Exact Nonlinear Filtering and Prediction in Process Model-Based Prognostics

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ABSTRACT

Opportunities exist to apply nonlinear filtering to model-based prognostics in order to provide a systematic way of dealing with the propagation of system damage at some future time, whenever imprecise diagnostic information is obtained. Central to the prognostics problem is the ability to properly capture and manage uncertainties when predicting remaining useful life of a particular component of interest. The goal of this paper is to present a foundation for prediction and filtering of the failure process using nonlinear prognostic models and exact (finite-dimensional) filters. Specifically, we consider the use of nonlinear filters to represent the uncertainty distributions exactly for certain classes of nonlinear systems, given a statistically-representative process model of remaining useful life. One such filter, known as the Beneš filter, is derived in this paper for a certain class of prognostic process model. The filter is applied to crack growth data and is shown to perform reasonably well in the context of the 1-D hyperbolic model. Although directly applicable to certain prognostic systems, the techniques descibed provide a theoretical foundation for approximate but less model-restrictive techniques for dynamic model-based prognostics such as particle filtering.

1 INTRODUCTION

One of the most important goals of prognostics and health management (PHM) algorithms is to provide the end-user with the capability to probabilistically forecast the future health of a given component so that changes in operation and condition-based maintenance tasks can be planned effectively. Failure prognostics (forecasting) requires extrapolation into the future, and imprecisions in the damage state accumulate with time, causing uncertainties in the prognostic result to grow substantially over moderate time intervals. Therefore it is imperative to represent the prediction step as accurately as possible by correctly accounting for uncertainties the damage state and measurements as well as in the parameters and model structure. Moreover, when health information is present, it is essential to update the current state based on the availability of this data in a statistically-meaningful manner. The dynamics of all damage processes are inherently nonlinear process with uncertainties that are often non-Gaussian, therefore propagating these effects through time becomes a challenging and error-prone task. Data-based regression techniques are applicable to many systems, but the form of the regression curves are quite general, and are not guaranteed to extrapolate well when failure. Dynamic models, on the other hand, allow a number of systematic tools to be applied for prediction in the presence of noisy diagnostic information, loading profiles, and modeling uncertainties. The paradigm offers flexibility to be combined with damage-mitigating control systems in order to extend the remaining useful life. In model-based prognostics, it is important to employ techniques that provide a reasonable representation of the prognostic result over a long prediction horizon in order to reliably determine perform maintenance on a component based on its current health. Linear filtering techniques, such as the Kalman filter, do provide one way to solve the prediction and updating problem exactly, but these of course do not provide adequate levels of accuracy when making predictions within a nonlinear modeling framework. This motivates the exploration of nonlinear methods to perform the prognosis task.

The key elements of the model-based prognostic problem are illustrated in Fig. 1. Each time measured data is received, the data is filtered to form a posterior distribution based on the likelihood formed by the incoming data. Afterward, a prediction is performed to project the state distribution to next time step (or an arbitrary future time step). The recursive computation of the state's probability density from a measurement update step and a prediction step is known as the *filtering problem*. The prediction step can be repeated at several points in time until a critical threshold is reached at which logistical or maintenance actions are ordered; this is a long-term prediction process. The potential danger with any non-deterministic prediction technique is that any errors or approximations in the initial probability density function (pdf), however small, can accumulate and grow over certain time horizon, and can severely distort the probability distribution over a long time frame. Hence, the problems of uncertainty representation and uncertainty management become improtant problems. Here, the *uncertainty representation* problem focuses on how well an algorithm can propagate the true probability distribution of remaining useful life (RUL) through time. In contrast, the *uncertainty management* problem is focused on how one can extract the most information from available measurements to reduce the system uncertainties. Clearly, both of these problems benefit from consideration of the wealth of dynamic/stochastic systems approaches in existence to treat such problems. The purpose of this paper is therefore to examine the uncertainty representation problem by exploring exact (finite-dimensional) nonlinear filtering strategies to reduce the approximations made when forecating the states. By "exact," we mean that the uncertainty distributions in the damage profile or RUL can be represented exactly. It should be noted that the solution is only exact in the context of the chosen model, which may or may not provide a good representation of reality.

