On Applying the Prognostic Performance Metrics

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ABSTRACT

Prognostics performance evaluation has gained significant attention in the past few years. As prognostics technology matures and more sophisticated methods for prognostic uncertainty management are developed, a standardized methodology for performance evaluation becomes extremely important to guide improvement efforts in a constructive manner. This paper is in continuation of previous efforts where several new evaluation metrics tailored for prognostics were introduced and were shown to effectively evaluate various algorithms as compared to other conventional metrics. Specifically, this paper presents a detailed discussion on how these metrics should be interpreted and used. shortcomings identified, Several while applying these metrics to a variety of real applications, are also summarized along with discussions that attempt to alleviate these problems. Further, these metrics have been enhanced to include the capability of probability incorporating distribution information from prognostic algorithms as opposed to evaluation based on point estimates only. Several methods have been suggested and guidelines have been provided to help choose one method over another based on probability distribution characteristics.

These approaches also offer a convenient and intuitive visualization of algorithm performance with respect to some of these new metrics like prognostic horizon and α - λ performance, and also quantify the corresponding performance while incorporating the uncertainty information.

1 INTRODUCTION

Prognostics being an emerging research field, most of the published work has naturally been exploratory in nature, consisting mainly of proof-of-concepts and oneoff applications. Prognostic Health Management (PHM) has by-and-large been accepted by the engineered systems community in general, and the aerospace industry in particular, as the direction of the future. However, for this field to mature, it must make a convincing case in numbers to the decision makers in research and development as well as fielded applications. It is as Prof. Thomas Malone, an eminent management guru, said, *"If you don't keep score, you are only practicing"*.

In research, metrics are not simply a means to evaluate the quality of an approach, they can be useful in a variety of different ways. One of the most direct uses is reporting performance both internally and externally with respect to the research organization. Metrics can create a standardized language with which technology developers and users can communicate their findings with each other and compare results. This aids in the dissemination of scientific information as well as decision making. Metrics may also be viewed as a feedback tool to close the loop on research and development by using them as objective functions to be minimized or maximized, as appropriate, by the research effort.

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The multifarious uses notwithstanding, metrics can be a double-edged sword. An off repeated quote in the management world goes, "Be careful what you measure—you might just get it." What this saying means is that we mostly set goals based on what we can measure, and then we set about achieving those goals without heed to the inherent value of the work. It is usually much harder to ascertain value than it is to evaluate metrics. Thus, the careful choice and design of a metric to reflect the intended value is of paramount importance, especially in a nascent research area like prognostics.

1.1 Recent Developments

Recently there has been a significant push towards crafting suitable metrics to evaluate prognostics performance. Researchers from academia and industry are working closely to arrive at useful performance measures. For instance, in (Leao et al, 2008) authors propose some prognostics metrics and compare them with diagnostics metrics. However, these metrics are mostly derived from metrics used for prediction tools in finance as opposed to being specifically tailored for prognostics. In (Saxena et al., 2008) we categorized various forecasting applications based on their different characteristics and pointed out that there are notable differences between forecasting in other domains and the task of remaining life prediction for PHM. It is therefore, important to develop metrics that directly address the problem at hand. On the flipside it becomes quite challenging to reshape the mindset around using metrics from other forecasting domains. (Wang and Lee, 2009) propose simple metrics in their paper from the classification discipline and also suggest a new metric called "Algorithm Performance Profile" that tracks the performance of an algorithm using the accuracy score each time an estimated Remaining Useful Life (RUL) is estimated. In (Yang and Letourneau, 2007), authors present two new metrics for prognostics; in particular they define a reward function for predicting the correct time-to-failure that also takes into account prediction and fault detection coverage. They also propose a cost-benefit analysis based metric to quantify how much an organization could save by deploying a given prognostic model. Thus, in general, efforts are being made to evaluate prognostics from different end-user point of views.

This paper is a thematic continuation of previous works that surveyed metrics in use for prognostics in a variety of domains (Saxena *et al.*, 2008) in order to come up with a list of metrics to assess critical aspects of RUL predictions and showed how such metrics can be used to effectively assess the performance of prognostic algorithms (Saxena *et al.*, 2009). This paper will focus on the design and choice of parameters for metrics that are specifically designed for prognostics beyond the conventional ones being used for diagnostics and other forecasting applications. These metrics have been introduced in (Saxena *et al.*, 2008) and their implementation discussed in (Saxena *et al.*, 2009). Here we discuss the ways in which these metrics may be interpreted and used, and even misused or abused depending upon specific application scenarios. Furthermore, some enhancements over the original definitions have been presented to incorporate issues observed while applying these metrics to real applications.

The next section motivates our detailed analysis on prognostics metrics. This analysis is first presented in the context of prognostic horizon and then extended to other metrics in section 3. Section 3 also presents enhancements on these metrics to include prediction distributions and methods to implement them. Finally, the paper discusses future directions in section 4 followed by conclusions in section 5.

2 MOTIVATION

In this paper we discuss the twofold benefits of performance metrics (see Figure 1).





