# Bayesian Reliability Prognosis for Systems with Heterogeneous Information

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**ABSTRACT**

A Bayesian methodology for prognosis of system reliability with heterogeneous reliability information is presented. Available information may be in the form of physics-based or experiment-based mathematical models, historical reliability data, or expert opinion. Such information typically describes the failure rates of individual components of the system and does not provide information on dependencies between them. The Bayesian methodology presented in this paper addresses this concern by learning the conditional probabilities in the Bayes network as observations about the system are made. First, the component and system faults are defined and the failure event tree is established. Bayesian priors for probabilities of both individual failure events and the conditional probabilities between them are established using various types of experimental data, expert opinion, or simulation data. Both the priors and conditional probabilities are updated as new data is collected, leading to an updated prognosis of system reliability. The methodology is demonstrated on an automobile startup system.[1]

## 1 INTRODUCTION

Reliability and prognosis are measures of how likely a system will be able to perform a prescribed task and may be given in terms of the system's probability of failure. Commonly used methods for analyzing system reliability are failure modes and effects analysis (FMEA) (Høyland & Rausand, 1994), failure modes and effects and criticality analysis (FMECA) (Høyland & Rausand, 1994), fault and event trees (Høyland & Rausand, 1994; Stapelberg, 2009), reliability block diagrams (RBD) (Kumar, 2000), Ishikawa diagrams (Sallis, 2002), root cause analysis (Andersen & Fagerhaug, 2006), and Bayes networks (Bearfield & Marsh, 2005; Marsh & Bearfield, 2007).

Bayes networks have a wide array of uses and have been applied to many fields including cognitive assessment (Martin & VanLehn, 1995), medicine (Sierra, Inza, & Larrañaga, 2000), social networking (Min, Jang, & Cho, 2009), sensor diagnosis and validation (Mengshoel, Darwiche, & Uckun, 2008), and crack modeling (Patrick et al., 2007). In a Bayes network, an observation at one component in the system allows the entire system to be updated. An important part of this process for a Bayes network with discrete events is populating the conditional probability tables with accurate conditional probabilites. Naturally, the effect of inaccurate conditional probabilities is to decrease the accuracy of the results obtained from the Bayes network. The consequences of this can be quite severe.

Toyota has in the past few years struggled to find the cause of potentially lethal unintended acceleration. As of 2007, the automaker blamed faulty floormats (Whoriskey, 2010). Finally, in September 2009, Toyota issued a massive recall to fix the floormat problem only to have to issue another recall in 2010, as unintended acceleration was still reported, and finally resolved to install brake override systems on all future models (Allen & Sturcke, 2010; McCurry, 2010; "Toyota Consumer Safety Advisory: Potential Floor Mat Interference with Accelerator Pedal," 2009). The initial misdiagnosis has undermined confidence in the Toyota brand and had a sharp financial impact on the company.

To safeguard against such a debacle when a Bayesian network is the analytical tool, conditional probability tables should be filled with the most accurate information possible. Methodologies exist for making such estimates using existing data or expert opinion. In a six step algorithm for learning Bayes nets, Lucas (Lucas, 2002) uses existing data to determine conditional probabilities. Heckerman (Heckerman, 1995) discusses using supervised learning methods which utilize existing data to determine the values in the conditional probability tables. Das (Das, 2004) argues that the databases of information for supervised learning are few and far between and instead proposes a methodology invoking expert opinion.

Pearl (Pearl, 1988) states that new evidence will change the conditional probabilities in the Bayesian network. Conditional probabilities therefore require updating as new evidence is obtained. Unfortunately, this requires great computational effort. Conditional probabilities are assumed constant instead.

When using a Bayes net to estimate system reliability, data from sources such as the Government-Industry Data Exchange Program (GIDEP), will often be in the form of failure rates for particular components and will not have the conditional failure probabilities for unique engineering systems. The expert opinion alternative is likely to provide uncertain information. It is necessary to have an approach which supplements the current insufficient information about the system with new information to accurately describe system behavior and reduce uncertainty. If it is possible to collect observations about the complete system, the conditional probabilities can be updated using a Bayesian approach.

