Enhanced Multivariate Based Approach for SHM Using Hilbert Transform

Rafik Hajrya, Nazih Mechbal, Michel Vergé

Process and Engineering in Mechanics and Materials Laboratory (PIMM)
Arts et Métiers ParisTech
Paris, France
Rafik.HAJRYA-7@etudiants.ensam.eu
nazih.mechbal@ensam.eu
michel.verge@ensam.eu

ABSTRACT

In structural health monitoring, features extraction from measured data plays an important role. In order to enhance information about damage, we propose in this paper, a new damage detection methodology, based on the Hilbert transform and multivariate analysis. Using measurements given by distributed sensors of a smart composite structure, we apply the Hilbert transform to calculate an envelope matrix. This matrix is then treated using multivariate analysis. The subspaces associated to the envelope matrix are used to define a damage index (DI). Furthermore, from perturbation theory of matrices, we propose a bound associated to this DI, by inspecting this bound, decision on the health of the structure is generated. Experimentation on an actual composite smart structure will show the effectiveness of the proposed approach.

1. INTRODUCTION

Composite structures have been increasingly adopted by the aviation community to provide high performance, strength, stiffness and weight reduction. One of the major concerns associated with composites is the susceptibility to impact damage,(Staszewski 2002). Impact damage may occur during manufacture, service or maintenance. Low-velocity impacts are often caused by bird strikes, runway stones and tool-drops during maintenance. Impacts can induce serious damage to composites such as delamination, matrix and fiber cracking. Faced with these various damages, a structural health monitoring system (SHM) is needed and if possible in real time.

SHM methods are implemented on structures known as "smart structures", (Giurgiutiu et al. 2002). These structures consist of a network sensors and actuators and offer a monitoring capability for real-time application. Recently emerged piezoceramic patches have the potential to improve significantly developments of structural health monitoring systems. These patches offer many advantages, among of them: lightweight properties, relative low-cost and can be produced in different shapes. Recently, (Su et al. 2006) have developed a sensor network for SHM using printed circuit to embed piezoceramic patches into a composite structure.

Damage is a structural state which is different from a reference state that is healthy. A damage event is not meaningful without comparisons between two different structural states. The greatest challenge is to ascertain what changes are sought in the signal after the presence of damage. Features extraction is therefore a key step in the processing of signal sensor. In SHM, feature extraction is the process of identifying damage-sensitive properties derived from the measured response data of a smart structure; it serves an indicator to describe the damage and its severity. These extracted features are termed as damage index (DI). Recently, the method of empirical mode decomposition (EMD) and Hilbert transform have been applied in SHM, (Huang et al. 1998). By applying EMD and Hilbert transform in a measured data, (Yang et al. 2004) have developed a method to detect the damage time instant and damage location, in addition they propose in others works the identification of linear structure using the EMD and Hilbert transform, (Yang et al. 2003a; Yang et al. 2003b).

In recent years, techniques based on multivariate statistics have been also applied in SHM. As the name implies, multivariate analysis is concerned with the analysis of multiple measurements from sensors and treats them as a single entity. There are two major multivariate techniques in SHM, principal components analysis (PCA) and independent components analysis (ICA). These techniques serve two purposes, namely order reduction and feature extraction by revealing structure hidden in the measurement, (Kerschen et al. 2005). By applying a PCA on the sensor time responses, (De Boe and Golinval 2003) have developed a damage index based on angle between subspace to detect and locate damage, in addition (Hajrya et al. 2011) have applied the same principle and they propose a bound based on correlation coefficient that automatically decides if a composite structure is in healthy or damaged state. Using independent component analysis combined with artificial neural network, (Zang et al. 2004) have used a mixing matrix which is extracted from ICA to detect and locate damage.

In this work, we propose an original damage index (DI) based on the calculation of an envelope matrix. This matrix is built using the Hilbert transform of time response matrix measurements. Furthermore, from perturbation theory of matrices, we define a bound that automatically decides if the composite structure is in healthy or damaged status.

The paper is organized as follows: In the next section the experimental test is presented. In section 3, the mathematical formulation of the Hilbert transform and the multivariate analysis are briefly described. In section 4, our methodology for damage detection is presented. In section 5, the proposed damage detection scheme is applied on an experimental laboratory test bench. Finally, conclusions and further directions will be drawn in section 6. Main terms, table and figures are illustrated at the end of the paper before the references.

