

Identification of Equivalent Damage Growth Parameters for General Crack Geometry

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ABSTRACT

Analytical damage growth equations, such as Paris law, need the stress intensity factor for predicting damage growth. Analytical expressions for the stress intensity factor are available only for simple crack locations, geometries and loading conditions. Therefore, actual damage growth requires numerical solution, such as by finite elements. However, for estimating the uncertainty in remaining useful life (RUL), thousands of simulations of crack growth must be undertaken, which is computationally expensive. Here, an estimate of the error associated with RUL estimation based on an analytical stress intensity factor that does not consider the effects of boundary conditions, crack location or complex geometry is introduced. An effective damage parameter is identified which, although different from the true value, results in accurate damage growth prediction. Actual damage growth is simulated using the extended finite element method (XFEM) to model the effects of crack location and geometry on the relationship between crack size and stress intensity factor. The XFEM data are then perturbed with noise to simulate measurements. The damage growth parameter is then identified using least square filtered Bayesian (LSFB) method. The identified parameter can then be used with the model to estimate the RUL. Examples include center and edge cracks in a plate that experiences both horizontal and vertical finite effects and stress concentrations caused by the presence of holes. For these examples, it is found that the RUL estimates are accurate even when an inaccurate stress intensity factor model is used.*

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1. INTRODUCTION

Intuitively, model-based prognosis for a structure's health management requires accurate estimation of model parameters. Here, model parameters which are different from the true values are identified which result the same prediction in the model-based prognosis model. Once these model parameters are identified, they can be used to predict the future behavior of the system. However, many physical models are limited to simple conditions. For example, the Paris model (Paris, 1999) describes the rate of crack growth in terms of material properties and the stress intensity factor. The simplest available expression for the stress intensity factor is the infinite plate with a through-the-thickness center crack. In reality the stress intensity factor is a complicated function of applied loading, boundary conditions, crack position, geometry, and material properties. Although there are many correction factors for taking into account for finite plate size or edge cracks (Mukamai, 1987), still they are limited in representing complex engineering systems.

The objective of this paper is to demonstrate that in model-based identification, one can use simple models to predict the remaining useful life even if they do not model well actual behavior. This is accomplished through the identification of an equivalent damage growth parameter that compensates for the difference between the model and the true stress intensity factor.

A square plate is chosen as the geometry for the problem. The addition of cracks and holes to the plate causes the crack tip state of stress to experience finite plate effects in both the horizontal and vertical directions as well as stress concentrations caused by the addition of holes to the plate. As no solution is known to the authors which considers the vertical effects, the damage growth is simulated using the extended finite element method (XFEM) for calculating "true" stress intensity factors and Paris law is used to grow the crack. XFEM (Moës, 1999) allows for discontinuities to be modeled independently of the finite element

mesh, which avoids costly remeshing as the crack grows. The stress intensity factors which are the driving force for crack growth are calculated using the domain form of the contour integrals (Shih, 1998).

In practice, the actual damage sizes are measured using structural health monitoring systems in which on-board sensors and actuators are used to detect damage location and size. In this paper, instead of using actual measurement data, synthetic data are generated to demonstrate the insensitivity of RUL to errors in the stress intensity model. First, the true values of Paris model parameters are assumed. Then, the true crack will grow according to the given parameters and prescribed operating and loading conditions. Thus, the true crack size at every measurement time is known. With the true crack size, the remaining useful life is defined when the crack size reaches the critical crack size, which is a function of material, operating, and loading conditions. It is assumed that the measurement instruments may have a deterministic bias and random noise. These bias and noise are added to the true crack sizes, which are denoted as synthetic measured crack sizes. Then, these data are used to predict the effective damage growth parameters and thus the remaining useful life. In this way, it is possible to evaluate the accuracy of prognosis method.

Of the many methods available for parameter identification, the least-square-filtered Bayesian method (LSFB) (Coppe, 2009) is used to identify damage growth parameters using the synthetic data. This method applies nonlinear least-square method to the measurement data, so that the magnitude of noise can be reduced, followed by Bayesian inference, (Sheppard, 2005) to identify a probability distribution for model parameters. The identified distribution of damage growth parameters can then be used to predict the distribution of remaining useful life.

An important question that is explored in this paper is whether or not a simple stress intensity model can be used for general crack geometries for the purpose of prognosis. The key concept in this paper is that the Paris model can be considered as an extrapolation tool. Thus, even if the actual crack growth behavior is different from that obtained with simplified stress intensity expressions, Bayesian inference will identify equivalent damage growth parameters, different from the true ones, such that the model accurately predicts future damage growth behavior.

The paper is organized into the following sections. In Section 2, the crack growth model is introduced. In Section 3, the least-square-filtered Bayesian method is summarized. Results are presented in Section 4, three problems with increasingly complicated geometry, in the sense that the center crack in an infinite plate model is an increasingly bad predictor of the actual state of

stress at the crack tip. Concluding remarks and future work are presented in Section 5.

