Using Condition Based Maintenance to Improve the Profitability of Performance Based Logistic Contracts

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ABSTRACT

This paper outlines a scheduling algorithm which leverages Condition-Based Maintenance (CBM) data to determine when maintenance should be performed. The objective of the scheduler is to reduce the cost associated with Performance-Based Logistics contracts, which ultimately improves the profit margins of Product Support Providers.

An example consisting of 50 aircraft for which regular recurring maintenance and CBM actions are required is analyzed as a representative problem both in term of complexity and scale. The results indicate that significant cost savings can be achieved by utilizing a CBM scheduling algorithm. In addition, to the maintenance cost savings, the CBM scheduling algorithm is also able to identify potential resource limitations within the maintenance organization.*

1. INTRODUCTION

One of the driving forces behind awarding a Performance-Based Logistics (PBL) contract is that a single Product Support Provider (PSP) will ensure operational readiness for a fixed price. In essence, the PSP is responsible for managing the supply chain and establishing the necessary support structure to meet the performance goals specified by the contracting agency. The PSP is therefore not just responsible for coordinating repairs, but must also manage consumable procurement and establish depot repair requirements.

The PBL contract will essentially guarantee that the cost associated with maintaining serviceable high-value components will be defined upfront given a pre-specified component utilization profile. From the contracting agency's perspective, there are several obvious advantages associated with PBL contracts which include:

1. Fixed fleet operational cost.
2. Performance guarantees.
3. Continuous aircraft system/component modernization.
4. Improved maintenance response time in contingency situations.

Items 1 and 2 tend to be explicitly outlined in the contract, whereas items 3 and 4 are typically secondary benefits associated with entering into such contracts. For instance, it is in the PSPs best interest to improve the design and the reliability of the aircraft components such that it becomes as durable as possible thereby improving the PSP's profitability under the PBL contract.

From the PSP's perspective the revenue associated with PBL offerings is generally lower than the revenue associated with legacy offerings. Hence, the PSPs must strive to reduce the cost associated with maintaining the fleet to achieve an attractive profit margin. Consequently, the challenge for the PSPs is to minimize the cost associated with the PBL contract while simultaneously satisfying the government's readiness requirements. There are several factors that can be optimized to make a PBL contract more profitable such as reducing the repair time, optimizing the repair process and manage the volume of repairs being performed.

To achieve these improvements, implementing Condition-based Maintenance (CBM) on high-valued components (such as occurs in the aviation industry) can be very beneficial (B. Haan). By applying CBM to systems covered by a PBL contracts, it is possible to assess, track and forecast the health of the components such that potential component failures can be identified early. Additionally, the system health forecasting allows the PSPs to schedule maintenance early thereby not only avoid potential surges in the number of repairs the maintenance facilities have to complete but also improve the supply chain management by having a timely and accurate estimate of the parts needed to perform the maintenance.

In this paper, we outline an approach that leverages the benefits a CBM system and philosophy can provide to optimize the maintenance schedule for an aircraft fleet. In particular, the approach utilizes remaining useful life (RUL), future usage and maintenance scheduling constraint information to derive a cost effective maintenance schedule. The resulting schedule can then be used to minimize the spare parts on hand, and thereby address the “bullwhip” effect (see R. Warburton) within the supply chain, while simultaneously incorporate appropriate safeguards against unexpected failures without incurring unreasonable cost overhead.

2. PAPER OVERVIEW

The paper is structured as follows: In section 3, a short overview of research into effective scheduling algorithms is provided. In section 4, the assumptions associated with the problem formulation and the resulting complexity is discussed and a simple scheduling scenario is described. Section 5 provides an overview of the scheduling algorithm along with a short discussion on how visibility into the restrictive constraints can aid in identifying points of potential improvements within the maintenance organization. In section 6, the results achieved by simulating the scenario described in section 4 are shown followed by a short discussion on how the results impacts the cost of a PBL contract. Following the discussion, section 7 provides a few concluding remarks.

