

# Condition Based Maintenance Optimization for Multi-component Systems

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## ABSTRACT

Most existing condition based maintenance (CBM) work reported in the literature only focuses on determining the optimal CBM policy for single units. Replacement and other maintenance decisions are made independently for each component, based on the component's age, condition monitoring data and the CBM policy. In this paper, a CBM optimization method is proposed for multi-component systems, where economic dependency exists among the components subject to condition monitoring. In a multi-component system, due to the existence of economic dependency, it might be more cost-effective to replace multiple component at the same time rather than making maintenance decisions on components separately. Deterioration of a multi-component system is represented by a conditional failure probability value, which is calculated based on the predicted failure time distributions of components. The proposed CBM policy is defined by a two-level failure probability threshold. A method is developed to obtain the optimal threshold values in order to minimize the long-term maintenance cost. An example is used to demonstrate the proposed multi-component CBM method.

## 1. INTRODUCTION

Condition based maintenance (CBM) generally aims to determine an optimal maintenance policy to minimize the overall maintenance cost based on condition monitoring information. The health condition of a piece of equipment is monitored and predicted via collecting and analyzing the inspection data, such as vibration data, acoustic emission data, oil analysis data and temperature data. Various CBM policies and optimization methods have been proposed (Banjevic et al, 2001, Jardine et al, 2006). However, most existing condition based maintenance (CBM) work reported in the literature only focuses on determining the optimal CBM policy for single units. Replacement and other

maintenance decisions are made independently for each component, based on the component's age, condition monitoring data and the CBM policy.

For multi-component systems which involve multiple components, economic dependency exists among the components subject to condition monitoring. For example, in the replacement of bearings on a set of pumps at a remote location, the fixed maintenance cost, such as sending a maintenance team to the site, is incurred whenever a preventive replacement is performed. Thus, for multi-component systems, it might be more cost-effective to replace multiple component at the same time rather than making maintenance decisions on components separately. Tian and Liao (2011b) developed a proportional hazards model based approach for CBM of multi-component systems. In this paper, we proposed an approach which can utilize prediction information from more general prediction tools. More specifically, the proposed CBM can be used as long as the prediction tool can produce predicted failure time values and their associated uncertainty information. The cost evaluation method is presented. An example is used to illustrate the proposed approach.

## 2. COMPONENT HEALTH CONDITION PREDICTION

The output of component health condition prediction is the predicted failure time values and the associated uncertainty information. That is, at a certain inspection point, health condition prediction tools can generate the predicted failure time distribution. In this section, we present a general framework for generating the predicted failure time distribution.

Suppose at a certain inspection point where the age of the component is  $t$ , the predicted failure time is  $T_{n,t}$ , and the actual failure time of the component is  $T_m$ . The prediction error is defined in this paper as the  $e_{n,t} = (T_{n,t} - T_m)/T_m$ . We also define the life percentage as  $p_t = t/T_m$ . The

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prediction error is a measure of the prediction accuracy. To obtain the predicted failure time distribution, Tian et al (2010, 2011a) developed a method to calculate the standard deviation of the predicted failure time, while using the artificial neural network (ANN) prediction model. The basic idea is that the ANN life percentage prediction errors can be obtained during the ANN training and testing processes, based on which the mean,  $\mu_p$ , and standard deviation,  $\sigma_p$ , of the ANN life percentage prediction error can be estimated. These values can be used to build the predicted failure time distribution at a certain inspection point. Suppose the component age is  $t$  and the ANN life percentage output is  $P_t$ , then the predicted failure time will be  $t/(P_t - \mu_p)$ , and the standard deviation of the predicted failure time will be  $\sigma_p \cdot t/(P_t - \mu_p)$ . That is, the predicted failure time  $T_p$  at the current inspection point follows the normal distribution as:

$$T_p \sim N\left(t/(P_t - \mu_p), \sigma_p \cdot t/(P_t - \mu_p)\right). \quad (1)$$

It is assumed that the ANN life percentage prediction errors follow normal distribution, and due to this assumption, the predicted failure time at a certain inspection point also follows normal distribution. It is also assumed that the standard deviation of the ANN life percentage prediction errors is constant and does not change over time.

### 3. THE MULTI-COMPONENT CBM APPROACH

In this section, we present the CBM policy for multi-component systems, and the cost evaluation method for the CBM policy.