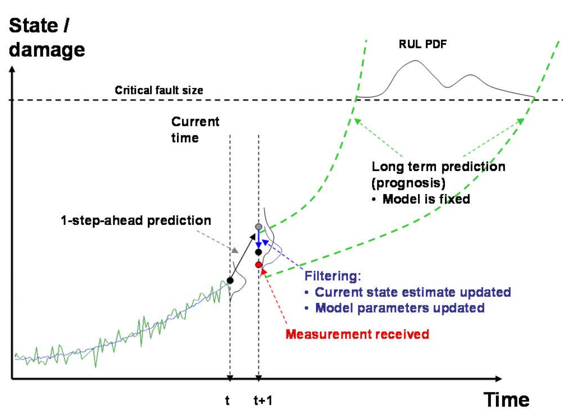


Figure 1: Concept of process model-based prognostics.

Sample-based filtering techniques such as sequential Monte Carlo filters (a.k.a. particle filters) have increasingly been applied to the prognostics problem (Orchard *et al.*, 2008), (Goebel *et al.*, 2008). Particle filters are attractive to practitioners in various disciplines because they offer a unified means for the nonlinear filtering problem for general classes and structures of nonlinear systems excited by non-Gaussian noise. These methods are extremely powerful tools when used within the filtering and target tracking disciplines, but deserves some foundational development in order to gain confidence as a viable tool in the prognostics arena. Particle filters, when applied to the prognostic problem, must be tuned to reduce the effects on sample impoverishment and loss of prognostic fidelity. There are few rules governing the resampling step in the absence of measurements, so past work has focused on establishing design considerations, such as resampling methods and kernel modifications, to mitigate these effects (Orchard *et al.*, 2008). This motivates a sound theoretical basis for model-based prognostics and the exploration of exact filtering techniques to properly represent and manage the uncertainty in the prognostic horizon. It is important to understand which applications such filters are and are not appropriate by framing the development on an actual system. For example, exact filters can certainly be used to calibrate and tune particle filters by minimizing the "predictive erosion" with time due to the sample-based approximation when obtaining long-term predictions. Such a statistically-grounded theory is especially important since it is extremely costly and time consuming to validate such techniques with statistically-significant ground truth failure data. We explore one such filter in this paper, the Beneš filter, where exact solutions are obtained provided a reasonable nonlinear model of the physics-of-failure phenomenon exists that satisfies the Beneš condition (Beneš, 1981). The key to this filtering problem is in the exact solution to the Fokker-Planck-Kolmogorov equation, which provides a complete stochastic description of the prediction process. We then apply the technique to a gear carrier plate example problem, which exhibits nonlinear failure progression characteristics. It is important to note that the proposed methods are seen as complementary to approximation techniques, and therefore a direct comparison between finite-dimensional filters sample-based filters is beyond the scope of this paper.

The remainder of the paper is structured as follows. Section 2 outlines some preliminaries of the nonlinear filtering problem, including the problem formulation, assumptions, formulation of the Fokker-Planck-Kolmogorov partial differential equation (PDE), and the formulation of Bayesian updating for the computation of conditional densities. Section 3 details the finite-dimensional filtering strategy studied herein that admits closed-form solutions to the Fokker-Planck-Kolmogorov equation. Section 4 treats the necessary step of model and parameter selection, both for calibration and online model adaptation to incoming data. Section 5 outlines an example gear carrier plate problem to demonstrate this unified framework for prognostics evaluated in the context of established performance metrics.

2 PROBLEM FORMULATION

To set up the exact filtering problem, it is necessary to introduce the form of the process and measurement equations considered and to describe the general methodology for solving the filtering problem of such processes. From there, we can proceed to an exact solution method for a specific class of models, based on the underpinning assumption that a reasonable prognostic model exists that belongs to this class.

2.1 Diffusion Model

The dynamic system under consideration is known as a drift/diffusion process, which provides a general problem set-up for many filtering problems. For $x \in \mathbb{R}^n$, the system of interest is written:

$$dx(t) = f(x,t)dt + g(x,t)dW(t) \tag{1}$$

where $f(x,t)$ is the process equation, $g(x,t)$ is the diffusion term, $\{W(t)\}$, $t > 0$ is a Wiener process. The physical process is often represented as the drift term

$$D(x,t) = f(x,t) + \frac{1}{2}\frac{\partial g(x,t)}{\partial x}Qg(x,t) \tag{2}$$

where the process noise generated from $\{W(t)\}$ is Gaussian with covariance Q. Note that the drift equation is the same as the process equation when the diffusion $g(x,t)$ is constant.