3. RELATED RESEARCH

The problem of optimally scheduling tasks has been an active area of research for a couple of decades. The problem has received particular attention from researchers that strive to optimize the resource utilization within multi-processor systems. Even though the CBM enabled scheduling problem is more complex than the multi-processor scheduling problem, many of the solution algorithms can be utilized in both application domains. In many cases, the multi-processor scheduling problem is described by a directed graph that effectively captures the task interdependencies. Once such a graph has been established, several different methods can be used to assign subsets of tasks to the individual processors in an attempt to ensure most effective utilization of the processors. N. Amato et al. provides an overview of a few such methods including Wave Front Method (WFM), the Critical Path Method (CPM) and the Dominant Sequence Clustering (DSC) methods.

Other approaches that do not rely on this assumption generally rely on Dynamic Programming (DP) and Branch and Bound (BB) techniques to solve more general scheduling problems. In particular H. Yau et al. extended research done by T. Abdul-Razaq et al and F. Wourd et al. to solve single machine earliness-tardiness problems using DP and BB; however as noted by H. Keller et al, the scheduling problems involving multiple machines is NP-hard. In particular the solution scheme suggested by H. Keller et al. relies on finding the solution to the NP-complete knapsack problem.

Due to the inherent complexity of multi-asset and multi-objective scheduling problems, researchers have utilized non-deterministic solution schemes. One such technique, Simulated Annealing (SA), was successfully utilized by T. Loukil et al. to solve a production scheduling problem. They achieved promising results; however, as with any such solution schemes the performance cannot be guaranteed. The scheduling problem investigated in this paper incorporate additional complexity to the scheduling problem by incorporating prognostic information into the scheduling framework. In essence, the scheduling problem is expanded to not only schedule the currently known tasks, but also the expected future tasks.

4. PROBLEM OVERVIEW

Optimizing when and where maintenance is performed can greatly reduce the cost of performing maintenance and thereby make PBL contracts more attractive to PSPs. To effectively illustrate this, an algorithm which leverages CBM information to optimize the maintenance schedule was implemented and compared to a simple algorithm that schedules tasks as they arise with little or no preplanning. To capture many of the issues that must be considered when scheduling maintenance tasks, several types of resource constraints were incorporated into the problem description.

4.1. Assumptions and Constraints

Scheduling maintenance tasks is generally a very complex problem due to the wide variety of issues that must be considered. In this simple example we are considering scheduling three types of tasks:

1. Corrective Maintenance Tasks.
2. Recurring Maintenance Tasks.
3. CBM Tasks.

Corrective Maintenance Tasks are tasks that address unanticipated fault or failures that have occurred (such as a failure during a flight) that require corrective action before the aircraft regains operational availability. Recurring Maintenance Tasks are tasks that must be performed on a regular basis, either based on a prescribed time interval or the load the system is experiencing. Recurring tasks may be required for safety and/or failure prevention on non-CBM components/systems. Finally, CBM Tasks are maintenance tasks that are scheduled based on an estimate of actual condition or health state of the system. This may be state of charge on a battery, likelihood of fatigue crack, degree of degradation in a rudder or a flap actuator, etc.. To effectively schedule these tasks, it is important to have accurate diagnostics and prognostics technologies deployed on the equipment.

The main difference between systems with CBM and the ones without CBM is the third task type. In essence, a CBM enabled system reduces the number of corrective maintenance tasks by transforming a subset of them into CBM tasks. It should be noted that secondary damage effects associated with running equipment until it reaches a failure mode is not modeled.

There are several constraints that expresses the capacity of a given overhaul/repair facility. Instead of explicitly specifying how many serviceable components of a specific type a maintenance facility can repair, an indirect approach is taken. The indirect approach considers the available maintenance personnel and their certifications, the number of available tools/workbenches, and the number of facilities that can perform the requested repairs. This indirect approach significantly increases the complexity; however, this granular approach not only models the actual facility capacity much more accurately but also ensures that underutilized resources can be readily identified. This information allows a process engineer to make the appropriate changes to the maintenance organization.

