

Optimization of fatigue maintenance strategies based on prognosis results

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ABSTRACT

A general approach to determine the optimal set of maintenance alternatives for fatigue safety is introduced in this paper. The optimal maintenance alternatives are the solutions to maximize the fatigue reliability of aircrafts fleet subject to maintenance budget. A novel equivalent stress transformation model and the first-order-reliability method (FORM) are adopted to determine the failure probability or reliability associated with future fatigue loading. The equivalent stress transformation model is capable to transform future random loading to an equivalent constant loading, and does not require cycle-by-cycle simulation. First-order-reliability-method can resolve the computational complexity. Optimal maintenance solution can be efficiently found considering the future fatigue loading. Numerical examples are performed to demonstrate the application of the proposed approach.

1 INTRODUCTION

Most structures and components, e.g. aircrafts and rotorcrafts, are experiencing cyclic loading throughout their service life. These cyclic loading results in many failure modes, and fatigue failure is one of most common failure modes. There is an increasing interest to enhance the durability, reliability and safety of the structures with limit budget. Scheduling of inspection and repair activities can effectively mitigate the fatigue detrimental effects (Y. Garbatov & C. Guedes Soares, 2001) (D. Straub & M. H. Faber, 2005).

To obtain a reasonable future maintenance plan, first of all, very good diagnostic techniques are required. There exist several non-destructive inspection (NDI) techniques, e.g. shearography (Y. Y. Hung, 1996), thermography (M. Koruk & M. Kilic, 2009), ultrasonics (R. Kazys & L. Svilainis, 1997), X-ray CT(G. Nicoletto, G. Anzelotti & R. Konecn) and so on. Furthermore, structures experience different loading spectrums during entire fatigue life. The applied fatigue

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cyclic loading (S. Pommier, 2003) is stochastic in nature. It is well-known that different loading sequences may induce different load-interaction effects (S. Mikheevskiy & G. Glinka), such as the overload retardation effect and underload acceleration effect. Due to the complicated and nonlinear nature of random loading interaction, a cycle-by-cycle simulation is generally required for each different loading history. Hence this approach is computationally expensive for fatigue safety optimization, which usually requires a large number of Monte Carlo simulations.

Prediction will provide valuable information for decision making in prognostics and health management (PHM). The most difficult part is how to accurately and effectively estimate the future health status of aircraft fleet. This estimation should be built on an efficient fatigue damage prognosis procedure. A novel equivalent stress transformation (Y. Xiang & Y. Liu, 2010) and reliability method have been adopted to reduce the complexity of fatigue damage prognosis. This equivalent stress transformation is using the statistical description of the random loading, such as the probabilistic distribution of applied stress range and stress ratio. The future variable amplitude loading problem is reduced to an equivalent constant amplitude problem, which greatly facilitates the integration for crack length prediction. The FORM have been developed and used for the reliability-based design optimization problem (A. Der Kiureghian, Y. Zhang & C.-C. Li, 1994).

This paper is organized as follows. First, basic problem for optimal maintenance alternatives will be formulated, and some key parts will be pointed out. Following this, the equivalent stress transformation is briefly discussed. After that, the first-order-reliability method will be introduced. Numerical example is used to demonstrate the application of the proposed method. Parametric study has been performed to investigate the effects of some important parameters. Finally, some conclusions and future work are given based on the current investigation.

2 Problem formulation

It is well-known that structures experience fatigue cyclic loading during their service life. Crack may

propagate until parts of some components fail. The structures may break suddenly in a few cycles, or survive for a long period of time. Hence, difference exists in fatigue duration due to the uncertainties. An appropriate fatigue maintenance plan is required to optimize the condition status.

First of all, very good diagnostic techniques are required to detect the current damage stage. Several advanced diagnostic techniques are available. The current diagnostic results are regarded as the baselines for future fatigue damage prognosis. This paper mainly focuses on the prognosis techniques, and diagnostic techniques are beyond the scope of this paper.

The future loading is a critical problem in fatigue maintenance alternatives optimization. The fatigue loading is usually stochastic in nature, and the loading sequence effects are big challenges in fatigue prognosis. Traditional fatigue prognosis models focus on different explanation of crack growth mechanism, and require cycle-by-cycle simulation. These models require a large number of Monte-Carlo simulation, and is computational expensive for fatigue maintenance optimization. An equivalent stress transformation (Y. Xiang & Y. Liu, 2010) has been proposed based on the statistical description of the random loading. The variable amplitude loading problem is reduced to an equivalent constant amplitude problem. Detailed derivation and explanation will be discussed in Section 3.

Apparently, fatigue prognosis will provide valuable information for decision making in PHM. Maintenance optimization under uncertainty can be formulated as a reliability problem. Therefore, some of the developed algorithms can be applied (e.g., FORM, subset simulation, etc.) In the current study, First-order-reliability-method will be applied to find the fatigue reliability of structures. Comprehensive derivation will be discussed in Section 4.

Fatigue maintenance problem can be formulated in different ways, e.g. minimizing the total cost subjected to reliability constraints and performance constraints. This kind of problem is quite common in real engineering application, since the best condition stage of structures are desired with least cost. There is another way to formulate the problem, such like maximizing the performance reliability subject to budget constraints (e.g., annual budget for maintenance is fixed). Basically the budget is limited and the desirable condition stage of structures is required. This paper is mainly focusing on fatigue performance maximization.

The first step in the fatigue performance optimization is to define several categories depending on the crack length. For example, the fatigue performance can be divided into six stages, excellent condition, very good

condition, good condition, fair condition, poor condition and very poor condition (or failure condition).

Secondly, diagnostic methods are used to determine the fatigue damage in the current stage. The performance transition matrix can be formulated using some existing fatigue prognosis models (Equivalent stress level mode) and diagnostic results.

Thirdly, a maintenance decision matrix should be defined to specify the maintenance method for each performance category. Then the cost function can be calculated associated with each category using different maintenance alternatives.