For the exact filtering problem, the diffusion process in Eq. (1) is observed through the measurement equation:

$$y(t) = H(t)x(t) + v(t) \tag{3}$$

where the underlying state x represents the system damage and the output y is the measured variable or diagnostic feature. Here, $v(t)$ is a white noise process with covariance R that characterizes the uncertainty in how well the diagnostic result correlates to actual damage as well as the uncertainty in the measurement or feature. The process noise $w(t)$ generated by $W(t)$ captures any additive uncertainties in the process model, due to parameters, structure or unmodeled dynamics.[1]

2.2 The Fokker-Planck-Kolmogorov Equation

The conditional probability density function may be obtained by solving a special stochastic partial differential equation known as the Fokker-Planck-Kolmogorov equation (FPKE). The FPKE has been well-studied and, in many cases, closed form solutions exist. Recently, there has been much interest in solving these equations both for transient and stationary solutions to chaotic dynamical systems (Paola and Sofi, 2002), (Wang and Zhang, 2000), (Fuller, 1969). In some finite-dimensional filters, the unnormalized pdf is obtained by solution to the Zakai equation, (Zakai, 1969) a more general form of the FPKE, the PDE encompasses a description of both the prediction and filtering problem. Since we apply the technique by Mitter (Mitter, 1983) and Daum (Daum, 1987) to separate the prediction part from the filtering part, the focus is on solving the FPKE directly for the pdf $p(x,t)$, which is described by the forward generator

$$\frac{\partial}{\partial t}p(x,t) = \mathcal{L}\left[p(x,t)\right] \tag{4}$$

with

$$\mathcal{L}(\cdot) = -\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left[J_i(x,t)(\cdot)\right] + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2}{\partial x_i \partial x_j}\left[K_{ij}(x,t)(\cdot)\right] \tag{5}$$

and

$$J(x,t) = f(x,t) + \frac{1}{2}\frac{\partial g(x,t)}{\partial x}Qg(x,t) \tag{6}$$

$$K(x,t) = \frac{1}{2}g(x,t)Qg^T(x,t) \tag{7}$$

In the FPKE, Eq. (6) is known as the drift term and Eq. (7) is known as the diffusion term.

Direct solutions to the FPKE include exact methods, approximate numerical methods (Galerkin projection, finite element analysis methods), and sample-based methods (Monte-Carlo methods). Numerical methods suffer greatly from the curse of dimensionality, but since prognostics problems are rarely required to be implemented online, these methods still hold merit. The remainder of this paper focuses on an exact special case solution to the FPKE, known as the Beneš filter.

3 EXACT FILTERING AND PREDICTION

Finite-dimensional filters where exact solutions to the FPKE have been discovered are rare, but those that exist are nonetheless applicable to realistic engineering problems. Exact filters referred to as "finite-dimensional" due to the fact that, as measurements arrive, the dimension of the filter remains finite even as more data enters. The Beneš filter (Beneš, 1981) is one example of a finite-dimensional filter whose conditional pdf is taken from the exponential family. Another notable exact filter is the Daum filter, (Daum, 1986) where the model is generalized to more completely include the Beneš filter and Kalman filter.

The main premise of this paper is that, if a reasonable failure progression model exists that yields an analytical solution to the measurement updating and prediction problem, then we can avoid using sample-based filtering methods to perform prognostics. In the Beneš filter, the conditional pdf can be solved exactly if a nonlinear process function $f(x,t)$ satisfies the Beneš condition

$$\mathrm{Tr}\left[\nabla_x f\right] + f^T f = x^T A x + \beta^T x + \gamma \tag{8}$$

where A, β and γ are of appropriate dimension and we assume that $f(x,t)$ is Lipschitz. Note that linear functions of the form $f(x,t) = Ax$ satisfy the condition, as do certain classes of nonlinear functions.

If a system is found that satisfies Eq. (8), then the solution always satisfies the well-known Generalized Fisher-Darmois-Koopman-Pitman Theorem (provided without proof):

Theorem 3.1. *Generalized FDKP Theorem (Daum, 1987) Given the nonlinear system of Eqs.* (1) *and* (3)*, where $x(t)$ has nowhere vanishing unconditional probability density $p(x,t)$ and given the process equation satisfying Eq.* (8) *and some $\mathbb{R}^M$-valued sufficient*

[1] Note that the system is posed in continuous time, which does not always satisfy prognostic models that are determined by cycle counts or discrete events. Naturally, one may use approximate discretization methods to transform the models into this form, without appreciable loss of fidelity.

statistic $\Psi(t)$, the unnormalized probability density is given by

$$p(x,t|Y_k) = \psi(x,t)\exp\left[\theta^T(x,t)\Psi(t)\right] \quad (9)$$

which belongs to the exponential family. Here, $Y_k = \{y_0, y_1, \ldots, y_k\}$ is the set of measurements up to and including time k.