Given current Technology Readiness Level (TRL) for the prognostics technology, lack of assessment about prognosability of a system and concrete uncertainty management approaches, managers of critical systems/applications have struggled to define concrete performance specifications. In most cases performance requirements are either derived from previous diagnostics experience or are very loosely specified. Prognostics metrics as proposed in (Saxena *et al.*, 2008) depend on various parameters that must be specified by the customer as requirements that an

algorithm should attempt to meet as specifications. The process of coming up with reasonable values for these parameters must consider a complex interplay between several other factors as discussed in this paper. We show, here, how these metrics can help users to come up with these specifications by taking such factors into account in a systematic framework. It is anticipated that in this manner these metrics will be useful for decision making in practical implementations of prognostics. On the other hand, providing feedback to algorithm developers and helping them improve their algorithms while trying to meet such specifications is yet another role these metrics are expected to play in a more conventional sense.

The new prognostics metrics developed in previous work require a change in thinking about what constitutes a good performance. More importantly the time varying aspect of performance, each time the estimates are updated, differentiates these metrics from other related domains. These metrics offer visual as well as quantitative assessment of performance as it evolves over time. The visual representation allows making several observations about the performance and it is necessary for us, now, to understand the capabilities and the limits of information these new metrics can provide. Therefore, we try to draw a scope where these metrics may be applicable, useful and also describe where the limitations may be.

With some initial experience we found ourselves under a dilemma between creating a comprehensive but complicated metric and a simple but less generic metric. The trade-off originates from the interplay between the ease of use, interpretability, and comprehensiveness. We determined that a more complicated metric has fewer chances of being adopted and more chances of breaking in many special cases that may not have been envisioned while formulating these ideas. Therefore, suggesting enhancements to the extent where these metrics are still simple enough to use and at the same time pointing out cases where these metrics are not expected to break is another objective of this paper. A significant enhancement presented in this paper is the ability of these metrics to incorporate uncertainty estimates available in the form of RUL distributions.

3 PROGNOSTIC PERFORMANCE METRICS

In this paper we discuss the four prognostics metrics Prognostic Horizon (PH), α - λ Performance, Relative Accuracy (RA), and Convergence that can be used for offline performance evaluation of the prognostic performance. Here to begin the discussion we first discuss the reasons why these metrics are more suitable for offline evaluation only and how they can ultimately lead to online performance evaluation. A detailed discussion on the methodology to use these metrics will subsequently follow.

3.1 Offline vs. Online Performance Metrics

Some confusion has prevailed regarding the concepts of online and offline performance evaluation for prognostics. This confusion arises from the fact that prognostic performance evaluation is an acausal problem that requires inputs from the events that are expected to take place in the future. Specifically, one needs to know the true end of life of the system to evaluate prediction accuracy. Another aspect that makes this evaluation further complicated is the paradox of prognostics in real applications that if something is sensed to break in the future it is immediately attended to prevent any downtime. This alters the original system and leaves no way to confirm whether the prediction about failure would have been accurate or not. Therefore, it has been a rather tricky proposition to assess long term prognostic results.

Keeping this situation in mind we proposed a gamut of performance metrics for offline performance evaluation. This would particularly help prognostic algorithm development by providing a way to measure performance in cases where one knows the true end of life and therefore provide appropriate feedback. Once these metrics are refined and fine tuned, further development will follow on extending these concepts for online performance evaluation. Online evaluation will have to incorporate methods to deal with uncertainties associated with future operating conditions in particular. This will require significant uncertainty advancements in representation, quantification and management methods, which renders the discussion on online performance evaluation appropriate for the future work.

3.2 Offline Performance Evaluation

The four prognostic performance metrics follow a systematic progression in terms of the information they seek (Figure 2).

First, the prognostic horizon identifies whether an algorithm predicts within a specified error margin (specified by the parameter α) around the actual end-oflife and if it does how much time it allows for any corrective action to be taken. In other words it assesses whether an algorithm yields a sufficient prognostic horizon and if not it may not even be meaningful to compute other metrics. Thus if an algorithm passes the PH test the α - λ Performance goes further to identify whether the algorithm performs within desired error margins (specified by the parameter α) of the actual RUL at any given time instant (specified by the parameter λ) that may be of interest to a particular application. This presents a more stringent requirement of staying within a converging cone of error margin as a system nears End-of-Life (EoL). If this criterion is also met, the next step is to quantify the accuracy levels relative to actual remaining useful life.



Figure 2: Hierarchical design of the prognostics metrics

These notions assume that prognostics performance improves as more information becomes available with time and hence by design an algorithm will satisfy these metrics criteria if it converges to true RULs. Therefore, the fourth metric Convergence quantifies how fast the algorithm converges provided it satisfies all the previous metrics. The group of these metrics can be considered a hierarchical test that yields several levels for comparison among different algorithms in addition to the specific information these metrics provide individually regarding algorithm performance.

Since these metrics share the attribute of performance tracking with time; we first develop our discussions using Prognostic Horizon as an example. These discussions are then extended to the rest three with additional details specific to individual metrics.

3.3 Prognostic Horizon

Prognostic Horizon is defined as the difference between the time index *i* when the predictions first meet the specified performance criteria (based on data accumulated until time index *i*) and the time index for EoL. The performance requirement may be specified in terms of allowable error bound (α) around true EoL.

$$PH = EoL - i \tag{1}$$

where:

 $i = \min\{j \mid (j \in \ell) \land (r_*(1-\alpha) \le r^l(j) \le r_*(1+\alpha))\}$ is the first time index when predictions satisfy α bounds

 ℓ is the set of all time indexes when a prediction is made

l is the index for l^{th} unit under test (UUT)

r_* is the ground truth RUL

Prognostic horizon produces a score that depends on length of ailing life of a system and the time scales in the problem at hand. The range of PH is between $(t_{EoL}-t_P)$ and $max[0, t_{EoL}-t_{EoP}]$. The best score for PH is obtained when an algorithm always predicts within desired accuracy zone and the worst score when it never predicts within the accuracy zone.