At last, the maximization of the performance under the budget constraints can be formulated. This maintenance optimization under uncertainty can be formulated as a reliability problem. Some of the developed algorithms can be applied (e.g., FORM, subset simulation, etc.)

In the maintenance optimization problem, some variables need to be clarified:

G = number of facility groups Group of aircrafts

T = number of missions in the planning horizon

Q_g = total quantity of facilities in group g

S = number of performance condition states;

M_g = number of possible maintenance alternatives for facilities in group g

C_{gm} = cost vector ($sx1$) of group g and maintenance alternative m

$D_{gt} = [d_{gt}^1 \ d_{gt}^2 \ d_{gt}^3 \ d_{gt}^4 \ \dots \ d_{gt}^S]$, condition vector of group g at beginning of mission t , each term represents the percentage. d_{gt}^s is the element on the diagonal of an $S \times S$ matrix.

D_{total} = the summation of the condition elements for G groups from the condition 1 to condition S , after T missions

$X_{gmt} = [x_{gmt}^1 \ x_{gmt}^2 \ x_{gmt}^3 \ x_{gmt}^4 \ \dots \ x_{gmt}^S]$, x_{gmt}^s is the maintenance decision matrix, percentage of facilities in group g and condition state s that had maintenance m in year t .

P_{gm} = transition probability matrix ($S \times S$) of group g when the maintenance m is implemented (from model prediction or existing database)

$$P_{gm} = \begin{vmatrix} P_{gm}^{1,1} & P_{gm}^{1,2} & \dots & P_{gm}^{1,S} \\ P_{gm}^{2,1} & P_{gm}^{2,2} & \dots & P_{gm}^{2,S} \\ & \dots & & \dots \\ P_{gm}^{S,1} & P_{gm}^{S,2} & \dots & P_{gm}^{S,S} \end{vmatrix}$$

The condition of facilities from group g at year t can be predicted using previous information.

$$D_{gt} = \sum_{m=1}^{m=Mg} X_{gm(t-1)} D_{g(t-1)} P_{gm} \quad (1)$$

The total cost function can be formulated as:

$$Cost = Q_g \sum_{m=1}^{Mg} X_{gmt} D_{gt} C_{gm} \quad (2)$$

From above derivation, the maximization problem can be easily built as:

$$D_{total} = \sum_{s=1}^S S \sum_{g=1}^G \sum_{t=1}^T d_{gt} \quad (3)$$

For a certain mission, the budget $Budget_i$ is limited after each year i , and may be different from one year to another. The total budget $Budget_{total}$ during year t is also limited. And the budget constraints can be built as Eq. (4) and Eq. (5):

$$\sum_{g=1}^G Q_g \sum_{m=1}^{Mg} X_{gmt} D_{gt} C_{gm} \leq Budget_i \quad (4)$$

$$\sum_{t=1}^T \sum_{g=1}^G \sum_{m=1}^{Mg} X_{gmt} D_{gt} C_{gm} \leq Budget_{total} \quad (5)$$

For some cases, the reliability constraints are required. For example, the percentage of facilities in condition s should be less than a value R^s . The reliability constraints can be formulated as Eq. (6) :

$$d_{gt}^s \leq R^s \quad (6)$$

Following the above procedures, the fatigue maintenance problem can be easily formulated. However, there are some problems existing: first, the transition probability matrix reliability of each future mission is complex problem, due to measurement uncertainties (NDI testing) modeling uncertainties. The future loading dominates the transition probability matrix. The Equivalent stress transformation is proposed for the future loading. First order reliability method (FORM) can be used to calculate the probability transformation matrix

3 Equivalent stress level

Fatigue cyclic loading is a random process in nature. Proper inclusion of loading interaction effects is a big challenge, and is very important for future mission reliability. Traditional models focus on different explanation of fatigue crack growth mechanism, and require cycle-by-cycle simulation. Therefore, a large number of simulations is required and is time-consuming.

Equivalent stress transformation model has been proposed transformation (Y. Xiang & Y. Liu, 2010). This objective is to transform a random loading to an equivalent constant loading, which does not require a cycle-by-cycle simulation and can facilitate the integration. The basic idea of the equivalent stress level can be shown as below:

For an arbitrary random future loading, the statistics of stress range and stress ratio can be obtained. After a

series of calculation, the random loading can be transformed to an equivalent constant loading, which can be directly used for fatigue damage prognosis. Because this transformation is not the focus of this study, only the brief idea is illustrated. The details of the derivation and validation of the equivalent stress transformation can be found in the referred paper (Y. Xiang & Y. Liu, 2010).

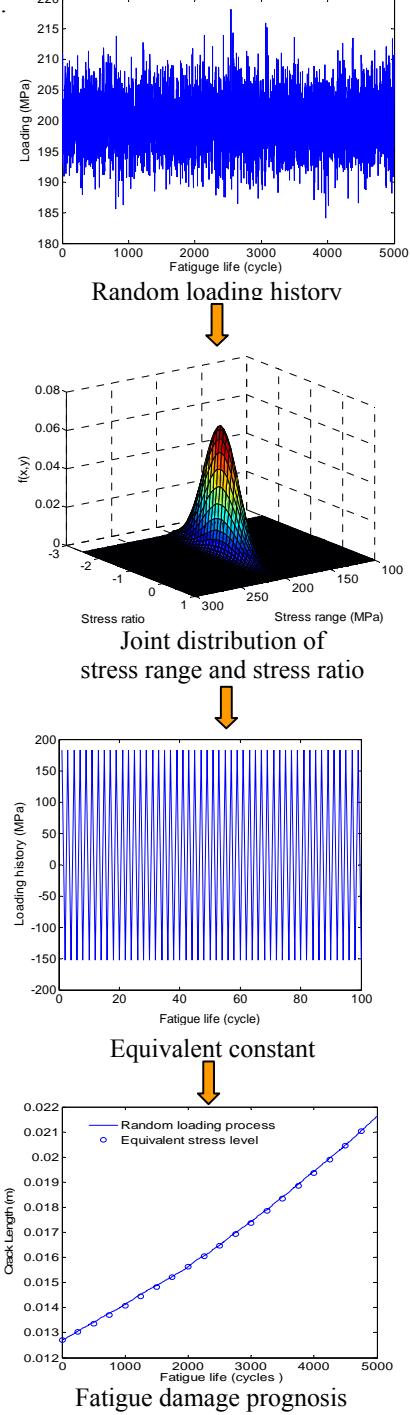


Figure. 1 basic principle of equivalent stress level

There are several different fatigue crack growth models, such as Forman's model (N. E. Dowling, 2007), Nasgro model, and EIFS-based fatigue crack growth model (Y. Liu & S. Mahadevan, 2009b). Different models focus on different aspects and will give different predictions. A generic function of crack growth rate curve can be expressed as