In short, this theorem states that the exponential family is the only nonvanishing family of distributions where the dimension of the sufficient statistic remains bounded as the sample size increases. Moreover, the estimation problem is separable into the prediction part to determine both the density $\psi(x,t)$ and the parameter vector $\theta(x,t)$, which may be computed offline, and the update part, which is computed online.

If we let $F(x) = \int_0^x f(s)ds$, then the conditional probability density function of Eq. (9) is expressed as

$$\begin{aligned} p(x,t|Y_k) &= \exp\left[F(x) - \frac{1}{2}(x-m(t))^T \right. \\ &\qquad \left. \cdot P^{-1}(t)(x-m(t))\right] \\ &= \psi(x,t)\exp\left[-\frac{1}{2}(x-m(t))^T \right. \\ &\qquad \left. \cdot P^{-1}(t)(x-m(t))\right] \end{aligned} \quad (10)$$

where $m(t)$ and $P(t)$ are the two sufficient statistics required to parameterize the exponential distribution; it is important to point out that these are not necessarily the mean and covariance. Note that the prediction and measurement update parts are separable and hence computed independently of one another, then combined to form the unnormalized pdf. Within this framework, the solution to the FPKE (assuming g constant with respect to x)

$$\begin{aligned} \frac{\partial\psi(x,t)}{\partial t} &= -\frac{\partial\psi(x,t)}{\partial x}f - \psi(x,t)\mathrm{Tr}\left(\frac{\partial f}{\partial x}\right) \\ &\quad + \frac{1}{2}\mathrm{Tr}\left(g(t)Qg(t)^T\frac{\partial^2\psi(x,t)}{\partial x x^T}\right) \end{aligned} \quad (11)$$

is given by

$$\psi(x,t) = \exp(F(x)) \quad (12)$$

The filtering problem is achieved through the recursive relationships:

$$m_k = \bar{m}_k + P_k H_k^T R_k^{-1}(y_k - H_k\bar{m}_k) \quad (13)$$

$$P_k = \bar{P}_k - \bar{P}_k H_k^T(H_k\bar{P}_k H_k^T + R_k)^{-1}H_k\bar{P}_k \quad (14)$$

where $\bar{m}_k$ and $\bar{P}_k$ are solved through the following ordinary differential equations:

$$\frac{d\bar{m}(t)}{dt} = -2\bar{P}(t)A\bar{m}(t) - \bar{P}(t)\beta \quad (15)$$

$$\frac{d\bar{P}(t)}{dt} = -2\bar{P}(t)A\bar{P}(t) + Q(t) \quad (16)$$

In this filter, the conditional pdf comes from the exponential class of distributions with the two sufficient statistics m_k and P_k.

3.1 Beneš Filter: Scalar Case

In this section, we examine the special case where the process equation is described by a scalar state. It turns out that, if $A = \beta = 0$, then the Beneš condition of Eq. (8) is satisfied for the following forms of the process equation:

$$f(x) = \frac{e^x - ce^{-x}}{e^x + ce^{-x}} \quad (17)$$

where c is an arbitrary constant. If, in addition, we set $\gamma = 1$, then $f(x) = \tanh(x)$. The form in Eq. (17) is referred to as the hyperbolic process.

For a scalar hyperbolic process where only prediction is considered (by removing the innovations terms), Eqs. (13)–(16) reduce to:

$$\bar{m}_k = m_k \quad (18)$$

$$\bar{P}_k = P_k \quad (19)$$

and

$$\frac{d\bar{m}(t)}{dt} = 0 \quad (20)$$

$$\frac{d\bar{P}(t)}{dt} = Q \quad (21)$$

which yields $\bar{m}_k = \bar{m}_0$ and $\bar{P}_k = Q(t_k - t_0) + \bar{P}_0$. The subscript "0" represents the initial value. The solution to the FPKE is

$$\psi(x,t) = \frac{c_1}{2}(e^x + ce^{-x}) \quad (22)$$

where c_1 is an arbitrary constant. Substitution of Eq. (22) into Eq. (10) results in the conditional pdf