#### 3.1 The CBM policy

In multi-component systems, the conditional probability  $Pr^*$  is used to determine not only when and also which components should be preventive replaced at each inspection time. The CBM policy for multi-component systems are proposed as below:

- 1) Identify number of components in multi-component systems.
- 2) Regularly inspect these components which subjected to condition-based monitoring. Calculate the predictive failure probability of each component at each inspection time based on the prediction method.
- 3) When a component's predicted failure probability  $Pr$  exceeds the level-1 threshold value  $Pr_1^*$ , preventively replace the component.
- 4) When a component fails, replace it by a new one.
- 5) When there is a preventive replacement or a failure replacement performed on any component in the system, simultaneously replace other components

if their  $Pr$  values exceed the level-2 threshold value  $Pr_2^*$ .

At each inspection time, one of the following events takes place exclusively for each component  $i$ :

1. Component  $i$  reaches  $Pr_1^* \rightarrow$  a preventive replacement is performed on  $i$ .
2. Component  $i$  reaches  $Pr_2^*$  if there is a failure replacement or a preventive replacement that needs to be performed on one of the components in the multi-component systems  $\rightarrow$  preventively replace component  $i$  simultaneously.
3. Component  $i$  fails  $\rightarrow$  a failure replacement is performed, the component is replaced by a new one.
4. None of the above  $\rightarrow$  component  $i$  continues its normal operation.

#### 3.2 A simulation method for cost evaluation

In our research, a simulation method is used to find the optimal condition failure probability threshold value which corresponds to the minimum expected replacement cost. We assume that there are  $N$  components in the multi-component systems. The procedure of the simulation method for CBM policy cost evaluation is shown in Figure 1, and is discussed in details as follows.

Step 1: Define the maximum simulation iteration.

Set the maximum simulation iteration  $NT$ , for example, 100,000 inspection points. It means we start from inspection point 0 and end with inspection points 100,000. Between each inspection point, there is a fixed inspection interval  $L$ , like 20 days.

Step 2: Generate a random failure time as the actual failure time of each component.

At the starting point of a new life cycle of component  $i$ , generate a random failure time,  $FT_i$ , which follows Weibull distribution with the parameters  $\alpha, \beta$ .

Step 3: Generate a random predicted failure time of a component.

At inspection point  $k$  ( $k = 0, \dots, NT$ ), generate a random predicted failure time for component  $i$  based on ANN RUL prediction error. In a simulation process, this random predicted failure time simulate the predicted result based on ANN model using condition monitoring data at each inspection time. The predicted lifetime is denoted by  $PT_{ki}$  and follows normal distribution:

$$PT_{ki} \sim N(\mu, \sigma^2) \quad (k = 0, \dots, NT; i = 1, \dots, N) \quad (2)$$

where  $\mu = FT_i$ ,  $\sigma = \sigma_p \times FT_i$ ,  $\sigma_p$  is standard deviation of the remaining useful life prediction error.

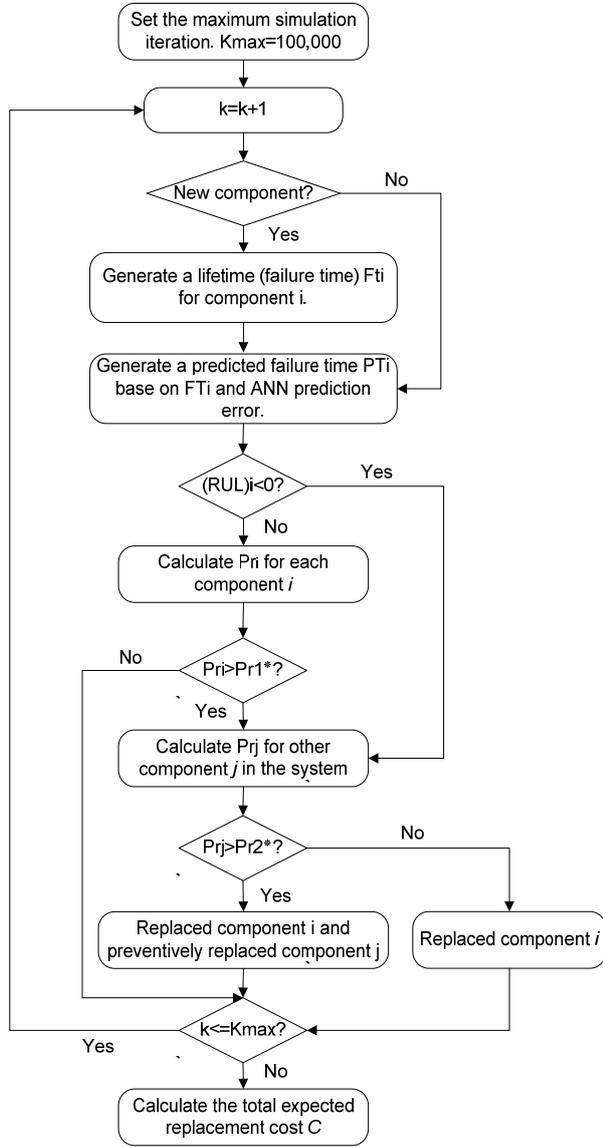


Figure 1. The procedure of the simulation method for cost evaluation in multi-component

Step 4: calculation of predicted failure probability.

