finally is introduced.

1. INTRODUCTION

Maintenance involves preventive and corrective actions carried out to retain a system in or restore it to an operating condition. Optimal maintenance policies aim to provide optimum system reliability/availability and safety performance at lowest possible maintenance costs, (Pham & Wang, 1996). In the literature, perfect maintenance actions (or replacement actions) which can restore the system operating condition to as good as new have been considered in various maintenance models. The implementation of perfect maintenance policies seems quite simple, however, perfect maintenance actions are often expensive. Imperfect maintenance implying that the system condition after maintenance is somewhere between the condition before maintenance and as good as new has grown recently as a popular issue to researchers as well as industrial engineering, see for example (Castro, 2009; Kijima, Morimura, & Suzuki, 1988; Labeau & Segovia, 2010; Levitin & Lisnianski, 2000; Nakagawa & Yasui, 1987; Liu & Huang, 2010). From a practical point of view, imperfect maintenance can describe a large kinds of realistic maintenance actions (Pham & Wang, 1996). Furthermore, imperfect maintenance is usually cheaper than perfect maintenance. Various methods and optimal policies for imperfect maintenance are summarized and discussed in (Pham & Wang, 1996; Wu & Zuo, 2010). In such maintenance models, preventive maintenance decision is however based on the system age and on the knowledge of the statistical informations on the system lifetime. As a consequence, the realistic operating conditions of the system over time can not be taken into account.

To face this issue, condition-based maintenance (CBM), for which preventive maintenance decision is based on the observed system condition, is recently introduced. Thank to rapid development of monitoring equipments which can provide accurately information about the system condition over time, CBM becomes nowaday an interesting approach for maintenance optimization. Various CBM policies have been proposed and applied for many industrial systems, see for example (Ghasemi, Yahcout, & Ouali, 2007; Grall, Dieulle, Bérenguer, & Roussignol, 2002; Neves, Santiago, & Maia, 2011; Noortwijk, 2009; Tan, Cheng, Guo, & Gong, 2010). It is recently shown in (Meier-Hirmer, Riboulet, Sourget, & Roussignol, 2008; Nicolai, Frenk, & Dekker, 2009; Ponchet, Fouladirad, & Grall, 2011) that CBM is specially suited to imperfect maintenance since according to the observed condition of the system, an optimal maintenance action represented by an optimal intervention gain is preventively carried out. However, in such maintenance policies, only imperfect preventive or imperfect repair actions are considered and the system is assumed to be imperfectly maintained an infinite number of times. From a practical point of view, this assumption may not always be relevant since, in variety of engineering and service applications, systems can be maintained only a limited number of times due to technical or economical reasons (Kurt & Kharoufeh, 2010). Furthermore, as mentionned in (Nicolai et al., 2009), each imperfect maintenance action may make the system more susceptible to future deterioration. To this end, a fixed number of allowable imperfect maintenance actions is introduced in maintenance models in (Kurt & Kharoufeh, 2010; Do Van & Berenguer, 2012) and considered as a decision parameter. However, the value of variable is arbitrary chosen and they do not describe how the imperfect repair actions affect the deterioration evolution of the system.

The aim of this paper is to propose a CBM model considering both perfect and imperfect maintenance actions for a deteriorating system. Imperfect maintenance is investigated with both positive and negative sides. Positive impact means that it can reduce the deterioration level of the system with reduced maintenance cost. Negative impact implies that each imperfect preventive action may accelerate the speed of the system's deterioration process. Moreover, in CBM practice, inspections are usually performed at regular intervals. However, it may not be always profitable to systematically inspect the system, especially when the inspection procedure is costly. The present paper proposes to use an aperiodic inspection policy which is based on the residual useful life (RUL) of the system, see (Cui, Xie, & Loh, 2004; Gebraeel, Lawley, Li, & Ryan, 2005; Yang & Klutke, 2001).

This paper is organized as follows. Section 2 is devoted to the description of the system characteristics and related assumptions. Imperfect maintenance actions and their related cost model are also described and discussed. Section 3 focuses on the proposed imperfect maintenance policy. To illustrate the proposed maintenance policy, a simple numerical example is introduced in Section 4. Some numerical results are in addition discussed here. Finally, the last section presents the conclusions drawn from this work.

2. Model description and assumptions

2.1. General assumptions

Consider a system in which the condition (deterioration level) at time t can be summarized by an observable random scalar variable X_t. This random variable can represent for example a crack length or defect products. In the absence of repair or replacement actions, the evolution of the system deterioration is assumed to be strictly increasing. The process $(X_t)_{t\geq 0}$ is then an increasing stochastic process. Moreover, we suppose that the following assumptions are verified.