$$\begin{aligned} p(x_k|x_0) &= \frac{1}{2}(e^{x_k} + ce^{-x_k})\exp\left[-\frac{1}{2}\frac{(x_k - m_0)^2}{Q(t_k - t_0) + \bar{P}_0}\right] \\ &= \frac{1}{2}\exp\left[x_k - \frac{1}{2}\frac{(x_k - m_0)^2}{Q(t_k - t_0) + \bar{P}_0}\right] \\ &\quad + \frac{c}{2}\exp\left[-x_k - \frac{1}{2}\frac{(x_k - m_0)^2}{Q(t_k - t_0) + \bar{P}_0}\right] \end{aligned} \quad (23)$$

This equation captures the pdf evolution of the state based only on the initial pdf described by m_0 and P_0. This prediction process is adopted when forecasting the damage state at future time instants where no measurements are available.

For the case where diagnostic updates are available, we pose the problem as a combined prediction/filtering problem. In this case, we assume that the state is measured directly ($H_k = 1$). Then, Eqs. (13) and (14) are represented by

$$m_k = \bar{m}_k + P_k(y_k - \bar{m}_k)/R \quad (24)$$

$$P_k = \bar{P}_k - \bar{P}_k^2/(\bar{P}_k + R) \quad (25)$$

and

$$\frac{d\bar{m}(t)}{dt} = 0 \quad (26)$$

$$\frac{d\bar{P}(t)}{dt} = Q \quad (27)$$

where $\bar{m}_k = \bar{m}_{k-1} = x_{k|k}$ and $\bar{P}_k = QT + \bar{P}_{k-1}$, for a uniform time step $T = t_k - t_{k-1}$. In this case, the conditional pdf is given by

$$\begin{aligned} p(x_k|Y_k) &= \frac{1}{2}(e^{x_k} + ce^{-x_k}) \exp\left[-\frac{1}{2}\frac{(x_k - m_k)^2}{P_k}\right] \\ &= \frac{1}{2}\exp\left[x_k - \frac{1}{2}\frac{(x_k - m_k)^2}{P_k}\right] \\ &\quad + \frac{c}{2}\exp\left[-x_k - \frac{1}{2}\frac{(x_k - m_k)^2}{P_k}\right] \end{aligned} \tag{28}$$

where the solution $\psi(x,t)$ is identical to the one in Eq. (22). By including measurements in the problem formulation, the distribution is of the same type as in the prediction-only case, with the exception that the statistical moments now include measurements in their propagation formulas.

4 MODEL STRUCTURE SELECTION AND PARAMETER IDENTIFICATION

Identification of an appropriate dynamic model is a necessary (and sometimes overlooked) step in the development of an adequate prognostic algorithm. In the field of prognostics, one may be confronted with a choice between physics-of-failure models and data-driven models. In the classical sense, data-driven models (often synonomous with data regression) are usually employed when a population of data exists and physics-of-failure is either unknown or not considered. Conversely, dynamic system models are used only if one has knowledge of the physics-of-failure and are used less often when data exists. Physics-of-failure mechanisms that describe transition to failure are widespread to different applications, but these are oftentimes not appropriate for a given piece of equipment under question. Such models include the Paris Law for fracture mechanics, Arrhenius models for thermal degradation, and exponential models for battery deterioration. In many realistic complex systems, the one failure mode targeted by the system models are often only a subset of the actual failure mode of the component. It is sometimes appropriate to employ semi-empirical models to effectively blend the two sources in a statistically meaningful manner, however, it is rare to have a physical model match the hyperbolic model.

4.1 Offline Model Selection and Calibration

Before employing an exact filter to the prognostic model, the hyperbolic model structure and model parameters should be deemed sufficient to describe certain failure physics phenomena (with respect to some "benchmark" physics-based model) or capture an enemble of damage progression data realizations. Indeed, it is necessary to qualify if a model of the form in Eq. (17) is suited (in a statistical sense) for the prognostics task. There is tremendous volume of literature that treat model structure selection and model calibration (see, for example (Chipman *et al.*, 2001)).