3.3.1 What can be inferred from the metric

The notion for Prognostic Horizon has been long discussed in the literature from a conceptual point of view. This metric indicates whether the predicted estimates are within the specified limits around the actual EoL so that the predictions can be considered trustworthy. It is clear that longer the prognostics horizon is more time becomes available to act based on a prediction. Therefore, while comparing algorithms, an algorithm with longer prognostic horizon would be preferred.

As shown in Figure 3, the desired level of accuracy with respect to the EoL ground truth is specified as $\pm \alpha$ -bounds. RUL values are then plotted against time for various algorithms that are being compared. The PH for an algorithm is declared as soon the corresponding predictions enter the band of desired accuracy. As clearly evident from the illustration, the first algorithm has a longer PH.



Figure 3: Prognostic Horizon

3.3.2 Issues resulting in ambiguities

There are several cases where standard definition for PH breaks from a practical point of view and declaring a PH may not be straight forward. We discuss some such cases next and suggest possible ways to deal with them.

RUL trajectory jumps out of the accuracy zone: Based on our experience and feedback from fellow researchers while applying these metrics to several applications there are often cases where RULs jump in and out of the $\pm \alpha$ accuracy zone. In such cases it may not be appropriate to declare the PH at a time instant where RUL enters within $\pm \alpha$ accuracy zone for the very first time and then jumps out again. As illustrated in Figure 4, for both examples predictions at time instant cjump out of the accuracy zone and get back in at a later prediction step. In absence of such an anomaly the PHs for these algorithms would have been declared at times a and b. A situation like this results in multiple time indexes when RUL trajectory enters the accuracy zone. A simple approach to deal with this situation can be being more conservative and declaring PH at the latest time instant the predictions enter accuracy zone. Another option is to use the original PH definition and evaluate other metrics to determine if the algorithm satisfies other requirements. To avoid confusions we recommend using the original definition. This will encourage practitioners to go back to the algorithm development stage and improve their prediction process to incorporate capabilities to deal with such anomalies in their algorithms



Figure 4: Cases where RUL predictions do not stay consistently within the accuracy zone

Situations like these can occur due to various reasons as listed below and it is important to identify the correct one before computing a PH.

• Inadequate system model: Real systems often exhibit inherent transients at different stages of their lives. These transients get reflected as deviations from the true value for computed RUL estimates if the underlying model assumed for the system does not account for these behaviors. For example, in (Saxena *et al.*, 2009) authors describe an application of Li-ion battery health management where the capacity decay shows such transient behaviors in the beginning and the end phases of the battery life. Their examples show cases where RUL trajectories jump away from ground truth whenever such transient phases occur. Therefore, for situations as depicted in Figure 4 one must go back and refine their models to incorporate such anomalies.

- Operational transients: Another source of such behaviors can be due to sudden changes in operational profiles under which a system is operating. Prognostic algorithms may show a time lag in adapting to such changes and hence resulting in temporary deviation from the real values.
- **Uncertainties** prognostic in environments: Prognostics is inevitably surrounded bv uncertainties arising from a variety of sources. This makes prognostics inherently a stochastic process and hence the behavior observed from a particular run may not exhibit the true nature of prediction trajectories. This discussion assumes that all measures for uncertainty reduction have already been taken during algorithm development and that such observations are an isolated realization of the process. In that case these trajectories should be obtained based on multiple runs to achieve statistical significance or such that a more sophisticated stochastic analyses can be carried out.

Before one arrives at the final assessment for PH metric, a situation like the one discussed above helps pinpoint the exact reason for such behaviors. Whenever such behavior is observed one must go back and identify the most probable cause and try to improve the models, fine tune algorithms, or better the experimental design as the situation demands. A robust algorithm and a system model should be capable of taking care of transients inherent to the system behavior and operational conditions. Plotting the RUL trajectory in prognostic horizon plot provides clues regarding such deficiencies to algorithm developers. Once these deficiencies are taken care of there is a good chance that such behaviors disappear and a PH can be easily determined, otherwise the simple but conservative approach, as discussed earlier, may be used.

RUL trajectory jumps out close to EoL: Other situations that were reported included cases where one observes a well behaved converging behavior for the RUL trajectory for most of the ailing life except at the very end when they jump out of the accuracy zone (see Figure 5). In (Saxena *et al.*, 2009) authors attribute such behavior to system transients that were not modeled well by some of the data-driven algorithms that were used. To deal with situations we introduce a new concept of "useful predictions". All engineered systems undergo non-linear dynamics during fault progression, leading to a system failure at t_{EoL} . More

often than not these dynamics are difficult to model or learn from data as the system nears the failure point. Thus, while evaluating the PH metric for a particular algorithm, it is possible in a given application that the RUL curve deviates away from the error band near $t_{\rm EoL}$, after having entered it earlier during its trajectory. In such a case, it may be counterproductive to bias ourselves against an algorithm which has a very small or no PH, since we would be ignoring the algorithm's performance elsewhere on the RUL curve. Consequently it may be prudent to evaluate PH on a error band that is limited in extent on the time x-axis by the time instant t_{EoUP} , which denotes the End-of-Useful-Predictions (EoUP), such that we ignore the region near t_{EoL} , within which it is impossible to take any corrective action based on the RUL prediction and these predictions are of little or no use practically. The value of $t_{\rm EoUP}$ chosen is dependent upon the application, the time and cost for possible redress actions in that domain. In other words EoUP determines the lower limit on acceptable range for PH in a given application. While we do not intend to specifically define the constituents of t_{EoUP} , there may be different connotations in different cases. For instance, in cases where a continuous uptime of the system is required t_{FoUP} would represent Mean Time to Repair (MTTR). In contrast the cases where it is required to safely abort the mission to avoid any catastrophe, t_{EoUP} would represent the time to take necessary actions in doing so.