In the following sections, a method for updating conditional probabilities as data is obtained and estimating system reliability is developed. Typical implementations of Bayes networks keep the conditional probabilities constant, and only update the probabilities of the individual nodes. In the proposed methodology, as data is collected, the system state is noted and conditional probabilities in the Bayes network and the system reliability estimate are updated via an algorithm little more complex than typical propagation through a Bayesian network. The methodology works on any Bayesian network whose nodes may be described by Bernoulli variables. It is demonstrated on a Bayesian Network for estimating the reliability of part of an automobile starter and ignition system.

## 2 SYSTEM RELIABILITY

The reliability of a system depends on the probability of events occurring at the component and subsystem levels that lead to the system failure event. The collection of events which leads to failure is easily represented by a Bayes network, which is described in detail by Pearl (Pearl, 1988). A Bayes network clearly displays the dependencies between components in a system through conditional probabilities and provides a simple method for inferring information about the entire system when observations are made in one part of the system.

### 2.1 Bayes Networks

A Bayes network is a directed acyclic graph whose nodes represent random variables and whose edges indicate conditional probabilities between the nodes. Conditional probabilities with inputs from parent nodes determine the probability of the random variable represented by the child node. A simple Bayes network is shown in Figure 1.

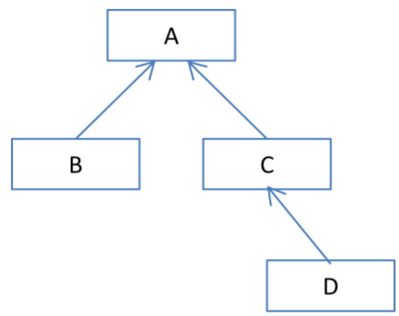


Figure 1: A simple Bayes network

Each node in a Bayes network with discrete events has a conditional probability table. Table 1 shows sample conditional probabilities at each of the four nodes.

Observations about one node may be propagated through the network to infer probabilities at other nodes using Bayes' theorem. The discrete form of Bayes' theorem is shown in Eq. 1, where $\theta$ is a discrete random variable, $\epsilon$ is an observed outcome, and $n$ is the number of possible values for $\theta$.

Table 1: Bayes network probability tables

| *P(A\|B,C)* | A=T | A=F |
|---|---|---|
| B=T, C=T | .98 | .02 |
| B=T, C=F | .95 | .05 |
| B=F, C=T | .75 | .25 |
| B=F, C=F | .5 | .5 |

| *P(C\|D)* | C=T | C=F |
|---|---|---|
| D=T | .67 | .33 |
| D=F | .91 | .09 |

| *P(B)* | B=T | B=F |
|---|---|---|
| | .96 | .04 |

| *P(D)* | D=T | D=F |
|---|---|---|
| | .8 | .2 |

$$P(\theta = \theta_i|\epsilon) = \frac{P(\epsilon|\theta = \theta_i)P(\theta = \theta_i)}{\sum_{i=1}^{n} P(\epsilon|\theta = \theta_i)P(\theta = \theta_i)} \quad (1)$$

The continuous formulation of Bayes' Theorem may be written similarly as

$$f''(\theta) = \frac{P(\epsilon|\theta)f'(\theta)}{\int_{-\infty}^{\infty} P(\epsilon|\theta)f'(\theta)} \quad (2)$$

where $f'(\theta)$ is the prior density function of $\theta$, $f''(\theta)$is the posterior density function of $\theta$, and $P(\epsilon|\theta)$ is a function of $\theta$ describing the probability of observing $\epsilon$ given $\theta$. $P(\epsilon|\theta)$ is also known as the likelihood function $L(\theta)$. The denominator $\int_{-\infty}^{\infty} P(\epsilon|\theta)f'(\theta)$ is a constant which ensures that $\int_{-\infty}^{\infty} f''(\theta)\, d\theta = 1$ and is a true density function.

To demonstrate the calculation in a Bayes network , consider the Bayes network in Figure 1. The probability of $A = T$ is

$$\begin{aligned} P(A = T) = \\ P(A = T|B = T, C = T)P(B = T)P(C = T) \\ +P(A = T|B = T, C = F)P(B = T)P(C = F) \\ +P(A = T|B = F, C = T)P(B = F)P(C = T) \\ +P(A = T|B = F, C = F)P(B = F)P(C = F) \end{aligned} \quad (3)$$

