Advanced Weibull Modelling with Outliers

Yipeng Pang[1], Guoqiang Hu[2], and Sungin Cho[3]

[1,2] *Nanyang Technological University, Singapore, 639798*
yppang@ntu.edu.sg
gqhu@ntu.edu.sg

[3] *SP PowerGrid Ltd (SPPG), Singapore, 349277*
chosungin@spgroup.com.sg

Abstract

This paper presents a comprehensive process for the advanced Weibull modelling with potential outlier inclusions. In this process, an algorithm is designed to identify if there exist any outliers (*i.e.*, failures with different failure modes from the majority) in the failure data of the equipment of interest. Depending on the conditions of the identified outliers, a suitable statistical model is developed. To validate the model, it is compared with the estimated empirical distribution function with the inclusion of new failure data. It is shown that the proposed advanced Weibull model outperforms the two-parameter Weibull model in terms of fitting, and hence a better accuracy is achieved in the failure statistical analysis. Case study in the application of power systems is conducted to illustrate its effectiveness.

1. Introduction

The outlier issue commonly exists in statistical analysis. There are often cases where a few data may not be homogeneous to the rest of the data due to a variety of reasons, and hence can be treated as outliers (Barnett & Lewis, 1994). In power systems, the outliers could be failures with different failure modes from the majority, such as the early failures due to defects as compared to the failures due to aging process. In this sense, the outliers can introduce significant errors in the model estimation, and hence impact the accuracy of the developed model. Identifying outliers is important and has drawn great attention over the decades. For example, Pettit (1988) presented a Bayesian approach to the modelling of outliers and examined its use in the members of the exponential family. Nasiri and Pazira (2011) considered outliers in the samples from the generalized exponential distribution and derived a Bayes estimator for the parameter estimation. Banerjee and Iglewicz (2007) proposed a simple univariate outlier identification procedure based on boxplot outlier-labeling rule, which could be used for various distributions, such as normal, t, gamma, etc. In terms of the Weibull distribution, Dixit (1994) adopted a Bayesian approach to obtain the predictive distribution for the samples from the two-parameter Weibull distribution in the presence of outliers. Recently, Gupta and Singh (2017) studied both classical and Bayesian estimation of Weibull model assuming the outliers are generated from an exponential distribution. However, these works rely on the prior knowledge on the number of outliers. It would be better if the algorithm could automatically identify the number of outliers and their locations in the distribution (Fung & Paul, 2007). More recently, Zhang et al. (2022) presented an estimation method for three-parameter Weibull based on the outlier detection to model the capacity distribution of Li-ion batteries. The outliers were identified based on the obtained Weibull parameters and excluded from the sample data. In some cases, the outliers may provide useful information with the in-depth understanding of outlier (Shu, Qin, Chen, & Yin, 2018).

In this work, we will propose an advanced Weibull modelling process, and demonstrate the case where the outliers will be exploited in the Weibull model development. Hence, the parameter estimation may be based on the data with or without the outliers depending on the conditions of the outliers. The major contributions of the work are summarized as follows. 1) An outlier detection algorithm is developed, which does not need the primary knowledge on the number of outliers. The algorithm can identify if there exist any outliers and output the set of outliers (if exist) to the users for any potential investigations. An evaluation check algorithm is designed to select the appropriate Weibull model. 2) An advanced Weibull modelling process is proposed. Depending on the conditions of the identified outliers, a suitable Weibull model is developed where the information of outliers will be exploited. 3) The proposed comprehensive process will be implemented based on the actual data in SP Group Power Grid in Singapore. It is shown that the proposed model outperforms the two-parameter Weibull model in terms of fitting.

The rest of the paper is organized as follows. Section 2 introduces some preliminary results on standard two-parameter Weibull, competing risk model with Weibull distribution, and R squared index. Section 3 presents the details on outlier detection, which is followed by the advanced Weibull modelling process with the inclusion of outlier detection in Sec. 4. Finally, the case study in the application of power systems is discussed in Sec. 5. Section 6 concludes the paper.