2. EXPERIMENTAL TEST BENCH

The structure employed consists of a piece of composite fuselage; it was manufactured by INEO DEFENSE which is a partner in the MSIE research program. The structure consists of a carbone-epoxy composite plate with dimensions: $(400 \times 300 \times 2\text{mm})$ and it is made up of 16 layers. The layers sequences are: [$0°_2$,$+45°_2$,$-45°_2$, $+90°_2$, $-90°_2$, $-45°_2$, $+45°_2$, $0°_2$]. The properties of the composite plate are detailed in table 1. Using a modal approach, we have performed in a previous work, (Hajrya et al. 2010), an optimal placement of ten piezoceramic patches (figure 2), with dimensions $(30 \times 20 \times 0.2\text{ mm})$. The piezoceramic patches are made on lead zirconate titanate (PZT). Figure 1 is a diagram and it shows the positions of the ten PZT in the composite plate. It is to be noted that in our work, only nine PZT are used (PZT 6 is not taken into account in the **damage detection** methodology). Sensor PZT 6 will be used in another work for **sensor fault detection**.

Figure 2 shows the experimental smart composite plate and it was used as baseline for damage detection. In order to develop a damage detection methodology, we have used a second composite plate with the same dimensions and numbers of PZT (at the same location), but, in this plate, impact damage was produced throwing a ball at high velocity: the damage is located at the middle of the plate. Figure 3 shows the location of this impact damage.

The input excitation generation and the data acquisition were made using a commercial system dSPACE ®. The input excitation consists in a signal pulse with 1ms width. Signals were acquired with sampling frequency $f_s = 100\ kHz$, time duration was $T = 0.65s$ and $N = 2^{16}$ time samples were recorded for each channel: one corresponding to the excitation applied to the PZT actuator and the others concern the measurements collected by the PZT sensors. Figure 4 shows the time responses of sensor PZT 7 in the case of the healthy and damaged plate while we have used PZT 10 as actuator, *i.e.* (Path PZT 10-PZT7): only the 512 first samples are displayed.

3. MATHEMATICAL FORMULATION

3.1 Hilbert transform

The Hilbert transform of an arbitrary signal $y(t)$ is defined as, (Bendat and Piersol 2000):

$$\tilde{y}(t)=\mathcal{H}[y(t)] = \int_{-\infty}^{+\infty} \frac{y(u)}{\pi(t-u)} \quad (1)$$

Equation (1) is the convolution integral of $y(t)$ and $(1/\pi t)$ and it performs a $90°$ phase shift or quadrature filter to construct the so-called analytic $z(t)$ expressed by:

$$z(t)=y(t) + j\tilde{y}(t) \quad (2)$$

Equation (2) can also been written as follow:

$$z(t)=e(t) \cdot e^{j\theta(t)} \quad (3)$$

where

$e(t)$ is called the envelope signal of $y(t)$ and $\theta(t)$ is called the instantaneous phase signal of $y(t)$, we have the relations:

$$e(t) = \sqrt{y^2(t) + \tilde{y}^2(t)}$$
$$\theta(t) = tan^{-1}\left[\frac{\tilde{y}(t)}{y(t)}\right] \quad (4)$$

The envelope $e(t)$ depicts the energy distribution of $y(t)$ in the time domain.

In practice, the data are discretized in time, let:

$\underline{\boldsymbol{y}}(k)$ be a discretized measurement vector at instant k from n_y PZT sensors, that are instrumented in the composite smart structure:

$$\underline{\boldsymbol{y}}(k) = \left[y_1(k) \cdots y_i(k) \dots y_{n_y}(k)\right]^T \quad (5)$$

The data matrix of measurements $\mathbf{Y} \in \mathbb{R}^{n_y \times \mathrm{N}}$ gathering N samples $\underline{\boldsymbol{y}}(k)(k = 1, \dots, N)$ is defined as follows:

$$\mathbf{Y} = \left[\underline{\boldsymbol{y}}(1) \cdots \underline{\boldsymbol{y}}(k) \cdots \underline{\boldsymbol{y}}(N)\right] \quad (6)$$

In our case of study, we have $n_y = 8, N = 2^{16}, n_y \ll N$.

The matrix **Y** has been autoscaled by subtracting the mean and dividing each line by its standard deviation.

For sensor i and instant k, the analytic signal $z_i(k)$, the envelope signal $e_i(k)$ and the instantaneous phase $\theta_i(k)$ are given by:

$$z_i(k) = y_i(k) + j\tilde{y}_i(k) \tag{7}$$

$$e_i(k) = \sqrt{y_i^2(k) + \tilde{y}_i^2(k)} \tag{8}$$

$$\theta_i(k) = \tan^{-1}\left[\frac{\tilde{y}_i(k)}{y_i(k)}\right] \tag{9}$$

Using Eq. (8), we define the envelope vector $\underline{\boldsymbol{e}}(k)$ at instant k for the n_y sensor:

$$\underline{\boldsymbol{e}}(k) = \left[e_1(k) \ \cdots e_i(k) \ldots e_{n_y}(k)\right]^T \tag{10}$$

For example, the corresponding envelope signal of sensor PZT 7 in the case of healthy and damaged structures are depicted in figure 5, only the 512 first samples of the envelope signals are displayed.