Figure 5: Prediction behavior after EoUP is practically inconsequential and hence need not affect the PH

As an example, Figure 6 shows the results for 4 different algorithms in predicting battery life. All algorithms except Relevance Vector Machines (RVM) deviate away from the true RUL whereas RVM reaches very close to true RUL near EoL. But at the same time RVM results in a shorter PH. Depending on how much time may be needed to repair/replace the battery, in

such cases, one would determine the EoUP to implement a better definition of PH.



Figure 6: An example showing different cases in a real application (Saxena *et al.*, 2009)

3.3.3 Guidelines for using the metric

The main idea behind these metrics is not only to compare different algorithms for performance evaluation but also to help management decide on specifications and requirements on prognostics algorithm in the fielded applications. The outcome of the metric depends directly on the values chosen for input parameters like α . To that end, we describe how α can be chosen for a specific application.

Prognostic horizon emphasizes the time critical aspects of prognostics. For a catastrophic event a prediction ahead of time is meaningful only if a corrective action can be completed before the system fails. Keeping the description generic, there are systems that involve different levels of criticality when they fail. In a mission critical scenario a failure may be catastrophic and hence a limited number of false positives may be tolerable but no false negatives. In other cases cost of acting on false positives may be prohibitively high, e.g. unnecessary maintenance and overhaul of aircraft engines. There are even cases where it is more cost effective to accept several false negatives as opposed to reacting to a false positive. In many such cases it is acceptable even if the system runs to failure once in a while, e.g. maintenance of copier machines.

There are several factors that determine how critical it may be to make a correct prediction. These factors combined together should dictate the choice of α while specifying the PH requirements for performance evaluation. Although not always necessary, we expect a reasonable prognostic algorithm to improve (converge to predicting true RULs) as time progresses. Keeping in mind such converging trend we expect a lower PH for a smaller value of α as it represents the tolerable error bounds on the predictions. This suggests that we cannot arbitrarily choose a low value of α , instead this selection should involve an assessment on how well it is possible to predict and how much confidence can be established in the prediction algorithm itself based on its uncertainty management capabilities. This leads into uncertainty representation and management techniques incorporated into the prognostics algorithm, which is beyond the scope of this paper. Some of the other important factors that should be considered are listed below.

1. *Time for problem mitigation*: The amount of time to mitigate a problem or start a corrective action when critical health deterioration of a component/system has been detected is a very important factor. As mentioned earlier, very accurate predictions at a time when no recovery action can be made is not useful. Hence, a tradeoff between error tolerance and time for recovery from fault should be considered. The time for problem mitigation will vary between system to system and involves multiple factors including logistic efficiency and availability of maintenance equipment.

2. Cost of mitigation: Cost of the reparative action is an important factor in all management related decisions and hence should be considered while determining α . This assessment should include a study on implications of false positives based on prediction uncertainties associated with an algorithm and the costs incurred in the maintenance operations.

3. Criticality of system or cost of failure (false negative): In time-critical applications, resources should be directed towards more critical and important components in order to efficiently maintain overall health of the system. Hence, if health assessment is being performed on multiple units in a system, α for the different units should be chosen based on a prioritized list of criticality. Assessment of criticality is usually done based on severity and frequency of occurrence statistics available from Failure Modes, Effects, and Criticality Analysis (FMECA) studies (MIL-STD-1629A, 1980). Another perspective to assess criticality is based on cost-benefit analysis where cost of failures is incorporated to assess the implications of false negatives (Banks and Merenich, 2007), (Feldman et al., 2008).

It should be noted that the factors mentioned above are not arranged based on any order of importance and users should utilize them based on characteristics of their systems and may skip a few of them. We denote the combination of factors used in determining α as "recovery cost" or *t*_{recovery}. These factors account for the requirements from logistics point of view in the health management and does not take into account the factor of *prognosability* of the system. There may be limitations on how well a prognostics algorithm may be able to manage and reduce uncertainty and hence is limited by a lower bound on the best achievable precision. The preferred approach should be to improve the algorithm to meet specified α requirement wherever possible. If due to the nature of the problem no algorithm can meet the specifications one needs to relax the specifications by choosing a larger α .

3.3.4 Incorporating probability distributions

In previous works (Saxena et al., 2008) and (Saxena et al., 2009) we presented the definitions and examples of performance metrics considering that the prognostics algorithm provides a RUL prediction r(k) represented by a single point. This assumed that such prediction is deterministic or that an algorithm includes additional reasoning to compute a single point estimate of the prediction distribution. Given that there are multiple sources of uncertainties inherent to the prognostics problem, it is expected/required that a prognostics algorithm provides information about the confidence around the prediction. This confidence can be represented in several ways. There are algorithms that provide an approximation of the probability distribution of the RUL, r(k) at any point k by providing a set of discrete samples of r(k) with their corresponding probabilities (Orchard and Vachtsevanos, 2009). Other algorithms that rely on Gaussian assumptions describe the uncertainty by providing the mean and variance of a normally distributed r(k) prediction (Goebel *et al.*, 2008). In some cases where multimodal distributions are obtained, an approximation with mixture of Gaussians has been considered to derive the distribution characteristics (Saha et al., 2009).

