

A Review of Time Synchronous Average Algorithms

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ABSTRACT

Time Synchronous Average (TSA) is an essential algorithmic tool for determining the condition of rotating equipment. Given its significance to diagnostics, it is important to understand the algorithms performance characteristics. This paper addresses four topics in relation to the TSA performance characteristics. The first topic is the evaluation of the performance (measured against gear fault detection) of 6 different TSA algorithms. The second topic is quantifying the ergodicity/noise reduction as a function of the number of revolutions in the TSA and show that noise reduction is $1/\sqrt{\text{number of revolutions}}$. The third topic examines TSA techniques when no tachometer signal is available. The final topic shown is the distribution of the magnitude of TSA orders associated with fault and nominal components are Rice and Rayleigh distributed.

1 INTRODUCTION

Mechanical diagnostics of drivetrain systems attempts to reduce maintenance cost and improve reliability/safety through monitoring. Vibration data is processed to determine the condition of components in the drivetrain. One of the most powerful algorithmic tools for vibration analysis is the time synchronous average (TSA). Time synchronous average resamples the vibration data synchronously with a shaft, and is the basis of numerous gear and shaft condition indicator (CI) algorithms.

Given the importance of TSA to gearbox analysis, a number of questions arise:

- What is the best TSA algorithm for fault detection?
- Noise reduction is modeled as an average: $1/\sqrt{\text{rev}}$, where *rev* is the number of average revolutions. Is this a good model?
- Many shaft and gear CIs are based on the magnitude of the TSA spectrum. What is the distribution under nominal or faulted cases?

- How is a TSA performed without phase information normally provided by a tachometer signal?

This paper we address these questions by investigating the performance of six TSA algorithms on test stand data and resolves experimentally some of these fundamental TSA performance questions. Both time domain and frequency domain TSA are explored.

2 TIME SYNCHRONOUS AVERAGING

Time synchronous averaging is a signal processing technique that extracts periodic waveforms from noisy data. The TSA is well suited for gearbox analysis, where it allows the vibration signature of the gear under analysis to be separated from other gears and noise sources in the gearbox that are not synchronous with that gear. Additionally, variations in shaft speed can be corrected, such that the spreading of spectral energy into an adjacent gear mesh bin. In order to do this, a signal phased-locked with the angular position of a shaft within the system is needed.

This phase information can be provided through a *n* per revolution tachometer signal (such as a Hall sensor or optical encoder, where the time at which the tachometer signal crosses from low to high is called the zero crossing) or through demodulation of gear mesh signatures ((Comber, 2007)). If *n* is 1 for the shaft under analysis, a balance solution can be achieved. If *n* is greater than one, or the shaft/gear under analysis is not the shaft generating the tachometer signal, then no balance solution is available (phase relative to the imbalance vector is ambiguous).

2.1 Vibration Model

The model for vibration in a shaft in a gear box was given in [3] as:

$$x(t) = \sum_{k=1}^K X_k (1 + a_k(t)) \cos(2\pi k f_m(t) + \phi_k(t) + \Phi_k) + b(t) \quad \text{eq(1)}$$

where:

X_k is the amplitude of the *k*th mesh harmonic

$f_m(t)$ is the average mesh frequency

$a_k(t)$ is the amplitude modulation function of the *k*th mesh harmonic.

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$\phi_k(t)$ is the phase modulation function of the k th mesh harmonic.

Φ_k is the initial phase of harmonic k , and

$b(t)$ is additive background noise.

The mesh frequency is a function of the shaft rotational speed: $f_m = Nf$, where N is the number of teeth on the gear and f is the shaft speed.

This vibration model assumes that f is constant. In most systems, there is some wander in the shaft speed due to changes in load or feedback delay in the control system. This change in speed will result in smearing of amplitude energy in the frequency domain. The smearing effect, and non synchronous noise, is reduced by resampling the time domain signal into the angular domain:

$$m_x(\theta) = E[x(\theta)] = m_x(\theta + \Theta) \quad \text{eq 2}$$

here Θ is the period of the cycle to which the gearbox operation is periodic, and $E[\]$ is the expectation (e.g. ensemble mean). Equation 2 makes the assumption that $m_x(\theta)$ is stationary and ergodic. If this assumption is true, than non-synchronous noise is reduce by $1/\sqrt{n}$, where n is the number of cycles measured for the TSA. This assumption will be tested.

2.2 TSA Techniques

Two basic techniques for resample are presented. In the Angular resampling technique (ref [1,2,3]), the number of data points in one shaft revolution (r_n) are interpolated into m number of data points, such that:

- For all shaft revolutions n , m is larger than r ,
- And $m = 2$ to of the ceiling ($\log_2(r)$) (typical for radix 2 Fast Fourier Transform).

A number of interpolation techniques could be used. Linear, bandwidth linear, and spline interpolation techniques are tested in the paper. Additionally, for linear and spline interpolation, two different zero crossing techniques are evaluated (interpolated zero cross time and bandwidth limited zero cross time (to remove jitter)).

In spline interpolation, a curve is fitted to a cubic equation through the sampled data points, and an interpolated point is found that has a smooth first derivative and a continuous second derivative (ref[5]).