The conditional probabilities used in Eq. (3) are given in Table 1. However, the probabilities for $P(C = T)$ and $P(C = F)$ must be calculated as

$$\begin{aligned} P(C = T) = \\ P(C = T|D = T)P(D = T) \\ +P(C = T|D = F)P(D = F) \end{aligned} \quad (4)$$

and $P(C = F) = 1 - P(C = T)$. Substituting into Eq. (3),

$$P(A = T) = 0.959858 \quad (5)$$

This is the prior probability for $P(A = T)$. Upon observing the evidence $\epsilon$ that $D = T$, the posterior of $P(A = T)$ is found by substituting $P(D = T) = 1$ and $P(D = F) = 0$ in Eq. (4)

$$P(A = T|\epsilon) = .957996 \quad (6)$$

If $A = T$ is observed as $\epsilon$ instead, the network may be updated using Eq. (1). The posterior probability for $D = T$ is

$$P(D = T|\epsilon) = \frac{P(\epsilon|D = T)P(D = T)}{P(\epsilon|D = T)P(D = T) + P(\epsilon|D = F)P(D = F)} \quad (7)$$

Entering the probabilities into Eq. (7)

$$P(D = T|\epsilon) = .989157 \quad (8)$$

The probabilities for nodes $B$ and $C$ may be updated similarly. The updating process gives Bayes networks their power, as an observation at any node can be used to infer probabilities at other nodes.

## 3 UPDATING CONDITIONAL PROBABILITIES

The Bayes network updates the probability of an event occurring based on the occurrence of another event elsewhere in the network. This works under the assumption that the tabulated conditional probabilities are correct. However, conditional probabilities are difficult to obtain and thus there is uncertainty in their value.

The information on which conditional probabilities are based may be heterogeneous in form and in source. It could consist of a simple point estimate from an expert source or lower and upper bounds derived from experiments. Since the uncertainty in the information may provide incorrect conditional probabilities, it is preferable to update the conditional probabilities as more information is obtained instead of using a constant conditional prior probability.

If observations are made at a node $A_i$ and at its parent nodes in the Bayes network (the normal case is an observation at only node $A_i$), the evidence no longer pertains to the total probability of occurrence of the event at node $A_i$. It instead relates to a conditional probability at node $A_i$ (or probability of an initiating event). Using the example in Section 2, if $C = T$ is observed and simultaneously it is known that $D = T$, $B = F$, and $A = T$, the evidence $\epsilon$ does not correspond to $C = T$ but to $C = T \mid D = T$. Thus, $P(C = T \mid D = T)$ should reflect this new information. In other words, if the state of the system is known, this evidence can be used to update the conditional probabilities.

Thus the process for updating conditional probabilities has two major steps. First, the prior distributions of the conditional probabilities of failure are established based on existing information. Next, the entire system is observed, resulting in evidence which is used to update the conditional probability distributions.

### 3.1 Framework for Updating Conditional Probabilities

Observations of discrete events in a Bayes network may be thought of as observations of Bernoulli trials. This is true under the assumption that the trials are not correlated. In a Bayes network with $k$ discrete events $A_1, A_2, \ldots, A_k$, each event is treated as a discrete variable which takes on a value of either true or false (occurs or does not occur). The observation of true occurs with probability $p$. Thus, for an event $A_i$ in a Bayes network, a binomial distribution describes the number of occurrences of $A_i$ = *true* in $n$ Bernoulli trials. When the state of the system is known, $p$ is one of the $j$ conditional probabilities for $A_i$ ($j \geq 1$), here denoted $p_i^j$, and is listed in the conditional probability table for $A_i$.

The probability of failure $p$ used in the binomial distribution may be described by the beta distribution. Schlaifer (Schlaifer, 1978) has shown that the beta distribution is a reasonable prior distribution for a probability $p$. For added convenience, it can be bounded between zero and one by letting $a = 0$ and $b = 1$. The beta distribution is

$$f(x) = \frac{1}{B(q,r)} \frac{(x-a)^{q-1}(b-x)^{r-1}}{(b-a)^{q+r-1}} \quad (9)$$

where $a \leq x \leq b$ are the bounds of the distribution, $q$ and $r$ are shape parameters, and $B$ is the beta function. $B(q,r)$ may be taken as

$$B(q,r) = \int_0^1 x^{q-1}(1-x)^{r-1}dx \quad (10)$$

The expected value of the beta distribution is

$$E(X) = a + \frac{q}{q+r}(b-a) \quad (11)$$

and the variance is

$$Var(X) = \frac{qr}{(q+r)^2(q+r+1)}(b-a)^2 \quad (12)$$

For each Bernoulli trial, observations are made about every event in the Bayes network and its parent nodes. The beta distribution parameters are updated to reflect the observations. Schlaifer (Schlaifer, 1978) shows that $q$ and $r$ with a uniform prior distribution (i.e. $q = r = 1$) may be taken as

$$\begin{aligned} q &= 1 + m \\ r &= 1 + n - m \end{aligned} \quad (13)$$

where $m$ is the number of occurrences in $n$ trials.