An important additional assumption relates to the cost of performing maintenance. In order to identify the ideal maintenance options, the estimated cost of performing a given maintenance task must effectively differentiate between the desirable and less desirable maintenance options. For instance, assume that a critical maintenance check is scheduled to be executed in a month for more than one aircraft. It is likely that the maintenance facilities will be experiencing an increased load in the month of the event, and therefore it is more beneficial to perform maintenance as early as possible. In such cases, it may be advantageous to setup the cost of performing maintenance such that a potential significant increase in maintenance requests will also result an even higher increase in cost. This will encourage a uniform distribution of workload throughout the planning horizon, and thereby ensure that excess capacity is available such that unexpected maintenance tasks can be handled effectively.

For the CBM enabled scheduling algorithm, a couple of additional assumptions are made. First, it is assumed that an estimate of a component's probability of failure vs. time is provided. This estimate is often not just a function of the current component state, but also a function of its expected future usage (i.e. loading). In addition, once a repair action has been performed, a post-maintenance state estimate is also considered known.

4.2. Example Scenario

Consider that a PSP has been awarded a PBL contract to maintain a particular serviceable component on a fleet of 50 aircraft. The PSP has devised and implemented a CBM solution for this component such that costly failures can be detected early; however, parts of the service agreement also includes two recurring maintenance actions that must be performed every 600 hours of operation on each component. Each of the 50 aircraft will operate in one of two different environments: 1) Remote Location Routes – results in increased maintenance cost, 2) Main Base Routes – standard maintenance cost. Additionally, an acceptable failure probability is explicitly stated in the contract.

The problem is modeled as follows. Given an m-dimensional binary option vector x, the purpose of the scheduling algorithm is to minimize:

$$J = f(x, p, F, s) \tag{2}$$

where p is the available personnel, F the available facilities and s the number of spare parts. For a given maintenance option x_i it is assumed that there exist a mapping Ψ: $(x,p,F,s) \rightarrow c \in \Re^m$, that is, for each option a unique cost value can be specified. In addition, a schedule S must be specified for each aircraft, such that the available facilities can be identified. In the simple example problem, two maintenance facilities are provided, one at the main base with standard cost and one at a remote location with higher cost. In addition, each of the facilities is manned by 4 aircraft maintenance technicians with differing certifications and labor costs. The recurring tasks are selected such that they will have to be performed 2 to 3 times per aircraft over the specified 1 year planning horizon. Additionally, the CBM monitored systems will require a significant amount of maintenance, on average about 3 maintenance events a year.

The CBM enabled solution is compared to a scenario in which the maintenance is performed only when the systems fail. An additional 10% cost increase is added to the cost of performing such maintenance task over the cost of performing the CBM maintenance tasks.

4.3. Complexity Considerations

From a complexity standpoint, task scheduling algorithms are somewhat peculiar. For a simple scheduling problem such as scheduling tasks that must be performed at one facility and that only have strict deadline constraint, the simple "earliest deadline first" algorithm is not just effective, but also optimal (T. Cormen et al.). See the appendix for an outline of the proof.

However, once additional facilities and constraints are added to the problem, the complexity of the solution algorithms also increases rapidly (E. Shchepin et al.) In fact, the scheduling problem outlined in the previous section falls into the category of NP-hard problems (see appendix for a short discussion on P, NP, NP-complete and NP-hard problems). Consequently, due to the inherent complexity involved in identifying the globally optimal solution, it is necessary to include additional simplifying assumptions to ensure that the solution algorithm will arrive at an acceptable answer within a reasonable time horizon.