According to Eq. (10), we define the envelope matrix $\mathbf{E} \in \mathbb{R}^{n_y \times N}$ of the matrix measurements $\mathbf{Y} \in \mathbb{R}^{n_y \times N}$ by:

$$\mathbf{E} = \left[\underline{\boldsymbol{e}}(1) \cdots \underline{\boldsymbol{e}}(k) \cdots \underline{\boldsymbol{e}}(N)\right] \tag{11}$$

This envelope matrix **E** gathers $\boldsymbol{N}$ samples $\underline{\boldsymbol{e}}(k), (k = 1, \ldots, N)$:

3.2 Multivariate analysis

As stated in section 1, multivariate analysis concerns the analysis of multiple measurements from sensors and treats them as a single entity. In our work, the single entity concerns the envelop matrix $\mathbf{E} \in \mathbb{R}^{n_y \times N}$. One way to study the matrix **E** is to use the singular value decomposition (SVD), (Golub 1983):

The matrix $\mathbf{E} \in \mathbb{R}^{n_y \times N}$ admits two orthogonal matrices:

$$\begin{aligned} \mathbf{U} &= \left[\underline{\boldsymbol{u}}_1, \cdots, \underline{\boldsymbol{u}}_{n_y}\right] \in \mathbb{R}^{n_y \times n_y} \\ \mathbf{V} &= \left[\underline{\boldsymbol{v}}_1, \cdots, \underline{\boldsymbol{v}}_{n_y}\right] \in \mathbb{R}^{N \times n_y} \end{aligned} \tag{12}$$

such that

$$\begin{gathered} \boldsymbol{\Gamma} = \mathbf{U}^T \cdot \mathbf{Y} \cdot \mathbf{V} = \mathrm{diag}(\sigma_1, \cdots, \sigma_p) \\ p = min\{n_y, N\} = n_y \\ \mathbf{U}^T \cdot \mathbf{U} = \mathbf{I}_{n_y}, \mathbf{V}^T \cdot \mathbf{V} = \mathbf{I}_{n_y} \end{gathered} \tag{13}$$

where $\boldsymbol{\Gamma} \in \mathbb{R}^{n_y \times n_y}$ is the matrix of singular values, the columns of the matrix $\mathbf{U} \in \mathbb{R}^{n_y \times n_y}$ contain the left singular vectors and the columns of the matrix $\mathbf{V} \in \mathbb{R}^{\boldsymbol{N} \times n_y}$ contain the right singular vectors.

The SVD of the matrix **E** provides important insight about the orientation of this set of vectors, and determines how much the dimension of **E** can be reduced, (Kerschen et al. 2005). One way to reduce the dimension of **E** is to take the sum of all singular values then to delete those singular values that fall below some percentage of that sum, (De Boe and Golinval 2003). In our work, we have decided to fix a percentage sum of 98%.

According to this, the SVD of matrix **E** take the following form:

$$\begin{aligned} \mathbf{E} &= [\mathbf{U}_1 \quad \mathbf{U}_2] \cdot \begin{bmatrix} \boldsymbol{\Gamma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Gamma}_2 \end{bmatrix} \cdot [\mathbf{V}_1 \quad \mathbf{V}_2]^T \\ &= \mathbf{E}_1 + \mathbf{E}_2 \end{aligned} \tag{14}$$

where:

$\mathbf{U}_1 \in \mathbb{R}^{n_y \times n_{POM}}$,$\boldsymbol{\Gamma}_1 \in \mathbb{R}^{n_{POM} \times n_{POM}}$,$\mathbf{V}_1 \in \mathbb{R}^{N \times n_{POM}}$,

$\mathbf{U}_2 \in \mathbb{R}^{n_y \times (n_y - n_{POM})}$,$\boldsymbol{\Gamma}_2 \in \mathbb{R}^{(n_y - n_{POM}) \times (n_y - n_{POM})}$,

$\mathbf{V}_2 \in \mathbb{R}^{N \times (n_y - n_{POM})}$,

n_{POM} is the retained dimension after reduction.

The columns of the matrix $\mathbf{U}_1$ are called the principal left singular vectors and the columns of the matrix $\mathbf{V}_1$ are called the principal right singular vectors. Analogously, the columns of the matrix $\mathbf{U}_2$ are called the residual left singular vectors and the columns of the matrix $\mathbf{V}_2$ are called the residual right singular vectors.

4. DAMAGE DETECTION METHODOLOGY

The presence of damage in the structure cause change in the stiffness and mass matrices. Consequently, damage will introduce change in the response of the measurement sensor and the matrix measurements **Y**, see (Hajrya et al. 2011) for the demonstration. Hence, the envelope matrix **E** is also modified. Figure 5 depicts the corresponding envelope signal of sensor PZT 7 and one can see that there is a significant difference in the envelope signal of the healthy and damaged structures.