2. CRACK GROWTH MODEL

The Paris model (Paris, 1999) gives the fatigue crack growth rate as a function of material properties C and m and the stress intensity factor range ΔK as

$$\frac{da}{dN} = C(\Delta K)^m. \quad (1)$$

This model is created from experimental observation. For a center crack in an infinite plate in Mode I loading, the stress intensity factor range ΔK is given as

$$\Delta K = \sigma\sqrt{\pi a} \quad (2)$$

where σ is the applied stress and a is the characteristic crack size. The characteristic crack length a_i at the i^{th} cycle derived from Eqs. (1) and (2) is given as

$$a_i = \left[N_i C \left(1 - \frac{m}{2} \right) (\sigma\sqrt{\pi a})^m + a_o^{1-\frac{m}{2}} \right]^{\frac{2}{2-m}} \quad (3)$$

where a_o is the initial crack size and N_i the number of cycles at the i^{th} measurement. Similarly, the number of cycles to failure for a center crack in an infinite plate can be derived by integrating Eq. (1) as

$$N_f = \frac{a_c^{1-\frac{m}{2}} - a_o^{1-\frac{m}{2}}}{C \left(1 - \frac{m}{2} \right) (\sigma\sqrt{\pi})} \quad (4)$$

where a_c is the critical crack size. Note that N_f is uncertain because the initial crack size and damage growth parameters are uncertain. Although the critical crack size can be uncertain, it can be specified by airliner as a criterion to fix the damage.

In general, the accuracy of Eq. (2) depends on geometrical effects, boundary conditions, crack shape, and crack location. A more general expression (Mukamai, 1987) is

$$\Delta K = f(\lambda)\sigma\sqrt{\pi a} \quad (5)$$

where $f(\lambda)$ is the correction factor, given as the ratio of the true stress intensity factor to the value predicted by Eq. (2). The value of λ is given in terms of the geometry and characteristic crack size and is problem dependent. An example of the effect that the correction factor $f(\lambda)$ can have on the stress intensity factor curve for a range of crack sizes is shown in Figure 1 for a center crack in an infinite plate, center crack in a

finite plate, and an edge crack in a finite plate (Mukamai, 1987). For this case the assumed plate width for the finite models was 0.2 m.

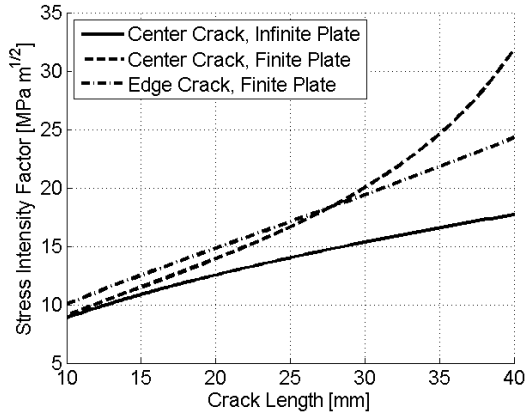


Figure 1. Comparison of stress intensity response for some correction factors and crack sizes. Plate width is 200 mm.

For complex geometry, analytical expressions as given in Eqs. (4) and (5) do not exist. In such a case, numerical methods can be used to calculate the stress intensity factor ΔK and Eq. (1) can be numerically integrated to calculate the crack size as a function of the number of cycles as is done in Appendix B.

3. LEAST SQUARE FILTERED BAYESIAN (LSFB) METHOD

Bayesian updating and least square fit are often used for identifying unknown model parameters and present advantages and limitations, but they appear to be complementary. Least square fit's ability to identify the bias and reduce the noise makes it a useful tool to process the data in order to identify the distribution of RUL using Bayesian updating. Note that we chose to update m here but similar results could be obtained by updating C or both parameters together.

The LSFB method processes information collected at every cycle by least square fit in order to reduce the noise, and identify the bias, b . The least square problem is expressed as

$$\min_{a_0, m, b} \sum_i (a_i^{meas} - b - a_i)^2 \quad (6)$$

where a_i^{meas} are the synthetic measured crack sizes with noise model to simulate measurement data.

The LSFB method assumes in this paper that the ΔK for the characteristic crack size a is given by Eq. (2), and an effective value of m is identified resulting in the same solution to Eq. (1) as though the true ΔK were known. The identified values of a_0 , m and b are

then used to generate a new estimate of the damage size at the i^{th} cycle using Eq. (3), they are referred to as filtered data. Those data are then used in Bayesian updating in order to narrow down the distribution of m and obtain a more accurate prognosis. The identified a_0 and b are considered as deterministic, and only uncertainty in m is considered in the Bayesian update.