- The initial state X_0 is 0;
- The system is failed if its deterioration level is greater than a level L. The threshold L can be seen as a deterioration level which must not be exceeded for economical or security reasons. We assume that *(i)* inspection and maintenance actions can be only performed at discrete times and *(ii)* the system failure is self-announcing.

The system degradation behavior and corresponding states are illustrated in Fig. 1.

To avoid failure occurrence of the system, preventive maintenance is considered. It is assumed that both imperfect and

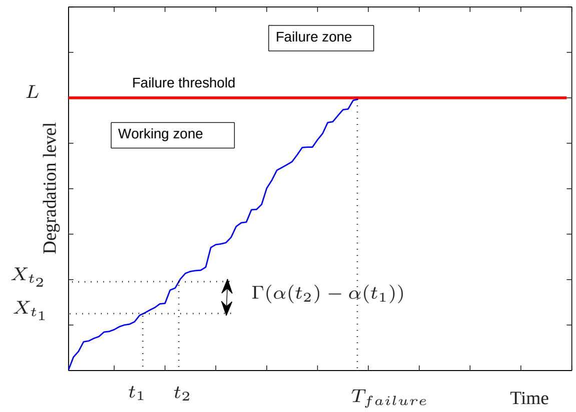

Figure 1. Illustration of the system degradation evolution and its state.

perfect preventive maintenance actions, which incur respectively perfect maintenance cost C_p and imperfect maintenance cost C_p^k, are possible.

2.2. Deterioration modelling

Gamma processes have been widely used to describe the degradation of systems (Noortwijk, 2009; Grall et al., 2002; Do Van & Berenguer, 2012). A characteristic of this process is that it is clearly monotone increasing which is the behavior observed in most physical deterioration processes. Moreover, its paths are discontinuous and it can be thought as the accumulation of an infinite number of small shocks. Following this spirit, it is assumed that the deterioration of the system between the kth and the $(k+1)$th maintenance actions evolves like a Gamma stochastic process $(\tilde{X}_t)_{t\geq 0}$, with the following characteristics:

- $\tilde{X}_0 = X^k$, (X^k represents the deterioration level of the system after the kth maintenance action);
- $(\tilde{X}_t)_{t\geq 0}$ has independent increments;
- for all $0 \leq l < t$, the random increment $\tilde{X}_t - \tilde{X}_l$ follows a Gamma probability density (pdf) with shape parameter $\alpha_k(t-l)$ and scale parameter β:

$$f_{\alpha_k(t-l),\beta}(x) = \frac{1}{\Gamma(\alpha_k(t-l))}\beta^{\alpha_k(t-l)}x^{\alpha_k(t-l)-1}e^{-\beta x}\mathcal{I}_{\{x\geq 0\}},$$

  where:
  - $\mathcal{I}_{\{x\geq 0\}}$ is an indicator function $\mathcal{I}_{\{x\geq 0\}} = 1$ if $x \geq 0$, $\mathcal{I}_{\{x\geq 0\}} = 0$ otherwise;
  - $\alpha_k = v_k/\beta$ with v_k being the mean deterioration speed of the system between the kth and the $(k+1)$th maintenance actions.

After a corrective or perfect preventive maintenance action, the system becomes as good as new (the deterioration level is set to 0 and the deterioration behavior evolves with time according to the nominal speed $v_0 = \alpha_0/\beta$). Imperfect maintenance actions can reduce the system's deterioration level with reduced maintenance costs. However, as mentioned in (Nicolai et al., 2009; Kurt & Kharoufeh, 2010), imperfect maintenance actions may affect the evolution of the system's deterioration process. The impacts of imperfect maintenance actions will be described in the next section.

2.3. Imperfect maintenance actions and related costs

2.3.1. Impact of imperfect actions on the deterioration level

It is shown in the literature that maintenance gains, defined as the reductions of the deterioration level of the system due to imperfection maintenance actions, could be random, see for instance (Castro, 2009; Meier-Hirmer et al., 2008; Do Van & Berenguer, 2012).