During a lifetime of component  $i$ , calculate conditional failure probability  $P_{r_{ki}}$  in each inspection point by using equation below:

$$P_{r_{ki}} = \frac{\int_{t_i}^{t_i+L} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t_i-\mu)^2}{2\sigma^2}} dt}{\int_{t_i}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t_i-\mu)^2}{2\sigma^2}} dt}$$

( $k = 0, \dots, NT; i = 1, \dots, N$ )

Where  $t_i$  is cumulated inspection time of component  $i$  in one life cycle,  $L$  is the constant inspection interval,  $\mu$  is the predicted failure time of different component at different inspection point of time  $PT_{ki}$ , and  $\sigma = \sigma_p \times FT_i$ ,  $\sigma_p$  is standard deviation of ANN RUL prediction error.

If  $P_{r_{ki}}$  is greater than the level-1 condition failure probability threshold  $Pr_1^*$ , preventively replace the component at inspection point  $k$ . If there is no preventive replacement performs during a lifetime of the component  $i$ , perform failure replacement at the inspection point just past the generated failure time  $FT_i$ . When there is a preventive replacement or a failure replace took place at inspection time  $k$ , check other components in the system, if  $P_{r_{kj}}$  ( $j = 1, \dots, N$ ) is greater than the level-2 failure probability threshold  $Pr_2^*$ , perform preventive replacement on component  $j$  simultaneously.

We also introduce two variables to represent the stature of the component  $i$  in the multi-component systems:

$$\Delta_{pki} = \begin{cases} 1 & \text{Component } i \text{ was preventively replaced;} \\ 0 & \text{No preventive replacement} \end{cases}$$

$$\Delta_{fki} = \begin{cases} 1 & \text{Failure replacement on Component } i \\ 0 & \text{No failure replacement on component } i \end{cases}$$

If  $\Delta_{pki} = 0$  &  $\Delta_{fki} = 0$ , component  $i$  continues its normal operation.

Step 5: New life cycle starts.

Start a new life cycle of component  $i$  after a preventive or a failure replacement takes place, go back to Step 2 and set the cumulated inspection time,  $t_i$ , equals to 0. The iteration would not stop until maximum simulation iteration is reached.

Step 6: Estimate total expected replacement cost.

The expected replacement cost for multi-component system can be obtained by the following equation:

$$C_r = \frac{\text{Cost\_total}}{\text{Time\_total}} = \frac{\sum_{k=0}^{NT} C_k}{NT \times L} \quad (\$/\text{day}) \quad (3)$$

where  $C_k$  is the total cost occurs at inspection point  $k$ ,  $NT$  is the total inspection point of the simulation process, and  $L$  is the inspection interval.

$$C_k = C_f \cdot \sum_{i=1}^N \Delta_{fki} + C_p \cdot \sum_{i=1}^N \Delta_{pki} + I(\Delta_{pki}) \cdot C_{p0} \quad (4)$$

where  $I(\Delta_{pki}) = 1$ , when  $\sum_{i=1}^N \Delta_{pki} \geq 1$  &  $\sum_{i=1}^N \Delta_{fki} = 0$ ; otherwise  $I(\Delta_{pki}) = 0$ .  $N$  is number of components under condition monitoring,  $C_{p0}$  is fixed preventive replacement cost and  $C_p$  is variable preventive replacement cost,  $C_f$  is failure replacement cost at a time.

At inspection point  $k$ ,  $C_k$  can be in one of three possible circumstances as follows:

$C_k = C_{p0} + nC_p$ , ( $1 \leq n \leq N$ ), there is at least one preventive replacement needed but no failure replacement;

$C_k = mC_f + nC_p$ , ( $1 \leq m \leq N, 0 \leq n \leq N - 1$ ), there are at least one failure replacement and  $n$  preventive replacement perform;

$C_k = 0$ , there is neither preventive replacement nor failure replacement needed.

Step 7: Determine the optimal CBM policy for multi-component systems.

The two level predicted failure probability are decision variables in the CBM policy for multi-component systems. The minimum calculated replacement cost corresponding to the predicted failure probability threshold value  $Pr_1^*$  and  $Pr_2^*$ . So once  $Pr_1^*$  and  $Pr_2^*$  are determined, the CBM policy is determined.