Bayesian inference is based on the Bayes' theorem on conditional probability. It is used to obtain the updated (also called posterior) probability of a random variable by using new information. In this paper, since the probability distribution of m given a is of interest, the following form of Bayes' theorem is used (An, 2008)

$$f_{updt}(m) = \frac{l(a|m) f_{ini}(m)}{\int_{-\infty}^{+\infty} l(a|m) f_{ini}(m) dm} \quad (7)$$

where f_{ini} the assumed (or prior) probability density function (PDF) of m , f_{updt} the updated (or posterior) PDF of m and $l(am)$ is called the likelihood function, which is the probability of obtaining the characteristic crack length a for a given value of m , the derivation of the likelihood function can be found in Appendix A.

The likelihood function is designed to integrate the information obtained from structural health monitoring (SHM) measurement to the knowledge about the distribution of m . Instead of assuming an analytical form of the likelihood function, uncertainty in measured crack sizes is propagated and estimated using the Monte Carlo simulation (MCS). Although this process is computationally expensive, it will provide accurate information for the posterior distribution.

Once the distribution of m has been identified at cycle N_i , it can be used to predict the remaining useful life (RUL). The distribution of RUL is calculated at every SHM measurement cycle N_i using MCS and the RUL is estimated using Eq. (4) derived from Paris' law. This allows us to estimate the distribution and from there obtain the 5th percentile.

The 5th percentile of N_f samples is used as a conservative estimate of RUL in order to have a safe prediction. Since random noise is added to the synthetic data, the result may vary with different sets of data. Thus, the above process is repeated with 100 sets of measurement data and mean plus and minus one standard deviation intervals are plotted.

In order to show the value of the LSFB method the RUL calculated using the distribution of m_{LSFB} and the distribution (mean \pm one standard deviation) of the 5th percentile of the distribution of RUL obtained using the updated distribution of m at each inspection are compared.

4. RESULTS

For each example an aluminum 7075 square plate with edge lengths of 0.2 m and thickness 2.48 mm and an initial crack size of 0.01 m is used. Aluminum 7075 has Young's modulus E of 71.7 GPa, Poisson's ratio ν of 0.33, critical mode I stress intensity factor K_{IC} of $30 \text{ MPa}\sqrt{\text{m}}$, Paris Law constant C of $1.5\text{E-}10$, and an assumed, deterministic Paris Law exponent m of 3.8. The plate is assumed to be an aircraft panel with radius 3.25 m, which undergoes pressurization cycles of amplitude 0.06 MPa. The relatively large initial crack size is chosen because many SHM sensors cannot detect small cracks. In addition, there is no significant crack growth when the size is small. However, this size is still too small to threaten the safety of an airplane.

True crack growth data was calculated using the extended finite element method using stress calculated from the pressurization model. XFEM simulations were performed on a structured mesh of square linear quadrilateral elements with characteristic length of 1 mm. Each cycle of fatigue crack propagation was modeled until the equivalent mode I stress intensity factor exceeded K_{IC} . The characteristic crack length at each iteration was then used in the identification of an equivalent Paris Law exponent through the use of the least-square-filtered Bayesian method with the simplified stress intensity formula, Eq. (2).

4.1 Center crack in a finite plate

The first problem considered is that of a center crack in a finite plate as shown in Figure 2. Only the right half of the plate was modeled with XFEM through the use of symmetry.

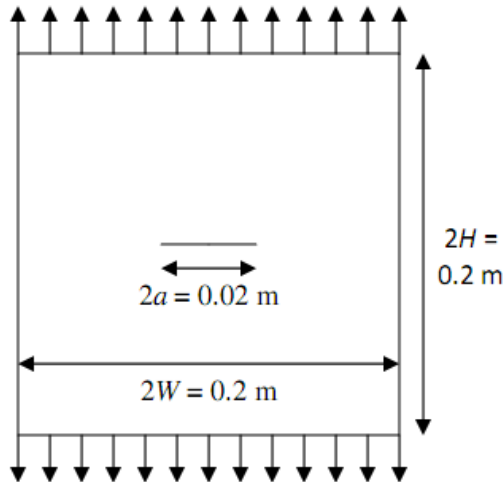


Figure 2. A center crack in a finite plate.

The corresponding curve of the correction factor $f(\lambda)$ which this edge crack represented is given in Figure 3. For this case, it was found that failure

occurred at 2070 cycles with a corresponding crack length of 37.5 mm.

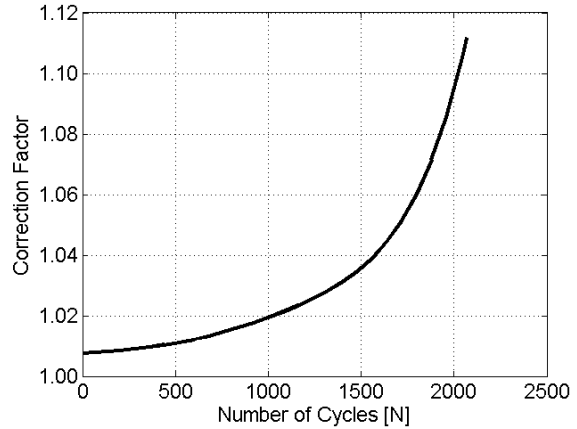


Figure 3. Correction factor for center crack.