5. SOLUTION APPROACH

The approach outlined in this section adheres to the constraints and assumptions described in section 4.1 with some important additions. First, it is assumed that the future is uncertain, and therefore we extend the solution space to include schedules that are not globally optimal but only optimal in the short term. Secondly, the scheduling algorithm will rely on a maintenance deadline and an associated ‘slack’ variable to define a scheduling time window for each task. If a task cannot be scheduled within that timeframe, the algorithm will return a partial solution.

5.1. Algorithm Outline

The algorithm consists of four main functional modules as depicted in Figure 1. The algorithm executes module 1-4 until all tasks have been scheduled. The first block analyzes the individual maintenance tasks and quickly eliminates options that do not satisfy the specified constraints. The module excludes maintenance options that do not fall within the required maintenance time window; options that would require utilizing maintenance personnel who do not have the necessary certifications etc. The module performs this task in an earliest deadline first fashion until the maximum allowable number of maintenance options is reached. The maximum number of maintenance options is determined based on the available computational resources and the required algorithm execution time.

From the generated set of tasks and options, the second module will determine the most cost effective

rithm will attempt to assign the next maintenance tasks constrained by the ones that have already been assigned a maintenance option. By considering CBM enabled tasks in the scheduling loop, it is possible to schedule tasks based on an acceptable level of risk. Hence, given an acceptable probability of failure, the scheduling algorithm will minimize the overall cost of performing maintenance.

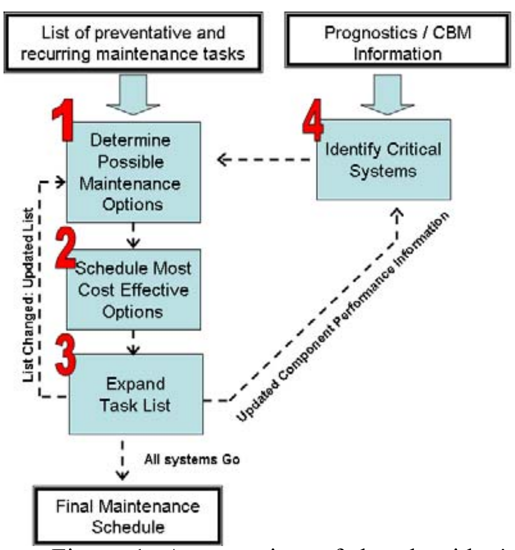

Figure 1. An overview of the algorithm's functional blocks and the flow of information between them.

Once the final maintenance task has been scheduled the system outputs the final schedule along with an estimate of the minimized cost of performing the maintenance.

combination of maintenance options. The combination is derived by using a combination of Integer Linear Programming (ILP) and a simple gradient ascend algorithm.

Once a maintenance option has been identified for each of the maintenance tasks, the solution is passed to the third module. The purpose of this module is to capture the effects of the scheduled maintenance tasks and spawn additional future recurring maintenance tasks. For the CBM tasks, the module updates the expected future aircraft component health state under the assumption that the scheduled maintenance tasks are performed and the updated component performance information is passed back to the final module.

The purpose of the fourth module is to identify CBM justified maintenance tasks. For less complex systems this could simply be determining if a given component will degrade to an unacceptable health state within the planning horizon, preventing an airliner from operating an aircraft. For more complex systems, arriving at a list of critical components can become more cumbersome due to potential time varying load profiles or internal component redundancies.

Once the optimal maintenance actions have been selected, the algo

5.2. Maintenance Action Selection

Identifying the combination of maintenance options that minimizes the overall cost is performed in two steps. First, an integer linear program described by equation 1 must be solved.

$$\begin{aligned} J &= \min \vec{c}^T \cdot \vec{x} \\ A_1 \cdot x &= b_1 \\ A_2 \cdot x &\le b_2 \end{aligned} \tag{1}$$

To do this, the selection cost ‘c’ associated with each of the selections that the binary selection vector ‘x’ describes must be provided; however, to ensure that only a single option is selected for each maintenance task and that trivially exclusive options are not selected simultaneously, equality and inequality constraints must be specified.