In the Frequency Domain TSA (Vachtsevanos 2006), the discrete Fourier Transform (DFT) is taken for r data points between each tachometer zero crossing, then m data of each of n DFT are taken, where

- For all shaft revolutions, m is he minimum of the floor($r/2$)-1.

Table 1 gives the TSA techniques used for each of the Six cases.

Table 1 TSA Cases Tested

Case	Zero Cross Technique	TSA Technique
1	Interpolated ZC Time	Spline Interpolation
2	Bandwidth Limited Interpolation	Spline Interpolation
3	Interpolated ZC Time	Linear Interpolation
4	Bandwidth Limited Interpolation	Linear Interpolation
5	Interpolated ZC Time	Frequency Domain TSA
6	Bandwidth Limited Interpolation	Bandwidth Limited Interpolation

2.3 TSA Psuedo Code

In this study, both time and frequency domain TSA algorithms where explored. Example Matlab(c) code is given in the Appendix. For the resample TSA method, the difference is in the interpolation method: Linear, Bandwidth Limited Linear, or Spline. The example code presents a linear or polynomial interpolation. For additional details on spline interpolation sea (Press 1992). For the bandwidth limited case the optimal interpolation filter was chosen: a linear-phase FIR filter is constructed that minimizes the weighted, integrated squared error between an ideal piecewise linear function and the magnitude response of the filter over a set of desired frequency bands. The generalized filter is (eq (3)):

$$h(t) = \frac{\sin(\pi t/T)}{(\pi t/T)} \times \frac{\cos(Rt/T)}{(1 - 4R^2 t^2/T^2)} \quad \text{eq (3)}$$

where R is the roll off factor and T is the symbol period.

The number of points per revolutions N , is derived form equation 4, where $\max(n)$, where n is the number of points between two subsiquent zero crossing indexes is

$$N = 2^{\text{ceiling}(\log_2(\max(n)))} \quad \text{eq(4)}$$

Ceiling is a round up function. Equation 4 assumes a radix 2 FFT.

The pseudo code for the TSA is:

1. Calculate the Zero Cross Times (ZCT)
2. For the Shaft of Interest, Interpolate the current ZCT_i and ZCT_{i+1} based on the Gear Ratio
3. Calculate the time between ZCT_i and ZCT_{i+1} , $dZCT$
4. Calculate the resample times: $dZCT/N$, where N is from eq 4.
5. Interpolate,
6. Accumulate the resample time domain data
7. Repeat 2 through 6 for M number of revolutions,
8. Divided the accumulated N point vector by M

Figure 1 plots the 6 TSA techniques.

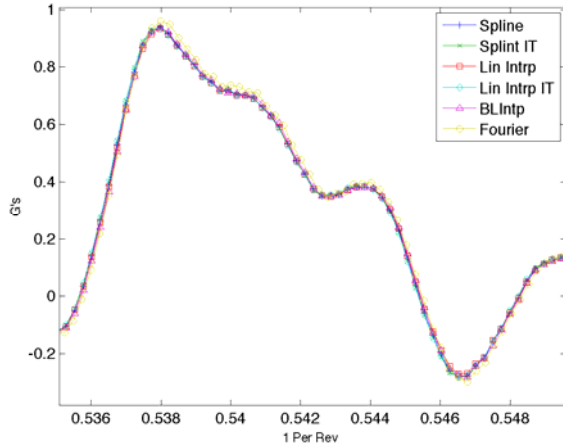


Figure 1 Plot of the 6 TSA Techniques

Both time and frequency domain representation of the TSA techniques are very similar. In fact, under casual scrutiny it is difficult to tell one technique from the other: the main difference being processing time. This makes selection of the best TSA algorithm difficult, as was noted in ref[3]. Since gear fault data was available, this allows selection of TSA technique based on fault detection.

2.4 The Evaluation of TSA Performance

The performance metric for TSA was the ability to detect a known gear fault, where statistically one can measure the ability to distinguish between a good and bad gear. Because the initial results were so close, the performance was measured over a sample of CIs and gear conditions. A total of 32 experiments where conducted: 16 with no gear fault, and 16 with a chipped gear tooth.

For each experiment, 30 condition indicators (CI) where collected. A condition indicator is an algorithm that extracts a feature that may be sensitive to gear damage. Of the 30 CI, the 5 most responsive CIs where chosen: Narrowband Peak to Peak, Narrowband Kurtosis, Narrowband RMS, Residual RMS and Signal Average RMS. For a complete description of the CI algorithms see (Vecer 2005).