As the LSF analysis results in a final distribution of m the predicted crack lengths for this distribution are plotted and compared directly to the XFEM data in Figure 4. The XFEM data fall within the bounds of the LSF identification.

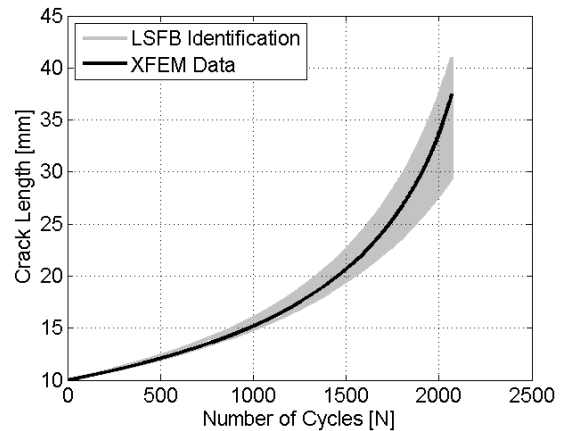


Figure 4. Comparison of XFEM crack growth data with crack growth predicted from LSF analysis.

Figure 5 shows in grey the distribution (mean \pm one standard deviation obtained from 100 sets of different measurements) of 5th percentile of RUL discussed in Section 2 for that geometry, compared to the actual remaining useful life for an arbitrarily chosen deterministic critical damage size a_c of 25 mm.

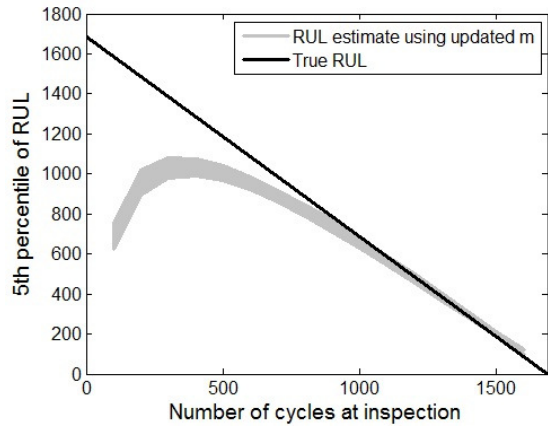


Figure 5. Distribution (mean \pm one standard deviation) of 5th percentile of RUL for a center crack.

It can be observed that the estimate of RUL converges to the actual remaining useful life from the conservative side.

4.2 Edge crack in a finite plate

Next, an edge crack in a finite plate was considered as shown in Figure 6. For this case the boundary conditions were fixing the lower right hand corner and allowing the top right corner to only move in the vertical direction.

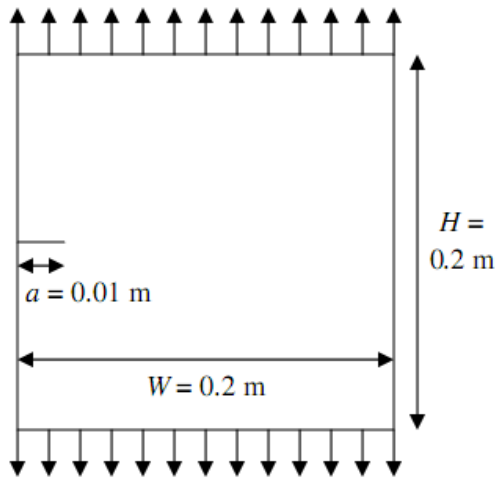


Figure 6. Edge crack in a finite plate.

The correction factor corresponding to the finite effect which this edge crack represented is given in Figure 7. For this case, it was found that failure occurred at 1018 cycles with a corresponding crack length of 27.2 mm.

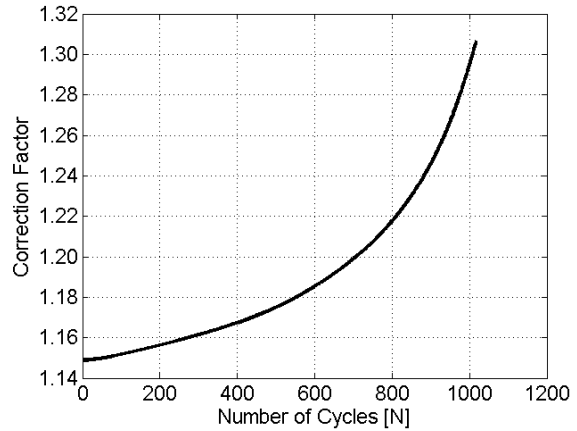


Figure 7. Correction factor for edge crack.

As the LSF analysis results in a final distribution of m the predicted crack lengths for this distribution are plotted and compared directly to the XFEM data in Figure 8. The XFEM data fall within the bounds of the LSF identification. Compared to the case of a center crack in an infinite plate the range of the identified distribution of m is wider, which is likely caused by the increased difference between the actual and assumed models for the stress intensity factor.