For instance, the following simple example indicates how the problem could be formulated:

$$\begin{gathered} \vec{c}^T = \begin{bmatrix} 3.2 & 3.0 & 2.8 & 3.15 & 2.9 & 4.0 & 3.0 & 3.1 \end{bmatrix} \\ A_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ A_2 = \begin{bmatrix} 2 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 3 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 5 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \\ 3 \\ 1 \\ 2 \\ 5 \end{bmatrix} \end{gathered} \tag{2}$$

where the A_1 matrix and the b_1 vector ensure that the first four maintenance options associated with the same maintenance task are not selected simultaneously. The A_2 matrix and b_2 vector are selected such that mutually exclusive options cannot be selected simultaneously. For instance if the first maintenance option is selected the first row in the A_2 vector indicates that the mutually exclusive fifth and seventh options cannot be selected.

It should be noted that depending on the ILP implementation, the system will only be able to consider a few hundred maintenance options simultaneously; however, due to the earliest deadline first type of approach taken, the problem solved at each iteration can be selected such that the ILP can be solved rapidly.

Even with the inequality constraints included in the ILP problem formulation, it is still possible that the derived solution is not feasible. The problem arises when multiple tasks have to be inserted into the same segment of time that may be larger than two individual maintenance options. As a simple illustration of this, imagine that the ILP algorithm indicates that three maintenance options that each require 1.2 hours to complete should be scheduled within a 3 hour period at a given facility. Since the tasks are not pair wise mutually exclusive (and therefore not captured by the inequality constraint) the combination of all three tasks cannot fit within the specified time frame. It should at this point be noted that the problem of fitting the tasks into the allocated time period is similar to the NP-complete ‘Knapsack’ problem.

To solve this problem, a simple gradient ascend algorithm is used to post-process the ILP output. The algorithm first assigns the maintenance options for which no other maintenance options are available, and then assigns the remaining options based on how much additional cost is incurred by not allowing the task to be scheduled using the ILP prescribed option.

5.3. Constraint Tracking

One of the advantages of the maintenance action selection process is the additional information that can be extracted from the algorithm. By tracking the active constraints it is possible to single out factors such as manpower and spare part availability that limit the overall maintenance performance. This information can then in turn be leveraged to improve the profit margin of the PBL contract. For instance, consider the scenario in which for a given aircraft a particular consumable is not available. Not only is it costly to procure the item, but it will also postpone the maintenance action and thereby incur additional overhead. In an environment where CBM enabled scheduling is performed most such issues can be avoided simply by analyzing the resulting schedule and incorporate appropriate safety margins.

5.4. Possible Algorithm Extension

The approach outlined above is applicable in situations where the planning horizon is long enough such that there may be several factors that cannot be accounted for. Therefore, the underlying assumption that it is more important to optimize the schedule in the immediate future is reasonable in such scenarios. However, if such an assumption cannot be made, a dynamic programming extension should be used, in which the scheduling is performed backwards rather than forwards, that is, the scheduling should be performed in accordance with the Bellman optimality principle.

6. RESULTS

A simulation of the example scenario outlined in section 4.1 was performed, and the CBM enabled maintenance cost was compared to a system which only deploys corrective maintenance. The health status indicator for one of the 50 modeled aircraft is depicted in Figure 2. During the 365 day planning horizon, the health of the system transitions from being healthy (green) to a warning state (yellow) to a failure state (red) assuming an expected future load profile. The system health status is obtained by determining when the health status indicator crosses two predetermined thresholds. These thresholds are set such that a minimum acceptable remaining useful life is available after the system has passed the alarm threshold. Hence, the scheduling algorithm will attempt to schedule the task during the warning period.