The acquisition system sampled at 66.667 KHz with a 10 point per revolution tachometer. The test stand was a SpectraQuest Gearbox Dynamic simulator (see figure 1)

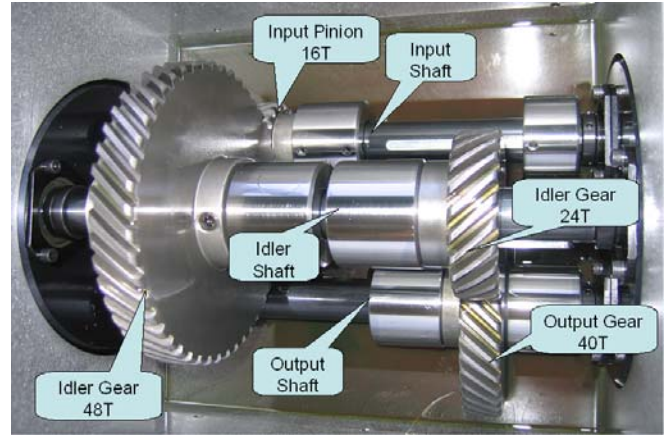


Figure 2 SpectraQuest Gearbox Dynamics Simulator: Chipped Gear

The 32 acquisitions, and subsequent CIs, where taken over a variety of input shaft RPM, load (brake on or off) and known fault. To remove the effects of rpm and load and reveal the fault, each CI, for each TSA method was modeled as a least square fit regression:

$$\mathbf{X} = [1 \quad rpm_i \quad torque_i \quad fault_i]$$

$$\mathbf{b} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y} \quad \text{eq(5)}$$

where \mathbf{X} is a matrix coding the rpm for trail i , the torque (1 if brake on, else 0) for trail i , and the fault (1 if chipped, else zero) for trail i , and \mathbf{Y} is a vector of the CI for trail i . The vector \mathbf{b} is then the least squares fit measure of the effect of \mathbf{X} . Thus, \mathbf{b}_4 (forth element in the vector \mathbf{b}) is the least squares estimator of the change in a CI because of the chipped gear. To normalize \mathbf{b}_4 for model noise, one can divide by the mean square error between the actual and estimated model:

$$\hat{\mathbf{Y}} = \mathbf{b}\mathbf{X}$$

$$\mathbf{r} = \mathbf{Y} - \hat{\mathbf{Y}} \quad \text{eq(6)}$$

$$mse = 1/n \sum_{i=1}^n r_i^2$$

The raw \mathbf{b}_4 and normalized \mathbf{b}_4

Table 2. CI values for Different TSA Techniques

	NB P2P	NB Kurt	NB RMS	Res RMS	SA RMS
Case 1	3.347	2.182	0.466	0.323	0.622
Case 2	3.347	2.181	0.466	0.323	0.622
Case 3	3.346	2.177	0.466	0.323	0.619
Case 4	3.346	2.181	0.466	0.323	0.620
Case 5	3.426	2.180	0.477	0.331	0.636

Case 6	3.345	2.181	0.466	0.324	0.622
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Table 3. Normalized CI values for Different TSA Techniques

	NB P2P	NB Kurt	NB RMS	Res RMS	SA RMS
Case 1	2.616	3.197	2.436	2.467	2.962
Case 2	2.616	3.196	2.437	2.468	2.964
Case 3	2.619	3.212	2.432	2.463	2.969
Case 4	2.617	3.196	2.437	2.468	2.974
Case 5	2.619	3.196	2.440	2.477	2.989
Case 6	2.611	3.192	2.433	2.475	2.964

From table 2 and 3, it is apparent that all 6 methods are relatively close in performance. A non-parametric comparison (scoring the TSA techniques by their rank order over all CIs) of the CIs and Normalized CIs clearly shows that one algorithm consistently performs the best (table 4)

Table 4. Non Parametric Rank Score

	CI	Normalized CI
Case 1	20	12
Case 2	23	16
Case 3	7	17
Case 4	13	20
Case 5	26	27
Case 6	16	16

Surprisingly, this indicates Case 5 (Fourier Domain TSA) gave the best discrimination in the detection of fault. A general observation is that bandwidth limiting the tachometer/zero crossing signal improved the performance of both the linear and spline interpolation TSA. In any case, the TSA algorithms worked well. Selection of TSA can be based on processing capability, in which case the linear interpolation is superior. The Fourier domain based TSA would be attractive if a DFT routine such as FFTW (Fastest Fourier Transform in the West) is available. A similar conclusion was reached in (Decker 1999).

3 TACHOMETER LESS TSA

A tachometer signal is used to define a shaft zero crossing times. The zero crossings times are needed for calculation of a TSA. TSA is an effective way to isolate the vibration signals associated with components on a given shaft. In general, a TSA resamples the time domain data into angular domain (e.g. once per revolution). As an example, the time between one shaft revolution might have 843 time domain samples. This, and subsequent data points between the zero crossing times, is then resampled into 1024 samples per revolution. For example, a two second acquisition on a 100

Hz shaft, includes 200 revolutions. The 200 resampled 1024 segments are averaged together. This reduces non synchronous noise by $1/\sqrt{\text{Number of Revolutions}}$.