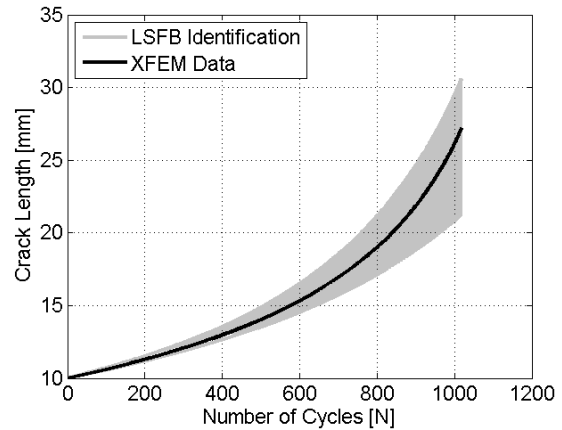


Figure 8. Comparison of XFEM crack growth data with crack growth predicted from LSF analysis.

Figure 9 shows the distribution of 5th percentile of RUL discussed in Section 2 for that geometry, compared to the actual remaining useful life. As for the previous geometry it can be observed that the estimate of RUL converges to the actual value from the conservative side.

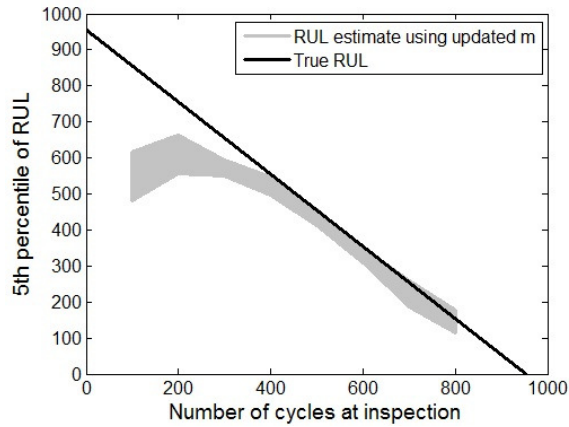


Figure 9. Distribution (mean \pm one standard deviation) of 5th percentile of RUL for an edge crack.

4.3 Center crack in a plate with holes

The final example considers differences between the actual and predicted model that may be caused by localized stress concentrations in structures. Four holes are inserted into the plate as shown in Figure 10. Only the right half of the plate was modeled with XFEM through the use of symmetry.

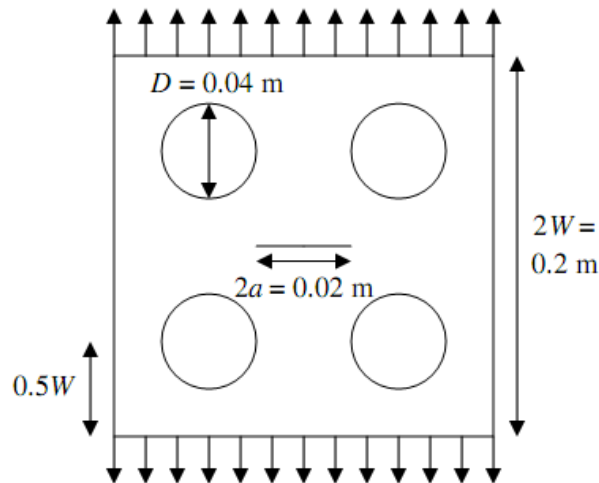


Figure 10. Center crack in a finite plate with holes.

Unlike the other examples presented, the authors are unaware of an approximation to $f(\lambda)$. This correction factor obtained from XFEM is shown in Figure 11. For this case, it was found that failure occurred at 625 cycles with a corresponding crack length of 24.2 mm.

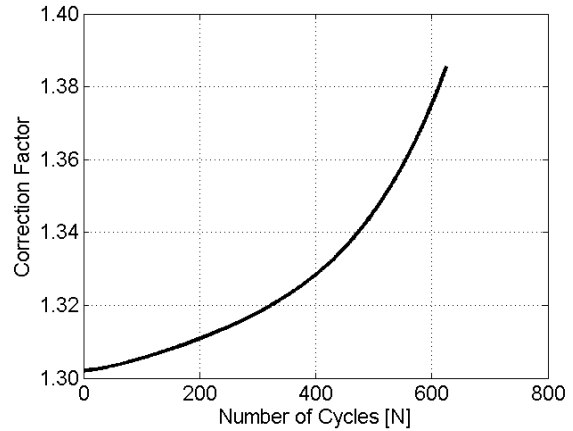


Figure 11. Correction factor for plate with holes.

As the LSF analysis results in a final distribution of m the predicted crack lengths for this distribution are plotted and compared directly to the XFEM data in Figure 8. The XFEM data fall within the bounds of the LSF identification. The identified crack size distribution is wider than others which corresponds to the model being increasingly far away from reality.