$$da/dN = f(\Delta\sigma, R, a) \quad (7)$$

Eq. (7) can be reformulated as

$$dN = \frac{1}{f(\Delta\sigma, R, a)} da \quad (8)$$

The total fatigue life N under arbitrary random loading history is the summation of N_i and can be written as

$$N_{total} = \sum_{i=0}^n N_i = \sum_{i=0}^n \int_{a_0}^{a_{i+1}} \frac{1}{f(\Delta\sigma_i, R_i, a)} da \quad (9)$$

where a_0 is the initial crack size and a_n is crack length at fatigue cycle N .

In this ideal crack growth process, the stress level is constant and is the proposed equivalent stress level (ESL). The equivalent stress level can be expressed as

$$N_{total} = \int_{a_0}^{a_{n+1}} \frac{1}{f(\Delta\sigma_{eq}, R_{eq}, a)} da \quad (10)$$

The equivalent stress level can be obtained by equaling Eq. (9) and Eq. (10) as

$$\int_{a_0}^{a_{n+1}} \frac{1}{f(\Delta\sigma_{eq}, R_{eq}, a)} da = \sum_{i=0}^n \int_{a_i}^{a_{i+1}} \frac{1}{f(\Delta\sigma_i, R_i, a)} da \quad (11)$$

Eq. (12) is the proposed equivalent stress level calculation and it can be applied to different types of crack growth models. For any arbitrary functions of $f()$. The analytical solution is not apparent and discussions of some special cases are given below.

In the current study, the simple Paris' equation is used as the $f()$. For a general case where both of stress range and stress ratio are random variables, a joint distribution of them is required for the derivation. The general equivalent stress can be expressed as

$$\begin{aligned} \Delta\sigma_{eq} &= \left(\sum_i^n \frac{N_i}{N_{total}} \frac{g(R_i)}{g(0)} \Delta\sigma_i^m \right)^{\frac{1}{m}} \\ &= \left(\sum_i^n p_i(R_i, \Delta\sigma_i) \frac{g(R_i)}{g(0)} \Delta\sigma_i^m \right)^{\frac{1}{m}} \end{aligned} \quad (12)$$

where $p_i(R_i, \Delta\sigma_i)$ is the joint distribution of stress range and stress ratio. $g()$ is a function of stress ratio. Eq. (12) is the generalized equivalent stress level expression without considering the loading interaction effect.

The above discussion did not consider the load interaction. It is well known that the "memory" effect exists for fatigue crack growth and coupling effect has to be considered. In this section, the previous developed equivalent stress model is extended to include the load interaction effect, such as the overload

retardation and underload acceleration. The modification is based on a recently developed small time scale formulation of fatigue crack growth and a load interaction correction function. The details of the small time scale model has been developed by Lu and Liu (Z. Lu & Y. Liu). This method is based on the incremental crack growth at any time instant within a cycle, and is different from the classical reversal-based fatigue analysis.

The equivalent stress level consider load interaction effect is defined as

$$\Delta\sigma_{eq}^* = \eta \Delta\sigma_{eq} \quad (13)$$

where $\Delta\sigma_{eq}^*$ is the equivalent stress level considering the load interaction effect and $\Delta\sigma_{eq}$ is calculated using Eq. (12) without considering the load interaction term. η is the coefficient for the load interaction effect and the details of derivation can be found in . (Y. Xiang & Y. Liu, 2010).

4 FORM methodology

The first-order reliability method is a widely used numerical technique to calculate the reliability or failure probability of various engineering problems (J. Cheng & Q. S. Li, 2009; S. Thorndahl & P. Willems, 2008; D. V. Val, M. G. Stewart & R. E. Melchers, 1998). Unlike the FORM method (A. Haldar & S. Mahadevan, 2000; Y. Liu, Mahadevan, S, 2009), the inverse FORM method tries to solve the unknown parameters under a specified reliability or failure probability level, which is more suitable for probabilistic life prediction (i.e., remaining life estimation corresponding to a target reliability level).

Limit state function is required for the analytical reliability method. A generic limit state function is expressed as Eq. (14a) as a function of two sets of variables x and y . x is the random variable vector and represents material properties, loadings, and environmental factors, etc. y is the index variable vector, e.g., time and spatial coordinates. The limit state function is defined in the standard normal space in Eq. (14a). The limit state function definition is similar to the classical FORM method (A. Haldar & S. Mahadevan, 2000). The solution for the unknown parameters needs to satisfy the reliability constraints, which are described in Eq. 14b) and Eq. (14c). β is the reliability index, which is defined as the distance from origin to the most probable point (MPP) in the standard normal space. The failure probability P_f can be calculated using the cumulative distribution function (CDF) Φ of the standard Gaussian distribution. Numerical search is required to find the optimum solution, which satisfies the limit state function (Eq. (14d)). Details of the general FORM method and

concept can be found in (A. Der Kiureghian et al., 1994).