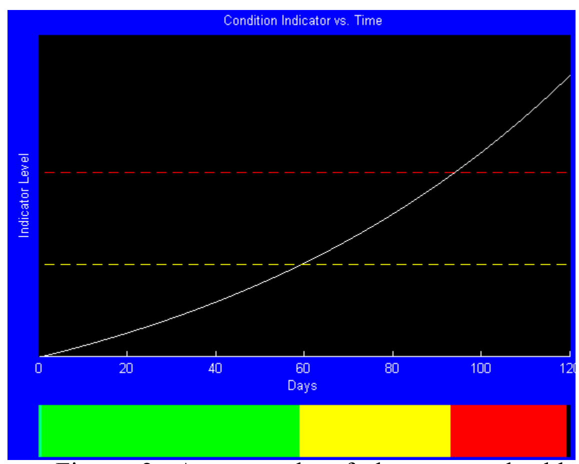

Figure 2. An example of the system health indicator before maintenance is performed.

Figure 3 shows the health status of the system after CBM maintenance has been performed. It should be noted that in this case the system health status never reaches the failure state, and only briefly enters the warning state. It should also be noted that the component health returns to a nominal/repaired health state after maintenance has been performed.

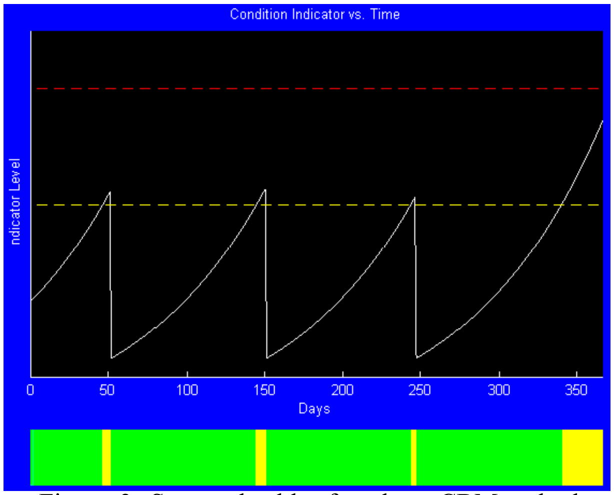

Figure 3. System health after three CBM tasks have been scheduled.

The cost of performing maintenance depends greatly on the resources being assigned to the particular maintenance task. Within the CBM scheduling framework, there is a significant amount of flexibility in selecting personnel or location to perform the maintenance, simply by shifting the time at which a maintenance task is performed.

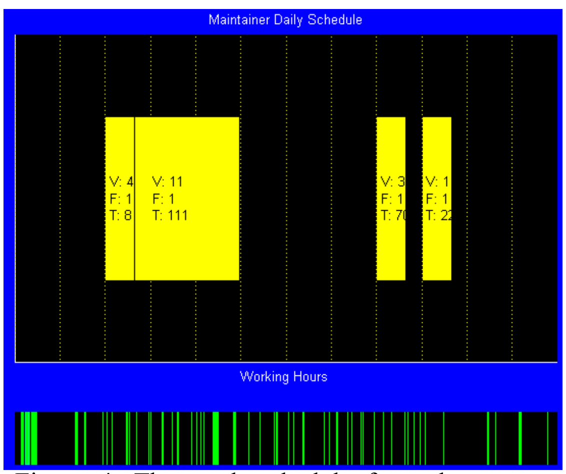

Figure 4. The work schedule for a less expensive maintainer: The top part of the figure depicts the maintainer's daily schedule while the bottom part depicts how busy he is throughout the year.

Figure 4 and Figure 5 show an example of utilizing the cheaper solution when available. In this case, two different maintainers are able to complete the same maintenance task, but the one that has the lower hourly rate will be selected more often for the maintenance task. The top section of the two Figures depicts how busy each maintainer is through out a single day while the bottom plot indicates how utilized they are throughout the year. The more expensive maintainer has more certifications, so for this particular scenario it can be argued that the demand for maintainers with the additional certifications is relatively low. Therefore it is recommended that the maintenance organization should recruit additional maintainers with the lower number of certifications.