4.1 Damage index

Let $\mathbf{E}^s, \mathbf{E}^u \in \mathbb{R}^{n_y \times N}$ be respectively the envelope matrices of the healthy and unknown structures. According to section 3.2, there SVD is defined as follow:

$$\begin{aligned} \mathbf{E}^s &= [\mathbf{U}_1^s \quad \mathbf{U}_2^s] \cdot \begin{bmatrix} \boldsymbol{\Gamma}_1^s & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Gamma}_2^s \end{bmatrix} \cdot [\mathbf{V}_1^s \quad \mathbf{V}_2^s]^T \\ &= \mathbf{E}_1^s + \mathbf{E}_2^s \end{aligned} \tag{15}$$

$$\begin{aligned} \mathbf{E}^u &= [\mathbf{U}_1^u \quad \mathbf{U}_2^u] \cdot \begin{bmatrix} \boldsymbol{\Gamma}_1^u & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Gamma}_2^u \end{bmatrix} \cdot [\mathbf{V}_1^u \quad \mathbf{V}_2^u]^T \\ &= \mathbf{E}_1^u + \mathbf{E}_2^u \end{aligned} \tag{16}$$

We suppose that the dimensions of all components in Eq. (15) and (16) are equals to those in Eq. (14).
In our methodology, we are interested in studying the principal left and right singular vectors.
Let:
$\mathbf{U}_1^s = [\underline{\boldsymbol{u}}_{11}^s \cdots \underline{\boldsymbol{u}}_{1i}^s \cdots \underline{\boldsymbol{u}}_{1n_{POM}}^s] \in \mathbb{R}^{n_y \times n_{POM}}$, be the principal left singular vectors of the healthy smart structure,
$\mathbf{V}_1^s = [\underline{\boldsymbol{v}}_{11}^s \cdots \underline{\boldsymbol{v}}_{1i}^s \cdots \underline{\boldsymbol{v}}_{1n_{POM}}^s] \in \mathbb{R}^{N \times n_{POM}}$, be the principal right singular vectors of the healthy smart structure,
$\mathbf{U}_1^u = [\underline{\boldsymbol{u}}_{11}^u \cdots \underline{\boldsymbol{u}}_{1i}^u \cdots \underline{\boldsymbol{u}}_{1n_{POM}}^u] \in \mathbb{R}^{n_y \times n_{POM}}$, be the principal left singular vectors of the unknown smart structure,
$\mathbf{V}_1^u = [\underline{\boldsymbol{v}}_{11}^u \cdots \underline{\boldsymbol{v}}_{1i}^u \cdots \underline{\boldsymbol{v}}_{1n_{POM}}^u] \in \mathbb{R}^{N \times n_{POM}}$, be the principal right singular vectors of the unknown smart structure.
We define the angle between $\underline{\boldsymbol{u}}_{1i}^s$ and $\underline{\boldsymbol{u}}_{1i}^u$ and the angle between $\underline{\boldsymbol{v}}_{1i}^s$ and $\underline{\boldsymbol{v}}_{1i}^u$ as, (De Boe and Golinval 2003):

$$\begin{aligned} |\cos\psi_i| &= |\langle \underline{\boldsymbol{u}}_i^s | \underline{\boldsymbol{u}}_i^u \rangle| = \left| (\underline{\boldsymbol{u}}_i^s)^T \cdot \underline{\boldsymbol{u}}_i^u \right| \\ \psi_i &= \cos^{-1}|\cos\psi_i|, \qquad \psi_i \in \left[0, \frac{\pi}{2}\right] \\ |\cos\varphi_i| &= |\langle \underline{\boldsymbol{v}}_i^s | \underline{\boldsymbol{v}}_i^u \rangle| = \left| (\underline{\boldsymbol{v}}_i^s)^T \cdot \underline{\boldsymbol{v}}_i^u \right| \\ \varphi_i &= \cos^{-1}|\cos\varphi_i|, \qquad \varphi_i \in \left[0, \frac{\pi}{2}\right] \end{aligned} \tag{17}$$

According to this, we define two angle vectors $\underline{\boldsymbol{\psi}}$ and $\underline{\boldsymbol{\Phi}}$ by :
$\underline{\boldsymbol{\psi}} = [\psi_1 \cdots \psi_i \cdots \psi_{n_{POM}}]^T, \underline{\boldsymbol{\Phi}} = [\varphi_1 \cdots \varphi_i \cdots \varphi_{n_{POM}}]^T$

We propose the following new damage index DI:

$$\mathrm{DI} = \sqrt{\left\|\sin \underline{\boldsymbol{\psi}}\right\|_2^2 + \left\|\sin \underline{\boldsymbol{\Phi}}\right\|_2^2} \tag{18}$$

Theoretically, when the current state is healthy, then the damage index DI is null, but if the current state is damaged, then the damage index is different from zero.
In order to improve the damage detection methodology under experimental conditions, we define in the next subsection a bound associated to the DI and it is based on the work of Wedin, (Wedin 1972).