2. Preliminaries

In this section, we present some preliminary results on Weibull distribution including two-parameter Weibull, competing risk model with Weibull distribution and R squared index for the sake of completeness.

2.1. Two-Parameter Weibull

The widely used two-parameter Weibull distribution has two parameters: shape and scale parameters. The scale parameter (denoted by η) quantifies the characteristic life, defined as the value at 63.2% percentile in the unit of time (t). The shape parameter (denoted by β) characterizes the shape of the distribution, and is also known as the slope when viewing from a linear cumulative distribution function (c.d.f) called Weibull probability plot (WPP). The c.d.f of the two-parameter Weibull distribution is given by

$$F(t) = 1 - e^{-(\frac{t}{\eta})^{\beta}}. \tag{1}$$

The fitting of data into a Weibull distribution can be viewed using WPP. WPP is a plot of the empirical c.d.f $\hat{F}(t)$ of the data on special axes where y-axis is $\ln(-\ln(1-F(t)))$ and x-axis is $\ln t$. As can be seen from (1), the c.d.f of the two-parameter Weibull distribution is affine in WPP due to $\ln(-\ln(1-F(t))) = \beta \ln t - \beta \ln \eta$.

2.2. Competing Risk Model with Weibull Distribution

The competing risk model with Weibull distribution is a statistical model consisting of a comination of two or more Weibull distributions that represent failure modes which are competing to end the life of the equipment. Specifically, for a competing risk model with two Weibull distributions, where there are two sets of shape and scale parameters (denoted by β_1, η_1, β_2 and η_2). The survival probability at time t, denoted by $S_{\text{cr}}(t)$ is the product of the survival probability of two Weibull distributions at time t, denoted by $S_1(t)$ and $S_2(t)$ respectively, *i.e.*,

$$S_{\text{cr}}(t) = S_1(t) \times S_2(t) \tag{2}$$

where $F_1(t) = 1 - S_1(t) = e^{-(\frac{t}{\eta_1})^{\beta_1}}$ is the c.d.f of the first Weibull distribution, and $F_2(t) = 1 - S_2(t) = e^{-(\frac{t}{\eta_2})^{\beta_2}}$ is the c.d.f of the second Weibull distribution.

2.3. R Squared Index

The R squared index (also known as coefficient of determination), denoted by R^2 characterizes how well the actual data points fits the predictions made by the theoretical model. Mathematically, given the actual data $\{y_i\}_{i=1}^N$ and the model predictions $\{\hat{y}_i\}_{i=1}^N$, the R squared index is computed by

$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}}, \tag{3}$$

where $SS_{\text{res}} = \sum_{i=1}^N (y_i - \hat{y}_i)^2$ is the residual sum of squares quantifying the prediction error, $SS_{\text{tot}} = \sum_{i=1}^N (y_i - \bar{y})^2$ is the total sum of squares quantifying the data variance, and $\bar{y} = \frac{1}{N}\sum_{i=1}^N y_i$ is the average. As can be seen from (3), R^2 closer to 1 indicates a better fitting.

3. Outlier Detection & Evaluation

In this section, we present two algorithms, where the first one is to identify if there exist any outliers in the failure data of the equipment of interest, and the second one is to evaluate the conditions of the set of outliers produced in the first algorithm and output a flag for further procedures.

3.1. Outlier Detection

To identify the potential existence of the outliers, the idea is to exclude each failure data one at a time, and check whether the fitting of the rest of the data has significantly improved. The proposed outlier detection algorithm takes two parameters – R squared difference threshold δ and minimum R squared threshold r_l. These two parameters jointly set the stopping criteria for the algorithm. Details of the algorithm are discussed as follows.