In this way, if the kth imperfect maintenance action is performed at inspection time T_i, the intervention gain is then assumed to be described by a continuous random variable Z^k. Z^k is restricted, $0 \le Z^k \le X_{T_i}$ where X_{T_i} is the deterioration level of the system at T_i. In fact, it is shown in (Do Van & Berenguer, 2012) that Z^k can be distributed according to a truncated normal distribution with density:

$$g_{\mu,\sigma,a,b}(x) = \frac{\frac{1}{\sigma}\phi(\frac{x-\mu}{\sigma})}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}\mathcal{I}_{[a,b]}(x), \qquad (1)$$

where:

- $\mathcal{I}_{[a,b]}(x) = 1$ if $a \le x \le b$ and $\mathcal{I}_{[a,b]}(x) = 0$ otherwise;
- $\phi(\xi) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\xi^2\right)$ is the probability density function of the standard normal distribution and $\Phi(\cdot)$ is its cumulative distribution function;
- $\mu = X_{T_i}/2$ and $\sigma = X_{T_i}/6$;
- $a = \mu - 3\sigma = 0$ and $b = \mu + 3\sigma = X_{T_i}$,

According to this distribution, it is clear that $a \le Z_{T_i} \le b$, i.e. $0 \le Z^k \le X_{T_i}$ is satisfied. The mean intervention gains is $\mathbb{E}(Z^k) = \mu$ and the variance is $\mathbb{VAR}(Z^k) = 0.973\sigma^2$, see (Ross, 1996). Thank to the imperfect preventive action, the deterioration level of the system after maintenance is set to $X^k = X_{T_i} - Z^k$. The illustration of the system deterioration evolution and random maintenance gain are shown in Fig. 2.

2.3.2. Impact of imperfect actions on the deterioration speed

To model the impact of imperfect actions on the evolution deterioration of the system, it is assumed in this work that each imperfect preventive action affects the speed of the system deterioration process. This can be found in variety of business sectors, e.g. removing several components for maintenance actions may accelerate the deterioration evolution of other components; spare parts may be reusable components

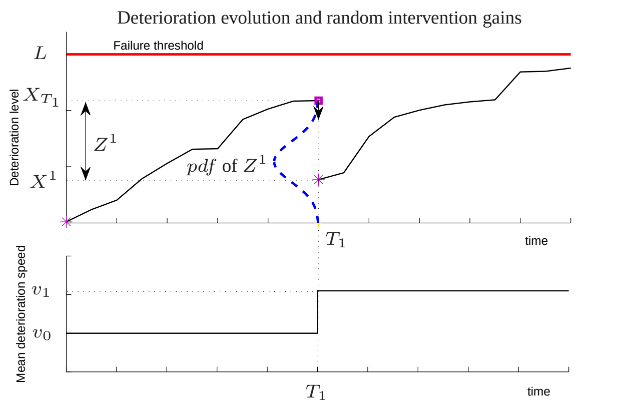

Figure 2. Illustration of deterioration evolution and impacts of imperfect maintenance.

or low quality components, as a consequence, after maintenance the deterioration level of the system can be reduced however the deterioration speed may be increased. The impact of an imperfect maintenance action on the system deterioration speed can be described by non-negative continuous random variable ϵ which follows an exponential distribution with density probability:

$$h(x) = \gamma e^{-\gamma x}\mathcal{I}_{\{x \ge 0\}},$$

where γ is a non negative real number. The mean value of ϵ is $\mathbb{E}[\epsilon] = \gamma$.

By this modelling, if the kth maintenance action is a corrective or perfect preventive maintenance, the mean deterioration speed of the system after maintenance is reset to $v_k = v_0 = \alpha_0/\beta$. If the kth maintenance action is an imperfect preventive one, the mean deterioration speed of the system after maintenance is:

$$v_k = v_{k-1} + \epsilon. \qquad (2)$$

An example of increasing of the deterioration speed due to an imperfect maintenance action is illustrated in Fig. 2. A case study on the sensitivity to the effect of imperfect maintenance actions will be discussed in Section 4.

2.3.3. Imperfect preventive maintenance cost

In general, each maintenance action incurs a cost and an imperfect maintenance action often incurs a reduced maintenance cost, namely imperfect maintenance cost, which may be independent of maintenance gain and bounded by perfect maintenance cost, see for instance (Castro, 2009; Labeau & Segovia, 2010; Meier-Hirmer et al., 2008). From a practical point of view, in most cases, the quality of the maintenance action increases with the level of ressources allocated to it, and hence with its cost, see (Lie & Chun, 1986; Liu & Huang, 2010; Mettas, 2000). To model imperfect maintenance actions in the context of deteriorating systems, the degradation improvement factor, defined as the ratio of the improvement gain divided by the deterioration level of the system before maintenance, has been recently introduced, see (Do Van & Berenguer, 2012). Based on the improvement factor, imperfect maintenance costs can be evaluated and considered as a function of the improvement factor. In this way, it is assumed in this work that when the kth imperfect preventive action is performed at inspection time T_i, we have to pay a maintenance cost which is defined as:

$$C_p^k = C_p^0.u(T_i)^\eta, \tag{3}$$

where:

- $u(T_i) = \dfrac{Z^k}{X_{T_i}}$ is the degradation improvement factor;
- C_p^0 is imperfect preventive cost incurred when the deterioration level of the system is reduced to 0 with imperfect maintenance action. This cost is usually lower than a perfect preventive cost ($C_p^0 \leq C_p$);
- η is a non-negative real number.

