A Simple Remaining Useful Life Algorithm Using the Quadratic

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ABSTRACT

The goal of predictive maintenance (PdM) is to facilitate on-condition maintenance or reduce/eliminate unscheduled maintenance events. For critical systems such as aircraft, PdM improves safety while increasing operational readiness. Aircraft operators can order the parts and ensure the correct skills and tools are available to avoid unplanned downtime. An enabler for PdM is the need to estimate the remaining useful life (RUL). For RUL to be accurate, there needs to be an assessment of the current component health, a threshold for when it is appropriate to do maintenance, and a degradation model. This model could be based on some physical processes, such as high-cycle fatigue failure. However, often the exact fatigue process is unknown. In this paper, a quadratic RUL model is used to calculate RUL using a state estimator. The proposed process allows for model validation of the RUL state estimator itself. This is demonstrated using a bearing fault, a gear fault, and oil debris example.

1. INTRODUCTION

The purpose of scheduled maintenance (see MSG-3, 2018) is to ensure the realization of the inherent safety and design reliability of, say, an aircraft. Additionally, maintenance goal are to:

- Restores safety and reliability when deterioration has occurred, and
- To accomplish these goals at a minimum of total cost.

Even in the context of moving maintenance to "On Condition," or extending the time between overhauls (TBO) as recommended in AIR6334 (2020), the objective is to reducing cost while increasing reliability and enhancing safety.

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In either the case of scheduled or moving to on-condition maintenance, PdM facilities lowering costs by predicting when maintenance should be performed. Estimating a remaining useful life (RUL) allows replacement components to be marshaled, the scheduling of the right personnel for the maintenance action, and better asset management. Asset management ensures that operators can continue to support their commitments and generate revenue.

RUL estimation, arguably, requires four components:

- The current component health,
- A threshold (the future component health where maintenance should occur),
- A fault propagation model, and
- An estimate of future load to drive the model.

In Orchard (2009), the RUL was estimated using a particle filter based on the Paris' law, which models high cycle fatigue. In Bechhoefer and Dube (2020), three contending models were tested. The models were based on some knowledge of how, physically, the fatigue crack is propagated.

For example, in a Mode 1 crack (opening mode), the crack surface moves directly apart. In a Mode 2 crack, there is an edge sliding mode, where the crack surfaces move normal to the crack front while remaining in plane. Finally, in a Mode 3 (shear mode), the crack surface moves parallel to the crack front and remains in the crack plane.

A Mode 1 failure can be modeled by a Linear Elastic Model (Beer, 1992),

$$\frac{da}{dN} = D(\Delta K)^m \tag{1}$$

where

- *da/dN* is the rate of change in the half crack length per cycle.
- *D* is a material constant,
- *m* is the crack growth exponent, typically 3 to 5, and *K* is proportional to strain.

A Mode 2 can be modeled by Head's Theory. Here it is assumed that the material near the crack is treated as an array of independent elastic tensile bars of modulus E, each carrying the remotely applied stress σ, which transmits the load to the bars both directly and through shear. The model for an applied stress σ, is:

$$\frac{da}{dN} = \frac{2E\sigma^3 a^{3/2}}{3E_w(\Delta\sigma)D} \propto K^3 \tag{2}$$

When the crack loading is in the anti-plane strain (e.g., Mode 3), the plastic zone at the crack tip can be represented as a continuously distributed array of small dislocations on the crack plane. The crack growth occurs when the accumulated plastic strain distribution at the crack tip exceeds some critical value and continues as this value is exceeded at the crack tip. The rate at which the crack grows per stress cycle in terms of displacement leads to:

$$\frac{da}{dN} = \frac{a^2\sigma_{max}^4}{DE\sigma^3} \tag{3}$$

In equations (1, 2, 3), the RUL is the time (cycles), N, which can be calculated by taking the inverse (e.g., solving for *dN/da)* and integrating.

However, in this paper, a more data-driven approach is taken. The RUL is calculated by solving the quadratic equation.

2. THRESHOLD SETTING

Online condition monitoring systems measure a feature that is proportional to damage. In the case of oil debris monitoring (ODM), it is typically the cumulative mass of ferrous material over time. In the case of vibration monitoring, acceleration data is operated on by an algorithm to develop a condition indicator (CI). In the case of bearing defects, the CI is usually calculated using the Envelope analysis (see Abboud et. al. 2017). For shaft or gear damage, analysis is based on a feature derived from the time synchronous average (Bechhoefer, Butterworth, 2019).

