Physical Reservoir-Based Health Monitoring of a Structure with Nonlinear Attachments

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Abstract

The purpose of this work is to discuss the possibility of the concept of physical reservoir computing (PRC) in the field of structural health monitoring (SHM) by regarding the target structure of SHM as the physical reservoir. To this end, the dynamics of the structure, which is assumed extrinsically linear, is tailored to be strongly nonlinear by installing nonlinear attachments. Our purpose is then to detect the change occurred in this augmented physical reservoir. As one possible methodology to achieve this, we propose in this study to train the output layer to learn a specific nonlinear mapping of the input so that the increase of the error may indicate the change of the reservoir. Numerical experiments are presented to examine the validity of the proposed concept.

Keywords: physical reservoir computing, structural health monitoring, damage detection, nonlinear dynamics

1. Introduction

The maintenance and management of infrastructure and industrial facilities is one of the most critical issues to be addressed to ensure the sustainability of advanced social life. Giving self-diagnostic abilities to structures, i.e. the ability to continuously monitor their own healthiness, is an essential technology for efficient, labor-saving, and sustainable maintenance and management of structures. Such technology is called structural health monitoring (SHM) (Farrar & Worden, 2007) and has been researched and developed for over 20 years. Particularly in the last decade, SHM has made significant progress at a practical level, thanks to the rapid development of IoT and artificial intelligence technologies.

Most SHM systems use wireless sensor networks to collect dynamic data, such as vibrations, from various parts of the structure, and then use high-performance computing resources to process the multidimensional data and perform diagnostic algorithms to make decisions. The wireless sensor networks utilized in SHM typically have a star or tree topology, through which large amounts of data are aggregated in a single location and fed to the diagnostic algorithms. Currently, sensor densities of several hundred to one thousand nodes per structure can be achieved, but further increases in sensor density will reach their limits due to saturation of communication capacity and increased computational costs caused by data concentration.

We intend to overcome the above-mentioned challenges in SHM sensor networks by introducing the concept of physical reservoir computing (PRC) (Tanaka et al., 2019), specifically mechanical reservoir computing (Hauser, Ijspeert, Füchslin, Pfeifer, & Maass, 2011; Caluwaerts, D'Haene, Verstraeten, & Schrauwen, 2013; Nakajima, Hauser, Li, & Pfeifer, 2015; Coulombe, York, & Sylvestre, 2017) to SHM. The idea is to utilize the dynamics of the target structure as a physical reservoir (PR) to perform some or most of the data processing and computation in SHM by using the structure itself as a computational resource. As a preliminary study, a simple nonlinear oscillator network was used as a model of the target structure in the previous paper (Masuda, Takashima, & Sakai, 2023), and a PRC system was defined by attaching input and output layers to the structural model as a PR, then a toy problem of learning a nonlinear function was conducted. It was demonstrated that changes in mechanical properties due to structural damage can be detected as an increase in the PRC output error.

In this study, we conduct similar investigation on a different target structure that is modified from the one used in the previous study (Masuda et al., 2023) to be slightly more realistic. There are two points of modification: first, it was assumed in the previous paper that the target structure consists of multiple oscillators with strong nonlinearity connected in a

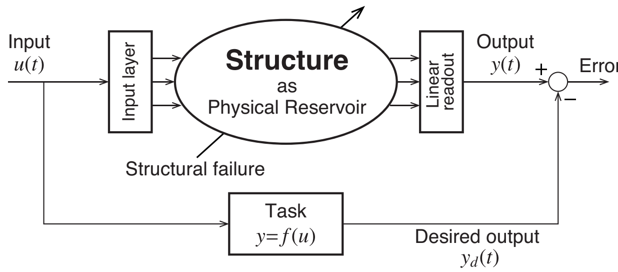


Figure 1. Conceptual drawing of PR-based SHM.

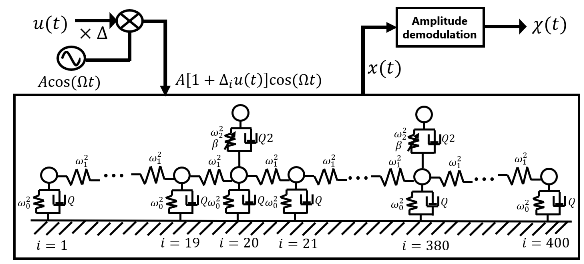


Figure 2. Schematic drawing of target structure with nonlinear attachments, and the PR built on it.

line, whereas the actual object structure is to be linear. Thus, in this study, we adopt a linear target structure composed of multiple linear oscillators connected in a line, some of which have secondary devices attached to them to grant strong nonlinearity. Second, in the previous study, the input signal was given to the PRC as an amplitude modulation signal produced by multiplying the input signal by a carrier wave, which was simultaneously applied to randomly selected approximately half of the oscillators as excitation force. This specific configuration was adopted as it was just quoted from the reference (Coulombe et al., 2017). In the PRC in this study, we apply the input force only to the oscillator where the nonlinear devices are added. This change comes from the assumption that the forcing device to excite the structure is integrated with the nonlinear device.