According to this cost model, different kinds of maintenance cost function can be found depending on the value of η. More precisely:

- when $\eta = 0$, imperfect maintenance cost is constant ($C_p^k = C_p^0$);
- when $0 < \eta < 1$, imperfect maintenance cost C_p^k is a concave function: the maintenance cost increases more than the improvement gain when performing the maintenance;
- when $\eta = 1$, C_p^k is a linear function which implies that the maintenance cost is proportional to the improvement level gain;
- when $\eta > 1$, C_p^k is a convex function: the maintenance cost increases less than the improvement gain.

Figure 3 illustrates these three different shapes of the imperfect maintenance cost function. A case study of the proposed imperfect maintenance policy with different kinds of imperfect maintenance cost functions will be presented in Section 4.

3. Maintenance policy

In the framework of CBM optimization, a maintenance policy relies essentially on two main decisions: when to take (preventively/correctively) maintenance actions and when to inspect. The system degradation can be used to make the decision on the inspection time and on the maintenance action to be performed, (Grall et al., 2002; Noortwijk, 2009; Do Van & Berenguer, 2010).

The maintenance decision is herein based on both the system deterioration level at inspection time and the potential evo-

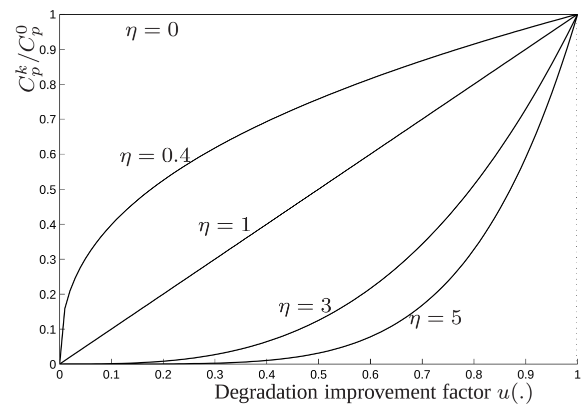

Figure 3. Illustration of imperfect maintenance cost function.

lution of the system's deterioration process. More precisely, according to the degradation level X_{T_i} at inspection time T_i, the maintenance decision is the following:

- if $X_{T_i} < M$, the system is in a working state, no maintenance action is performed. M is called the preventive maintenance threshold and it is a decision variable to be optimized.
- if $L > X_{T_i} \geq M$, the system is still functioning, however its deterioration level is considered as "high". A preventive maintenance action is immediately carried out. Without loss of generality, it is assumed that this preventive maintenance action is the kth preventive maintenance action from the last perfect maintenance of the system. If $k = K$ (K is called the imperfect threshold and it is a decision variable to be optimized), the kth preventive maintenance action is a perfect one. Contrarily, if $k < K$ the kth preventive maintenance action is an imperfect one. This imperfect preventive maintenance action may not restore completely the system but it leads the system to be a better state at which the degradation level is lower or equal to the current deterioration level of the system. The impacts of imperfect preventive maintenance actions and their related cost have been described in Section 2.3.
- if $X_{T_i} \geq L$, the system is failed, then a corrective replacement action is performed and a cost C_c is incurred. An additional cost is incurred by the time d^j elapsed in the failed state at a unavailability cost rate C_d which may correspond to, for example, production loss per unit of time. After a corrective maintenance action, the system is considered as good as new.

We assume finally that maintenance durations are neglected and all the necessary maintenance ressources to execute preventive or corrective maintenance actions are always available. The decision process of the maintenance policy is illustrated in Fig. 4.