$$\left\{ \begin{array}{l} (a) : g(x, y) = 0 \\ (b) : \|x\| = \beta_{target} \\ (c) : p_f = \Phi(-\beta_{target}) \\ (d) : \begin{cases} (1) x + \frac{\|x\|}{\|\nabla_x g(x, y)\|} \nabla_x g(x, y) = 0 & (P_f < 50\%) \\ (2) x - \frac{\|x\|}{\|\nabla_x g(x, y)\|} \nabla_x g(x, y) = 0 & (P_f \geq 50\%) \end{cases} \end{array} \right. \quad (14)$$

The overall objective of the FORM method is to find a non-negative function satisfying all constraint conditions specified in Eq. (14). Thus, the numerical search algorithm can be used to find the solutions of the unknown parameters. Numerical search algorithm is developed to iteratively solve the Eq. (14). The search algorithm is expressed as Eq. (15) after k iterations.

$$\begin{cases} X_{k+1} \\ y_{k+1} \end{cases} = \begin{cases} X_k + f_k \\ y_k \end{cases} = \begin{cases} X_k + (a_1 f^{1k} + a_2 f^{2k}) \\ y_k \end{cases} \quad (15)$$

where f^{1k} and f^{2k} are the search directions corresponding to different merit functions.

The convergence criterion for the numerical search algorithm is

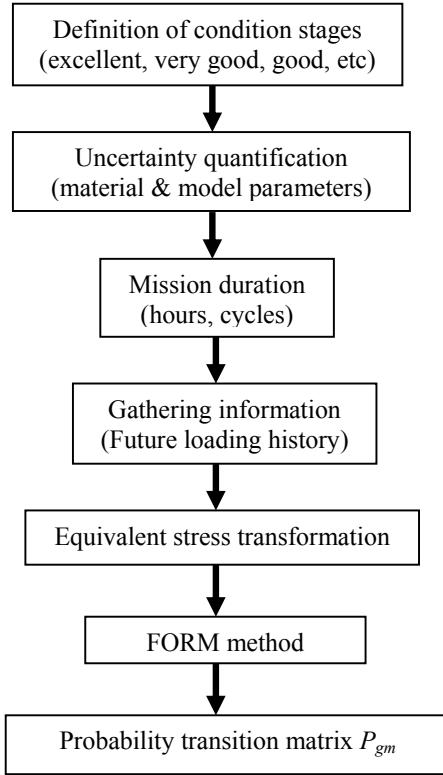
$$\frac{(\|x_{k+1} - x_k\|^2 + \|y_{k+1} - y_k\|^2)^{\frac{1}{2}}}{(\|x_{k+1}\|^2 + \|y_{k+1}\|^2)^{\frac{1}{2}}} \leq \varepsilon \quad (16)$$

where ε is a small value and indicates that the relative difference between two numerical solutions is small enough to ensure the convergence.

5 Transition probability matrix

Transition probability matrix is used to determine the future condition stage, based on the current observed fatigue damage. Calculation of the transition probability matrix P_{gm} is the key point in the fatigue safety optimization.

The general procedures to calculate P_{gm} is shown in flowchar.1. The first step is to define the condition stages, such as excellent, very good, good, etc. Following this, quantify the uncertainties in the current fatigue problem. Then obtain the information about future loading history, for example, the joint distribution of stress range and stress ratio. After equivalent stress transformation, the obtained equivalent constant loading can be directly used. The mission duration (cycles or hours) can be obtained. This information is the input data to the FORM method. The probability transition matrix can be directly calculated using FORM method.



Flowchart. 1 General procedure to calculate P_{gm}

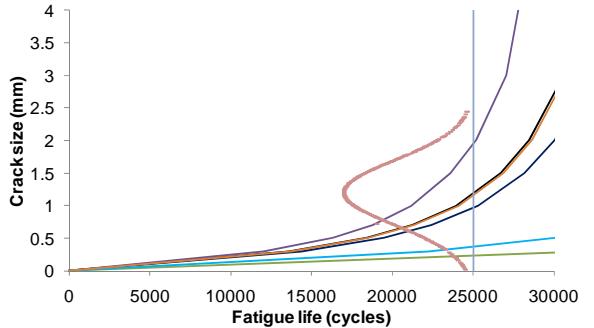


Figure. 2 Fatigue crack growth prognosis

Table 1. Statistics of random variables

Material	stress ratio	parameter	Mean	Std.
Al 7075-T6	R = -1	M_c	1.64E-10	3.86E-11
		M_m	2.3398	0.3122
		a_i	0.05mm	0.006mm

Huge uncertainties exist in the fatigue damage prognosis model, e.g., the model properties C , m and the initial crack size a_i . To determine the future condition status, these three parameters are random variables and are assumed to follow log-normal

distribution. With an initial fatigue damage (a_i) around 0.05mm, after some mission (e.g. 25000 cycles), the probability can be calculated from excellent stage to the other stages. The statistics of these three random variables are shown in Table 1. Suppose there exists a constant loading history with $S_{\max}=150$ MPa, $S_{\min}=15$ MPa. With different combinations of the three random variables, unlimited fatigue crack growth curves can be drawn, as shown in Fig. 2.

It can be easily observed that, after a certain mission (25000 cycles) the cracks reach different conditions: very few remain the same level, and most of them increase between 0.5~2 mm. The transition probability matrix from initial condition stage (around 0.05 mm) to other condition stages can be easily obtained. The above discussion is a simple case. In this case, there are only three random variables and a constant amplitude loading history. A numerical example has been discussed for more general cases in Section 6.

6 Numerical example and parametric study

A numerical example is demonstrated in this section. In this example, there are 10, 9 and 9 aircrafts in three different groups A , B and C respectively. The total number of future mission is 10.