Then, the system with the above modifications is used as the target structure to define the same toy problem as the previous study, and to examine how the changes in the structural parameters such as stiffness and damping can be detected and localized by the proposed PRC-based SHM.

2. Concept of PR-based SHM

Figure 1 illustrates the concept and entire workflow of the PR-based SHM proposed by the authors (Masuda et al., 2023). The most important aspect of this concept is to utilize the target structure as a computing resource to reduce the local concentration of information and computation costs. In other words, the target structure is regarded as a PR. Since the target structure is inherently linear, nonlinear devices are artificially attached to the structure to fulfill the nonlinearity required by the structure to perform as PR. Then, by implementing appropriate excitation and sensing points to define input and output layers, a PRC system is parasitically built on the target structure, which is trained to learn a task to reproduce a specific nonlinear input-output mapping.

Then, the structural damage to be detected can be regarded as a change of the dynamic property of the PR. Therefore, if the PRC is trained a specific task while it is in its intact state, the structural damage can be detected as the increase of the prediction error.

3. Modified toy problem for concept validation

3.1. Definition of target structure and damage

Figure 2 shows the schematic drawing of target structure with nonlinear attachments used in this study, and the PR built on it. The target structure is a mass-spring network consisting of $N = 400$ linear oscillators, each of which has a unit mass, linear stiffness of ${\omega_0}^2$, and linear damping ratio of $1/(2Q_1)$, all aligned in a line connected each other with stiffness of ${\omega_1}^2$. In order to augment the target structure as a strongly nonlinear system, Duffing oscillators with mass ratio of 1/10, linear stiffness of ${\omega_2}^2$, cubic stiffness coefficients of β, and linear damping ratio $1/(2Q_2)$ are introduced as nonlinear attachments mounted on every 40 oscillators, i.e., $i = 20, 60, 100, \ldots, 380$. The mass of the oscillator to which the nonlinear device is attached is excited by an amplitude modulated sinusoidal force with a mean amplitude A modulated by the input signal $u(t)$, scaled by a scaling factor Δ_i. The input $u(t)$ is supposed to be a temporal sequence, of which value changes at $t = kT$, where k is an integer and T is a period.

Thus, the equation of motion for the ith oscillator with the nonlinear attachment is given by

$$
\begin{aligned}
&\ddot{x}_i(t) + \frac{\omega_0}{Q_1}\dot{x}_i(t) + {\omega_0}^2 x_i(t) \\
&+{\omega_1}^2[-x_{i-1}(t) + 2x_i(t) - x_{i+1}(t)] \\
&+\delta_i\{{\omega_2}^2[x_i(t) - z_i(t)] + \frac{\omega_2}{Q_2}[\dot{x}_i(t) - \dot{z}_i(t)] \\
&+\beta[x_i(t) - z_i(t)]^3\} = A[1 + \Delta_i u(t)]\cos(\Omega t)
\end{aligned} \tag{1}
$$

where $x_i(t)$ is the displacement of the ith oscillator, δ_i takes one when the ith oscillator has the nonlinear attachment, otherwise zero, and $z_i(t)$ is the displacement of the mass of the nonlinear attachment mounted on the ith oscillator. The forth term of the left-hand side of equation (1) is replaced by ${\omega_1}^2[x_1(t) - x_2(t)]$ when $i = 1$, and by ${\omega_1}^2[-x_{N-1}(t) + x_N(t)]$ when $i = N$. The equation of motion of the nonlinear

Table 1. Design parameter values

Description	Symbol	Value
Number of oscillators	N	400
Linear stiffness of oscillator	ω_0	1.3
Q factor of oscillator	Q_1	60
Linear stiffness of connecting spring	ω_1	1.5
Cubic stiffness coefficient	β_i	1
Linear stiffness of attachment	$\omega_2 = \sqrt{10}\omega_0$	4.111
Q factor of attachment	Q_2	60
Frequency of excitation force	Ω	1.5
Mean amplitude of excitation force	A	8
Input scaling factor	Δ_i	0.7
Input period	T	65

Table 2. Definition of structural damage

Failure mode	Parameter altered	Amount
Loss of stiffness in oscillator	ω_0 of 50th oscillator	1, 5, 10 % decrease
Loss of stiffness in connection	ω_1 on both sides of 50th oscillator	5, 10, 20 % decrease

attachment is given by

$$\begin{aligned}&\ddot{z}_i(t) + {\omega_2}^2(z_i(t) - x_i(t))\\&+\frac{\omega_2}{Q_2}(\dot{z}_i(t) - \dot{x}_i(t)) + \beta_i(z_i(t) - x_i(t))^3 = 0\end{aligned} \tag{2}$$

The values of the parameters are summarized in Table 1.