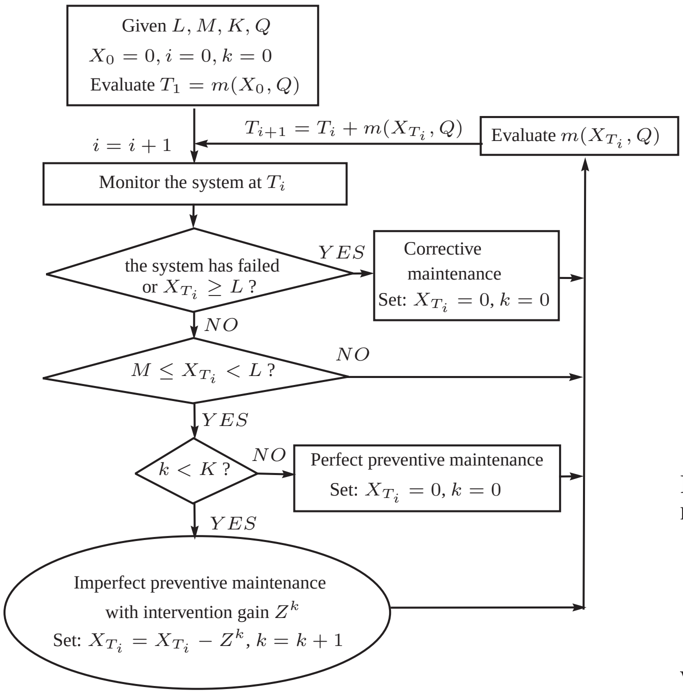

Figure 4. Decision process of the maintenance policy.

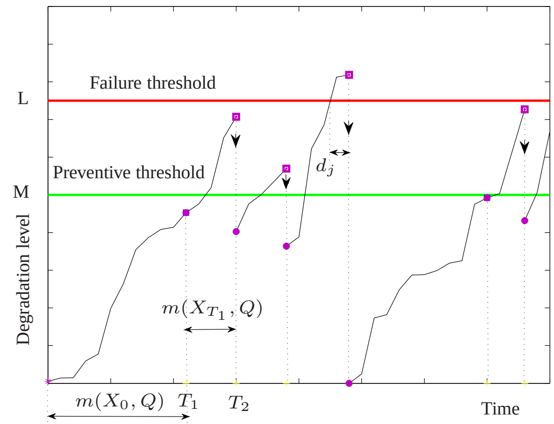

Figure 5. Illustration of degradation behavior and maintenance policy.

3.1. RUL based inspection

Different inspection policies, which aim to optimize the inter-inspection interval length, have been introduced in literature. In fact, the inter-inspection interval length can be fixed regardless of the degradation level, e.g. (Ponchet et al., 2011), or aperiodic and deteriorating-dependant via an inspection scheduling linear (Grall et al., 2002), or non-linear (Barker & Newby, 2009) function with respect to the deterioration level. Residual Useful Life (RUL) based inspection has been recently introduced, see (Cui et al., 2004; Gebraeel et al., 2005; Yang & Klutke, 2001). The latter seems very promising especially in the context of condition-based maintenance.

The main idea of the RUL based inspection is that the next inspection time is chosen such that the probability of the failure of the system before the next inspection remains lower than a limit Q ($0 < Q < 1$ and it is a decision variable to be optimized). If we let T_i denote the time at which the system is inspected, the corresponding degradation level of the system is X_{T_i}, the next inspection time is then determined by:

$$T_{i+1} = T_i + m(X_{T_i}, Q), \tag{4}$$

with,

$$\begin{aligned} & m(X_{T_i}, Q) \\ & = sup\{\Delta T : \mathbb{P}(X_{T_i+\Delta T} \geq L | X_{T_i}) \leq Q\}, \end{aligned} \tag{5}$$

where:

$$\begin{aligned} \mathbb{P}(X_{T_i+\Delta T} \geq L | X_{T_i}) = \mathbb{P}(X_{T_i+\Delta T} - X_{T_i} \geq L - X_{T_i}) \\ = \int_{L-X_{T_i}}^{\infty} f_{\alpha_k \Delta T, \beta}(x) dx \\ = 1 - \int_{0}^{L-X_{T_i}} f_{\alpha_k \Delta T, \beta}(x) dx. \end{aligned} \tag{6}$$

It is clear that $m(X_{T_i}, Q)$ depend on the current degradation level of the system, the failure threshold L and the parameter Q (Gebraeel et al., 2005). The illustration of inter-inspection interval is shown in Fig. 5. The integration of $m(X_{T_i}, Q)$ in the maintenance decision process is illustrated in Fig. 4.

Finally, the inspections are assumed to be instantaneous, perfect and non-destructive. When an inspection is performed, a cost C_i is incurred.

According to this inspection policy, the reliability of the system between two inspection times interval remains higher or equal to $(1 - Q)$. This means that the proposed maintenance policy can provide an optimal maintenance planning with a given reliability level. From a practical point of view, this result seems to be very interesting since in many industrial systems, the reliability of the system may be an important constraint due to technical and/or economical reasons, see for example (Do Van, Vu, Barros, & Berenguer, 2012).

3.2. Optimization of the maintenance policy

To evaluate the performance of the maintenance policy, the long-run expected maintenance cost rate including the unavailability cost is used herein as the main criterion.

