# Change Point-based Spatio-temporal Process Modeling of Image Degradation for Manufacturing Process

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## ABSTRACT

As an advent of smart factory technology, data-driven condition-based maintenance (CBM) is developed to automatically control the production process in engineering field. CBM usually focuses on diagnosing the production status based on real-time data from the sensors. In general manufacturing field, the performance of production equipment gradually decreases due to the wear or deterioration of equipment. To determine if the process is incontrol, degradation modeling of observed data from the equipment and its statistical inference is conducted. In this paper, we propose image-based degradation modeling and change-point detection using spatio-temporal process (CP-STP). To describe the deteriorating patterns of image observation, degradation based on spatial and temporal relationship is conducted. At the same time, change-point is estimated to distinguish the degradation under normal and abnormal production status. Through the application to the image stream in real industry, the proposed monitoring scheme effectively conduct the bi-phase representation providing the change-point of manufacturing processes.

### **1. INTRODUCTION**

In manufacturing industries, production tools continuously degrade according to the operating time. However, the degradation of equipment influences the yield of manufacturing process and the quality of products. To improve the yield in the manufacturing process, various technologies to capture more information from the equipment have been developed. Recently, real-time monitoring and maintenance based on product photography in production is conducted according to the advancement of sensing technology.

For the image-based maintenance, research on the definition of equipment status using image data is suggested. Firstly, the fault prediction using statistical feature extraction was suggested, by extracting the important features for an image. To name a few, Demidenko [1] introduced Kolmogorovsmirnov statistics for testing the homogeneity of images.

Lopes et al. [2] also proposed Kullback-Leibler divergence to estimate the distance with two matrices. However, these approaches compress images into a few indices leading to the loss of information. The second approaches are based on mathematical and deep learning method, where the algorithm is trained and calculated by data-driven methods. For instance, 2D CNN and LSTM learners are integrated by Shi et al. [3] to predict the time-series 2D data. The research of Fang et al. [4] provides tensor-based regression for residual lifetime estimation by introducing the mathematical matrix calculation. Still, there is a limitation that they require numerous parameters for data learning.

To address the problem from statistical feature extraction and data-driven approaches, spatio-temporal process (STP) is developed. Representatively, Sigrist et al. [5] suggested stochastic partial differential equation (SPDE) to link the physical and spatio-temporal degradation of images. STP has the advantage in that the degradation modeling with respect to space and time domain is possible, which provide the precise result with less parameters.

Still, STP only focuses on fitting the single degradation model regardless the abnormality of target equipment, although the degrading patterns at normal and abnormal status have different aspects. As previously mentioned, the maintenance aims to distinguish the normal from abnormal status and timely detect the changing-pattern to prevent the failure. To utilize the STP model for the prognostic maintenance more effectively, the detection of transition time to abnormality and multi-phase modeling based on the time point is required.

In this paper, we propose a change-point (CP) STP for optimal maintenance of time-series images. According to the literature from Majumdar et al. [6], CP is widely utilized in engineering industry to estimate the reliability when manufacturing processes are changed to out-of-control state. By introducing the concept of CP into STP modeling, the

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degradation of image streams at both of in- and out-of-control status can be modeled and interpreted. Moreover, the alternating time for the multi-phase model is estimated, which can be utilized for the maintenance plan.

The remainder of this article is organized as follows. Section 2 provides an introduction of STP modeling, with a brief review of general formulation. To model the transition of spatio-temporal degradation more effectively, a change-point analysis based on STP is suggested in Section 3. Then, a statistical inference to test the inflective degradation and detect the change-point of two-dimensional image stream based on STP is explained. Finally, some concluding remarks are presented in Section 4 with a discussion about future research topics.

#### 2. SPATIO-TEMPORAL PROCESS

STP is a stochastic process to model the diffusion or degradation of images with complex spatial and temporal correlation structures. With a space-time covariance structure, the changes of image streams can be explained. According to the research of Sigrist et al., the degrading observations at time t (i+1) is presented by stochastic partial differential equation (SPDE) as

$$\boldsymbol{w}(t_{i+1}) = \sum_{p=1}^{P} \boldsymbol{\beta}_{p} \cdot \boldsymbol{x}_{p}(t_{i+1}) + \boldsymbol{\xi}(t_{i+1}) + \boldsymbol{\nu}(t_{i+1})$$

where the STP consists of a linear parametric regression of spatio-temporal coefficients  $\boldsymbol{\beta}_p$  and covariates  $\boldsymbol{x}_p(t_{i+1})$ , and Gaussian process (GP)  $\xi(t_{i+1})$  representing the structured spatio-temporal variation, and unstructured residuals  $v(t_{i+1})$  following  $N(0, \tau^2 \mathbf{1})$ . To improve the computation speed, the Gaussian process is represented using Fourier transform as

$$\boldsymbol{\xi}(t_{i+1}) = \boldsymbol{\Phi} \cdot \boldsymbol{\alpha}(t_{i+1}),$$

with respect to the Fourier operator  $\Phi$  and Fourier coefficient  $\alpha(t_{i+1})$ . Due to the property of STP, the following equation  $\boldsymbol{\alpha}(t_{i+1}) = \boldsymbol{G} \cdot \boldsymbol{\alpha}(t_i) + \boldsymbol{\epsilon}(t_i),$ 

is satisfied for the propagator matrix  $\boldsymbol{G}$  and the residual error  $\epsilon(t_i) \sim N(0, \widehat{\boldsymbol{Q}}).$ 