Each time the set of observations $O$ for the entire Bayes network is collected, the parameters $q$ and $r$ are subsequently updated as shown in Eq. (13). Every $p_i^j$ has a corresponding $m$ and $n$, which are initially taken as zero and denoted $m_i^j$ and $n_i^j$ as well as a corresponding $q$ and $r$, denoted as $q_i^j$ and $r_i^j$.

The algorithm for conditional probability updating at a node $A_i$ may be summarized as follows. For each discrete point in time,

1. Observe whether parents of $A_i$ are true or false. If no parents exist, move to 3.
2. Locate corresponding conditional probability, identified by $j$.
3. Update number of trials
   $n_i^j(t) = n_i^j(t-1) + 1$
4. Observe if Ai is true
5. Update number of success
   $m_i^j(t) = m_i^j(t-1) + 1$ if $A_i$ = *true*
6. Update $q_i^j = 1 + m_i^j(t)$
7. Update $r_i^j = 1 + n_i^j(t) - m_i^j(t)$
8. Repeat for nodes $A_1, A_2, \ldots, A_k$

Initial values of $m_i^j$ and $n_i^j$ are taken as zero, with exception given to instances where prior information is available.

### 3.2 Establishing Priors with No Information Available

When no information is available for the conditional probability $p_i^j$, a uniform prior for $p_i^j$ is appropriate because it lets all values of $p_i^j$ be equally likely, initially. By setting beta distribution parameters $q = 1$, $r = 1$, $a = 0$, and $b = 1$, a uniform prior bounded by zero and one is assumed for $p_i^j$.

### 3.3 Establishing Priors with Some Information Available

It may be the case that some information about $p_i^j$ is known. This information may come in the form of confidence intervals, upper and lower bounds, or a point estimate provided by data, simulation, or opinion. This may be the case if some in situ testing of a component has been performed or if say a seasoned operator of particular piece of machinery can describe its performance characteristics. The information is used to determine the shape parameters of the beta distribution (instead of starting with $q = r = 1$) and is akin to estimating initial values for $n$ and $m$. The better the available information is, the stronger the prior will be, resulting in faster convergence for $p_i^j$ and the entire Bayes network.

Translating expert opinion or experimental data into priors for the beta distribution is not always simple.

The most straightforward situations assume a uniform distribution between an upper and lower bound. However, if a confidence interval or just a point estimate is provided, some decisions must be made about how to incorporate this information best into the prior.

If a beta distributed confidence interval [LB, UB] with significance level $\alpha$ is given, it is possible to solve for the prior $q$ and $r$ by estimating the mean and variance of the data using Eq. (14) and Eq. (15), which are derived using the methods of moments.

$$q = \hat{\mu}\left(\frac{\hat{\mu}(1-\hat{\mu})}{\hat{\sigma}^2} - 1\right) \tag{14}$$

$$r = (1-\hat{\mu})\left(\frac{\hat{\mu}(1-\hat{\mu})}{\hat{\sigma}^2} - 1\right) \tag{15}$$

Unfortunately, in this case with a confidence interval given, the mean and variance are unknown so cannot be calculated. Since the beta distribution for $p$ may converge to a distribution with relatively normal properties, the confidence interval is taken as a normal confidence interval. The estimate for the mean is

$$\hat{\mu} = \frac{UB + LB}{2}. \tag{16}$$

The variance may be found by

$$\hat{\sigma}^2 = \left(\frac{(UB-\hat{\mu})}{z_{1-\alpha/2}}\right)^2 \tag{17}$$

where $z_{1-\alpha/2}$ is the standard normal variable.

When a complete confidence interval is unavailable, such as when only upper and lower bounds are given or just a point estimate of $p$ is given, it is possible to obtain estimates for $q$ and $r$ again by assuming normality. A significance level and/or bounds which reflect the amount of uncertainty in the information must also be assumed. The condition that $q$ and $r$ are greater than zero must be enforced.