4.2 Definition of a bound for the damage index

Wedin have studied the perturbation of matrices using subspaces. Our contribution in this subsection is to extend the theoretical work developed by Wedin in the case of experimental SHM system.
Define first a new envelope matrix $\tilde{\mathbf{E}}^s \in \mathbb{R}^{n_y \times N}$ of the healthy smart structure:

$$\tilde{\mathbf{E}}^s = \mathbf{E}^s + \delta\mathbf{E}^s \tag{19}$$

where
$\delta\mathbf{E}^s \in \mathbb{R}^{n_y \times \mathrm{N}}$ is a matrix which reflects the effect of noise in an experiment.
According to subsection 3.2, the SVD of $\mathbf{E}^s$ and $\tilde{\mathbf{E}}^s$ are defined as follow:

$$\begin{aligned} \mathbf{E}^s &= [\mathbf{U}_1^s \quad \mathbf{U}_2^s] \cdot \begin{bmatrix} \boldsymbol{\Gamma}_1^s & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Gamma}_2^s \end{bmatrix} \cdot [\mathbf{V}_1^s \quad \mathbf{V}_2^s]^T \\ &= \mathbf{E}_1^s + \mathbf{E}_2^s \end{aligned} \tag{20}$$

$$\begin{aligned} \tilde{\mathbf{E}}^s &= [\tilde{\mathbf{U}}_1^s \quad \tilde{\mathbf{U}}_2^s] \cdot \begin{bmatrix} \tilde{\boldsymbol{\Gamma}}_1^s & \mathbf{0} \\ \mathbf{0} & \tilde{\boldsymbol{\Gamma}}_2^s \end{bmatrix} \cdot [\tilde{\mathbf{V}}_1^s \quad \tilde{\mathbf{V}}_2^s]^T \\ &= \tilde{\mathbf{E}}_1^s + \tilde{\mathbf{E}}_2^s \end{aligned} \tag{21}$$

Let $\underline{\boldsymbol{\psi}}^s$ and $\underline{\boldsymbol{\Phi}}^s$ the two angle vectors, repectively between the left singular vectors of $\mathbf{E}^s$ and $\tilde{\mathbf{E}}^s$and the right singular vectors $\mathbf{E}^s$ and $\tilde{\mathbf{E}}^s$, these angle vectors are calculated using Eq. (17) .
According to (Wedin 1972), we define two residual matrices $\mathbf{R}_{11}, \mathbf{R}_{21}$ as:

$$\begin{aligned} \mathbf{R}_{11} &= \mathbf{E}^s \cdot \tilde{\mathbf{V}}_1^s - \tilde{\mathbf{U}}_1^s \cdot \tilde{\boldsymbol{\Gamma}}_1^s = \left(\mathbf{E}^s - \tilde{\mathbf{E}}^s\right) \cdot \tilde{\mathbf{V}}_1^s \\ &= -\delta\mathbf{E}^s \cdot \tilde{\mathbf{V}}_1^s \end{aligned} \tag{22}$$

$$\begin{aligned} \mathbf{R}_{21} &= (\mathbf{E}^s)^T \cdot \tilde{\mathbf{U}}_1^s - \tilde{\mathbf{V}}_1^s \cdot \tilde{\boldsymbol{\Gamma}}_1^s \\ &= \left((\mathbf{E}^s)^T - \left(\tilde{\mathbf{E}}^s\right)^T\right) \cdot \tilde{\mathbf{U}}_1^s \\ &= -(\delta\mathbf{E}^s)^T \cdot \tilde{\mathbf{U}}_1^s \end{aligned} \tag{23}$$

Given, the aforementioned definitions, Wedin's theorem sates:

Theorem

If $\exists\ \alpha \geq 0$ and $\eta > 0$ such that
$\min\ \sigma(\tilde{\mathbf{E}}_1^s) \geq \alpha + \eta$ and $\max\ \sigma(\tilde{\mathbf{E}}_2^s) \leq \alpha$
And let $\mu = \max\ \sqrt{\|\mathbf{R}_{11}\|_2 + \|\mathbf{R}_{21}\|_2}$, then

$$\begin{cases} \left\|\sin \underline{\boldsymbol{\psi}}^s\right\|_2 \leq \dfrac{\mu}{\eta} \\ \left\|\sin \underline{\boldsymbol{\Phi}}^s\right\|_2 \leq \dfrac{\mu}{\eta} \end{cases}$$

According to this theorem, we define a bound $\mathcal{B}$ as:

$$\mathcal{B} = \sqrt{2}\frac{\mu}{\eta} \tag{24}$$

In order to improve the bound $\mathcal{B}$, we make n experimental tests of the healthy smart structure and we calculate the mean of the bound:

$$\mu_{\mathcal{B}} = \frac{1}{n}\sum_{j=1}^{n} \mathcal{B}_j \tag{25}$$

The detection procedure is as follow
If $\mathrm{DI} < \mu_{\mathcal{B}}$ then the unknown smart structure is in healthy state,
Else the unknown smart structure is in damaged state.
To summarize the damage detection methodology, we use the following steps:

Damage detection methodology

1. Measure acquisition of the healthy smart structure $\mathbf{Y}^s$,
2. Repeat n times the experiment for the healthy smart structure: $\tilde{\mathbf{Y}}_j^s, j = 1 \ldots n$,
3. Center the data matrices $\mathbf{Y}^s$, $\tilde{\mathbf{Y}}_j^s$ and normalize them using the standard deviation,
4. Using Eq. (8) and (11), calculate the envelope matrix $\mathbf{E}^s$and $\tilde{\mathbf{E}}_j^s$**,**
5. Using Eq. (13), applied the SVD for matrices $\mathbf{E}^s$ and $\tilde{\mathbf{E}}_j^s$,
6. Reduce the dimension if possible,
7. Using the Wedin' theorem and Eq.(24), calculate the bound $\mathcal{B}_j, j = 1 \cdots n$,
8. Calculate the mean bound $\mu_{\mathcal{B}} = \frac{1}{n}\sum_{j=1}^{n} \mathcal{B}_j$,
9. Measure acquisition of the unknown smart structure $\mathbf{Y}^u$,
10. Center the data matrix $\mathbf{Y}^u$ and normalize it using the standard deviation,
11. Using Eq. (8) and (11), calculate the envelope matrix $\mathbf{E}^u$,
12. Using Eq. (13), applied the SVD for the matrix $\mathbf{E}^u$,
13. Reduce the dimension if possible,
14. Using Eq. (17), calculte cos $\underline{\boldsymbol{\psi}}$ and cos $\underline{\boldsymbol{\Phi}}$
15. Calculate sin $\underline{\boldsymbol{\psi}}$ and sin $\underline{\boldsymbol{\Phi}}$,
16. Using Eq. (18), calculate the damage index DI between the healthy envelope matrix $\mathbf{E}^s$ and the unknown envelope matrix $\mathbf{E}^u$,
17. If :
    DI $< \mu_{\mathcal{B}}$: Then the unknown smart structure is in healthy state,
    Else the unknown smart structure is in damaged state.

5. APPLICATION TO THE COMPOSITE SMART STRUCTURE

The damage detection methodology described previously is applied to detect the impact damage of the composite plate presented in section 2. In the first step of our application, we were interested by using PZT 10 as an actuator while the others PZT are sensors (PZT 6 is not taken into account in the damage detection). Following the methodology developed, we have performed six measurements for the healthy composite plate and one measurement for the damaged composite plate. Using these measurement matrices, the envelope matrix for each healthy and damaged state was calculated. Before the calculation of the damage index DI and its associated bound $\boldsymbol{\mathcal{B}}$, we have search for each state of the composite plate to reduce the dimension of the envelope matrices. According to the 98% percentage sum of singular value fixed in subsection 3.2, we see using figures 6 and 7 that the dimension of the envelope matrices cannot be reduced, those the dimension remain: $\mathbf{E}^s, \mathbf{E}^u \in \mathbb{R}^{8\times 2^{16}}$. Using the six experiments of the healthy composite state, the mean value of the bound was first calculated: $\boldsymbol{\mu_{\mathcal{B}} = 0.40}$. The damage index between the healthy and damaged composite plates defined in Eq. (18) is: $\mathbf{DI = 3.37}$. One can we see that the DI is upper than the mean value of the bound, then damage is detected. In order to illustrate the efficiency of the damage detection methodology in term of false alarms, we have done another experiment of the healthy structure which is strictly independent from the others done previously, in this case, $\mathbf{DI = 0.26}$ and it is lower than $\mu_{\mathcal{B}} = 0.4042$.

In second step of our application, we have used PZT 7 as actuator, according to the same methodology, we have obtained the result depicted in table 3 a damage index $\mathbf{DI = 3.30}$, one can we see that the DI is upper than the mean value of the bound $\boldsymbol{\mu_{\mathcal{B}} = 0.54}$. No false alarms were detected.

6. CONCLUSION

In this paper, a damage detection methodology was developed to enhance feature information about damage. This methodology is based on the calculation of a damage index which consists on comparing subspaces of the healthy and damaged state of envelope matrix. This DI was associated with a bound.

The efficiency of the proposed approach was successively applied to detect experimentally impact damage in the composite smart plate. The proposed method presents a cheap computational cost and seems to be well adapted for structural health monitoring in real time application.

For the work under progress, we are investigating the localization of the impact damage in damaged composite plate.