For a finite set of K models $\mathcal{M} = \{M_1, \ldots, M_K\}$, the model selection proceeds by determining the model that best fits statistically-significant historical run-to-failure data Y. The posterior is determined by application of Bayes rule

$$p(M_k|Y) = \frac{p(Y|M_k)p(M_k)}{\sum_k p(Y|M_k)p(M_k)} \tag{29}$$

where the marginal likelihood of model M_k is

$$p(Y|M_k) = \int p(Y|\theta_k, M_k)p(\theta_k|M_k)d\theta_k \tag{30}$$

The posterior is dependent on the model parameters θ_k as well as the model M_k. Of course, the major consideration in determining the posterior is in arriving at a suitable set of priors $p(M_k)$ and $p(\theta_k|M_k)$. Since the parameters and model are often high-dimensional, posterior calculation takes place using numerical techniques such as Markov Chain Monte Carlo (MCMC) using noninformative priors.

4.2 Online Learning

When the model and parameters obtained from the offline learning process are now applied to a nonlinear filter, the parameters may be tuned to more accurately match the unit under question, given data acquired from that unit. Online learning is defined here as any adaptation process to immediately determine model parameters specific to a given unit each time diagnostic data is collected on that unit.

The online learning process is itself a stochastic process, and we therefore employ an extended Kalman filter parameter update scheme to determine the expected value of c_k when used in conjunction with the Beneš filter kernel. The parameter learning can be incorporated within the framework of the exact filter, but is separated in this work in order to illustrate that the updating is an outer loop process on the state estimation. Taking $z_k = [x_k \;\; c_k]^T$, the prediction step is:

$$\hat{z}_{k+1|k} = \Phi_k \hat{z}_{k|k} \tag{31}$$

$$\hat{P}_{k+1|k} = \Phi_k \hat{P}_{k|k}\Phi_k^T + \bar{Q}_k \tag{32}$$

and the filtering process is described by:

$$K_k = P_{k|k-1}H_k^T(H_k P_{k|k-1}H_k^T + \bar{R}_k)^{-1} \tag{33}$$

$$\hat{z}_{k|k} = \hat{z}_{k|k-1} + K_k(y_k - H_k\hat{z}_{k|k-1}) \tag{34}$$

$$\hat{P}_{k|k} = (I - K_k H_k)\hat{P}_{k|k-1} \tag{35}$$

where

$$\Phi_k = \begin{bmatrix} 1 + T^2\left(1 - \frac{e^{x_k} - c_k e^{-x_k}}{e^{x_k} + c_k e^{-x_k}}\right)^2 & -\frac{2e^{x_k}e^{-x_k}}{\left(e^{x_k} + c_k e^{-x_k}\right)^2} \\ 0 & 1 \end{bmatrix} \tag{36}$$

Here, $\bar{Q}_k$ and $\bar{R}_k$ correspond to user-definable process and measurement noise intensities corresponding to the learning algorithm. Essentially, this filter allows computation of the variable c_k with time.

5 RESULTS

In this section, we present some relevant results for the Beneš filter as an application as a prognostic algorithm to predict damage in helicopter planetary gear carrier plate. We first demonstrate the important result that the proposed model is suitable for representing existing run-to-failure datasets, then we apply the Beneš filter in tandem with the EKF-based online parameter learning process to predict remaining useful life.

5.1 Crack Propagation Model

The model under consideration in this study is a UH-60 gear carrier plate model from (Orchard *et al.*, 2008) The model assumes the form

$$x_s(k+1) = F(x_s(k), x_p(k), u(k), k) + \omega_1(k) \tag{37}$$

$$x_p(k+1) = x_p(k) + \omega_2(k) \tag{38}$$

$$y(k) = G(x_s(k), x_p(k), u(k), k) + v(k) \tag{39}$$

where x_s is the fault dimension, x_p is the parameter vector, ω_1 and ω_2 are the process noise terms and v is the measurement noise. The process equation used here is

$$F(x(k)) = x_s(k) + 3\times10^{-4}(0.05+0.1x_p(k))^3 \tag{40}$$

When fitted with the hyperbolic process of Eq. (17) to the above system, the model agreement is quite favorable, as verified by the posterior pdf value computed by the model selection process. Here, the state x represents the crack length. Application of maximum likelihood estimation (MLE) to three run-to-failure datasets produces an expected value of the c parameter in the process equation (Eq. (17)) of 1.16 at a variance of 0.0962. Diagnostic measurements are assumed to take place once every loading cycle and we assume that the crack length is measured, i.e. $y_k = x_k$. When measurements are available, Eqs. (24) - (28) are used to employ the exact filter. To obtain the prognostic forecast, Eqs. (18) - (23) are used to determine the damage estimate at future time intervals.