Alternatively, if upper and lower bounds are available, instead of assuming a significance level, the $a$ and $b$ bounds of the beta distribution may be set, still assuming a uniform prior.

### 3.4 Extensions Beyond the Instantaneous Case

The methodology can be extended from the case of an instantaneous event which is repeated over and over to a mission lasting a particular amount of time. In the case of a mission, the future state of the system is correlated to the current state. To capture this correlation, a dynamic Bayesian network is used and the system evaluated at each discrete point in time. The number of successes and failures is tracked and used to update the conditional probabilities.

The correlation between times $t$-1 and $t$ is accounted for in the structure of the dynamic Bayes net and does not undermine any of the independent trial assumptions required for updating conditional probabilities. When a particular state such as $A_1^{t-1} = F$ , $A_5 = T$ occurs, $A_3$ may be true with probability $p$ or false with probability $1 - p$. This is the case every time the state $A_1^{t-1} = F$ , $A_5 = T$ occurs. An observation about whether $A_3$ is true or false when the system state is $A_1^{t-1} = F$ , $A_5 = T$ is independent of previous observations under this system state because correlation is considered only between the current and past values of a specific node (which is handled by the Bayes net structure) and not between system states. Being a discrete variable which can be true or false and whose observations are independent of one another, the conditional probability is thus a binomial random variable. The probability $p$ associated with this binomial distribution may be taken as a beta distribution and updated as more observations are made about the value of $A_5$ when the state $A_1^{t-1} = F$ , $A_5 = T$ occurs.

Due to the amounts of data required, the extension of the methodology to a mission is best suited for applications where the mission is repeated. The more data that is collected, the more accurate the conditional probabilities will be.

A further extension of the methodology is the ability to include incipient faults. This is currently achieved by using a damage index comprised of several discrete values to rate the severity of the incipient fault.

## 4 EXAMPLE PROBLEM

For illustration of the methodology, the event where a car starter fails to crank the engine is considered. Rosenthal (Rosenthal, 2010) shows a decision tree for diagnosing and repairing a car that refuses to start. This tree shows qualitative causal relationships between events, allowing for adaptation into a Bayes network, which may be used to calculate the total probability of the car not starting. The events $A_i$ in the Bayes network (see Appendix) are discrete and are described by variables which may assume values of either true or false. Each node has a corresponding probability table.

The updating of probability tables began with observing the true or false values of the initiating events. The distributions which describe the probability of occurrence of the initiating events were then updated. A value was sampled from each updated distribution and used in updating distributions of any child nodes. The process is repeated until the distributions at all nodes have been updated. This required having an observation for every node telling whether that node was true or false. These observations formed the entire set of observations $O$ for the network. Once all distributions in the network had been updated, the system prognosis was updated via forward propagation through the Bayes network and the entire process was repeated with a new set of observations. The data was assumed to be collected by testing many independent realizations of the same type of car without replacement.

### 4.1 Results

Conditional probabilities were assumed to have uniform priors, except for the probabilities given in Table **2**, which have heterogeneous initial data from experiments and expert opinion. Several values of $p_i^j$ have complete confidence intervals given, which are assumed to have normal properties. Others have only bounds or just the mean given and missing information is marked as a "Guess." The corresponding initial $q_i^j$ and $r_i^j$ values for the prior beta distributions are shown in Table 3 - Table 5 along with the results after 20,000 observations of the entire state of the system.

After 20,000 complete observations of the system, the total probability of the system had a mean of 0.3655 and standard deviation of 0.0108. A histogram of the probability of failure of the system is shown in Figure 2. Figure 3 shows the evolution of the mean probability of system failure event (A0 = T) as more observations are made. The values in Figure 3 were obtained by using Eq. (11) to estimate the mean value of each beta distribution and propagating these mean values through the Bayes network.