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MAIN TERMS

$\tilde{y}(t)$ Hilbert transform of signal $y(t)$
$z(t)$ Analytic signal
$e(t)$ Envelope signal
$\theta(t)$ Instantaneous phase signal

n_y Number of sensors in the composite smart structure
N Number of samples

$\underline{\boldsymbol{y}}(k)$	Measurements vector at instant k
$\mathbf{Y}$	Matrix measurements
$\underline{\boldsymbol{e}}(k)$	Envelope vector
$\mathbf{E}$	Envelope matrix
$\mathbf{E}^s$	Envelope matrix of the healthy structure
$\tilde{\mathbf{E}}^s$	Envelope matrix of a second experiment of the healthy structure
$\mathbf{E}^u$	Envelope matrix of the unknown structure
$\mathbf{U}$	Matrix of left singular vectors
$\mathbf{V}$	Matrix of right singular vectors
$\underline{\boldsymbol{\psi}}$	Angle vector between the left singular vectors of the healthy matrix $\mathbf{E}^s$ and unknown matrix $\mathbf{E}^u$,
$\underline{\boldsymbol{\Phi}}$	Angle vector between the right singular vectors of the healthy matrix $\mathbf{E}^s$ and unknown matrix $\mathbf{E}^u$,
$\underline{\boldsymbol{\psi}}^s$	Angle vector between the left singular vectors of the two healthy matrices $\mathbf{E}^s$ $\tilde{\mathbf{E}}^s$
$\underline{\boldsymbol{\Phi}}^s$	Angle vector between the right singular vectors of the two healthy matrices $\mathbf{E}^s$ $\tilde{\mathbf{E}}^s$
DI	Damage index
$\mathcal{B}$	Bound of the damage index
$\mu_{\mathcal{B}}$	Mean value of the damage index
$\delta\mathbf{Y}^s$	Matrix of noise
$\mathbf{Y}^T$	Transpose of matrix $\mathbf{Y}$
j	Imaginary number
$\mathbb{R}$	Set of real number

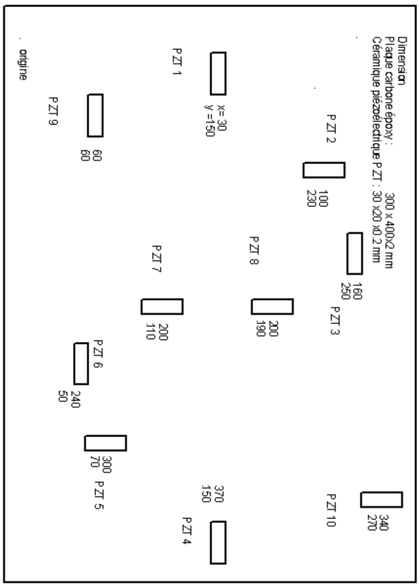

Figure 1: Placement of the PZT in the composite plate

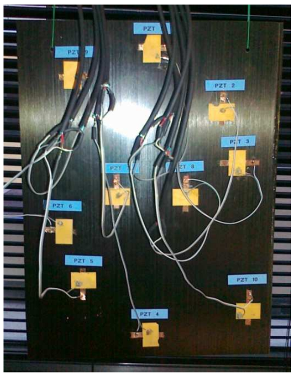

Figure 2 Healthy composite plate bonded with ten PZT patches

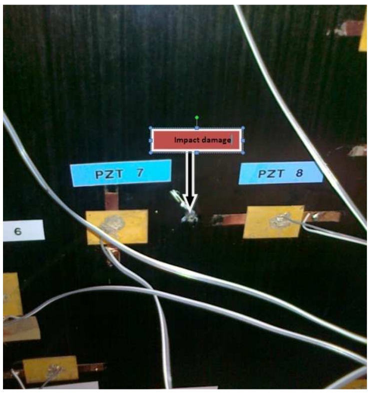

Figure 3: Impact damage in the composite structure

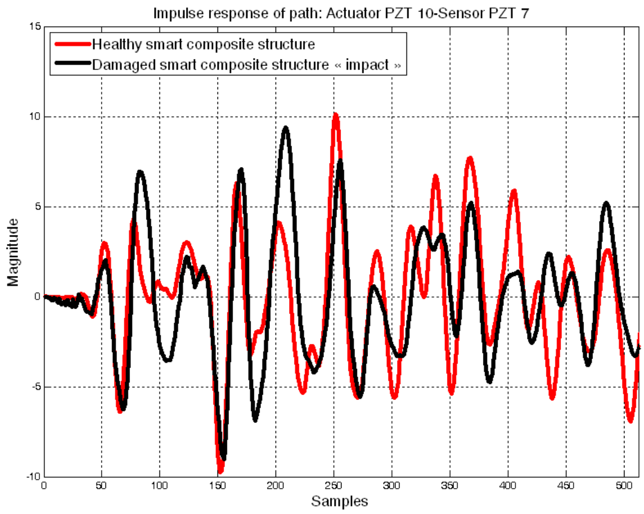

Figure 4: Impulse response of the healthy and damaged smart structures path: actuator PZT 10-sensor PZT7