Without a tachometer signal, an angular reference is needed to estimate the one per revolution. A time domain feature, such as a gear mesh, can be used. A bound on the phase error included in using a vibration base reference is given in ref[3]: that standard deviation of the phase error of a band passed signal is:

$$\sigma[\theta] = 1/\sqrt{2} * 10^{-SNR/20} \quad \text{eq(7)}$$

This gives a phase error of shaft angular position of:

$$\sigma[\theta_{shaft}] = \frac{\sigma[\theta]}{N_{GearTeeth}} \quad \text{eq(8)}$$

One algorithmic method for calculating phase is:

- Demodulate the vibration signal using the complex analytic signal with a frequency of operating shaft RPM times the number of teeth on the gear meshes of interest. In some instances the 2nd or 3rd harmonic has a larger SNR than the base gear mesh rate. If so, multiply by the appropriate harmonic value.
- Low pass filter the demodulated signal with a bandwidth which is the RPM error (e.g. 1%).
- Calculate the angle of the low pass filter with the quadrant corrected arctan function (such as atan2 in C++, matlab® or Fortran 77).
- Unwrap the angle (change angles greater than 2π to their 2π complement) and add in the nominal shaft angle: new angle = $\text{unwrap}(\text{angle}) + \text{rpm}/\text{grT}*(1:n)$
- Normalize to radian by dividing the low pass signal by the number of teeth on the gear.
- Take the Modulo of the normalized angle by 2π ;

The zero crossing time is the interpolated point at π radians. In the given example, a 32 T Gear is used to calculate the instantaneous frequency. The input shaft speed was 109 Hz and the sample rate was 100 KHz. Figure 3 shows approximately 10dB SNR. This gives a theoretical error of 0.4% (see figure 4).

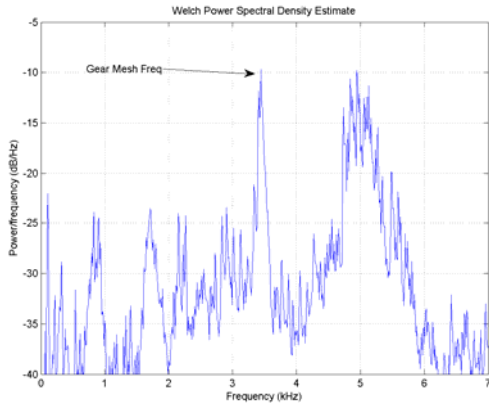


Figure 3: Spectrum of 32T Gear with 10dB SNR

Figure 4 shows that there are some outliers that are greater than the theoretical error.

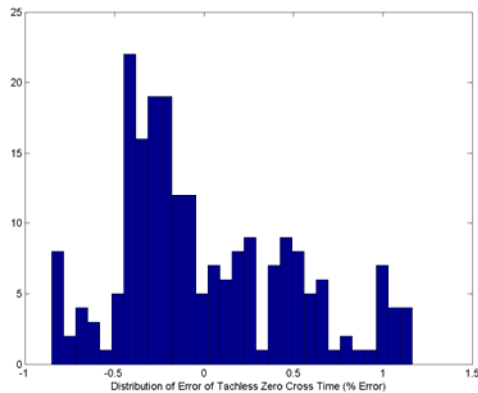


Figure 4: Histogram of Phase Error

On investigation, it was found that the error was periodic. This is easily seen in the auto correlation of the error (figure 5). Notice that the period of the error is every 15 revolutions.

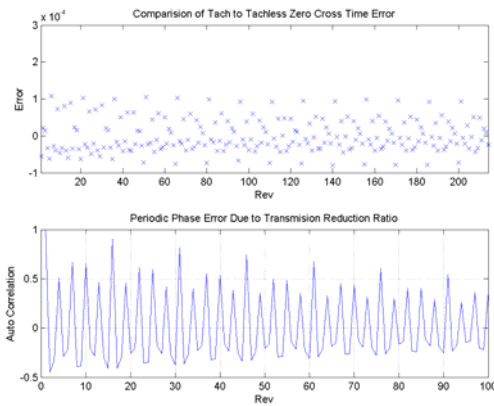


Figure 5: Plot of error vs. time and the autocorrelation of error

This corresponds to the product of the first shaft reduction (1/3) and the total gearbox reduction (1/5). Every 15 revolutions the phase noise is reinforced. This results in a poor TSA as measured in a comparison between the spectrum of a TSA with and without the tachometer signal.

In order to improve the performance, we reduced the phase noise using a polynomial smoothing technique (see Press 1992). This is not a low pass filter – it is non-causal and introduces no delay. In this example, it reduced the jitter error to $4.6e-7$ seconds (figure 5.).

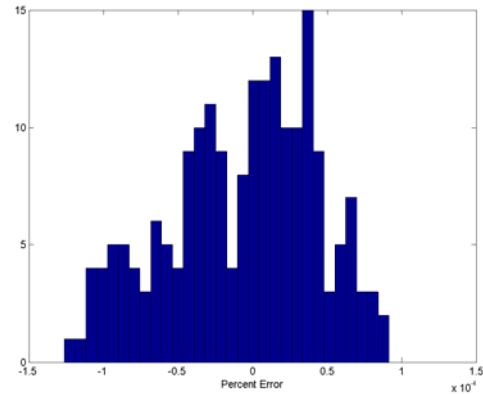


Figure 6: Histogram of the error after polynomial (Golay) smoothing.

Note that percent error is 10^{-4} percent scale – 10,000 times better than without polynomial smoothing.