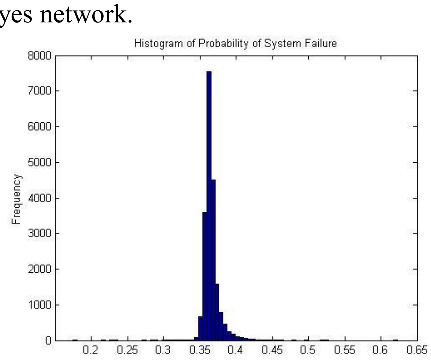


Figure 2: Histogram of Probability of System Failure After 20,000 Sets of Observations, $O$

Table 2: Initial Data

| | α | LB | UB | MEAN |
|---|---|---|---|---|
| P(A3\|A4=T, A5=F, A6 = F) | Guess .4 | - | Guess .7 | .5 |
| P(A3\|A4=T, A5=T,A6=F) | Guess .15 | .5 | .7 | .6 |
| P(A3\|A4=F, A5=T, A6=F) | .05 | .6 | .8 | .7 |
| P(A0\|A1=T,A2= F,A3=F) | .01 | .95 | .97 | .99 |
| P(A1) | .01 | .1 | .18 | .14 |
| P(A2) | .01 | .07 | .14 | .105 |
| P(A4) | .01 | .02 | .07 | .045 |
| P(A5) | .01 | .03 | .1 | .065 |
| P(A6) | .01 | .04 | .11 | .075 |

Table 3: Results for Initiating Causes

| | Prior | | Posterior | | |
|---|---|---|---|---|---|
| | | E(p) | | E(p) | True $p$ |
| A1=T | q = 40.3 r = 247.7 | .1400 | q = 3130 r = 17158 | .1543 | .15 |
| A2=T | q = 30.8 r = 262.9 | .1050 | q = 2000 r = 18294 | .0985 | .10 |
| A4=T | q = 11.8 r = 251.3 | .0450 | q = 1017 r = 19246 | .0502 | .05 |
| A5=T | q = 12.3 r = 177.3 | .0650 | q = 1243 r = 18946 | .0616 | .06 |
| A6=T | q = 16.2 r = 200.3 | .0750 | q = 1407 r = 18809 | .0696 | .07 |

Table 4: Results for Node A3

| | Prior | | Posterior | | |
|---|---|---|---|---|---|
| | A3 = T | E(p) | A3 = T | E(p) | True *p* |
| A4=T A5=T A6=T | q = 1 r = 1 | .5 | q = 2 r = 1 | .6667 | .9 |
| A4=T A5=F A6=T | q = 1 r = 1 | .5 | q = 47 r = 13 | .7833 | .8 |
| A4=T A5=F A6=F | q = 1.7 r = 1.7 | .5 | q = 513.7 r = 343.7 | .5991 | .6 |
| A4=T A5=T A6=F | q = 1 r = 1 | .5 | q = 44 r = 15 | .7458 | .7 |
| A4=F A5=T A6=T | q = 1 r = 1 | .5 | q = 56 r = 25 | .6914 | .65 |
| A4=F A5=F A6=T | q = 29.2 r = 19.5 | .6 | q = 835.2 r = 462.5 | .6436 | .65 |
| A4=F A5=F A6=F | q = 1 r = 1 | .5 | q = 830 r = 15814 | .0499 | .05 |
| A4=F A5=T A6=F | q = 55.7715 r = 23.9 | .7 | q = 854.8 r = 284.9 | .7500 | .75 |

Table 5: Results for Node A0

| | Prior | | Posterior | | |
|---|---|---|---|---|---|
| | A0 = T | E(p) | A0 = T | E(p) | True *p* |
| A1=T A2=T A3=T | q = 1 r = 1 | .5 | q = 51 r = 1 | .9808 | .99 |
| A1=T A2=F A3=T | q = 1 r = 1 | .5 | q = 425 r = 8 | .9815 | .98 |
| A1=T A2=F A3=F | q = 270.1 r = 8.4 | .97 | q = 2520 r = 86.4 | .9669 | .97 |
| A1=T A2=T A3=F | q = 1 r = 1 | .5 | q = 240 r = 8 | .9677 | .975 |
| A1=F A2=T A3=T | q = 1 r = 1 | .5 | q = 278 r = 8 | .9720 | .975 |
| A1=F A2=F A3=T | q = 1 r = 1 | .5 | q = 2296 r = 73 | .9692 | .97 |
| A1=F A2=F A3=F | q = 1 r = 1 | .5 | q = 513 r = 12328 | .0400 | .04 |
| A1=F A2=T A3=F | q = 1 r = 1 | .5 | q = 941 r = 516 | .6458 | .65 |