- First, a failure set $\mathcal{F}$ and an outlier set $\mathcal{O}$ are created. Specifically, the failure set $\mathcal{F}$ is initialized with all the failure data, and the outlier set $\mathcal{O}$ is an empty set.
- Repeat the following steps 1-4 if the difference between the largest and smallest R squared value, $\Delta_r = r_{\max} - r_{\min}$ ($r_{\max}, r_{\min}$ to be defined in the following) is greater than or equal to the R squared difference threshold δ (*i.e.*, $\Delta_r \geq \delta$), or the smallest R squared value is smaller than or equal to the minimum R squared threshold r_l (*i.e.*, $r_{\min} \leq r_l$):
  1. For each failure data k in the failure set $\mathcal{F}$ (*i.e.*, $k \in \mathcal{F}$), do the following steps (a) and (b):
     (a) remove the failure data k from the failure set $\mathcal{F}$, and define the resulting failure set as $\mathcal{F}_e$, *i.e.*, $\mathcal{F}_e = \mathcal{F} \backslash \{k\}$,

(b) use all failure data in the failure set $\mathcal{F}_e$ and all non-failure data (*e.g.*, operating data) to obtain the empirical distribution function via Kaplan-Meier (KM) estimator and two-parameter Weibull via maximum likelihood estimation (MLE), which are compared in Weibull probability plot (WPP) and the corresponding R squared value r_k is obtained.

2. Compare all the obtained R squared values, record the largest and smallest R squared value (denoted by $r_{\max}$ and $r_{\min}$, respectively), and record the point (denoted by $k_{\max}$) which gives the largest R squared value.
3. Compare all the obtained R squared values, and record the difference between the largest and the smallest R squared value (denoted by Δ_r), *i.e.*, $\Delta_r = r_{\max} - r_{\min}$.
4. If the difference between the largest and smallest R squared value is greater than or equal to the R squared difference threshold (*i.e.*, $\Delta_r \geq \delta$) or the smallest R squared value is smaller than or equal to the minimum R squared threshold (*i.e.*, $r_{\min} \leq r_l$), then add the point $k_{\max}$ to the outlier set $\mathcal{O}$, *i.e.*, $\mathcal{O} = \mathcal{O} \cup \{k_{\max}\}$, and exclude the point $k_{\max}$ from the failure set $\mathcal{F}$, *i.e.*, $\mathcal{F} = \mathcal{F} \backslash \{k_{\max}\}$; otherwise, the algorithm continues with no changes.

- Output the outlier set $\mathcal{O}$.

The proposed outlier detection procedures are summarized in Algorithm 1.

3.2. Evaluation Check

With the obtained set of outliers produced in Algorithm 1, we proceed with checking the conditions of these outliers, where the output will determine the choice of Weibull models in the modelling process (see Sec. 4).

The evaluation check algorithm takes one parameter, the maximum number of outliers in the tail, $n_{\max}$. Detailed procedures are discussed as follows.

- The tails of the outliers need to be extracted first. Hence, a normal failure set is defined by removing the outlier set from the original failure set, *i.e.*, $\mathcal{F}_n = \mathcal{F} \backslash \mathcal{O}$.
- Find the smallest age $t_{\min} = \min_{i \in \mathcal{F}_n} t_i$ and the largest age $t_{\max} = \max_{i \in \mathcal{F}_n} t_i$ in the normal failure set $\mathcal{F}_n$. Now the lower and upper tails of the outliers could be extracted, where the lower tail includes the outlier whose age is younger than $t_{\min}$, *i.e.*, $\mathcal{O}_{\text{low}} = \{i \in \mathcal{O} | t_i < t_{\min}\}$, and the upper tail includes the outlier whose age is older than $t_{\max}$, *i.e.*, $\mathcal{O}_{\text{high}} = \{i \in \mathcal{O} | t_i > t_{\max}\}$.
- Find the number of outliers in the lower and upper tails $N_{\text{low}} = |\mathcal{O}_{\text{low}}|$, $N_{\text{high}} = |\mathcal{O}_{\text{high}}|$ respectively, where $|\mathcal{A}|$ denotes the number of elements in the set $\mathcal{A}$.