Firstly, the condition stages are defined into 6 stages: excellent (crack<0.05), very good (0.5<crack<0.6), good (0.6<crack<0.8), fair(0.8<crack<1.2), poor (1.2<crack<1.5) and very poor (crack>1.5). Three maintenance alternatives are available: do nothing, repair method I, repair method II. The cost of maintenance alternatives are shown as:

conditionstate	1	2	3	4	5	6
do_nothing	0	0	0	0	0	0
repair_method_I	0	400	600	800	1600	1800
repair_method_II	0	0	1600	1600	3000	3000

At the initial stage, the initial condition stage D_{gt} should be defined. In this numerical example, the initial condition stage is randomly generated and just for demonstration. In really engineering cases, the condition stage needs to be defined with help of advanced diagnostic techniques.

[0.1638 0.1896 0.1900 0.1627 0.1090 0.1849;
 0.2618 0.1326 0.2039 0.0498 0.2056 0.1462;
 0.0293 0.3242 0.3518 0.0086 0.2609 0.0251;]

Secondly, the uncertainties are quantified. The material used in the structure is Al-7075 and the random variables are calibrated from experimental fatigue crack growth data shown in Fig. 3. A summary of the properties for the collected experimental data are listed in Table 2& 3.

Thirdly, the mission duration needs to be clarified. Normally, the mission duration of a flight is about

10,000 fatigue cycles, which is used in the current study.

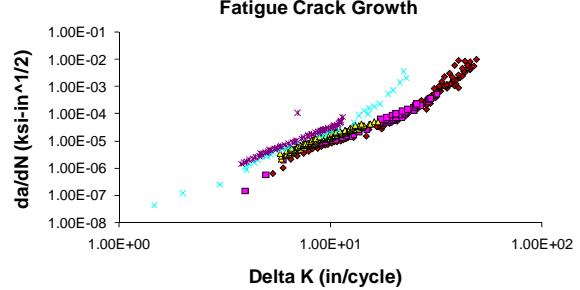


Figure. 3 Fatigue crack growth for Al-7075 under different stress ratios

Table 2 Stochastic coefficient of a and fatigue limit

Material	stress ratio	parameter	mean	std.
Al 7075-T6	0.03	M_c	7.72E-10	1.82E-10
		K_c	50	5
	0.05	M_c	7.96E-10	1.88E-10
		K_c	50	5

Table 3 Geometry and material properties of plate specimens

Specimen material	7075-T6
Ultimate strength σ_u (MPa)	575
Yield strength σ_y (MPa)	520
Modulus of elasticity E (MPa)	69600
Plate width (mm)	305
Plate thickness (mm)	4.1

The fourth step is gathering information about the future loading. Two blocks loading spectrum are used as the future loading in this numerical example. A schematic illustration of the f loading is shown in Fig. 4. p and n in Fig. 4 controls the number of cycles at the high amplitude (400MPa) and the low amplitude (250 MPa), respectively. $p=10$ and $n=50$ in the current study.

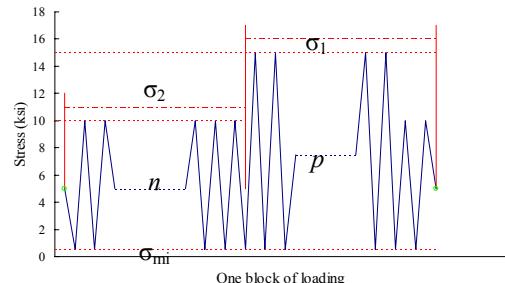


Figure. 4 Schematic illustration of the two blocks loading

It is assumed that, after some repair, the current condition stages (crack size) can be partially or fully changed to an ideal station. In another word, the fatigue crack size may follow a bi-normal distribution, for example:

$$a_i \sim A \times \log N(0.05, 0.006) + B \times \log N(0.25, 0.03) \quad (18)$$

A, B are two parameters. For different repair method I and repair method II, A and B take different value as shown in table 4.

Table 4 Model parameters in a bi-normal distribution

	Repair I	Repair II
A	0.7	0.3
B	0.9	0.1

The distribution of crack size after repair can be easily calculated using above information.

After the equivalent stress transformation, the above information can be inputted into FORM method. For example, the elements on the first row of P_{gm}^1 can be calculated by setting up the indexing vectors as condition limits in each condition stage in FORM method. The transition probability is shown below for three different maintenance alternatives:

$$P_{gm}^1 = \begin{bmatrix} 0.9956 & 0.0043 & 0.0001 & 0 & 0 & 0 \\ 0 & 0.8818 & 0.1065 & 0.0112 & 0.0004 & 0.0001 \\ 0 & 0 & 0.9246 & 0.0307 & 0.0253 & 0.0194 \\ 0 & 0 & 0 & 0.9161 & 0.0532 & 0.0307 \\ 0 & 0 & 0 & 0 & 0.9238 & 0.0762 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{gm}^2 = \begin{bmatrix} 0.8325 & 0.0875 & 0.0373 & 0.0344 & 0.0073 & 0.0010 \\ 0.8325 & 0.0875 & 0.0373 & 0.0344 & 0.0073 & 0.0010 \\ 0.8325 & 0.0875 & 0.0373 & 0.0344 & 0.0073 & 0.0010 \\ 0.8325 & 0.0875 & 0.0373 & 0.0344 & 0.0073 & 0.0010 \\ 0.8325 & 0.0875 & 0.0373 & 0.0344 & 0.0073 & 0.0010 \\ 0.8325 & 0.0875 & 0.0373 & 0.0344 & 0.0073 & 0.0010 \end{bmatrix}$$

$$P_{gm}^3 = \begin{bmatrix} 0.9406 & 0.0370 & 0.0114 & 0.0099 & 0.0010 & 0.0001 \\ 0.9406 & 0.0370 & 0.0114 & 0.0099 & 0.0010 & 0.0001 \\ 0.9406 & 0.0370 & 0.0114 & 0.0099 & 0.0010 & 0.0001 \\ 0.9406 & 0.0370 & 0.0114 & 0.0099 & 0.0010 & 0.0001 \\ 0.9406 & 0.0370 & 0.0114 & 0.0099 & 0.0010 & 0.0001 \\ 0.9406 & 0.0370 & 0.0114 & 0.0099 & 0.0010 & 0.0001 \end{bmatrix}$$

The budget constraints in each mission are shown as below:

$$\text{Budget} = [10000 \ 8000 \ 9000 \ 12000 \ 10000 \ 8000 \ 9000 \ 8000 \ 8000 \ 9000];$$

$$\text{The total budget} = \$65000.$$

The reliability constraint is built as: the percentage of aircrafts in very poor condition is no more than 5%.