5.2 RUL Prediction of a Gear Carrier Plate

In this work, two parameter adaptation cases were tested: the first is one where the parameters are updated aggressively (by using a large process-to-measurement noise ratio), while the other is one where the parameters are more slowly updated (by using a small ratio). Figure 2 shows the result of both cases. Noting that the parameter update process modifies the Bayesian calibration scheme used to parameterize the selected model, we seek an update process that has a gradual impact on the prognostic model. In the first case, the parameter is clearly heavily influenced by the measurements and drifts by about 15% over 1000 cycles. In contrast, the variation is nearly 2.5% over the same range for the slow learning case.

Choosing the slow parameter learning case, three snapshots of the prognostic result are shown in Fig. 3. It is evident that the system is, in fact, nonlinear since the trajectory rises with an almost linear slope, then later exhibits a steeper exponential profile. In each figure, the measured fault dimension is shown in blue and the 95% two-sided confidence interval is shown as the red line. The pdfs displayed on both axes are unnormalized. For the crack growth data, the hazard line is chosen to be 2.9 units and the true end-of-life (EOL) is 840 cycles. Figure 3(a) shows the response at 300 cycles. Clearly, the data lies within the confidence interval for a great majority of the dataset. Figure 3(b) shows the prognostic result at 740 cycles, just before a sharp discontinuity appears in the fault dimension (that is un-modeled by both the polynomial model and the hyperbolic model). This jump is a phenomenon that appears when the crack reaches a critical length, and is fairly repeatable. Since this is an unmodeled effect, the algorithm is not able to predict the jump, as is evident in the prediction. By cycle number 760 (Fig. 3(c)), however, the filtering process adjusts the prediction and the predicted remaining useful life (RUL) is again close to the actual EOL.

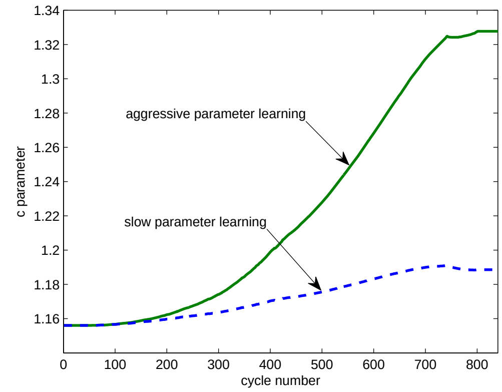


Figure 2: Parameter adaptation.

To quantify the performance of the filter, prognostic performance metrics were computed using a single evaluation dataset, following (Saxena *et al.*, 2009). This was done for both the aggressive and slow parameter learning cases. The relative accuracy is shown in Fig. 4. This is computed by

$$RA = 1 - \frac{|RUL^*(\lambda) - RUL(\lambda)|}{RUL^*(\lambda)} \tag{41}$$

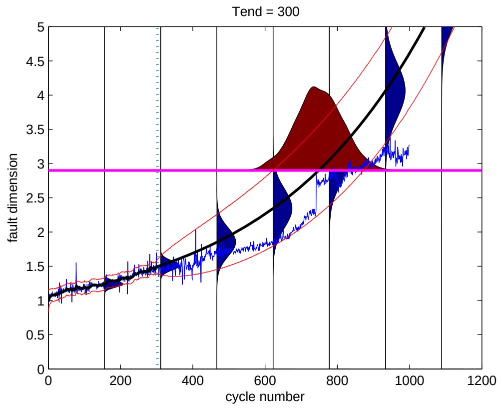
where λ is the time instant at which the prediction is made and "$*$" denotes the ground truth. Clearly, the two cases diverge before the jump in the dataset. The aggressive learning process clearly aggrevates the prognostic result, because the parameter adaptation tends to over-adjust the prognostic model in response to an ever-accumulating volume of unit-specific data. The α–λ performance is computed by

$$(1-\alpha)RUL^*(\lambda) \leq RUL(\lambda) \leq (1+\alpha)RUL^*(\lambda) \tag{42}$$

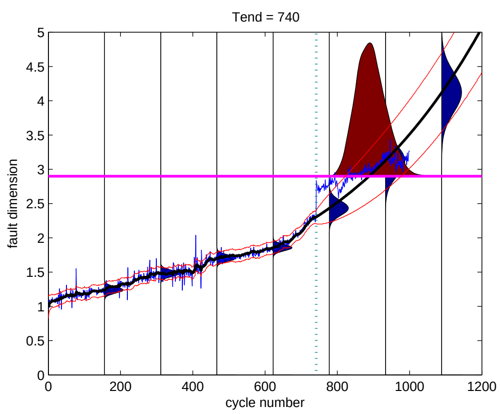
The α–λ performance is shown in Fig. 5 using an accuracy modifier α of 0.25. Clearly, these performance trends confirm the observations made in the relative accuracy evaluation, namely that the prognostic results for the slow learning case lie closer to the RUL cone than the results obtained for the case where aggressive learning is employed.