Generally, a common way to describe a distribution is based on the first two moments. The mean is an indication of central tendency or location and the variance is an indication of the spread of the distribution. These quantities completely summarize Gaussian distributions. For cases were normality cannot be established, one can rely on median as a measure of location and the quartiles or inter quartile range as a measure of spread (Hoaglin *et al.*, 1983).

Prognostics metrics like prognostic horizon and α - λ performance provide a great deal of visual information in addition to answers that one seeks at specific time instances. Therefore, incorporating enhanced visual representations for prediction distributions improves the efficacy of these metrics for performance comparison. For cases involving Normal distribution, including a confidence interval represented by an error bar around the point prediction is useful (Devore, 2004). For cases with non-Normal single mode distributions this can be done with an inter-quartile plot represented by a box plot (Martinez, 2004). This conveys how a prediction distribution is skewed and whether these skew should be considered while declaring prognostic horizon. Box plot also has provisions to represent outliers that may be useful to keep track of in risk sensitive situations (Figure 7).



Figure 7: Representations for distributions

In addition to visual enhancements, distribution information can be better utilized by computing total probability mass of a prediction falling within the specified α -bounds versus using a point estimate to compute the metric. This concept has been depicted in Figure 8 with original point prediction superimposed on box plots.



Figure 8: Enhanced representation for prognostic horizon incorporating distribution information

The original PH metric assumed a single point prediction for an output of the prognostics algorithm. This ignores uncertainty information even if algorithms provide this information in the form of distributions and does not allow a fair comparison for situations where a prediction is very close to the alpha bound but not quite inside it. To incorporate this situation we introduce a β criterion, which states that a prediction is

considered inside α -bounds only if the total probability mass of the corresponding distribution within the α bounds is more than a predetermined threshold β . In this manner the definition of PH will be modified by determining the index *i* in Eq.1 as,

$$i = \min\left\{j \mid (j \in \ell) \land \left(\pi[r(j)]_{\alpha^{-}}^{\alpha^{+}} \ge \beta\right)\right\},\$$

where

 $\pi[r(j)]_{\alpha^-}^{\alpha^+} = \int_{\alpha^-}^{\alpha^+} \phi(x) dx; \quad x \in \Re^+ \text{ is the total probability}$ mass of the prediction pdf within the α -bounds that are given by $\alpha^+ = r_* + \alpha \cdot EoL$ and $\alpha^- = r_* - \alpha \cdot EoL$.

This way, as shown in Figure 9, a modified PH can be obtained for the same case if the RUL distribution satisfies β criterion. Therefore, it is desirable to take advantage of the uncertainty information and use it to declare PH even if the point prediction does not fall within the bound explicitly.



Figure 9: Integrating the probabilities of RUL falling in α -bound can be used to decide declaration of PH

This formulation, however, raises another question of how to pick a suitable value of β . This parameter is also linked to the issues of uncertainty management and risk absorbing capacity of the system as mentioned earlier. In the most simple case we suggest using $\beta = 0.5$ that would correspond to making a decision based on the

mean value for a Gaussian distribution case, the approach that we had been following with the original definition of PH using means as point estimates. The user is of course free to choose any larger number for β in a more conservative setting. A higher value of β indicates that a larger portion of the prediction pdf is required to be within the α -bounds for declaration of the PH.

Method to compute the metric: As mentioned above, computing the metric now requires integrating the probability distribution that overlaps with the desired region to compute the total probability. For cases where analytical form of the distribution is available, like for Normal distributions, it can be computed analytically by integrating the area under the prediction pdf between the α -bounds (α^- to α^+). However, for cases where there is no analytical form available, a summation based on histogram obtained from the process/algorithm can be used to compute total probability. This procedure has been pictorially depicted in Figure 10.

An important question, which still remains to be answered, is what should one use as the representations for location and spread. For simple cases like Normal distributions this is straightforward, however, for other cases this may not be very clear. As outlined in Table 1, there are four main categories a distribution may fall under, which can be further classified under *parametric* and *non-parametric* subclasses. This subclassification mainly determines the method of computing the total probability, i.e. continuous integration or discrete summation. It is suggested to use box plots along with a dot representing the mean of the distribution, which will allow keeping the visual information in perspective with respect to original plots. For mixture of Gaussians case, it is suggested that a model with few (preferably $n \leq 4$) Gaussians is created and corresponding error bars plotted adjacent to each other.

$$\phi(x) \cong \omega_1 \cdot N(\mu_1, \sigma_1) + \dots + \omega_n \cdot N(\mu_n, \sigma_n); n \in I^+ \quad (2)$$

where:

- $\boldsymbol{\omega}$ is the weight factor for each Gaussian component
- $N(\mu, \sigma)$ is a Gaussian distribution with parameters μ and σ

The weights for each Gaussian component can then be represented by the thickness of the error bars. We do not recommend multiple box plots in this case as there is no methodical way to differentiate between samples, assign them to particular Gaussian components, and compute the quartile ranges for each of them. Also, to keep things simple we assume a linear additive model while computing the mixture of Gaussians.