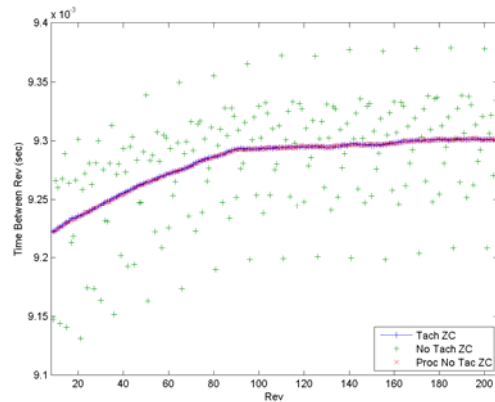


Figure 7: Comparison of tach-less to tach zero crossing

Essentially, the tachometer-less zero crossing is indistinguishable from the tachometer zero crossing (figure 8 and 9).

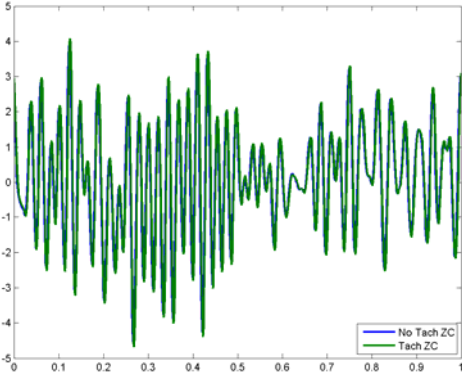


Figure 8 Comparison of TSA, No Tach to Tach ZC

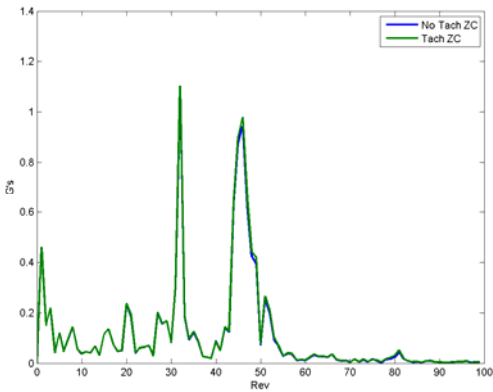


Figure 9 Spectrum of No Tach and Tach TSA

4 A TEST OF THE ASSUMPTION OF ERGODICITY

Under the assumptions of equation 2, the non synchronous noise of the TSA should be consistent and ergodic. If true, as the number of revolutions used in the TSA increases, the ensemble mean value should approach the actual synchronous waveform. Additionally, the standard deviation of the TSA should (and any measured frequency content) decrease as:

$$\sigma_k = \sigma \frac{1}{\sqrt{rev}} \quad \text{eq(9)}$$

where *rev* is the number of revolutions.

This can easily be tested experimentally on a gearbox test stand. One hundred acquisitions were taken for 1 second (sample at 66.667 KHz) with an input shaft speed of 100 Hz. Then the standard deviation of the input pinion gear mesh amplitude was calculated for 2 to 96 shaft revolutions. This was done using the spline interpolated TSA and the frequency domain TSA (see Figure 10)

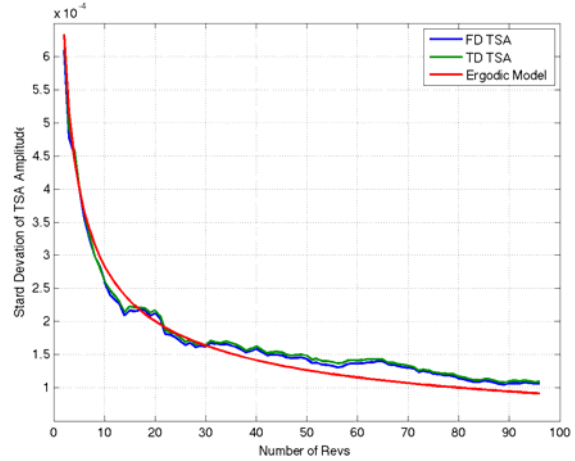


Figure 10: Measured Relationships between Noise and Number of Revolutions

It should be noted that estimating standard deviation is much more difficult than estimating mean. While one can not prove the standard deviation is the same as the model, one can test the hypothesis that the TSA standard deviation is equal to the ergodic model using an F test.

$$H_0: \sigma_{tsa}^2 = \sigma_{model}^2$$

$$H_a: \sigma_{tsa}^2 > \sigma_{model}^2$$

$$\text{The test statistic: } F = S_{tsa}^2 / S_{model}^2$$

The null hypothesis is rejected if $F > F_{\alpha}$. If α is .025 (a typical value), with 99 degrees of freedom in the numerator and denominator, the rejection region is: 1.49. The measured F statistics for 96 revs is $1.09e-4^2 / 9.2e-5^2$, or 1.28. The null hypothesis cannot be rejected and it must be assumed that the TSA is ergodic and that the noise is reduced by $1/\sqrt{revs}$.

5 MODELED AND ACTUAL DISTRIBUTION OF TSA VALUES

For thresholding and/or anomaly detection of shaft conditions, it is important that the distribution of the shaft is known. Defining a threshold with a probability of false alarm of 10^{-6} is very different between a Gaussian model, and some other non-Gaussian distribution.