As the structural damage, two damage modes are defined as described in Table 2, in which the structural parameters, ω_0 and ω_1 involved in the 50th oscillator are altered independently, to represent the deterioration of the target structure.

3.2. PRC design

In order to define the output layer of PRC, the displacement of every mass is measured with a sampling period of 0.1, and then amplitude demodulated using Hilbert transform to obtain the envelope, followed by low-pass filtering with fourth-order Butterworth filter. The resultant envelope signal $\chi_i(t)$ are used to define the PRC output signal as

$$y(t) = \sum_{i=1}^{i_2-i_1+1} w_{i+i_1-1}\chi_{i+i_1-1}(t) \tag{3}$$

where the output is defined as a linear sum of the envelope signal at the masses of $i{=}i_1, \ldots, i_2$. The linear weights w_i are determined in the training phase so that the output signal $y(t)$ becomes as close as possible to the desired signal $y_d(t)$ in terms of minimizing the least square error (supervised learning). This is done by a simple matrix calculation given by

$$\mathbf{w} = (\mathbf{\Xi}^\top\mathbf{\Xi} + \lambda\mathbf{I})^{-1}\mathbf{\Xi}^\top\mathbf{y_d} \tag{4}$$

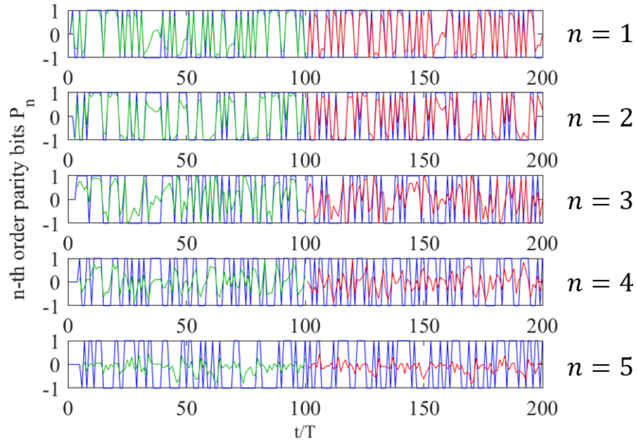


Figure 3. Output of PRC compared with desired output.

where $[\mathbf{w}]_i = w_{i+i_1-1}$, $[\mathbf{\Xi}]_{j,i} = \chi_{i+i_1-1}(t_j)$, and $[\mathbf{y_d}]_j = y_d(t_j)$, for $i{=}1, \ldots, i_2{-}i_1{+}1$ and $j{=}1, \ldots, J$; further, I is the identity matrix and λ is the parameter for Tikhonov regularization.

Multiple PRCs are defined for the same input by using partial subsets of masses to extract localized information about the structural healthiness. Each subset is organized from 40 contiguous masses, i.e., the mth subset consists of mth through $(m{+}39)$th masses. Note that the sampling is cyclic so that the number of the masses are equally 40 for all 400 subsets. The resultant 400 PRCs are trained independently to learn a specific task described next.

3.3. Learning Task

The learning task used in the presented toy problem is the same as the previous work (Masuda et al., 2023), which was quoted from the reference (Coulombe et al., 2017). The learning task is defined as the nth-order parity function.

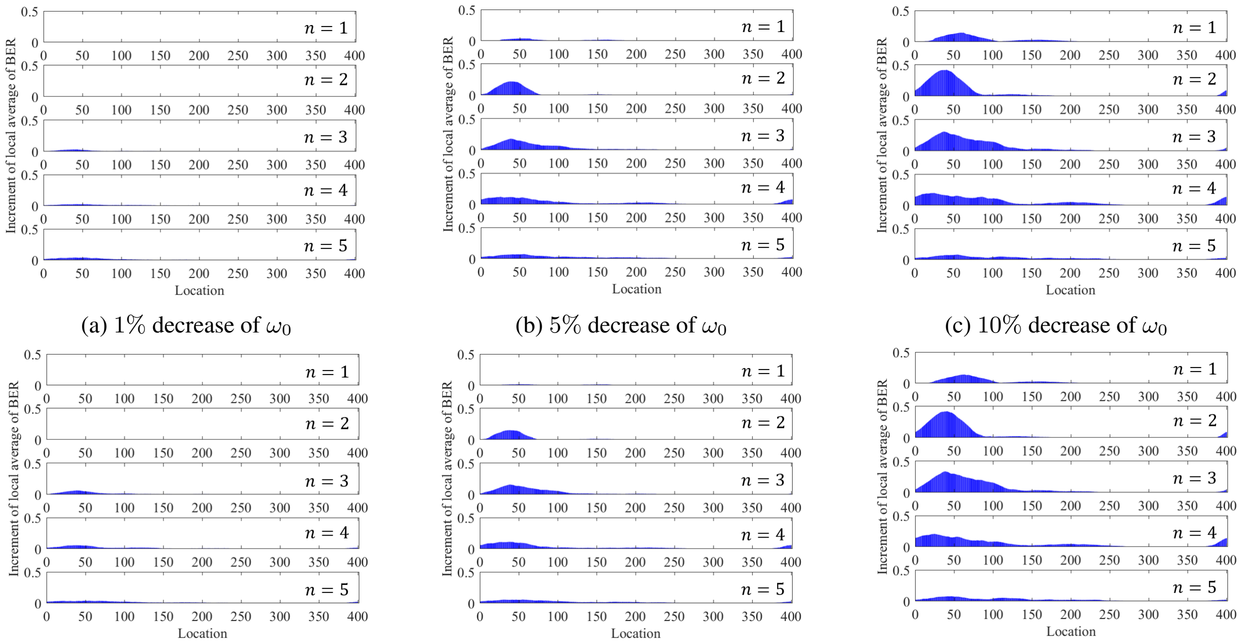
$$P_n(t) = \prod_{i=1}^{n} u(t - iT) \tag{5}$$