Algorithm 1 Outlier Detection

Input: all non-failure data and failure data $1, 2, \ldots, N$
Parameter: R squared difference threshold δ, minimum R squared threshold r_l
Output: outlier set $\mathcal{O}$

Initialize:
  failure set $\mathcal{F} = \{1, 2, \ldots, N\}$, outlier set $\mathcal{O} = \emptyset$, $\Delta_r = 1$, $r_{\min} = 1$
while $\Delta_r \geq \delta$ or $r_{\min} \leq r_l$ **do**
  $\Delta_r = 1$, $r_{\min} = 1$
  for each $k \in \mathcal{F}$ **do**
    exclude k from failure set: $\mathcal{F}_e = \mathcal{F} \backslash \{k\}$
    use all non-failure data and failure data from failure set $\mathcal{F}_e$ to plot WPP, fit into a two-parameter Weibull, and obtain the R squared index
  end for
  record the data point $k_{\max}$ which gives the largest R squared index $r_{\max}$
  record the smallest R squared index $r_{\min}$
  record the difference between the largest and smallest R squared index $\Delta_r = r_{\max} - r_{\min}$
  if $\Delta_r \geq \delta$ or $r_{\min} \leq r_l$ **then**
    add the data point to outlier set $\mathcal{O} = \mathcal{O} \cup \{k_{\max}\}$
    exclude the data point from the failure set $\mathcal{F} = \mathcal{F} \backslash \{k_{\max}\}$
  end if
end while

- If $N_{\text{low}} \leq n_{\max}$ and $N_{\text{high}} \leq n_{\max}$, then a flag 'PASS' will be returned; otherwise, a flag 'FAIL' will be returned.

The proposed evaluation check procedures are summarized in Algorithm 2.

Algorithm 2 Evaluation Check

Input: outlier set $\mathcal{O}$, failure set $\mathcal{F} = \{1, 2, \ldots, N\}$
Parameter: maximum number of outliers in the tail $n_{\max}$
Output: flag

define normal failure set by removing outlier set from the original failure set, $\mathcal{F}_n = \mathcal{F} \backslash \mathcal{O}$
find the number of elements in the outlier set, whose age is younger than $t_{\min}$, denoted by N_{low}
find the number of elements in the outlier set, whose age is older than $t_{\max}$, denoted by N_{high}
if $N_{\text{low}} \leq n_{\max}$ and $N_{\text{high}} \leq n_{\max}$ **then**
  flag = 'PASS'
else
  flag = 'FAIL'
end if

4. Advanced Weibull Modelling Process

In this section, we discuss the proposed advanced Weibull modelling process, where the workflow is as shown in Fig. 1.

First, an outlier detection algorithm (**1**) referred to Algorithm 1 is developed to identify if there exist any outliers in the failure data of the equipment of iterest. The obtained outlier set will then be proceeded for evaluation check (**2**) re-

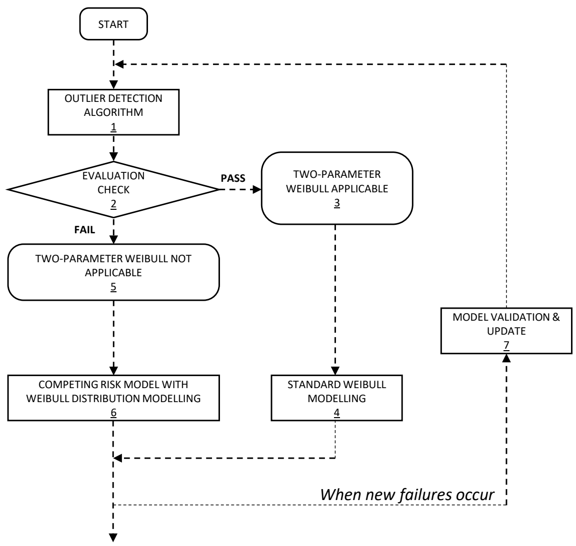


Figure 1. Workflow of the advanced Weibull modelling process.

ferred to Algorithm 2. Based on the returned flag, a suitable Weibull model will be selected for the modelling procedures.