While AI or other machine learning techniques could be used for threshold setting, here, the approach of a hypothesis test is used. That is, the observed (CIs) have a PDF. An operation is performed on the CI to define a health index (HI), which is then a function of distributions. The HI function in this application is the weighted norm of n CIs: the normalized energy of n CIs. The weights of the CIs is set by the Jacobian (the inverse covariance):

$$HI = {}^{0.5}/_{critical}\sqrt{\boldsymbol{Y}^T\boldsymbol{Y}} \tag{4}$$

where $\boldsymbol{Y}$ is the whitened, normalized array of CIs, and *critical*, is the critical value of the test. In a hypothesis test, the critical value is calculated from the inverse cumulative distribution function (ICDF) for a given probability of the false alarm. For Eq. (4), the ICDF is the Nakagami where η is the number of CIs in the array and = n, and ω = η/(2-π/2)*2.

A normalized HI > 0.50 for a component indicates that the Null Hypothesis is rejected. That is, the component is no longer nominal. Note that maintenance is not recommended until the HI > 1. This threshold process has been tested on numerous helicopters, wind turbines, and seeded fault testing on 60+ gearboxes. The level of damage for an HI of 1.00 is typically moderate visible damage.

It can be shown that a whitening solution can be implemented using Cholesky decomposition. The Cholesky decomposition of the Hermitian, positive definite matrix results in $\mathbf{A} = \boldsymbol{LL}^*$, where $\boldsymbol{L}$ is a lower triangular, and $\boldsymbol{L}^*$ is its conjugate transpose. Thus, by definition, the inverse covariance is positive definite Hermitian. It then follows that:

$$\boldsymbol{LL}^* = \boldsymbol{\Sigma}^{-1}, \text{ then } \boldsymbol{Y} = \boldsymbol{L} \times \boldsymbol{CI}^T \tag{5}$$

3. A QUADRATIC RUL ESTIMATE

Equations (1, 2, and 3) describe a time in terms of a measured crack length, a. This measurement is impractical in application. In the CI and resulting HI paradigm, the HI becomes the surrogate for the crack length a. It is assumed that the CIs and HI are corrupted by noise, where the HI is nearly Gaussian. Redefining variables, then at some acquisition index, i, a quadratic can describe the current HI as:

$$HI_{i+1} = 1/2\, a_i t^2 + b_i t + c_i \tag{6}$$

The variable a, here, represents d^2HI/dt^2 (second derivative of the HI), while b is dHI/dt, and c is the current estimate of the HI. The derivatives and estimate of the HI can be restricted using a Kalman filter or other Riccati equation. In our case, a fixed rate *Alpha-Beta-Gamma* filter was constructed to calculate the rates. The input states for the filter are `HI`, `dHIdt`, `d2HIdt2`, and the current measured HI, `mH`. The pseudo code to update the state is then:

```
HI = HI + dHIdt*dt + d2HIdt2*dt*dt/2;
rk = mHI - HI;
HI = HI + alpha * rk;
dHIdt = dHIdt + beta * rk / dt;
d2HIdt2 = d2HIdt2 + gamma * rk /
(2*dt*dt);
```

The process variance is defined as σw^2, and the plant noise σv^2. The *Alpha-Beta-Gamma* filter values are then calculated as:

```
lam = σw²/σv²*t*t;
b= lam/2-3;
c = lam/2+3;
p = c-b^2/3;
q = 2*b^3/27 - b*c/3 - 1;
v = sqrt(q^2 + 4*p^3/27);
z = -(p+v/2)^(1/3);
s = z - p/(3*z) - b/3;
```

```
alpha = 1 - s^2;
beta = 2*(1-s)^2;
gamma = b2^2/(2*a2);
```

Since we are solving the time (RUL), *t*, until HI is equal to 1, we can rearrange terms and:

$$1/2\, a_i t^2 + b_i t + c_t - 1 = 0 \tag{7}$$

The quadratic equation can now be solved for the positive value of *t,* the RUL as:

$$t = {-b \pm \sqrt{b^2 - 4ac}}/{2a} \tag{8}$$

3.1. Validation of the RUL

The RUL estimate itself is noisy and requires filtering. That said, let one assume that the RUL model (equation 7) is correct. Then at some give RUL, say 100, the machine is run for an hour. Then at the end of the hour, the RUL should be 99. A further hour of life is consumed, and the RUL should be 98. That is, for a valid model, the *dRUL/dt* should be approximately -1, and the second derivative should be close to zero (no acceleration).