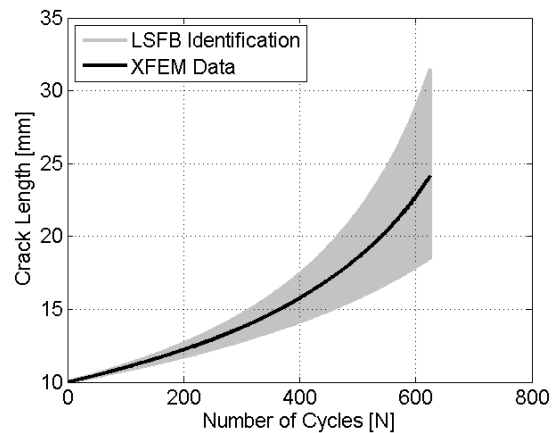


Figure 12. Comparison of XFEM crack growth data with crack growth predicted from LSF analysis.

Figure 13 shows the distribution of 5th percentile of RUL discussed in Section 2 for that geometry, compared to the actual remaining useful life for a critical damage size of 24 mm. As for the previous geometries it can be observed that the estimate of RUL converges to the actual value from the conservative side. It has to be observed that the estimation is not as accurate but this can be explained by the fact that the geometry is very different from the one assumed in the model and the number of cycles to failure is much smaller.

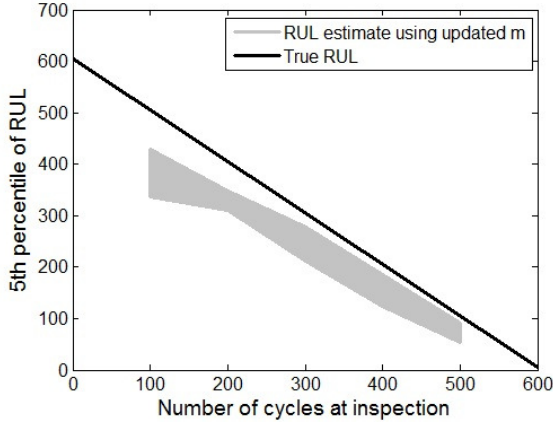


Figure 13. Distribution (mean \pm one standard deviation) of 5th percentile of RUL for a plate with holes.

5. CONCLUDING REMARKS

Effective damage growth parameters were identified using the LSF method for cases of finite and geometric effects. The stress intensity factor relationship was assumed to follow the center crack in an infinite plate and the Paris Law exponent m was identified which is correct for the incorrect stress intensity factor relationship. Damage growth was simulated at each loading cycle through the use of the extended finite element method with a reanalysis algorithm.

This represents the versatility of the proposed method in that it does not require *a priori* knowledge of the correction factor $f(\lambda)$. The mean value of the updated distribution of m and the RUL curves show good agreement with the simulated results. It is especially encouraging that the RUL converges from the conservative side.

The method is demonstrated here updating only one parameter, m , of Paris' law, the same idea can be applied to the parameters m and C together. This should allow for even more accurate results because it would allow for more flexibility in fitting the equivalent model. The feasibility of using XFEM in the calculation of the likelihood function will also be explored which may identify the true m and C .

APPENDIX A: LIKELIHOOD FOR BAYESIAN INFERENCE

The idea is to identify the damage parameters m or C from the measured half crack size that is contaminated by measurement errors. In order to do that, the measurements are compared to the simulated crack size defined above. In order to use the information in Bayes law, it is necessary to estimate the

likelihood $l(a|m)$ that for a given set of material properties m or C , $a_N^{meas} = a_N^{sim}$ or in other words:

$$d = a_N^{sim} - a_N^{meas} = 0 \quad (8)$$

If analytical expressions for the PDFs of a_N^{meas} and a_N^{sim} are available they can be used to obtain the PDF of d , then the value of this PDF at $d = 0$ is the likelihood function. Since this rarely happens, MCS will be used as the likelihood function

$$l(a|m) = P(|d| \leq \epsilon). \quad (9)$$

Note that the integration over ϵ is just a normalizing constant that is taken care of by the normalization in the Bayesian formulation.

If the likelihood $l(a|m)$ is calculated purely by sampling a_N^{meas} and a_N^{sim} then the tolerance ϵ needs to be large enough to include enough sample points to reduce the sampling error to acceptable levels. On the other hand if ϵ is large, errors will increase due to nonlinearity in the likelihood function.

It is assumed that the measurement error that controls a_N^{meas} is independent of the modeling errors that control a_N^{sim} . In that case, separable sampling can be performed by comparing all possible combinations of two individual samples.