If the returned flag from (**2**) is ‘PASS’, it implies that the standard two-parameter Weibull is applicable for modelling the life-time properties of the equipment (**3**). In this case, the standard two-parameter Weibull modelling procedures (**4**) will be proceeded. Denote the probability density function (p.d.f) and cumulative distribution function (c.d.f) of the standard two-parameter Weibull by $f_{2\text{p}}(t;\beta,\eta)$ and $F_{2\text{p}}(t;\beta,\eta)$, respectively. Further, let $\{t_i\}_{i=1}^{n_f}$ be the life-time for the failure data excluding the failures from outliers, and $\{t_i\}_{i=1}^{n_s}$ be the life-time for the operating data. Then, the likelihood function of the data, denoted by $L_{2\text{p}}(\beta,\eta|\text{data})$, can be obtained by

$$L_{2\text{p}}(\beta,\eta|\text{data}) = \prod_{i=1}^{n_f} f_{2\text{p}}(t_i;\beta,\eta) \times \prod_{i=1}^{n_s}(1 - F_{2\text{p}}(t_i;\beta,\eta)).$$

The estimates of the shape and scale parameters $\hat{\beta},\hat{\eta}$ can be obtained by maximizing the log-likelihood function $\log L_{2\text{p}}(\beta,\eta|\text{data})$, which completes the standard Weibull modelling (**4**). Fig.-(a) shows an example of the modelling results.

If the returned flag from (**2**) is ‘FAIL’, it implies that the standard two-parameter Weibull is **NOT** applicable for modelling the life-time properties of the equipment (**5**). In this case, the modelling procedures for competing risk model with two Weibull distributions will be proceeded.

The maximum likelihood estimation (MLE) method will be applied to estimate the four parameters in the competing risk model with two Weibull distributions. For the convenience of presentation, the p.d.f and c.d.f for the two Weibull distributions are respectively denoted by f_1, f_2, F_1 and F_2. Then the c.d.f for the competing risk model with two Weibull distributions, F_{cr} is obtained by

$$F_{\text{cr}} = 1 - S_{\text{cr}} = 1 - S_1 \times S_2 = 1 - (1 - F_1)(1 - F_2).$$

The p.d.f for the competing risk model with two Weibull distributions, f_{cr} is obtained by

$$\begin{aligned} f_{\text{cr}} &= \lambda_{\text{cr}} \times S_{\text{cr}} = (\lambda_1 + \lambda_2) \times S_1 \times S_2 \\ &= \left(\frac{f_1}{S_1} + \frac{f_2}{S_2}\right) \times S_1 \times S_2 \\ &= f_1(1 - F_2) + f_2(1 - F_1), \end{aligned}$$

where λ_1, λ_2 and λ_{cr} are the hazard rate functions of the two Weibull distributions and the competing risk model, respectively. Thus, the likelihood function of the data, denoted by $L_{\text{cr}}(\beta_1,\eta_1,\beta_2,\eta_2|\text{data})$, can be obtained by

$$\begin{aligned} L_{\text{cr}}(\beta_1,\eta_1,\beta_2,\eta_2|\text{data}) = &\prod_{i=1}^{n_f} f_{\text{cr}}(t_i;\beta,\eta) \\ &\times \prod_{i=1}^{n_s}(1 - F_{\text{cr}}(t_i;\beta,\eta)). \end{aligned}$$

The estimates of the two shape and two scale parameters $\hat{\beta}_1,\hat{\eta}_1,\hat{\beta}_2,\hat{\eta}_2$ can be obtained by maximizing the log-likelihood function $\log L_{\text{cr}}(\beta_1,\eta_1,\beta_2,\eta_2|\text{data})$, which completes the modelling of competing risk model with two Weibull distributions (**6**).