Figure 10: Procedure to compute total probability of RULs being within specified α -bounds

	Normal Distribution	Mixture of Gaussians	Non-Normal Distribution	Multimodal (non-Normal)
	Parametric		Non-Parametric	
Location (Central tendency)	Mean (μ)	Means: $\mu_1, \mu_2,, \mu_n$ weights: $\omega_1, \omega_2,, \omega_n$	Mean, Median, L-estimator, M-estimator	Dominant median, Multiple medians, L-estimator, M-estimator
Spread (variability)	Sample standard deviation (σ) , IQR (inter quartile range)	Sample standard deviations: $\sigma_1, \sigma_2,, \sigma_n$	Mean Absolute Deviation (MAD), Median Absolute Deviation (MdAD), Bootstrap methods, IQR	
Visualization	Confidence Interval (CI), Box plot with mean	Multiple CIs with varying bar width I	Box plot with mean	Box plot with mean

Table 1: Recipe to select location and spread measures along with visualization methods

In this section we have described in details various aspects of prognostics metrics in the context of prognostic horizon. Many of these concepts naturally transfer to the other metrics and therefore for the sake of conciseness and less repeatability, we will very briefly point out other salient features of the rest of the three metrics and frequently refer to the discussion above.

3.4 α - λ Performance

 α - λ Performance quantifies prediction quality by determining whether the prediction falls within specified limits at particular times. These time instances may be specified as percentage of total ailing life of the system. Any performance measure of interest may fit in this framework. So far in general, we have used accuracy as the main performance measure. In our implementation of α - λ accuracy we seek answer to the question whether the prediction accuracy is within α *100% of the actual RUL at specific time instance t_{i} , which is expressed as a fraction of time between the point when an algorithm starts predicting and the actual failure. For example, this metric determines whether a prediction falls within 10% accuracy (i.e., $\alpha = 0.1$) halfway to failure from the time the first prediction is made (i.e., $\lambda = 0.5$). Therefore, one needs to evaluate whether the following condition is met.

$$(1-\alpha) \cdot r_*(t) \le r'(t_\lambda) \le (1+\alpha) \cdot r_*(t) \tag{3}$$

where:

 α is the accuracy modifier

 λ is a time window modifier such that $t_{\lambda} = t_{P} + \lambda (EoL - t_{P})$

The output of this metric is binary (True or False) stating whether the desired condition is met at a given particular time. This is a more stringent requirement as compared to prognostic horizon as it requires predictions to stay within a cone of accuracy i.e. the bounds that shrink as time passes by. With the new enhancements, it is also possible to compute total probability overlapping with this cone to determine whether the criteria are met (Figure 11a). For easier interpretability α - λ accuracy can also be plotted as shown in Figure 11b. However, in this case the definition of *α*-bounds modified is as $\alpha^{+} = r_{*}(1+\alpha)$ and $\alpha^{-} = r_{*}(1-\alpha)$.

The concept of α - λ precision is further illustrated in Figure 12. The choice of precision measure may be application specific or based on the type of distribution. Definition of α is slightly different for the precision case. Here α is a function of time, where $\alpha(t_p)$ is the allowable upper bound for the precision measure at the beginning and $\alpha(t_{EoL})$ is the allowable upper bound at

EoL point. This plot shows how precision evolves with time and evaluates whether it satisfies a given level of precision at a specified time instant t_{λ} .



Figure 11: (a) α - λ accuracy with error bars (b) alternative representation of α - λ accuracy



Figure 12: Concept of α - λ precision

3.4.1 What can be inferred from the metric

 α - λ Performance metrics convey a great deal of information. First, it visually summarizes the performance of an algorithm over a length of time in a very concise manner. Although the metric itself is intended to be evaluated at a particular given time, the overall viewgraph provides a way to assess in what region a particular algorithm performs well. In our experience we have found that different algorithms model system dynamics in different stages of life differently. Some perform well in the early stages and some in the later stages. In such cases looking at the α - λ plot one could even decide to fuse outcomes of different algorithms in different stages of the system/component life. This decision may incorporate various factors like the computational intensity of the various algorithms, criticality of the system in different stages, performance of individual algorithms, etc. This approach seems useful in particular where fault evolution model follows complex trends.

Other inferences from the metric include the ability to establish performance limits for different algorithms. Therefore, given an α - λ plot for different algorithms (e.g. see Figure 6) one could compute the best values of α each algorithm can satisfy for a given value of β , either at any given time instant or a time duration. From the requirements specification point of view, it may also be useful to construct the α - λ plot with different values of α (multiple cones) against a benchmark algorithm (assuming there exited one) to assess what values may be more suitable for specifying the performance requirements. Further, if prognostics algorithms have the ability to incorporate uncertainty estimates and the corresponding risks on making decisions can be evaluated, requirements on the values of β can be specified. It may be noted that a larger value of β imposes more stringent limits on accuracy and precision of RUL prediction pdfs.

Whereas we have listed a few inferences that one could draw from this metric we are confident that there are more to follow as more experience is gained in the user community while applying these metrics to different applications and situations.