The fatigue maintenance problem is to optimize the maintenance design (X_{gmt}) to maximize the condition state, and satisfy the budget limits in each year, the

total budget for whole mission process, as well as the reliability constraints.

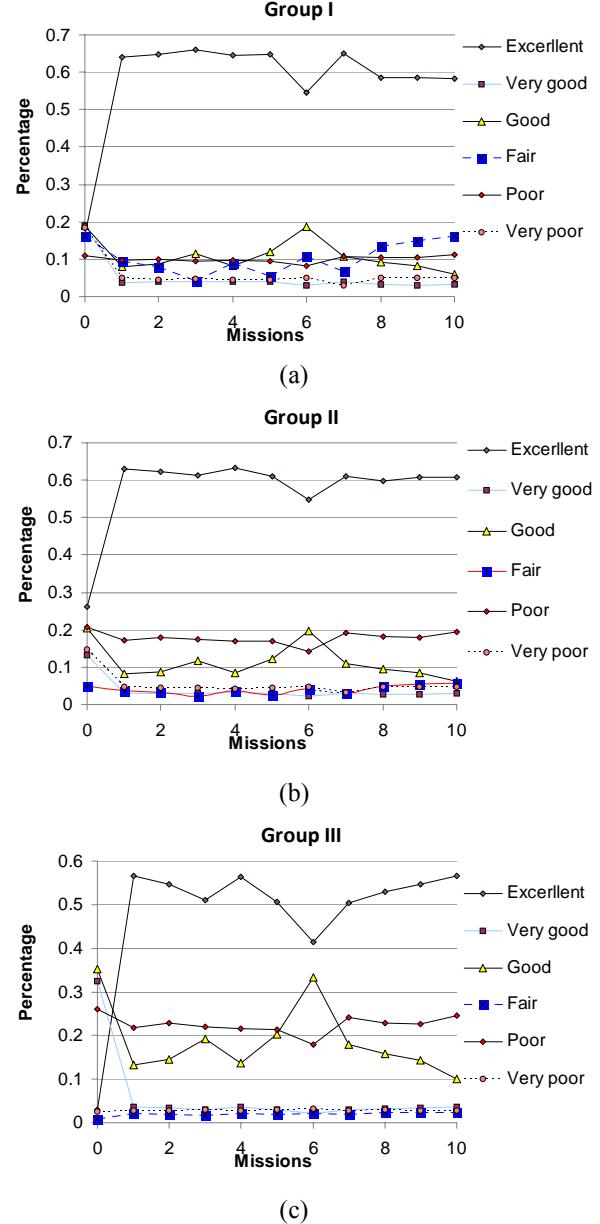


Figure 5 optimal results after each mission

The optimal results are shown in Fig. 5 for three different groups. At the very beginning, only about 16% aircrafts are in excellent condition. To maximize the total condition, more money should spend to repair as many aircrafts as possible, subjected to the first year budget. It can be easily observed that more than 60% aircrafts are in excellent condition after the first mission. And those in excellent condition remain at very high level throughout the 10 missions. Those aircrafts in very poor condition takes less than 5%. In

another word, the money spent on very poor condition does not change much. The real cost and maintenance budget limit for each mission is shown in Fig. 6. The cost at each mission is less than the budget limit and satisfies the budget constraint.