Consider a large population of shafts in nominal condition. Each shaft would have some variation in stiffness and center of mass. If we measured shaft order 1 for each shaft, we would expect that the phase of the vibration would be uniformly distributed between 0 and 2π , while the magnitude would be some positive value. Now imagine that if the vibration vectors were plotted in Cartesian coordinates, there would be points clustered on the X (real) and Y (imaginary) coordinates, the magnitude is then:

$$CI = \sqrt{X^2 + Y^2} \quad \text{eq(10)}$$

Assuming the distribution around the X and Y axis where Normal with zero mean, then,

$$f(X) = (1/2\pi\sigma^2) \exp(-X^2/2\sigma^2) \quad \text{eq(11)}$$

and

$$f(Y) = (1/2\pi\sigma^2) \exp(-Y^2/2\sigma^2) \quad \text{eq(12)}$$

In the nominally healthy case (no imbalance), X(t) and Y(t) are independent, such that P(X,Y) = P(X)P(Y). Then the joint probability function is:

$$f(X,Y) = (1/2\pi\sigma^2) \exp(-(X^2 + Y^2)/2\sigma^2) \quad \text{eq(13)}$$

This is function of a distribution in which SO1 is a function of X and Y. Define a change in variables of SO1 as a function of magnitude, M, instead of P(X,Y). If Φ as phase, then $X = M \cdot \cos(\Phi)$ and $Y = M \cdot \sin(\Phi)$. Transforming differential areas using $dXdY = M dM d\Phi$ gives the joint probability function:

$$f(M, \Phi) = M\Phi/2\pi\sigma^2 \exp(-M^2/2\sigma^2) \quad \text{eq(14)}$$

which is independent of phase:

$$f(M, \Phi) = P(M)P(\Phi) \quad \text{eq(15)}$$

The probability distribution function (PDF) for shaft magnitude is then:

$$f(M) = \int_0^{2\pi} f(M, \Phi) d\Phi = M/\sigma^2 \exp(-M^2/2\sigma^2) \quad \text{eq(16)}$$

The expected value, $E[M]$ of this continuous random variable can then be shown to be:

$$E[M] = \int_0^{\infty} Mf(M) dM = \sigma\sqrt{\pi/2} \quad \text{eq(16)}$$

The variance is then calculated as:

$$E[M^2] - E[M]^2 = \int_0^{\infty} M^2 f(M) dM - \sigma^2 \pi/2 = \sigma^2(2 - \pi/2) \quad \text{eq(17)}$$

This is the definition of a Rayleigh distribution.

If we assume that the distribution around the X and Y axis where Gaussian with non zero means, such as an imbalance:

$$X \sim N(m_1, s) \quad \text{eq(18)}$$

$$Y \sim N(m_2, s) \quad \text{eq(19)}$$

The real non-centrality is:

$$s = \sqrt{m_1^2 + m_2^2} \quad \text{eq(20)}$$

This is a non-central Rayleigh, or more correctly a Rice Distribution. The PDF is (see Proakis 1995)

$$P(M) = M/\beta^2 \exp\left(-\frac{(M^2 + s^2)}{2\beta^2}\right) I_0(Ms/\beta^2) \quad \text{eq(21)}$$

where I_0 is the 0-th order Bessel function of the first kind. The CDF is:

$$F(M) = 1 - Q_1\left(\frac{s}{\beta}, M/\beta\right) \quad \text{eq(22)}$$

where Q_1 is Marcum's Q function.

We can test the validity of these models by taking a large sample (100 acquisitions used in the previous example) of a shaft, take the TSA and calculate the magnitude of the shaft order. Now we can test the hypothesis on the distribution of the magnitude. In the first case, a small imbalance was generated by placing a bolt offset on the shaft, spun at 100 Hz. The hypothesis is that the distribution of SO1 would be Rician (see figure 11)

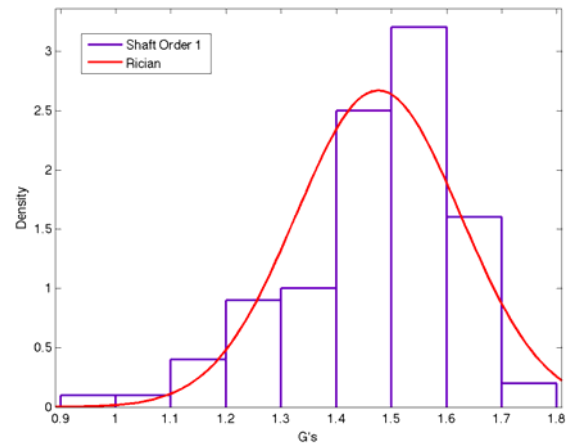


Figure 11 Distribution of Imbalance in SO1

Using the Komogorov-Smirnov test for Goodness of fit, the null hypothesis is that the distribution is Rician, vs. the alternative hypothesis that the distribution is not Rician. For the 100 samples, the KS statistic is 0.1245. For an $\alpha = .05$ rejection region, the cut off value is 0.134. Therefore we can not reject the null hypothesis.

If we measure SO3, which on this test stand should have small values (e.g. no misalignment, and strain on a coupling is small), the distribution should be near Rayleigh. Figure 12 is a plot of SO3.