According to the proposed model, the cumulative maintenance cost at time t is:

$$C^t(M,K,Q)$$
$$= C_i.N_i(t) + \sum_{k=1}^{N_{ip}(t)} C_p^k + N_p(t)C_p + \sum_{j=1}^{N_c(t)} C_c + C_d.d(t), \tag{7}$$

where: $N_i(t), N_p(t), N_{ip}(t), N_c(t)$ are respectively the number of inspections, of perfect preventive maintenance, of imperfect maintenance and of corrective replacement in $[0,t]$; $d(t)$ is the the total time passed in a failed state in $[0,t]$.

By using the renewal theory (Ross, 1996), the long run expected maintenance cost per unit of time is:

$$C^\infty(M,K,Q) = \lim_{t\to\infty} \frac{C^t(M,K,Q)}{t}. \tag{8}$$

Stochastic Monte Carlo simulation is used to evaluate this cost criterion. The optimal values of the decision parameters (M,K,Q) are obtained by minimizing the expected maintenance cost rate, e.i.,

$$C^\infty(M^*,K^*,Q^*) \tag{9}$$
$$= \min_{M,K,Q}\{C^\infty(M,K,Q), 0 \le M < L, 0 \le K, 0 < Q < 1\}. \tag{10}$$

According the maintenance policy, the interest of imperfect maintenance policy is represented by K^*. When $K^* = 0$, no imperfect action is considered, the proposed maintenance policy becomes a perfect one whose performance is investigated and proved in (Grall et al., 2002; Do Van & Berenguer, 2010). Besides, the higher K^* is, the more the interests of imperfect actions are.

4. NUMERICAL EXAMPLE

The purpose of this section is to show how the proposed maintenance policy can be used in maintenance optimisation through a simple example whose characteristics are described in Section 2.

Consider a deteriorating system in which its degradation behavior, when no maintenance is carried out, is assumed to be described by a Gamma process with scale parameter $\alpha_0 = 1$ and shape parameter $\beta = 1$. If the degradation of the system exceeds the failure threshold $L = 20$, the system is failed. Both corrective and perfect maintenances can restore completely the system to the 'as good as new' state. Besides, the deterioration level of the system can be improved by imperfect maintenance actions which however may affect the deterioration speed, see again Section 2.2. Table 1 reports the data related to inspection, maintenance costs, unavailability cost rate (all costs are given in arbitrary units) and the impact of imperfect maintenance actions on the deterioration speed.

C_i	C_c	C_p	C_p^0	C_d	η	γ
10	100	90	70	20	3	0.2

Table 1. Data of costs and impact of imperfect actions

To evaluate the mean maintenance cost per unit of time, the simulations are done on a very large interval of time so that t is assumed to tend to infinity. In order to find the optimal decision parameters (M,K,Q), the average of maintenance cost per unit of time $C^\infty(M,K,Q)$ is evaluated with different values of M ($0 \le M < L$), K ($K \ge 0$) and Q ($0 < Q < 1$) by using Equation (8). A numerical optimization scheme is used, the optimum values of the decision parameters are $M^* = 14$, $K^* = 4$ and $Q^* = 0.10$ for an optimal cost rate $C^\infty(M^*,K^*,Q^*) = 5.15$. To compare with a perfect maintenance policy (only the maintenance cost criterion is herein used), K is set to be 0 (as mentionned above, when $K = 0$, the proposed imperfect maintenance policy becomes the perfect RUL based maintenance policy which seems to be an efficient policy in the framework of perfect condition-based maintenance, (Grall et al., 2002; Do Van & Berenguer, 2010)), the minimum average maintenance cost rate is then 6.23 which is much more higher than the result obtained by the imperfect policy. Moreover, the sensitivity with respect to the number of imperfect actions within a life cycle is sketched in Fig.6. The results show that when $K \le 4$, the maintenance cost rate increases quickly if K is close to 0 and when $K \ge 4$ the maintenance cost rate increases slowly with respect to the increasing of K. According to these results, it is clear that imperfect maintenance actions seem to be more appropriate than perfect ones.

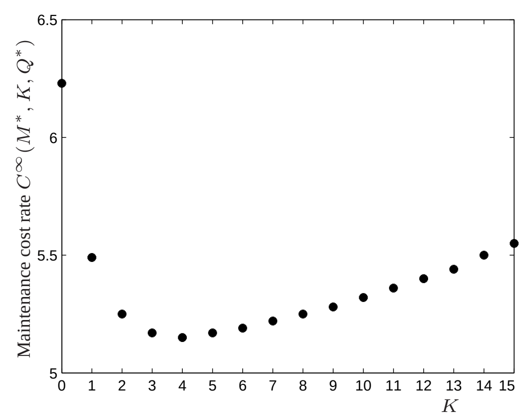

Figure 6. Mean maintenance cost rate as a function of K.