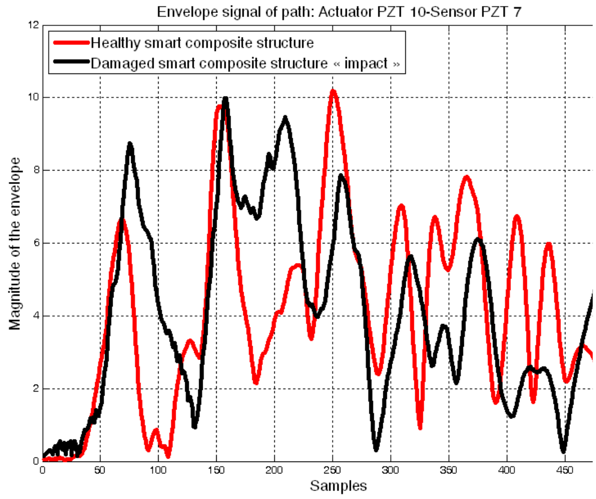

Figure 5: Envelope signal of the healthy and damaged structures: path: Actuator PZT 10-Sensor PZT7

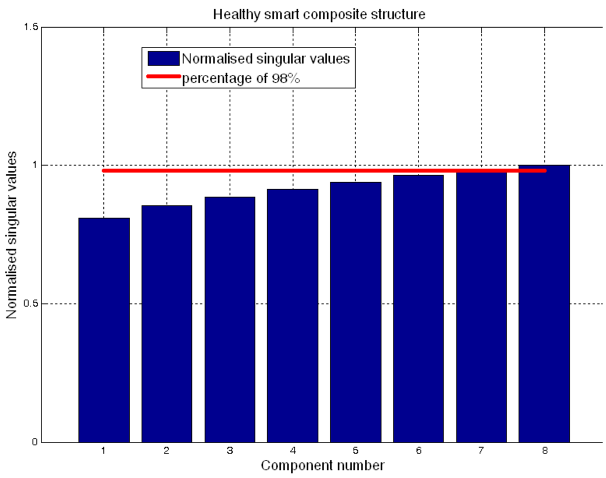

Figure 6: Order reduction of the healthy smart structure

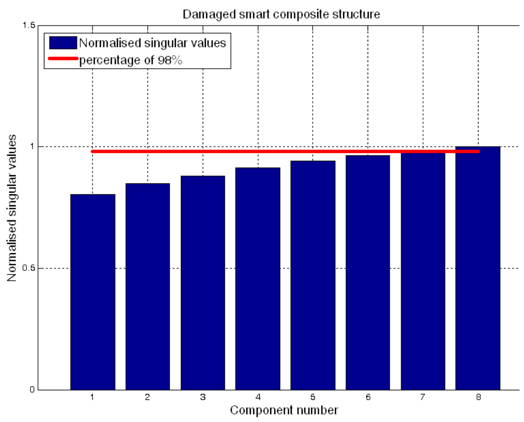

Figure 7: Order reduction of the damaged smart structure

Table 1 Mechanical property of the carbone-epoxy composite plate

Property	$E1$	$E2 = E3$	$G12 = G13$	$G23$	$\nu12 = \nu12$	ν_{23}	ρ
Unit	Gpa	Gpa	Gpa	Gpa	-	-	Kg/m^3
Value	127.7	7.217	5.712	2.614	0.318	0.38	1546

Table 2 Result of the damage detection in the case of the use of actuator PZT 10

	DI_{POD}	$\mathcal{B}$
Damage plate	**3.37**	**0.4042**
Safe plate	**0.2602**	**0.4042**

Table 3 Result of the damage detection in the case of the use of actuator PZT 7

	DI_{POD}	$\mathcal{B}$
Damage plate	**3.3056**	**0.5374**
Safe plate	**0.2190**	**0.5374**

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AUTHORS BIOGRAPHIES

Rafik Hajrya was born in Algeria, on 07 November 1984; actually, he is a PhD student at the laboratory of Processes and Engineering in Mechanics and Materials (PIMM-UMR CNRS) of Arts et Métiers ParisTech (Paris, France). He obtained: a Diploma in electronic-control engineering at the University of USTHB (Algiers, Algeria), a Master degree in Robotic and intelligent system at the University of Paris VI (Paris-France). His research interests focus on structural health monitoring of composite smart structure in particular damage detection using advanced signal processing.

Nazih Mechbal was born in Morocco, on 18 March 1971, he is an associate professor at the laboratory of Processes and Engineering in Mechanics and Materials (PIMM-UMR CNRS) at the engineering school Arts et Métiers ParisTech (ENSAM) of Paris, where he is member of the control and supervising team. He received his PhD degree in robotics from the ENSAM Paris in 1999. His research interests include structural health monitoring, robust fault detection and diagnosis, active control and robotics.

Michel Vergé was born in France, on 09 July 1950; he is a professor on control and supervising at the laboratory of Processes and Engineering in Mechanics and Materials (PIMM- UMR CNRS) of Arts et Métiers ParisTech (Paris, France). He obtained HDR at Nancy University (France) in 1991. His research interests focus on the fault detection methods and structural health monitoring.