6 CONCLUSION

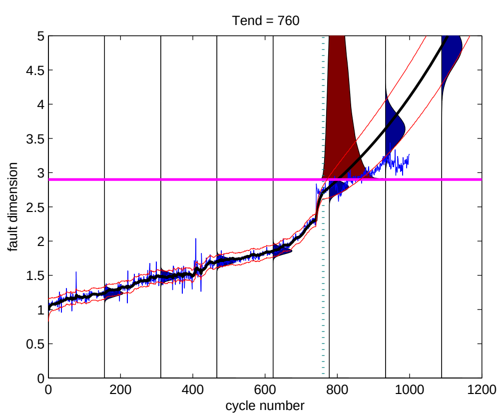
We have shown a promising theoretical foundation for an exact finite-dimensional nonlinear filter as a useful algorithm for practical prognostic problems. We have proposed an exact solution to the prognostic uncertainty representation, in the form of a Beneš filter, to underscore the utility that such an exact formulation has on the quality of the RUL prediction. We have also presented a Bayesian scheme that qualify the hyperbolic process model for application to a gear carrier plate example problem. When applying an on-line



(a)



(b)



(c)

Figure 3: Benes filter results for the slow parameter adaptation scenario.

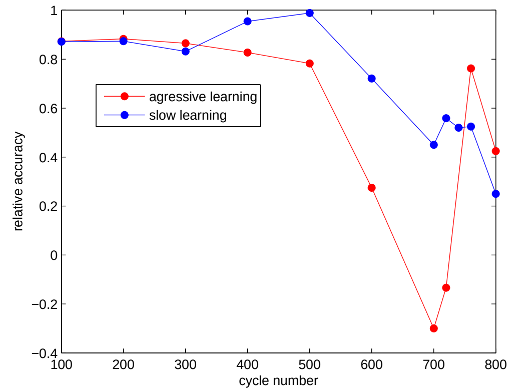


Figure 4: Relative accuracy performance metric.

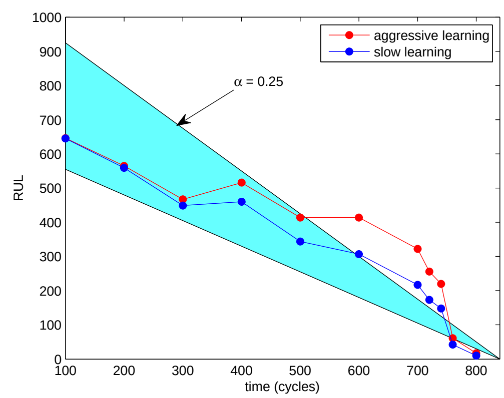


Figure 5: α-λ performance metric.

parameter learning scheme, the ability of the system to capture RUL in long-horizon predictions is remarkably promising. In addition to the possessing the uncertainty representation attributes of an exact nonlinear filter, the technique represents an improvement in computational efficiency over sample-based methods as well.

This paper only presents a small fraction of the potential solution methods available for application to process model-based prognostics. Future work will include applying the technique to other types of prognostic models for other systems (e.g., batteries, electromechanical actuators (EMA), etc.), and discovering a greater variety of models that satisfy the Beneš condition to apply model selection. A more complete, statistically-representative characterization of the algorithm's ability to match the RUL distribution as compared to a population of unit RULs is required. Refining the online adaptation process to unify the filtering and parameter updating, and expanding to incorporate online model structure or parametric uncertainties, is left as future work.

ACKNOWLEDGMENT

This work was partly supported by NASA Ames Research Center under contract #NNX09CC71P, Edward Balaban serves as the technical monitor. The input and suggestions from Dr. Marcos Orchard, Dr. Bhaskar Saha and Dr. Johann Reimann are appreciated.

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