4.1. Sensitivity analysis to the imperfect maintenance cost

The performance of imperfect maintenance actions may depend on their related cost which are herein by characterized by η, see Equation (3). Table 2 reports the optimum values of M, K and Q and the minimum value of $C^\infty(M,K,Q)$ for different values of η.

	Optimal decision parameters			
η	M^*	K^*	Q^*	$C^\infty(M^*,K^*,Q^*)$
0	16	0	0.15	6.23
0.4	16	0	0.15	6.23
1	16	0	0.15	6.23
2	14	2	0.13	5.72
3	14	4	0.10	5.15
5	12	6	0.08	4.44

Table 2. Optimal maintenance policy with a given η

The results show that when $\eta \leq 1$, the maintenance cost rate remains unchanged, the optimal maintenance policies corresponds to a perfect maintenance policy ($K = 0$). However, the maintenance cost rate decreases dramatically when $\eta > 1$. This means that, in this case study, the imperfect maintenance cost has a significant influence on the performance of the imperfect maintenance policy.

4.2. Sensitivity analysis to the impact of imperfect maintenance on the deterioration speed

To analyze the impact of imperfect maintenance actions, different values of γ are considered. For each value of γ, a maintenance policy characterized by the decision parameters(M, K, Q) is optimally found using Equation (9) and the obtained results are reported in Table 3.

	Optimal decision parameters			
γ	M^*	K^*	Q^*	$C^\infty(M^*,K^*,Q^*)$
0.05	12	13	0.06	4.31
0.1	13	5	0.07	4.78
0.2	14	4	0.10	5.15
0.5	15	2	0.11	5.69
1	15	1	0.12	5.99
2	16	0	0.15	6.23
3	16	0	0.15	6.23

Table 3. Optimal maintenance policy with a given γ

The results show that when the impact of imperfect actions on the deterioration speed of the system is small, the maintenance cost is relatively low. Oppositely, when the impact of imperfect actions on the deterioration speed is large, $\gamma \geq 2$, the maintenance cost is high and the proposed imperfect maintenance becomes a perfect policy ($K = 0$). This can be explained by the fact that imperfect maintenance actions are cheaper than perfect ones however they are indirectly penalized by their negative influence on the deterioration speed of the system. As a consequence, when their effect on the system's deterioration process is small, they become more appropriate and when this effect is large, perfect maintenance actions seems to be a better choice.

5. Conclusions

In this work, a condition-based maintenance (CBM) model considering both perfect and imperfect maintenances for a deteriorating system is proposed. Imperfect maintenance actions characterized by random intervention gains are studied and discussed with different types of their related cost which may be a concave, linear or convex function with respect to intervention gain. The impact of imperfect actions on the deterioration speed of the system is also investigated. Moreover, an adaptive maintenance policy with the residual useful life (RUL) based inspections is proposed. The proposed imperfect maintenance policy may optimally become a perfect policy (Grall et al., 2002; Do Van & Berenguer, 2010) for several cases, e.g., when the imperfect cost is high or/and imperfect actions largely affect the deterioration speed of the system. Finally, the performance of the proposed policy is illustrated and discussed through an example of deteriorating system. Different sensitivity analysis are investigated to show the interest of the proposed CBM model.

Our future research work will focus on the detailed development of the impacts of imperfect maintenance actions in the framework of CBM. Furthermore, the application of the proposed CBM model for realistic industrial systems will be investigated.

NOMENCLATURE

C_i	inspection cost
C_p	perfect (replacement) preventive maintenance cost
C_p^k	cost of the kth imperfect maintenance action
C_c	corrective (replacement) maintenance cost
C_d	unavailability cost rate of the system
$d(t)$	total time passed in failed state in $[0, t]$
$C^\infty(.)$	long-run expected maintenance cost rate
L	failure threshold
K	imperfect maintenance threshold
M	preventive maintenance threshold
$m(.)$	inter-inspection length
$N_i(t)$	number of inspection in $[0, t]$
$N_p(t)$	number of perfect preventive maintenance in $[0, t]$
$N_{ip}(t)$	number of imperfect preventive maintenance in $[0, t]$
$N_c(t)$	number of corrective maintenance in $[0, t]$
Q	failure probability between two inspection times
T_i	i th inspection time
X_t	system deterioration level at time t
Z^k	kth intervention gain
$u(.)$	degradation improvement factor
v_k	mean deterioration speed after the kth maintenance action
α_0, β	scale and shape parameters of the deterioration process when the system is as good as new
α_k	scale parameter of deterioration process after the kth imperfect maintenance action
η	a non-negative real number
γ	non-negative real number and represents the impact of imperfect maintenance actions on the deterioration speed of the system