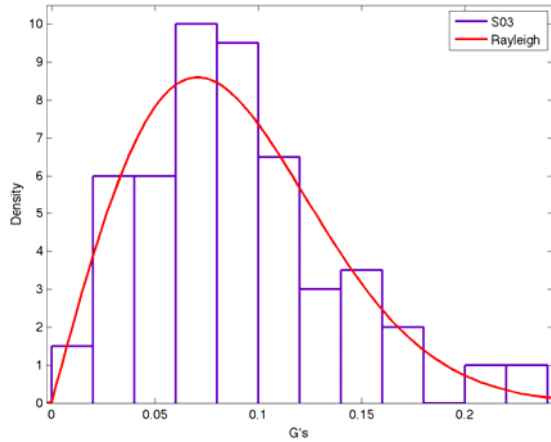


Figure 12 Nominal SO3 and Rayleigh Distribution

Using the Komogorov-Smirnov test for Goodness of fit, the null hypothesis is that the distribution is Rayleigh, vs. the alternative hypothesis that the distribution is not Rayleigh. From SO3, a beta of 0.0706 was estimated for a Rayleigh distribution. For the 100 samples, the KS statistic is 0.0742. For an $\alpha = .05$ rejection region, the cut off value is 0.134. Therefore, we can not reject the null hypothesis.

These experimental results support the hypothesis that the magnitude of a shaft order is Rayleigh when eccentricity is close to zero, and Rician when eccentricity is greater than zero. This information is important for modeling the threshold setting process in HUMS (see Bechhoefer 2007).

6 CONCLUSIONS

This paper is a review of TSA techniques and provides background on statistical properties of the TSA. This is important in the system design and threshold setting in HUMS applications. Of note:

- Both time and frequency domain TSA have similar performance.
- From a fault detection perspective, we found the frequency domain TSA techniques to have the greatest sensitivity to fault (see table 4).
- The TSA is ergodic: non-synchronous noise is reduced by $1/\sqrt{\text{number of revolutions}}$
- The statistical distribution of the amplitude of a shaft order is Rayleigh if no eccentricity is present (due to an imbalance or fault)
- The statistical distribution of the amplitude of a shaft order is Rician if eccentricity is present (due to an imbalance).

In the future, we wish to test the hypothesis that sampling at rates exceeding the resonance of the accelerometer improve fault detection.

REFERENCES

(Comber, 2007) Combet, L., Gelman L., “An automated methodology for performing time synchronous averaging of a gearbox signal without seed sensor”, *Mechanical Systems and Signal Processing*, Volume 21, Issue 6, August 2007, 2590-2606.

(McFadden 1987) McFadden, P., “A revised model for the extraction of periodic waveforms by time0domain averaging”, *Mechanical Systems and Signal Processing* 1 (1) 1987, pages 83-95

(Decker 1999) Decker, H., Zakrajsek, J., “Comparison of Interpolation Methods as Applied to Time Synchronous Averaging” ARL-TR-1960, MFPT, April 19-22, 1999

(Vachtsevanos 2006) Vachtsevanos, G., et al. Intelligent Fault Diagnosis and Prognosis for Engineering Systems, John Wiley & Sons, Inc., 2006, page 418-419.

(Press 1992) Press, W., et al. Numerical Recipes in C, Cambridge University Press, New York, 1992, page 113 – 116.

(Vecer 2005) Vecer, P., Kreidl, M., Smid, R., “Condition Indicators for Gearbox Condition Monitoring Systems” *Acta Polytechnica* Vol. 45, No 6/2005.

(Proakis 1995) Proakis, John, G., *Digital Communications*, McGraw-Hill, Boston MA, 1995, page 45-46

(Bechhoefer 2007) Eric Bechhoefer, Andreas Bernhard, “A Generalized Process for Optimal Threshold Setting in HUMS” *IEEE Aerospace Conference*, Big Sky, 2007.