To test the quality of the RUL model, another *Alpha-Beta-Gamma* model is run to generate, in real-time, to filter the RUL and to generate performance statistics to validate the performance of the RUL model.

4. TEST CASE: BEARING FAULT

This dataset was collected on a 2.1WM wind turbine. This data set was collected over 55 days, with one acquisition every 10 minutes (144 acquisitions per day). Note that the fault, a high-speed inner race fault, starts to propagate at approximately time -600 hours, which corresponds to high loads from a winter storm (Figure 1).

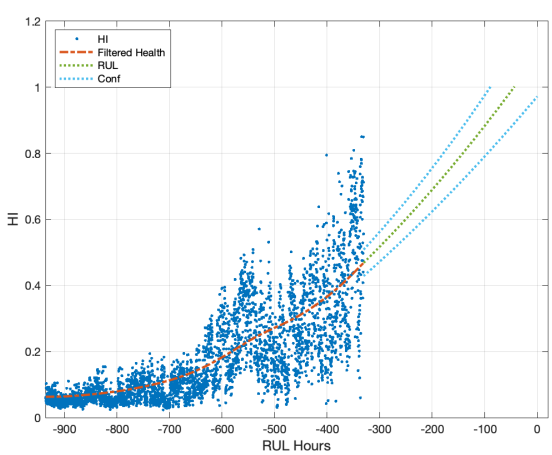

Figure 1 High-Speed Bearing Fault, Wind Turbine, with RUL

More details on the fault can be seen in Ref. Bechhoefer and Dube, 2020. In this paper, the RUL found the best performance was the linear elastic model. The current quadratic model's fit is not as tight. The linear elastic model had a converged/closer RUL at approximately -500 hours. However, the quadratic model does converge by -300 hours with good prognosability and trend.

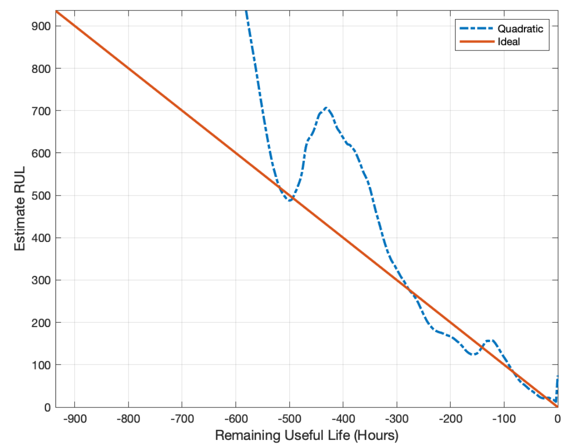

Figure 2 High Speed Bearing RUL

Figure 3 gives the first and second derivatives of the RUL.

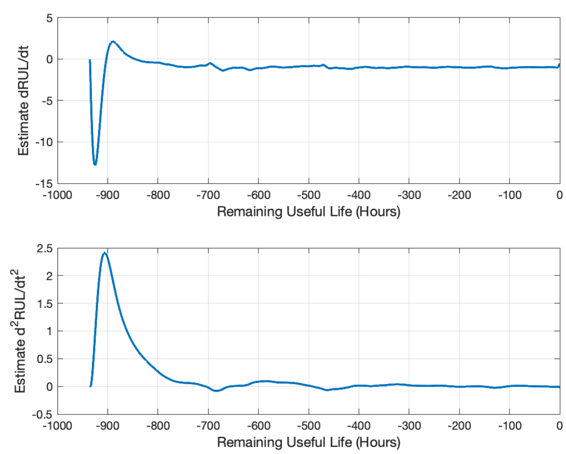

Figure 3 High-Speed Bearing Derivatives

The first derivative of the RUL converges to -1 by -400 hours while the second derivative approach zero -400 hours, as well. Note that when the model is not converged, early in the experiment (-950 to -700 hours or so), are not converged to -1, 0 respectively, indicating a poor model fit, as the bearing is not trending/faulted. This can be confirmed in Figure 2, where the RUL does not fit the -1 RUL slope until -400 hours or so. Of course, the RUL is also based on the usage of the machine, a function of wind loading.

5. TEST CASE: PLANET GEAR FAULT

In (Wang, Blunt, Kappas, 2023), an EDM notch was applied to the planet gear in an OH-58 gearbox. The gearbox was loaded to 125% torque, and acquisitions were taken every 3 minutes. The HT TSA was collected for a 100-second acquisition over four channels. Condition indicators were developed to take advantage of the hunting tooth phenomena. The resulting CIs were fused into an HI, as per Eq 4. In "Helicopter Main Gearbox Planet Gear Crack Propagation Test Dataset," it is stated that the initial nominal data is from acquisitions 1 to 146 (from -25 to -25 hours) while crack initiation is from acquisitions 147 to 241, and from acquisitions 242 to 526, the crack propagates. The HI algorithm indicated performing a repair at acquisition 465. An example of the trend and prognostic in Figure 4.