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Biographies

Phuc DO VAN is currently assistant professor at Lorraine University, Research Centre for Automatic Control (CRAN CNRS UMR 7039), France. He received his Ph.D. in Systems Optimisation and Dependability in 2008 from Troyes University of Technology (France) where he held an assistant professor position from 2009 to 2011. His research interests include stochastic modeling of systems deterioration, optimization of maintenance policies (condition-based maintenance, prognostics for maintenance decision-making, opportunistic and dynamic grouping maintenance), reliability importance measures and their related applications.

Alexandre VOISIN was born in Metz, France, in 1969, obtained an engineering degree in Electrical Engineering in 1992. In 1999, he received his Ph.D degree in Electrical Engineering from the Lorraine University. He is currently assistant professor at the Lorraine University. His primary research were in the field of fuzzy logic and information processing where he applied these techniques to subjective evaluation in the area of car seat comfort.

Since 2003 he is involved in a maintenance project, managed by Pr. B. Iung. His research deals with dependability, maintenance decision in a proactive maintenance strategy, prognostics and monitoring, e-maintenance. He is member of French and International projects/groups on e-maintenance such as the CNRS MACOD working group (Modeling and Optimization of Distributed vs. Collaborative Maintenance), the French scientific interest group 3SGS on ”Dependability of Complex Systems” in the project DEPRADEM (Degradation and Prognosis Modeling for Maintenance Decision Making), the French project BMCI (Condition monitoring for maintenance and Piloting of naval systems), the European Integrated Project DYNAMITE (Dynamic Decision in Maintenance), and the international project DEPEN-IMPRO (Modeling Policies for the improvement of Production Systems' Dependability). He is involved in industrial projects with EDF, DCN, ALSTOM. His main research interests deal with prognostic, maintenance, multi-criteria decision making, data analysis, subjective evaluation.

Eric LEVRAT received his Ph.D. in 1989 from the Universit H. Poincaré Nancy 1, where he currently holds the position of an assistant professor. He has been researcher at the Research Centre for Automatic Control of Nancy since 1990. Since 2003 he is involved in maintenance area, his research deals with dependability, maintenance decision in a proactive maintenance strategy, maintenance organisation, e-maintenance. He is member of French and International projects/groups on e-maintenance such as the CNRS MACOD working group (Modelling and Optimisation of Distributed vs. Collaborative Maintenance), the French scientific interest group 3SGS on ”Dependability of Complex Systems”, where he's leader of the project DEPRADEM2 (Degradation and Prognosis Modelling for Maintenance Decision Making), the Integrated Project DYNAMITE (Dynamic Decision in Maintenance), and the international project DEPEN-IMPRO (Modelling Policies for the improvement of Production Systems' Dependability). He is involved in several industrial projects with EDF, DCN, ALSTOM. His current research interests include prognosis (data driven and reliability driven prognosis), maintenance decision (opportunistic maintenance based on odds algorithm), dependability assessment, Integrated logistic support.

Benoit IUNG was born in 1962 in Nancy, France. He is currently full Professor at Lorraine University (France). He received his Ph.D. in Manufacturing Engineering (1992) and an accreditation to be research supervisor (2002), respectively, from this same University. Since 1988, he conducted researches at the Nancy Research Centre for Automatic Control (CRAN, UMR7039) where he is managing today a research group on Sustainable Industrial System Engineering. His research and teaching interests are related to dependability, prognostics, maintenance engineering and e-maintenance (PHM area). In relation to these topics he took scientific responsibility for the participation of CRAN in a lot of national, European (REMAFEX, DYNAMITE) and international projects (EIAM-IPE, iMaPla). He is now the chairman of the IFAC WG A-MEST on advanced maintenance, the chairman of the ESRA TC on Manufacturing, a member of the IFAC TC 5.1., a French Associate Member to CIRP, the co-chairman of the WG ”Prognostics” of DIAG21 (French PHM community) and a founding Fellow to the International Society for Engineering Asset Management. Benoit Iung has authored several books including the first e-maintenance book in Springer and has published more than 90 scientific papers, 35 of them in international journals (i.e. ARC, RESS, IJPR, JIM, PP&C, JRR, JQME ...). He developed about 15 keynote speeches in international conferences. He has supervised until now about 15 MA, 12 Ph.D. Students and 2 Post-Doctorate students.