Finally, the developed Weibull model in (**4**) and (**6**) can be validated when new failures occur (**7**). Specifically, the new data (new failures and/or new installations) combined with the existing dataset (with the exlusion of the outliers for standard Weibull, or with the inclusion of the outliers for the competing risk model with Weibull distributions) will be used to obtain the empirical distribution function via Kaplan-Meier (KM) estimator. This will be compared with the developed Weibull model to obtain the corresponding R squared index. A satisfactory R squared index implies that the proposed Weibull model is validated *i.e.*, the model is sufficiently good to quantify the life-time properties of the equipment. Otherwise, it may imply the existence of outliers in the new dataset, and investigations on the new failures may be needed. Meanwhile, the model will be returned to the start of the process for an automatic update with the new dataset.

5. CASE STUDY

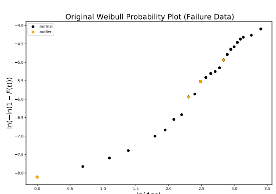
In this section, we conduct case studies on the transformer and cable based on the actual data provided by SP Group Power Grid in Singapore to demonstrate our proposed advanced Weibull modelling process. We suppose we were in 2019 and use data up to 2019 for the modelling purpose, and then use data in 2020 for the model validation purpose.

5.1. Transformer

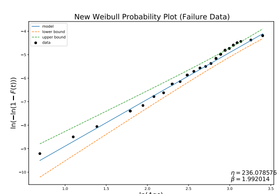
For the transformer, we have a dataset with over 99% censoring rate (i.e., over 99% of the fleet of equipment is still operating). We set the parameters $\delta = 0.01$ and $r_l = 0.9$ for the outlier detection algorithm (Algorithm 1). As can be seen in Fig. 2a, we have identified 8 outliers (marked in yellow dot) from the failure data: 3 in 1 year old, 2 in 10 years old, 2 in 12 years old and 1 in 17 years old. All information of these outliers can be extracted for the engineers to further analyze the status of the equipment, which is not demonstrated in this paper for the confidential purpose.

Next, we proceed with the evaluation check algorithm (Algorithm 2) to decide which Weibull model to use for the modelling purpose. We set $n_{\max} = 4$. Clearly, there is no upper or lower tail with the number of outliers larger than $n_{\max}$. Hence, a 'PASS' flag is output from Algorithm 2 indicating that the two-parameter Weibull is applicable for the modelling purpose. Figure 2b shows the Weibull modelling result based on our proposed advanced Weibull modelling process.

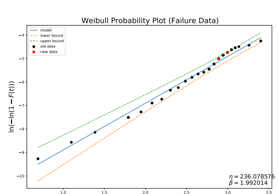
To validate the model, we now include the data in 2020 and plot the developed two-parameter Weibull model and all data up to 2020 in Fig. 2c, where the new failure data is marked in red dot. The overall fitting accuracy measured by R squared index is 0.986, which outperforms the two-parameter Weibull without consideration of outliers (having $R^2 = 0.854$).



(a) Outlier detection



(b) Advanced Weibull modelling



(c) Model validation

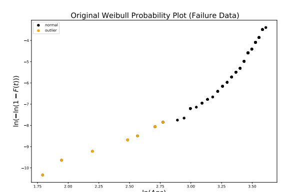
Figure 2. Advanced Weibull modelling with outliers for the transformer.

5.2. Cable

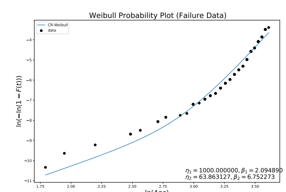
For the cable, we have a dataset with over 99% censoring rate. We set the parameters $\delta = 0.01$ and $r_l = 0.9$ for the outlier detection algorithm (Algorithm 1). As can be seen in Fig. 3a, we have identified 9 outliers (marked in yellow dot) from the failure data.