The PDF of a_N^{sim} is not available analytically, because it is obtained from propagation of uncertainties through an analysis code. On the other hand, the measurement errors are assumed rather than propagated, and they are here assumed to be uniform in a bounded region. It is investigated how to take advantage of the given distribution of a_N^{meas} and a_N^{sim} in order to improve the efficiency or accuracy of the sampling. In this case a_N^{meas} and a_N^{sim} are scalar, such that

$$\begin{aligned} l(a|m) &= P(|d| \leq \epsilon) \\ &= 1 - P(d + \epsilon \leq 0) - P(d - \epsilon \geq 0). \end{aligned} \quad (10)$$

Using conditional expectation on the second term on the right-hand side the following expression is obtained:

$$\begin{aligned} P(d - \epsilon \geq 0) &= P(a_N^{sim} - a_N^{meas} - \epsilon \geq 0) \\ &= \int_{a_N^{sim}} P(a_N^{sim} - a_N^{meas} - \epsilon \geq 0) f_{sim}(a_N^{sim}) da_N^{sim} \quad (11) \\ &= \int_{a_N^{sim}} F_{meas}(a_N^{sim} - \epsilon) f_{sim}(a_N^{sim}) da_N^{sim} \end{aligned}$$

where $f_{sim}(x)$ is the PDF of a_N^{sim} and $F_{sim}(x)$ is the CDF of a_N^{sim} . The last relation is obtained from the definition of CDF; i.e., by considering Δa_N^{meas} as the only random variable, $P(a_N^{meas} \leq a_N^{sim} - \epsilon) = F_{meas}(a_N^{sim} - \epsilon)P(\Delta a_N^{meas} \leq \Delta a_N^{sim} - \epsilon) = F_{meas}(\Delta a_N^{sim} - \epsilon)$. Similarly, the first term can be written as

$$P(d + \epsilon \geq 0) = \int_{a_N^{sim}} P(a_N^{sim} - a_N^{meas} + \epsilon \geq 0) f_{sim}(a_N^{sim}) da_N^{sim} \\ = \int_{a_N^{sim}} F_{meas}(a_N^{sim} + \epsilon) f_{sim}(a_N^{sim}) da_N^{sim} \quad (12)$$

By combining Eqs. (11) and (12), the likelihood can be written as

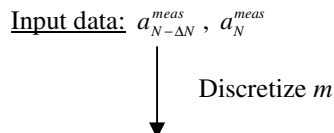
$$l(a|m) = \int_{a_N^{sim}} [F_{meas}(a_N^{sim} + \epsilon) - F_{meas}(a_N^{sim} - \epsilon)] f_{sim}(a_N^{sim}) da_N^{sim} \\ \approx 2\epsilon \int_{a_N^{sim}} f_{meas}(a_N^{sim}) f_{sim}(a_N^{sim}) da_N^{sim} \quad (13)$$

where the central finite difference approximation is used in the second relation, which becomes exact when $\epsilon \rightarrow 0$. As explained before, since the posterior PDF will be normalized, the coefficient 2ϵ can be ignored. The above expression is in particular convenient for separable MCS because the analytical expression of $f_{meas}(x)$ is known, and $f_{sim}(x)$ can be evaluated by propagating uncertainty through numerical simulation. Let M be the number of samples in MCS, the likelihood can then be calculated by

$$l(a|m) = \int_{a_N^{sim}} f_{meas}(a_N^{sim}) f_{sim}(a_N^{sim}) da_N^{sim} \\ \approx \frac{1}{M} \sum_{i=1}^M f_{meas}(a_{N,i}^{sim}) \quad (14)$$

In the literature (Li, 2009), the Gaussian function is often assumed for the likelihood function. In addition, the expression of this function remains unchanged throughout the entire process. However, the likelihood function is quite different from the Gaussian function and it varies at different crack sizes. Since the uncertainty structure of the posterior distribution strongly depends on the likelihood function in Bayesian inference, the error in the likelihood calculation directly affects the accuracy of the posterior distribution.

In the case presented here $f_{meas}(a_{N-\Delta N,i}^{sim})$ is the PDF corresponding to the uniform distribution of the measure damage size.



For every m_i :

M samples of: $a_{N-\Delta N}^{sim} = a_{N-\Delta N}^{meas} + v_i, a_{N-2\Delta N}^{sim} = a_{N-2\Delta N}^{meas} + v_i$

With $v_i \sim U(-V, V)$

$$a_N^{sim} = \left[\Delta NC \left(1 - \frac{m_i}{2} \right) (\sigma \sqrt{\pi})^{m_i} + (a_{N-\Delta N}^{sim})^{\frac{2-m_i}{2}} \right]^{\frac{2}{2-m_i}}$$

$$l(a|m) = \frac{1}{M} \sum_{i=1}^m f_{meas}(a_{N,i}^{sim})$$

APPENDIX B: EXTENDED FINITE ELEMENT METHOD

Modeling crack growth in a traditional finite element framework is a challenging engineering task. The finite element framework is not well suited for modeling crack growth because the domain of interest is defined by the mesh. At each increment of crack growth, at least the domain surrounding the crack tip must be remeshed such that the updated crack geometry is accurately represented. If a large number of cycles are to be considered, this repeated remeshing can consume a large amount of the computational time for the analysis.