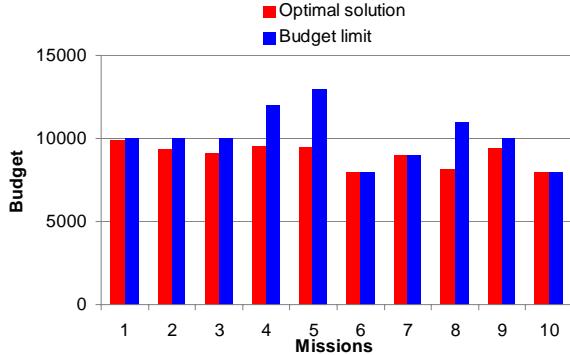


Figure. 6 Cost Vs Budget for each mission

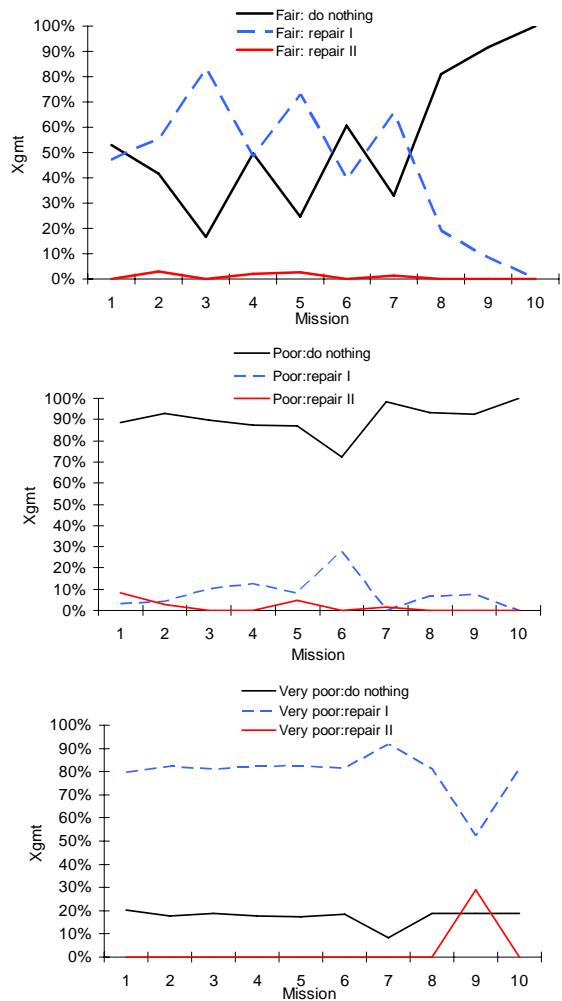
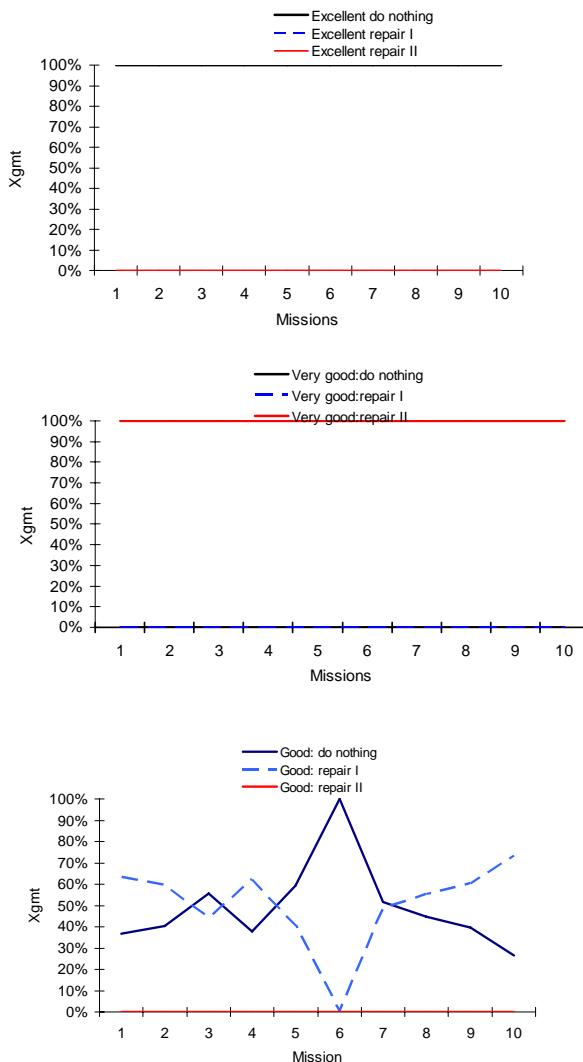


Figure. 7 Optimal solutions for maintenance alternatives

The optimal solution for maintenance alternatives are displayed in Fig. 7. For the excellent condition, no maintenance is required, which is reasonable. All the aircrafts in very good condition should take repair method II. For good and fair conditions, the aircrafts takes different combinations of maintenance alternatives. The best choice for those in poor condition is do nothing, but for those in very poor condition, repair method I is absolutely necessary.

Parametric study has been done to investigate effects caused by the variance of parameter C. In this case, the variance of parameter C takes four different values, 0.05, 0.1, 0.3, 0.5. From Fig. 8, it can be concluded that, as the variance increases, the total maximum condition value decrease. The best choice to maintain the maximum condition is to reduce the uncertainties materials properties.