APPENDIX

Matlab code for Time Synchronous Averaging based on resampling the time domain data. Note: Linear or Polynomial interpolation is given. It is a simple matter to change the Interpolate function to cubic spline or band limited filter (such a cosine filter)

```
function [tsadata, navgs,rpm]=tsa_intp(data,tach,sr,ratio,navgs)
%[tsadata, navgs,rpm, z]=tsa_intp(data,tachsignal,sr,ratio,navgs)
%Inputs:
% data:    time domain data in g's
% tach:    tachometer signal
% sr:      sample rate
% ratio:   gear ratio/pulse per revolution on the tach
% navgs:   desired number of averages
% find the portion of the tach vector that is zero or negative
%Output:
% tsadata: time synchronous average data
% navgs:   the number of averages in the TSA
% rpm:     mean shaft rpm
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dt=1/sr; %time bewteen samples.
n = length(tach);
tach = tach - mean(tach);
x=find(tach<=0); % find the zero crossings,
%eliminate the first value to insure that
i=x(find(tach(x(2:length(x))-1)>0)); % (i-1)>=1 {i.e. x(2:length(x))}
if i(end) == n,
    i(end) = [];
end
i1 = i + 1;
% now interpolate the zero crossing times
in = i+tach(i)./(tach(i)-tach(i1));
zct = in*dt';

rpm = mean(1./diff(zct))*60*ratio;

% Define the number of averages to perform
if nargin < 6,
    navgs = floor((length(zct)-1)*ratio);
end
% Determine radix 2 number where # of points in resampled TSA
% is at sample rate just greater than fsample
N=(2^(ceil(log2(60/rpm*sr))));

% now calculate times for each rev (1/ratio teeth pass by)
xidx = 1:length(zct);
% resample vibe data using zero crossing times to interpolate the vibe
yy = zeros(1,N); % data to accumulate the resampled signal once per rev
ya = yy; % ya is the resample signal once per rev

iN = 1/N; %resample N points per rev
tidx = 1; % start of zct index
ir = 1/ratio; %inverse ratio - how much to advance zct
zct1 = zct(tidx); %start zct time;
x = (0:length(data)-1)*dt; %time index of each sample
z = zeros(navgs,1);
for k = 1:navgs
    tidx = tidx + ir; % get the zct for the shaft
    stidx = floor(tidx)-1; %start idx for interpolation
    zcti = polint(xidx,zct, stidx, 2, tidx); %interpolated ZCT
    dtrev = zcti - zct1; % time of 1 rev
    dtic = dtrev*iN; % time between each sample

    for j = 1:N,
        cidx = floor(zct1*sr);
        ya(j) = polint(x,data,cidx,2,zct1); %interp. time domain sample
        zct1 = zct1 + dtic; %increment to the next sample
    end
    zct1 = zcti;
    % generate resampled vibe data and accumulate t
    % the vector values for each rev
    yy = yy + ya;
end
tsadata = yy/navgs; % compute the average

function y = interpolate(xa, ya, stidx, n, x)
%polynomial interpolation: n = 2 is linear,
%Numerical Recipies in C, 1995
%given vect xa and vect ya, and valuex, retures a value y.
% uses a N-1 degree polynomial
% such that P(xai) = pai, i = 1:n,
ns = 1;
c = zeros(n,1);
d = zeros(n,1);
dif = abs(x - xa(stidx+1));
for i = 1:n,
    dift = abs(x-xa(stidx+i));
    if dift < dif,
        ns = i;
        dif = dift;
    end
end

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c(i) = ya(stidx+i);
d(i) = ya(stidx+i);
end
y = ya(stidx+ns);
ns = ns-1;
for m = 1:n-1,
    for i = 1:(n-m),
        ho = xa(stidx+i) - x;
        hp = xa(stidx+i+m) - x;
        w = c(i+1) - d(i);
        den = ho-hp;

        if den == 0.0
            disp('Error in polint');
            return;
        end
        den = w/den;
        d(i) = hp*den;
        c(i) = ho*den;
    end
    if 2*ns < (n-m),
        dy = c(ns+1);
    else
        dy = d(ns);
        ns = ns-1;
    end
end

y = y + dy;
end

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function [tsa,navgs,rpm] = tsa_fft(v,tach,sr,ratio,navgs)
%[tsa,navgs,rpm] = tsa_fft(v,tach,sr,ratio)
%Frequency domain time synchronous average
%Inputs:
% v :vibration signal
% tach :tach signal
% sr :sample rate
% ratio :1/pulse per rev* reduction ratio
%Outputs:
% tsa :time synchronous average
% navgs :number of avearges
% rpm :shaft rpm
%Example: 10 pulse per rev tach, 5:1 reduction ratio, sr = 200000/3;
%[tsa,navgs,rpm]=tsa_fft(v,tach,sr,1/10*1/5);
%Eric Bechhoefer, April 10, 2009 for PHM Conference
%Adapted from: Vachtsevanos, G., et al., "Intelligent Fault Diagnosis and
% Prognosis for Engineering Systems" John Wiley and Sons, 2006, page
%418
if nargin < 5,
    navgs = 0;
end
n = length(tach);
t = tach-mean(tach);
x=find(t<=0);
% find the zero crossings
i=x(t(x(2:length(x))-1)>0);
if i(end) == n,
    i(end) = [];
end
i1 = i + 1;

% linear interpolation
idx = i+t(i)./(t(i)-t(i1));
rpm = mean(1./diff(idx))*60*sr*ratio;

% number of tach zero crossings give number of pulse per rev
% and reduction ration

nr = 1/ratio;
% number of averages to perform
avgs = floor(((length(idx)-1)/nr));
if navgs>avgs || navgs == 0,

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navgs = avgs;
end

% length of the fft;
minRL = min(diff(idx)*nr);
nF = floor(minRL/2)-1;
yy = zeros(navgs,nF);
cnt = 0;
for k = 1:nr:navgs*nr,
    % get the indexes between shaft zero crossings
    sidx = round(idx(k)):round(idx(k+nr))-1;
    % the signal for 1 revolution
    sv = v(sidx);
    % spectrum of one per rev vibration
    spec = fft(sv);
    % generate resampled vibe data and accumulate the vector values for
    each rev
    cnt = cnt + 1;
    yy(cnt,:)= spec(1:nF);
end
% compute the average
Y = mean(yy);
fd = [Y 0 conj(fliplr(Y(2:end)))];
% get the time domain representation and normalize
tsa = real(ifft(fd));
```