The extended finite element method (XFEM) allows discontinuities to be represented independently of the finite element mesh (Moës, 1999). Arbitrarily oriented discontinuities can be modeled by enriching all elements cut by a discontinuity using enrichment functions satisfying the discontinuous behavior and additional nodal degrees of freedom. For the case of a domain containing a crack and voids (Daux, 2000) the approximation is:

$$u^h(x) = V \sum_I N_I [u_I + H a_I + \Phi_\alpha b_I^\alpha] \quad (15)$$

where N_I are the finite element shape function, V is the void enrichment function, H is the Heaviside enrichment function, Φ_α are the crack tip enrichment functions, and $u_I, a_I,$ and b_I are the classical and enriched degrees of freedom (DOF).

To decrease the computational time for the repeated solutions, a reanalysis algorithm (Pais, 2010) is used which takes advantage of the large constant portion of the global stiffness matrix represented by $\mathbf{K}_{uu}, \mathbf{K}_{ua},$ and \mathbf{K}_{aa} .

The mixed-mode stress intensity factors K_I and K_{II} for the given cracked geometry were calculated using the domain form of the interaction integrals (Shih, 1988). The direction of crack growth was calculated using the maximum circumferential stress criterion (Shih, 1998). The effective stress intensity factor (Tanaka, 1974) given as

$$\Delta K_{eff} = \sqrt[4]{K_I^4 + 8K_{II}^4} \quad (16)$$

was used to convert the mixed-mode stress intensity factors into a single value for used in Paris law. The crack growth at a given cycle is given as

$$\Delta a = C (\Delta K_{eff})^m. \quad (17)$$

The implementation of XFEM used here was verified using the center crack in a finite plate problem given in Section 3.1. For this problem the theoretical finite correction factor based on the equations of elasticity for a center crack in a finite plate (Mukamai, 1987) is given as

$$f(\lambda) = \sqrt{\sec\left(\frac{\pi\lambda}{2}\right) \left(1 - \frac{\lambda^2}{40} + \frac{3\lambda^4}{50}\right)} \quad (18)$$

where $\lambda = a/W$ and a and W are the half crack length and half plate width. This model assumes that the plate is finite in the x-direction and infinite in the y-direction. A comparison of the crack lengths as a function of the number of cycles was first performed to ensure the accuracy of the XFEM data provided to the identification routine. As there is no closed form solution for the crack size as a function of N due to the finite correction factor given in Eq. (5) the forward Euler method with 10^4 steps was used. This step size represents less than 0.1 percent change from 10^3 steps. A comparison of the results is shown in Figure 14.

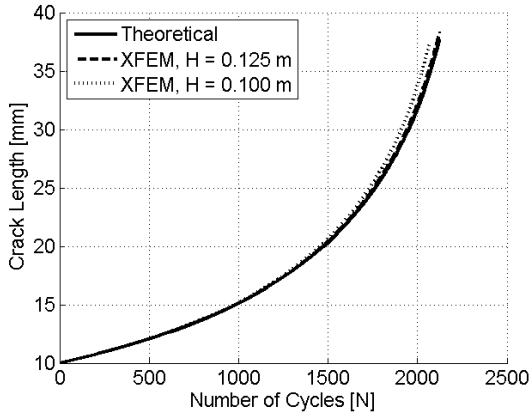


Figure 14. Theoretical and XFEM crack growth curves.

It was noticed that for the given plate geometry the finite and XFEM models predicted different crack growth curves. Increasing the height of the plate leads to good agreement with the theoretical values indicating that the chosen crack configuration has a finite effect from both the vertical and horizontal directions. The resulting difference in $f(\lambda)$ caused by the vertical finite effect as a function of the number of cycles is shown in Figure 15.

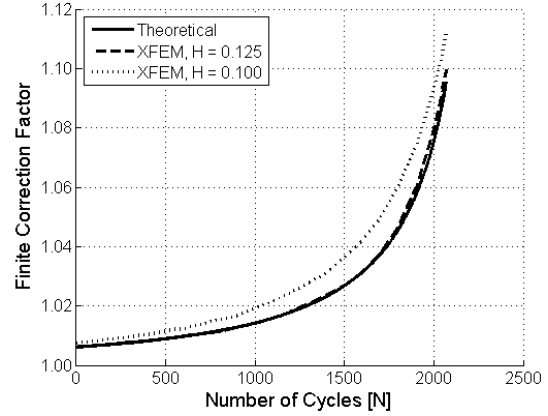


Figure 15. Theoretical and XFEM prediction of $f(\lambda)$.

ACKNOWLEDGMENT

This work was supported by the Air Force Office of Scientific Research under Grant FA9550-07-1-0018 and by the NASA under Grant NNX08AC334.

NOMENCLATURE

a	current crack length
a_C	critical crack length
a_i	crack length at cycle N
a_o	initial crack length
b	bias applied to crack size data
C	Paris Law constant
m	Paris Law exponent
m_{CRIT}	effective m value for critical crack length
m_{LSFB}	final LSF distribution of m
m_{LSQ}	effective m value from least square fit
m_{TRUE}	true value of m
ΔK	stress intensity factor range
K_{IC}	critical mode I stress intensity factor

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