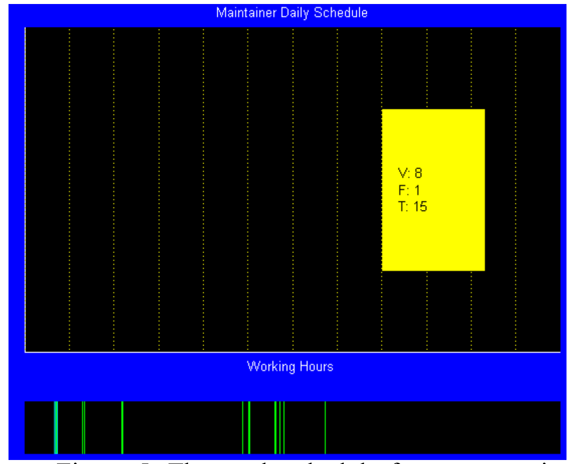

Figure 5. The work schedule for an expensive maintainer: The more expensive resource is utilized significantly less often.

The graph shown in Figure 6 represents the accumulated cost of performing maintenance throughout the 365 day planning horizon. The yellow dotted line shows the cost of performing CBM scheduled maintenance, whereas the solid white line indicate the cost of performing maintenance in a purely corrective manner.

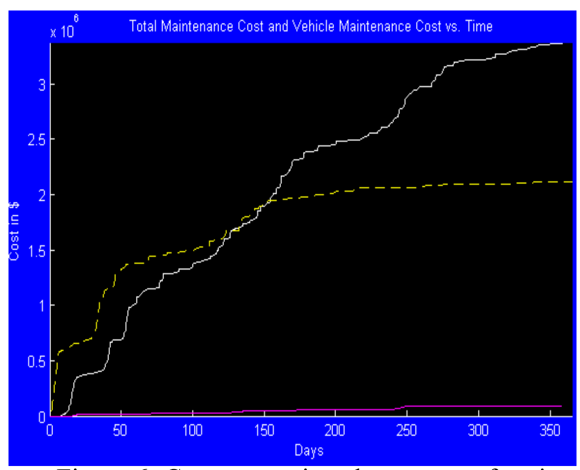

Figure 6. Cost comparison between performing maintenance with (yellow dotted line) and without (white solid line) CBM scheduling.

The large cost savings is achieved by a combination of performing less expensive maintenances and assigning the most cost effective resources. It should also be noted that the cost associated with performing CBM maintenance is higher up front due to the anticipation of future failures.

6.1. Discussion

The results shown in the previous section clearly indicate that the total cost of maintaining the modeled systems can be reduced by implementing CBM enabled scheduling. However, the estimates are somewhat conservative in that only incurring a 10% penalty for repairing a failed system is very low. Issues such as additional wear on connected components and the cost of aircraft down time is not explicitly modeled, and will ultimately result in additional overhead that is to a large extent prevented in a CBM enabled maintenance environment. Despite these conservative cost estimates, the cost of performing maintenance in the CBM enabled environment is reduced to approximately 70% of the nominal maintenance cost. An additional benefit of the CBM enabled task scheduler that is not explicitly captured in the simulation relates to the operational availability expressed in equation 3.

$$A_0 = \frac{OT + ST}{OT + ST + TCM + TPM + ALDT} \tag{3}$$

where OT is the operating time, ST is the standby time, TCM is the total corrective maintenance time, TPM is the total preventative maintenance time and ALDT is the administrative and logistic downtime (R. Stapelberg). Even if we assume that the corrective maintenance time remains the same with and without early problem detection, the ALDT will be reduced significantly since parts can be ordered early and equipment utilization can be managed effectively.