Next, we proceed with the evaluation check algorithm (Algorithm 2) to decide which Weibull model to use for the modelling purpose. We set $n_{\max} = 4$. Clearly, the number of outliers in the lower tail is larger than $n_{\max}$. Hence, a 'FAIL' flag is output from Algorithm 2 indicating that the two-parameter Weibull is **NOT** applicable for the modelling purpose. Thus, we proceed with the competing risk model with two Weibull distributions modelling procedures. Figure 3b shows the modelling result based on our proposed advanced Weibull modelling process.

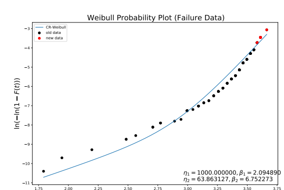
To validate the model, we now include the data in 2020 and plot the developed competing risk model with two Weibull distributions and all data up to 2020 in Fig. 3c, where the new failure data is marked in red dot. The overall fitting accuracy measured by R squared index is 0.965, which outperforms the two-parameter Weibull without consideration of outliers (having $R^2 = 0.662$).



(a) Outlier detection



(b) Advanced Weibull modelling



(c) Model validation

Figure 3. Advanced Weibull modelling with outliers for the cable.

The advanced Weibull modelling process has demonstrated significant improvement in terms of fitting in both transformer and cable cases. The potential impact brought by the improvement of the fitting can be seen as follows. The improved fitting accuracy suggests that the derived model provides a better representation of the failure property, which gives more accurate information on the reliability of the equipment. The improved model will benefit many related applications, such as failure number prediction and remaining useful life estimation, where the prediction and estimation accuracy is closely related to the performance of the maintenance planning and cost optimization. Moreover, the identified outliers may indicate an emerging failure mode. Hence, these outliers may potentially serve as some alert for the engineers to trigger any failure investigation process, and may bring changes to the proactive maintenance strategies and decision-making processes.

6. CONCLUSION

This paper has presented a comprehensive process for the advanced Weibull modelling with possible existence of outliers. An outlier detection algorithm has been designed to identify if there exist any outliers in the failure data. Based on conditions of the detected outliers, an evaluation check algorithm has been devised to select appropriate Weibull models. For model validation, the developed model has been compared with the empirical distribution function estimated by Kaplan-Meier model with the inclusion of new failure data. Through case study in the applications of transformer and cable systems, it has been shown that the proposed advanced Weibull model outperforms the two-parameter Weibull in terms of fitting. The potential impact brought by the improved modelling process has been discussed.

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REFERENCES

Banerjee, S., & Iglewicz, B. (2007). A simple univariate outlier identification procedure designed for large samples. *Communications in Statistics: Simulation and Computation*, *36*(2), 249–263.

Barnett, V., & Lewis, T. (1994). *Outliers in statistical data*. Wiley New York.

Dixit, U. J. (1994). Bayesian approach to prediction in the presence of outliers for Weibull distribution. *Metrika*, *41*(1), 127–136.

Fung, K. Y., & Paul, S. R. (2007). Comparisons of outlier detection procedures in weibull or extreme-value distribution. *Communications in Statistics - Simulation and Computation*, *14*(4), 895–917.

Gupta, P. K., & Singh, A. K. (2017). Classical and Bayesian estimation of Weibull distribution in presence of outliers. *Cogent Mathematics*, *4*(1).

Nasiri, P., & Pazira, H. (2011). Bayesian Approach on the Generalized Exponential Distribution in the Presence of Outliers. *Journal of Statistical Theory and Practice*, *4*(3), 453–475.

Pettit, L. I. (1988). Bayes Methods for Outliers in Exponential Samples. *Journal of the Royal Statistical Society: Series B (Methodological)*, *50*(3), 371–380.

Shu, C., Qin, T., Chen, X., & Yin, P. (2018). Study on Outlier Detection Method in Survival Analysis: Weibull Regression Outlier Model. *Journal of Biometrics & Biostatistics*, *9*(4).

Zhang, H., Gao, Z., Du, C., Bi, S., Fang, Y., Yun, F., ... Shen, X. (2022). Parameter estimation of three-parameter Weibull probability model based on outlier detection. *RSC Advances*, *12*(53), 34154–34164.