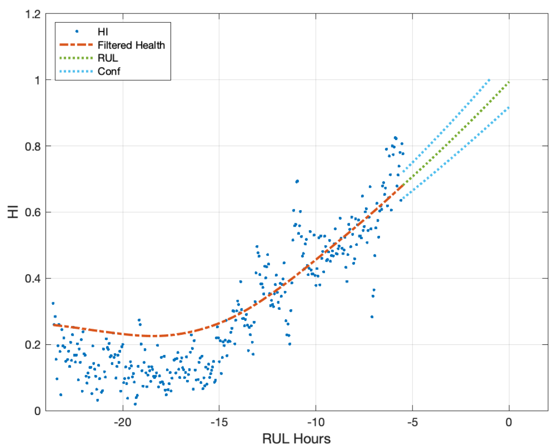

Figure 4 Planet Gear Fault, OH-58 Gearbox, with RUL

It should be noted that it was found that the best model for estimating RUL was a Mode 3 dislocation theory model. Figure 5 gives the RUL, while Figure 6 is the derivatives.

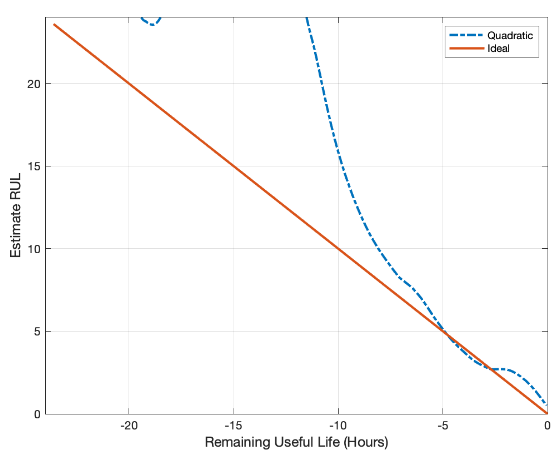

Figure 5 Planet Gear Fault RUL

Note that this testing was accelerated. In practice, HUMS (health and usage monitoring systems) would generate alerts 70 to 150 hours in the future. However, the DSTG did a remarkable job in determining the depth of the EDM notch and load needed to propagate the fault in a reasonable time. In Figure 6, it is seen that the first derivative did not converge to -1 until -10 hours RUL, while the second derivative did not approach zero until -7 hours. This is confirmed in Figure 5, that the RUL approach the ideal at approximately -7 hours. Using a dislocation theory model, the RUL had converged by -10 hours. Clearly, the dislocation theory model had better performance, yet the quadratic model did eventually fit the data and give good results.

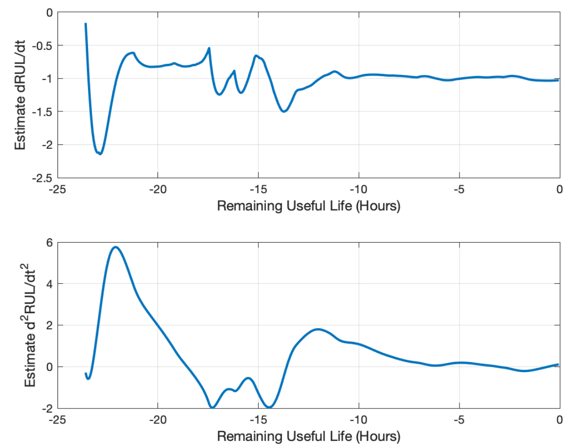

Figure 6 Planet Gear RUL Derivatives

Other metrics for RUL performance have been developed, as given by Dr. Coble (2010), who introduces the concept of prognosability, monotonicity and trendability as RUL performance metrics. However, these metrics are more appropriate for comparing different RUL vs. identifying when an RUL model is delivering valid data.

6. TEST CASE: OIL DEBRIS MONITORING BEARING FAULT

Oil debris monitoring systems detect ferrous and non-ferrous wear particles in the gearbox oil. It can determine the size so that the cumulative mass of material can be trended. In this experiment of spalling on a turbine engine bearing, the threshold was set at 400 mg (that is, the mass was divided by 400 to derive an HI, such that maintenance would be recommended at an HI of 1.0). Figure 7 gives an example of the oil debris HI and prognostics.