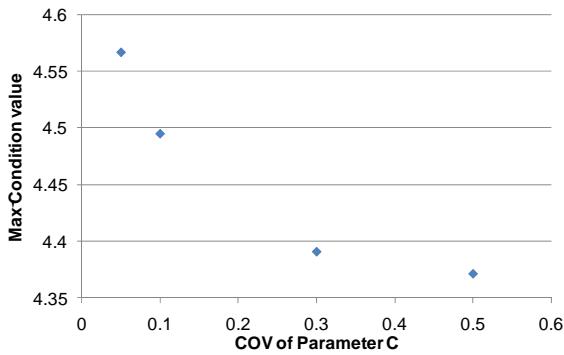


Figure. 8 Effects of the variance of parameter C

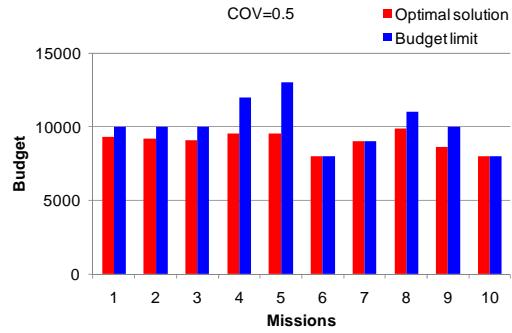
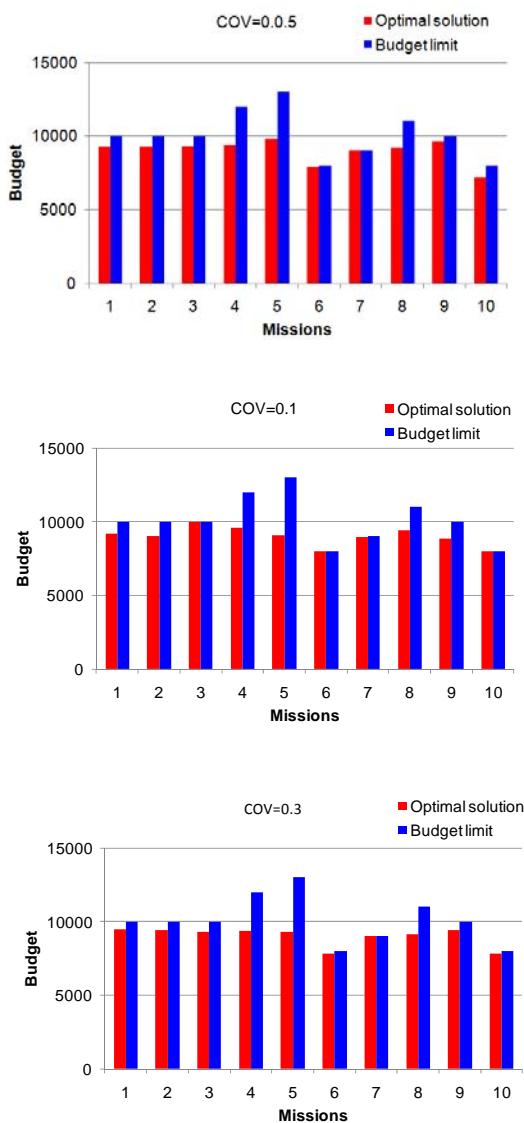
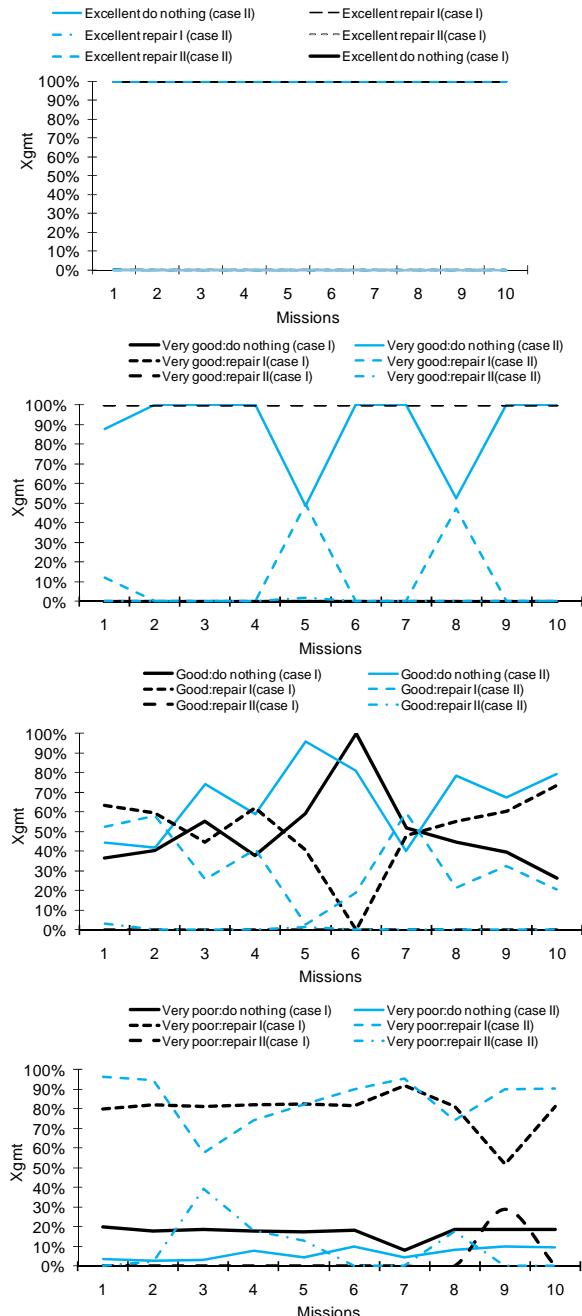


Figure. 9 Cost Vs Budget for different variance of parameter C



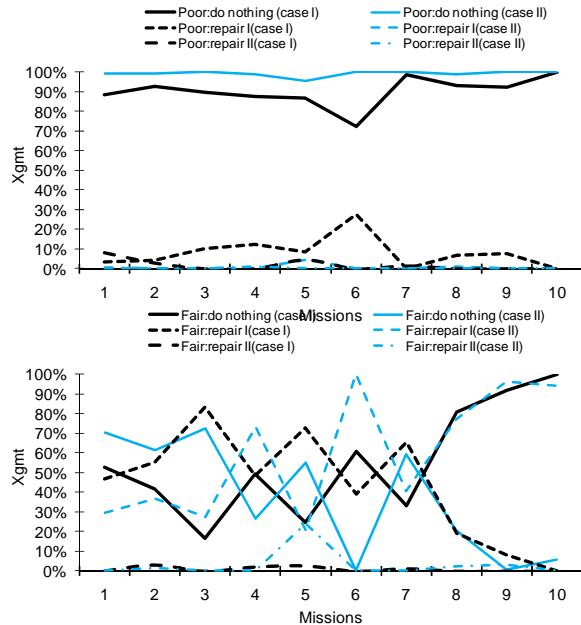


Figure. 10 Optimal solutions for maintenance alternatives

Fig. 9 shows the cost vs budget for each different variance of parameter C. No big difference can be observed. Fig. 10 displays the optimal maintenance alternatives for two cases, variance equaling to 0.05 and 0.5. Slightly difference can be observed for these two cases.

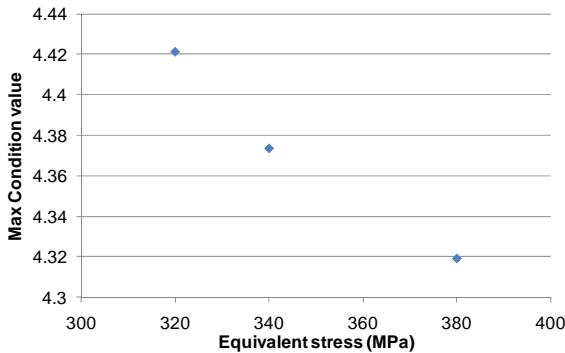


Figure. 11 Effect of equivalent stress level

The other parametric study investigates the effect of equivalent stress level. The maximum condition value decreases steadily with increase of equivalent stress level. This phenomenon is almost the same as expectation.

7 Conclusion

In this paper, a maintenance optimization framework using prognosis results is formulated. The proposed approach is based on a novel prognostic model. This

prognostic model is the combination of equivalent stress transformation and the FORM method. It is able to deal with the uncertainties in future loading. Optimization problem has been formulated based on the performance maximization under budget constraints and reliability constraint. An example with three group of facilities are considered. Parametric study has been done to investigate the effects of parameter C as well as the equivalent stress level. The results meet the expectation. Reliability constraints and other uncertainty effects are being investigated in the future study. More complicated case study is required.

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