### 3.3 The CBM optimization model

The objective of the CBM optimization is to determine the optimal failure probability threshold values to minimize the long-run expected replacement cost. The optimization model can be formulated as below:

$$\min C_r(Pr_1^*, Pr_2^*) \quad (5)$$

s. t.

$$C_r \leq C_0, Pr_1^* \geq Pr_2^* \geq 0$$

where  $C_0$  is the cost constraint value,  $Pr_1^*$  and  $Pr_2^*$  are Level-1 and Level-2 failure probability threshold values and also are the policy decision variables.

## 4. EXAMPLE

In this section, we present an example based on bearing vibration monitoring data collected from bearings on a group of Gould pumps at a Canadian kraft pulp mill company (Stevens 2006). We use totally 24 bearing histories which were examined in 8 pump locations, embracing 10 bearing failure histories and 14 suspension histories. For each pump, seven types of measurements were recorded: five different vibration frequency bands (8\*5), and the overall vibration reading (8\*1) plus the bearing's acceleration data (8\*1). So the original inspection data includes 56 (8\*5+8\*1+8\*1) vibration measurements at each time.

EXAKT was used to do the significance analysis for the 56 vibration measurements (Stevens 2006). Two of the variables were identified as significant influence on the health of bearings. Then we use these two measurements and the age time of the component as the inputs of the ANN model. 5 failure histories and 10 suspension histories are used as training ANN inputs and the other 5 failure histories are used as test. After comparing the predicted lifetime to the actual lifetime, we found that the prediction error follow the normal distribution. The mean of prediction error is 0.1385 and the standard deviation is 0.1429.

For multi-component systems, level-1 and level-2 probability thresholds are two decision variables to determine the optimal CBM policy, and therefore, the expected replacement cost of certain CBM policy can be evaluated by giving certain probability threshold values  $Pr_1^*$  and  $Pr_2^*$ . In this case, we consider a multi-component system consisting of 5 identical bearings which are operating in parallel and which are subject to random failures. The lifetimes of the individual components are independent random variables and are identically distributed as Weibull distribution with parameters  $\alpha = 1386.3, \beta = 1.8$ .

The simulation procedure is as follows:

Step 1: Set the maximum simulation inspection point is 100,000, same as in single unit policy. Between each inspection point, the fixed inspection interval,  $L$  equals 20 days.

Step 2: At the starting point of each iteration for component  $i$  ( $i = 1, \dots, 5$ ), set  $t_i$  equals 0, generate a random failure time,  $FT_i$ , of the component which follows Weibull distribution.

Step 3: At inspection point  $k$  ( $k = 0, \dots, 100,000$ ), generate a random predicted failure time,  $PT_{ki}$ , of the component  $i$ , based on the ANN RUL prediction error.  $PT_{ki}$  follows a normal distribution. In this case:  $\mu_i = FT_i$ ,  $\sigma = \sigma_p \times FT_i$ ,  $\sigma_p$  is standard deviation of ANN RUL prediction error. Thus, we have

$$PT_{ki} \sim N(FT_i, (0.1429 \times FT_i)^2) \quad (6)$$

Step 4: During the lifetime of component  $i$ , calculate the conditional failure probability  $Pr_i$  of each inspection point, thus we have:

$$Pr_{rki} = \frac{\int_{t_i}^{t_i+20} \frac{1}{0.1429\sqrt{2\pi}} e^{-\frac{(t_i-PT_{ki})^2}{2 \times 0.1429^2}} dt_i}{\int_{t_i}^{\infty} \frac{1}{0.1429\sqrt{2\pi}} e^{-\frac{(t_i-PT_{ki})^2}{2 \times 0.1429^2}} dt_i}$$

( $k = 0, \dots, 100,000; i = 1, \dots, 5; t_i \geq 0$ )

where  $t_i$  is cumulated inspection time in one life circle for component  $i$ .

At each inspection point  $k$ , if  $Pr_{ki}$  ( $i = 1, \dots, 5$ ) is greater than the given level-1 condition failure probability threshold  $Pr_1^*$  ( $0 < Pr_1^* < 1$ ), preventively replace the component at time point  $k$ . If there is no preventive replacement during the lifetime of component  $i$ , perform failure replacement at the inspection point just behind  $FT_i$ . When there is a preventive/ failure replacement occurs in time  $k$ , check other components, if  $Pr_{kj}$  ( $j = 1, \dots, 5$ ) is greater than the given level-2 failure probability threshold  $Pr_2^*$ , perform preventive replacement on component  $j$  simultaneously.