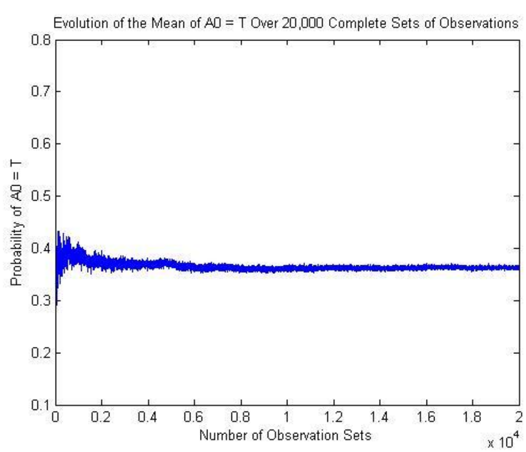

Figure 3: Evolution of the Mean of System Failure with Number of Observation Sets

The results show that reasonable estimates of the conditional probabilities within a Bayes network of discrete events may be obtained if enough observations about the system can be made. The quality of each $p_i^j$ estimate depends on the number of parent nodes $A_i$ has and its distance from initiating events, and the amount of data collected.

## 5 CONCLUSION

A Bayesian reliability prognosis methodology has been presented. To correctly model dependencies, the events in the Bayes network were treated as Bernoulli random variables. The distributions of these variables were assumed to be binomial and the beta distribution was used to describe the probability $p$ associated with each Bernoulli trial. The probability $p$ was equivalent to one of the conditional probabilities $p_i^j$ associated with the event $A_i$, as determined by the state of the system. Heterogeneous data including confidence intervals, upper and lower bounds, and a point estimate was used to inform the priors for the conditional probabilities $p_i^j$. The system prognosis given by the Bayes network was then updated to account for the latest observation. The methodology has been demonstrated on an example of a car starter. Future work will extend the methodology to include continuous time missions.

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**APPENDIX**

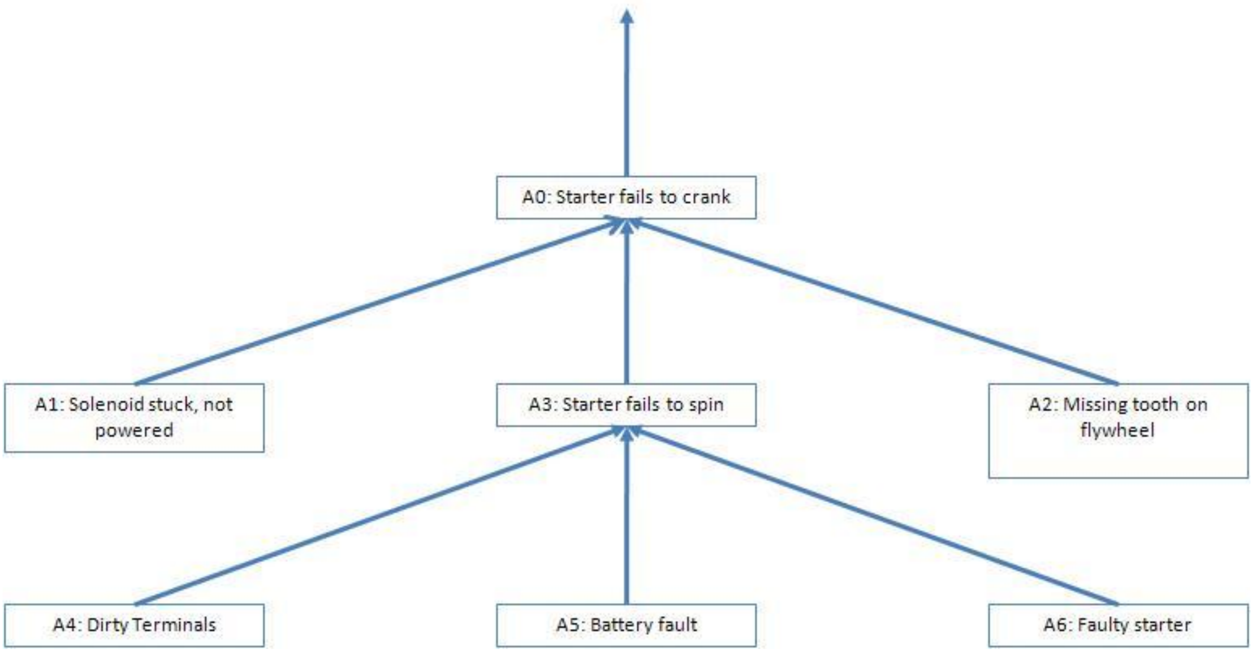


Figure 4: Bayes network for car starter failing to crank engine.