3.5 Relative Accuracy

Relative prediction accuracy is a notion similar to α - λ accuracy where, instead of finding out whether the predictions fall within given accuracy levels at a given time instant, we also quantitatively measure the accuracy. As shown in Figure 13, prediction trajectories from two algorithms are assessed against true RUL line at two different times. At a given time instant t_{λ} the RUL prediction error is computed and normalized by the true RUL at t_{λ} . The time instant is described as a fraction of the ailing life as before. An algorithm with higher relative accuracy is desirable. The range of values for RA is [0,1], where the perfect score is 1. It must be noted that if the prediction error magnitude grows beyond 100% RA gives a negative value. We do not consider such cases since they would not have passed the first two tests in the first place.

$$RA_{\lambda} = 1 - \frac{\left|r_{*}(t_{\lambda}) - r'(t_{\lambda})\right|}{r_{*}(t_{\lambda})} \tag{4}$$

where $t_{\lambda} = t_{P} + \lambda (EoL - t_{P})$



Figure 13: Schematic showing Relative Accuracy concept. At $\lambda = a$, Algorithm 1 has a better relative accuracy than Algorithm 2 but vice versa at $\lambda = b$.

3.5.1 How to use the metric

RA conveys information at a specific time. In particular, it measures the accuracy with respect to the time remaining before EoL. It is desired to obtain higher accuracy levels as system approaches EoL. While measures like PH and α - λ performance provide a great deal of visual information, RA provides a quantitative measure that can be used in automated implementation of post prognostic decision making.

To account for general behavior of the algorithm over time use of Cumulative Relative Accuracy (CRA) is proposed. Relative accuracy can be evaluated at all time instances where a prediction is made before t_{λ} . These numbers can be aggregated by defining Cumulative Relative Accuracy as a normalized weighted sum of relative prediction accuracies at a specific time instant t_{λ} .

$$CRA_{\lambda} = \frac{1}{\left|\ell_{\lambda}\right|} \sum_{i=1}^{\ell_{\lambda}} w(r^{i}) RA_{\lambda}$$
(5)

where:

 $w(r^{l})$ is a weight factor as a function of RUL at all time indices

 ℓ_{λ} is the set of all time indexes before t_{λ} when a prediction is made

 $|\ell_{\lambda}|$ is the cardinality of the set.

In most cases it is desirable to weigh the relative accuracies higher when closer to the EoL. In general it is expected that t_{λ} is chosen such that it holds some physical significance such as a time index that provides required prognostic horizon, or time required to apply a corrective action, etc. For instance RA evaluated at $t_{0.5}$ signifies the time when a system is expected to have consumed half of its ailing life, or in terms of damage

index the time index when damage magnitude has reached 50% of the failure threshold. This metric is useful in comparing different algorithms for a given λ to get an idea on how well a particular algorithm does at a given time when it is critical. Choice of t_{λ} should also take into account the uncertainty levels that an algorithm entails by making sure that distribution spread at t_{λ} does not cross beyond expected EoL by significant margins especially for critical applications. In a special case where $\lambda = 1$ (i.e. $t_{\lambda} = t_{EoL}$) and all RAs are equally weighted CRA yields an estimate of average RA for all predictions in a given application. While this estimate may not be of use for any practical purpose it can help guide the prognostic algorithm development by providing feedback for improvements.

On the issue of incorporating distribution information in RA, one can make an informed decision about choosing a good point estimate of the RUL distribution compared to choosing only the mean value. As pointed out in Table 1, the shape of RUL distribution should guide the selection of an appropriate location indicator. This choice should also consider the nature of the application. For instance a critical application where risk tolerance level may be low one should choose an indicator that weighs the tails importantly and even outliers in some cases. We will further limit our discussion on RA here since all other enhancements and concepts discussed in perspective of the previous two metrics directly apply to RA.

3.6 Convergence

Convergence is defined to quantify the manner in which any metric like accuracy or precision improves with time to reach its perfect score. As suggested earlier, our discussion assumes that the algorithm performance improves with time, i.e. has passed all previous tests. Convergence is a meta-metric in the sense that it is computed using other performance metrics computed from prognostic results. It goes a step further and estimates how fast an algorithm learns and improves with respect to a chosen metric as more data becomes available. For illustration of the concept we show three cases that converge at different rates (see Figure 14).



Figure 14: Schematic for the convergence of a metric

It can be shown that the distance between the origin and the centroid of the area under the curve for a metric quantifies convergence. Lower distance means a faster convergence. Let (x_c, y_c) be the center of mass of the area under the curve M(i). Then, the convergence C_M can be represented by the Euclidean distance between the center of mass and $(t_p, 0)$, where:

$$C_{M} = \sqrt{(x_{c} - t_{P})^{2} + y_{c}^{2}}, \qquad (6)$$
$$x_{c} = \frac{\frac{1}{2} \sum_{i=P}^{EoP} (t_{i+1}^{2} - t_{i}^{2}) M(i)}{\sum_{i=P}^{EoP} (t_{i+1} - t_{i}) M(i)}, \text{ and} \qquad (7)$$

$$y_{c} = \frac{\frac{1}{2} \sum_{i=P}^{EoP} (t_{i+1} - t_{i}) M(i)}{\sum_{i=P}^{EoP} (t_{i+1} - t_{i}) M(i)^{2}}.$$

M(i) is a non-negative prediction error accuracy or precision metric.

Convergence is a useful metric since we expect a prognostic algorithm to converge to the true value as more information accumulates over time. Further, a faster convergence is desired to achieve a high confidence in keeping the prognostic horizon as large as possible.