The RUL for the ODM data is an excellent fit, superior to either the linear elastic model, Head's, or dislocation theory. In fact, the motivation for developing the quadratic model was that the exponential models failed to perform well.

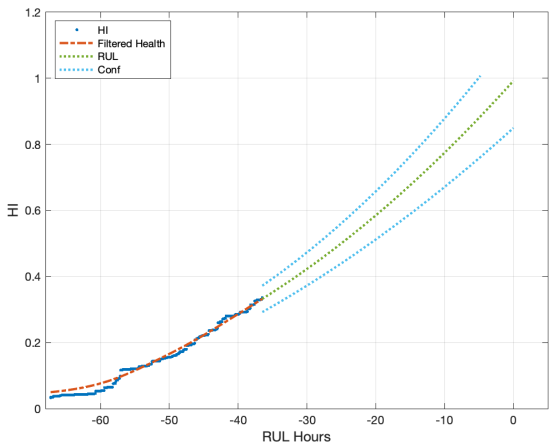

Figure 7 ODM, Bearing Fault with RUL

Figure 8 gives the RUL, which again shows a good fit for 75% of the run. This indicates that the release of debris material is, in fact, quadratic.

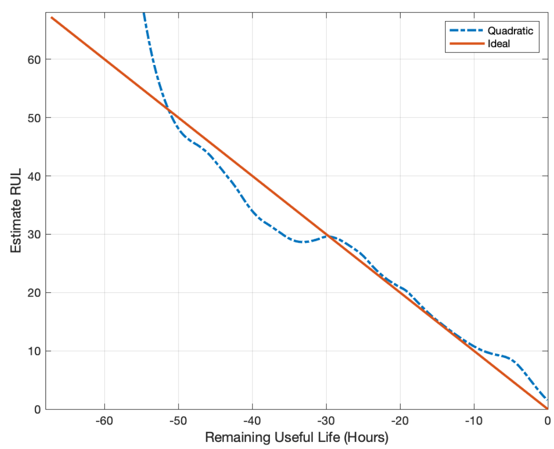

Figure 8 ODM Bearing Fault RUL

The derivative is given in Figure 8, which show that the first and second derivative had converged to -1 and 0 by -50 hours. Again, this analysis occurred under accelerated life testing. While only 70 hours of data, it is likely that in real-world applications, the RUL would be 2 or 3x, or perhaps 200+ hours.

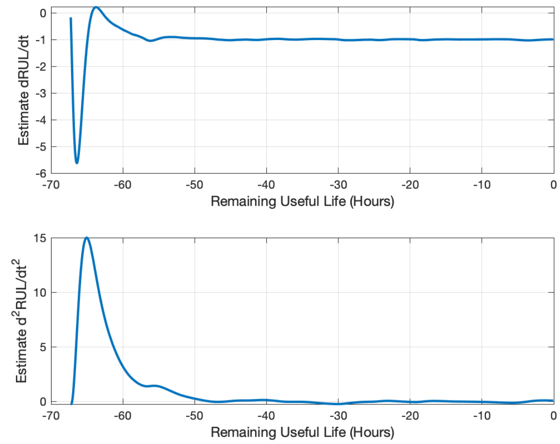

Figure 9 ODM First and Second Derivative of the RUL

7. CONCLUSION

An RUL calculation facilitates PdM, and its associated cost saving and increase in asset availability. However, RUL, in general, is difficult to calculate. It was observed that having knowledge of the degradation process is essential for a good model fit. That said, sometimes simple models give adequate results.

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BIOGRAPHIES

Eric Bechhoefer received his BS in Biology from the University of Michigan, his MS in Operations Research from the Naval Postgraduate School, and a Ph.D. in General Engineering from Kennedy Western University. He is a former Naval Aviator who has worked extensively on condition-based maintenance, rotor track and balance, vibration analysis of rotating machinery, and fault detection in electronic systems. Dr. Bechhoefer is a Fellow of the Prognostics Health Management Society, a Fellow of the Society for Machinery Fault Prevention Technology, and a senior member of the IEEE Reliability Society. Additionally, Dr. Bechhoefer is also a member of the SAE committee covering Integrated Vehicle Health Management and a member of the MSG-3 Rotorcraft Maintenance Programs Industry Group. Dr. Bechhoefer is the recipient of the 2023 Helicopter Association International "Salute to Excellence in Safety" award for his work in developing a light helicopter Health and Usage Monitoring System.