Based on the definition of STP, covariance matrices R and mean vectors m are derived using Kalman filter update process. Then, the log-likelihood function is defined as

$$\log L = \sum_{i=1}^{T} \log |\mathbf{R}_{t_i|t_{i-1}} + \tau^2 \mathbf{1}_N| + (\tilde{\mathbf{w}}(t_i) - \mathbf{m}_{t_i|t_{i-1}})^T (\mathbf{R}_{t_i|t_{i-1}} + \tau^2 \mathbf{1}_N)^{-1} (\tilde{\mathbf{w}}(t_i) - \mathbf{m}_{t_i|t_{i-1}}) + \frac{TN}{2} \log(2\pi),$$

for transformed observations  $\widetilde{w}(t_i)$ . T and N denotes the number of time points and pixel length, respectively.

Hereafter, the optimal parameter is estimated as the values maximizing the log-likelihood, where the parameter set through this process is defined as maximum likelihood estimators (MLE). Then, the changes in image streams are fitted and predicted using the parameter MLEs.

#### 3. CHANGE-POINT DETECTION BASED ON STP

To formulate the CP-STP modeling, Eq. \ref{stp} can be rewritten as

$$\boldsymbol{w}(t_l) = \begin{cases} \sum_{p=1}^{p} \boldsymbol{\beta}_p \cdot \boldsymbol{x}_p(t_l) + \boldsymbol{\xi}^{(1)}(t_l) + \boldsymbol{\varepsilon}(t_l), & \text{if } t_l \leq t_{\tau}, \\ \\ \sum_{p=1}^{p} \boldsymbol{\beta}_p \cdot \boldsymbol{x}_p(t_l) + \boldsymbol{\xi}^{(2)}(t_l) + \boldsymbol{\varepsilon}(t_l), & \text{if } t_l > t_{\tau}, \end{cases}$$

with respect to the specific change-point  $t_{\tau}$ , where  $\boldsymbol{\varepsilon}(t_l) \sim N(\mathbf{0}, v^2 \mathbf{1})$ . Compared to the general STP,  $\boldsymbol{\xi}(t_{i+1})$  is partitioned into two parts as  $\xi^{(1)}(t_l)$  and  $\xi^{(2)}(t_l)$ , denoting pre-change process, change-point process, and post-change process, respectively. Based on the equation, we can derive the SPDE with change-point as

$$\frac{\partial \xi^{(i)}(t_l, \boldsymbol{s}_n)}{\partial t_l} = -\boldsymbol{\mu}^{(i)T} \boldsymbol{\nabla} \xi^{(i)}(t_l, \boldsymbol{s}_n) + \boldsymbol{\nabla} \boldsymbol{\Sigma}^{(i)} \boldsymbol{\nabla} \xi^{(i)}(t_l, \boldsymbol{s}_n) - \zeta^{(i)} \xi^{(i)}(t_l, \boldsymbol{s}_n) + \epsilon^{(i)}(t_l, \boldsymbol{s}_n) = h^{(i)} \xi^{(i)}(t_l, \boldsymbol{s}_n) + \epsilon^{(i)}(t_l, \boldsymbol{s}_n)$$
  
for  $i = 1$  (pre-change), 2(post-change).

By transforming the image stream into Fourier coefficients. the log-likelihood can be redefined. To compute the parameters more efficiently, we introduced Kalman filter, where the likelihood function based on Fourier coefficients is updated through all of the time points. From the inputs of propagator matrix  $\boldsymbol{G}^{(i)}$  and innovation matrix  $\hat{\boldsymbol{Q}}^{(i)}$ , we can update the mean vector and variance matrix used for the estimation of MLE. Denoting the updated mean vectors and variance matrices as  $\boldsymbol{m}_{t_l|t_l-1}^{(i)}, \, \boldsymbol{m}_{t_l|t_l}^{(i)}, \, \boldsymbol{R}_{t_l|t_{l-1}}^{(i)}, \,$  and  $\boldsymbol{R}_{t_l|t_l}^{(i)}$ respectively, we can redefine the log-likelihood function for CP-STP as

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According to the updated log-likelihood, we can find the maximum likelihood estimates more efficiently.

#### 4. CONCLUSION

This paper provides an image-based degradation modeling and change-point detection based on spatio-temporal process for mass production. Through the image stream obtained from manufacturing process, the degrading pattern at production tools in spatial and time domain is modeled. To describe the degradation process in terms of spatial and temporal covariance structure, the degradation is modeled by STP, where the diffusion or convection from image streams are represented as a random field. Since the degradation of manufacturing tools at in- and out-of-control state provides different deterioration, we proposed bi-phase degradation process by introducing change-point model for STP. Based on the statistical derivation, we applied the proposed model to the engine block image stream in real manufacturing industry. The application results show that the proposed approach can express the decaying pattern of time-series images by detecting the proper change-point. For the future research, Bayesian inference can be introduced to improve the precision of the prediction result. Moreover, an estimation of optimal replacement time for the manufacturing tools can be also conducted for preventive maintenance.

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