3.6.1 Issues with Convergence Metric

As this metric was applied to various applications several issues came up that presented ambiguous situations. First, it was realized that convergence is not a normalized metric and that its value depends on the magnitudes and units of time and the chosen metric M. Even though this allows comparison between different algorithms within a specific application, it is difficult to compare performance of algorithms across different applications. Normalized measures, for instance on a scale of 0-1, give a quick rough idea about the performance in relative terms and how much further improvement may be needed. It is also easier to specify requirements in normalized terms. To tackle this issue one must normalize both axes of the convergence plot, i.e. the time axis and the metric axis. Therefore, we modify the horizontal axis by normalizing it by the total ailing life of the system, i.e. $t = (t_{EoL} - t_p)$. Normalizing the vertical axis is rather tricky. For comparisons across applications we recommend using only those metrics that are already normalized like RA and MAPE (Mean Absolute Percentage Error). Comparing algorithms within a single application does not, however, restrict one from using any other metric.

It must be noted that in practice t_p may be different for different algorithms. Recall that t_p is the first time instant an algorithm starts prediction. Depending on how quick a prognostic algorithm is tuned after the fault is detected t_p will differ. In such cases normalization should be carried out based on the smallest value of t_p among all algorithms. This will normalize all results in a uniform and comparable manner.

It should be noted that the discussion so far assumes that the best value for the normalized metric is 0, e.g. a measure of prediction error. However, there are metrics that have an ideal value of 1. To keep things simple we propose that such metrics be inverted by subtracting them from one and then use for computing the convergence.

Furthermore, there are reported cases for which a metric value starts diverging towards the end (Guan *et al.*, 2009). These cases have been reported where the algorithm is not able to model the complex dynamics of the process towards the end. In such cases it is recommended that convergence evaluation be done by replacing t_{EoL} by t_{EoUP} while normalizing and computing the center of mass to quantify convergence (see Figure 15). For more extreme cases where performance metric trajectories start diverging from the very beginning, it is certain that these cases do not pass the specifications on the previous three metrics and hence need not be tested for convergence.



Figure 15: Normalized Convergence Metric. Algorithm 1 diverges towards the end after t_{EoUP} .

Figure 15 illustrates the concepts discussed above for a modified convergence metric. The shaded region shows the area considered to compute convergence for algorithm 1 after discarding the predictions beyond t_{EoUP} . Corresponding C_M are shown by markers labeled by the number of predictions considered in computing them. Clearly algorithm 1 would have had a poorer convergence if all predictions were considered, represented by markers with dotted lines.

4 FUTURE WORK

We would like to conduct a quantitative analysis for comparison of various approaches suggested in this paper using a real application scenario and identify some key characteristics that may dictate the choice of one approach over another. We will investigate how performance estimates get affected by choosing different options of integrating the uncertainty estimates. This will allow us to identify the advantages and limitations of these techniques and their applicability towards a standardized performance evaluation method.

So far, the performance evaluation assumes that future loading conditions do not change or at least do not change the rate of fault growth. For offline studies this may be reasonable as we know the actual EoL index and can linearly extrapolate true RUL for all previous time indices to draw a straight line. However, for real-time applications this would not hold true as changes in operating conditions do affect the rate of fault evolution. Hence, we would also like to investigate how to incorporate effects of changes in the loading conditions that alter the RUL slope by changing the rate of remaining life consumption. Similar description will also support cases where maintenance actions prolong the lives of the system or the systems with self-healing characteristics.

We will continue to refine the concepts presented in this paper and apply them to a variety of applications in addition to developing more metrics. Developing more metrics like robustness and sensitivity, etc. remains on our research agenda.

5 CONCLUSION

In this paper we have presented a detailed analysis on how prognostics metrics should be used and interpreted. Based on feedback available from fellow researchers, who applied these metrics to a wide variety of applications, several refinements were carried out. Various cases were pointed out and discussed where these metrics may present ambiguous situations while making decisions. A detailed recipe was presented on how to select various parameters for these metrics on which the evaluation outcome depends. Furthermore, it was shown that these metrics are not only useful for algorithmic performance evaluation but also for coming up with performance specifications while keeping several critical constraints in mind. A detailed discussion was presented on how to include prediction distribution information for visual enhancements and more robust performance evaluation. It is expected that this paper will greatly enhance the understanding of these performance metrics and encourage a wider community to use these metrics to help standardize the prognostics performance evaluation.

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NOMENCLATURE

- α accuracy modifier
- α^+ maximum allowable positive error
- α^{-} minimum allowable negative error
- λ time window modifier s.t. $t_{\lambda} = t_{p} + \lambda (EoL t_{p})$
- β minimum desired probability mass threshold
- ω weight factor for a Gaussian component
- θ parameters of RUL distribution
- π total probability mass within α -bounds $[\alpha, \alpha^+]$
- φ non-parameterized probability distribution
- φ_{θ} parameterized probability distribution
- $t_{\rm EoL}$ time instant at End-of-Life (EoL)
- t_{EoUP} time for End-of-Useful-Prediction (EoUP)
- t_{repair} time taken by a reparative action for a system
- $t_{\rm P}$ time instant when the first prediction is made
- $t_{\rm D}$ time instant when a fault is detected
- *i* time index representing time instant t_i
- ℓ set of all prediction time indexes
- ℓ_{λ} set of all prediction time indexes before t_{λ}
- *l* is the index for l^{th} unit under test (UUT)
- r(k) predicted RUL at time index k
- $r_{*}(k)$ is the ground truth for RUL at time index k
- M(i) a performance metric of interest at time t_i
- $x_{c_y}y_c$ x and y coordinates for center of mass
- $C_{\rm M}$ center of mass as a measure of convergence

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