Furthermore, from a PBL contracting standpoint the additional performance assurance achieved by the PSP is the type of improvement that the government may reward through award term/fee incentives in which the PSP's contract is either extended or the PSP receives a larger compensation.

Another potential improvement that is not explicitly taken into account when evaluating the potential benefits of CBM scheduling is the improvements that can be made to the maintenance organization and the supply chain. The schedule provides not just a much more detailed view of how much work must be completed by maintainers with certain qualifications, but it also provides the necessary time to order consumable spares such that the amount available in a local storage facility can be reduced significantly.

7. CONCLUSION

This paper presents a framework whereby the benefits of CBM system are captured with a maintenance schedule optimization process to illustrate cost saving that may be realized by PBL contracts. The example presented is focused in the aviation industry and is clearly simplified in many ways but the benefit associated with the ability to schedule maintenance on the basis of a failure risk is substantive and only realizable with a CBM solution. Throughout the paper it is argued that leveraging the system health information that CBM enabled technologies provides to effectively schedule maintenance and manage resources ultimately reduces the cost of performing maintenance and allows for more effective management of the supply chain. This cost savings is vital to PBL contracts for which reducing maintenance cost has a direct effect on the profit margin. To demonstrate some of the expected advantages of CBM scheduling, a simplified scheduling problem was implemented and the derived simulation results confirmed the CBM cost-saving hypothesis.

NOMENCLATURE

A_1	Equality constraint matrix
A_2	Inequality constraint matrix
ALDT	Administrative and logistic downtime
b_1	Equality constraint vector
b_2	Inequality constraint vector
c	Cost vector
f	Generic function
F	Facility list
OT	Operating time
p	Personnel list
s	Spare parts
ST	Standby time
TCM	Total corrective maintenance
TPM	Total preventative maintenance
x	Binary selection vector

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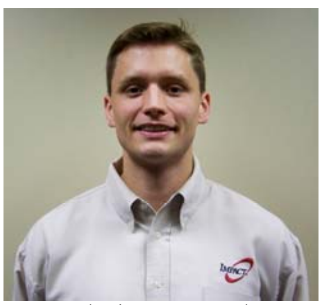

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APPENDIX A

Theorem 1: Given a set of n tasks each with an associated deadline, the earliest deadline first schedule is feasible (i.e. all the deadlines are met) if a feasible solution exist.

Proof:

Given a set **T** of n tasks each with a deadline t_i, $i = 1, 2,\ldots, n$. Let the schedule **S** be a permutation of the tasks in **T**, where $s_i \in \mathbf{S}$ is the i^{th} task performed in the schedule. Let us assume that there exist a feasible solution **S'** that is not isomorphic to the earliest deadline first schedule **S''**. Moreover, let us assume that **S''** is not feasible (otherwise we are done). Since $\mathbf{S''} \neq \mathbf{S'}$, there exist a pair of tasks $\{s_i,s_j\} \in \mathbf{S'}$ for which $i < j$ and $t_i > t_j$. Hence, if the two elements of **S'** is permuted, the resulting schedule $\mathbf{S}^+$ is still feasible. Continue to perform permutations until no pair exist (at most $O(n^2)$ times), and the resulting schedule $\mathbf{S}^+$ a feasible earliest deadline first schedule contradicting the assumption that the earliest deadline first schedule is not feasible.

■

P/NP Complexity Terminology:

Consider the abstract problem P which relates a set of problem instances I to the respective set of problem solutions S. A problem is considered polynomial time (or P) if there exist an algorithm A that solves the problem in polynomial time.

A problem P is considered NP if there is a polynomial time verification algorithm that can verify the solution in polynomial time, but no solution is necessarily known to solve the problem itself in polynomial time.

A problem P considered a part of the NP-complete class of problems if an algorithm that solves P also will be able to solve any other problem in the NP-complete class.

An NP-hard problem is one that requires at least as many operations to perform as the hardest problems in the NP class.

For a more thorough discussion see T. Cormen et al.