Step 5: When there is a preventive/ failure replacement took place on component  $i$ , start a new life circle of component  $i$  by setting  $t_i = 0$ , and back to Step 2. The iteration would not stop until  $k$  equals 100,000.

Step 6: Estimate cost rate. In this case, the fix preventive replacement cost  $C_{po}$  is 3,000 and the variable preventive replacement cost  $C_p$  is 1,800. We have:

$$C_r = \frac{\text{Cost\_total}}{\text{Time\_total}} = \frac{\sum_{k=0}^{100,000} C_k}{100,000 \times 20} \text{ (\$/day)} \quad (k = 0, \dots, 100,000)$$

where

$$C_k = C_f \cdot \sum_{i=1}^N \Delta_{fki} + C_p \cdot \sum_{i=1}^N \Delta_{pki} + I(\Delta_{pki}) \cdot C_{po}$$

$$\Rightarrow C_k = 16,000 \cdot \sum_{i=1}^5 \Delta_{fki} + 1,8000 \cdot \sum_{i=1}^5 \Delta_{pki} + 3,000 \cdot I(\Delta_{pki})$$

where

- $\Delta_{pki} =$ 
  - 1 Component  $i$  was preventively replaced;
  - 0 No preventive replacement
- $\Delta_{fki} =$ 
  - 1 Failure replacement on Component  $i$
  - 0 No failure replacement on component  $i$

If  $\Delta_{pki} = 0$  &  $\Delta_{fki} = 0$ , the component  $i$  continues its normal operation.

Step 7: find the optimal total expected replacement cost. By setting different value of  $Pr_1$  and  $Pr_2$ , the corresponding total expected replacement cost can be evaluated and the results are list in Table 1. The minimal cost value shows and the condition failure probability threshold value  $Pr_1^*$  and  $Pr_2^*$  can be determined.

The expected cost as a function of  $Pr_1^*$  and  $Pr_1^*/Pr_2^*$  is plotted in Figure 2. The optimal failure probability threshold values can be observed from this figure, where the lowest expected cost exists.

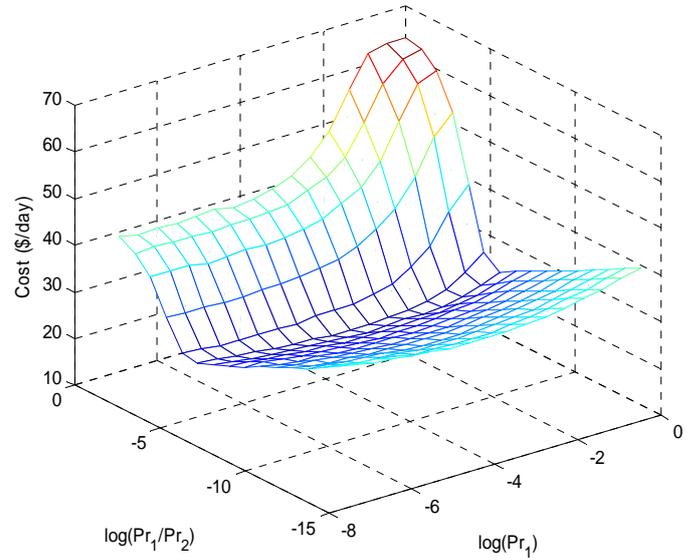


Figure 2. Cost versus two condition failure probability threshold values

The minimal expected cost for multi-component occurs when  $Pr_1^* = 0.100259$  and  $Pr_2^* = 4.0973 \times 10^{-4}$ , the expected maintenance cost for this multi-component system containing 5 components is \$17.5651/day.

The comparative results are showed in Table 1. Compare to the CBM policy for single component, the optimal cost is much lower when using multi-component CBM policy, cost savings in percentage is 27.21%.

Table 1. Comparison of cost between single unit and multi-component CBM policy

	Single Unit	Multi-component systems (5 components)
Cost (\$/day)	4.8264	17.5651
Cost for each component (\$/day)	4.8264	3.513
Cost savings (%)	27.21%	

This comparative study demonstrates that the proposed multi-component CBM policy can achieve a lower total expected replacement cost by taking advantage of economic dependency in the multi-component systems.

## 5. CONCLUSION

In this paper, a CBM optimization method is proposed for multi-component systems, where economic dependency exists among the components subject to condition monitoring. Deterioration of a multi-component system is represented by a conditional failure probability value, which is calculated based on the predicted failure time distributions of components. The proposed CBM policy is defined by a two-level failure probability threshold. A method is developed to obtain the optimal threshold values in order to minimize the long-term maintenance cost. An example is used to demonstrate the proposed multi-component CBM method.

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