where $u(t)$ is a binary signal that randomly switches between -1 and $+1$ whenever t is an integer multiple of the period T. This function is a multiplication of the past inputs, therefore, this task requires both memory and nonlinear computational capabilities.

4. Results and discussions

4.1. Training of PRC

Before performing anomaly detection, it is necessary to first confirm that the above PRC has been properly trained. We divided randomly generated input $u(t)$ into $100T$ for both training and validation data and calculated the desired signal $y_d(t)$ (in this toy problem, $P_n(t)$) by equation (5). The output layer



(a) 1% decrease of ω_0 (b) 5% decrease of ω_0 (c) 10% decrease of ω_0

(d) 5% decrease of ω_0 (e) 10% decrease of ω_0 (f) 20% decrease of ω_0

Figure 4. Increment of local average of BER by failures introduced at 50th location

weight w_i was calculated using the training data with $\lambda = 0.1$ by equation (4).

Figure 3 shows a comparison of the output signal $y(t)$ calculated by equation (3) and the desired signal $y_d(t)$, representing the training and validation phases from $n = 1$ to 5. The desired signal $y_d(t)$ is plotted in blue lines, whereas the outputs from 400 PRC are averaged and plotted in green lines for the training phase and in red lines for the evaluation phase. The estimation error in the evaluation phase was quantified by bit error rate (BER) given by

$$BER = \frac{D_h}{L} \tag{6}$$

where D_h is the Hamming distance between the estimated bits $y_{bit}(t)$ and the true bits $y_d(t)$, and L is the length of the data. The estimated bit is evaluated as

$$y_{bit} = \begin{cases} 1 & (y \geq 0) \\ -1 & (y < 0) \end{cases} \tag{7}$$

The BERs for the results shown in Figure 3 were 1%, 0%, 6%, 15%, and 37% for $n = 1, 2, 3, 4$, and 5, respectively. The fact that the estimation of first, second, and third parities exhibited satisfactory accuracy suggests the capability of the PRC to remember past input and to express nonlinear mapping.

The maximum amplitude of the relative displacement of $z_i(t) - x_i(t)$ in equation (2) for the case plotted in Figure 3 was examined to see how the nonlinear attachments worked during the task. The ratio of the equivalent stiffness to the linear stiffness ω_2^2 for each attachment was calculated to investigate how much the cubic nonlinearity of the attachment contributed. The ratios were 1.0507, 1.0456, 1.0456, 1.0461, 1.0458, 1.0459, 1.0458, 1.0496, 1.0456, respectively. These results suggest that the effects of nonlinearity on the behavior of the attachments may be quite limited. This is contrary to the common understandings that the physical reservoir has to be strongly nonlinear. The question how the nonlinearity affects the learning performance and what kind of nonlinearity can improve the learning of the physical reservoir must be addressed in the future study.

4.2. Damage detection and localization

Using the PRC trained in the above, damage detection tests were conducted. Using the same input, the BER of the output from 400 PRCs was calculated when there was no damage to the structure, and compared with the BER when there was damage to the structure. Figure 6 shows the increase in BER for damage shown in Table 2. The horizontal axis represents the position of the oscillator, and the height of the bar indicates the local average increase of the BER of the PRC associated with a specific position.

What can be seen from Figure 4 is that the larger the damage, the greater the error. The highest increase in error occurs near the damaged oscillator 50. Among the parities from the 1st to the 5th, it is found that the 2nd parity is the most sensitive to damage. This is the same as the result reported in the previous study (Masuda et al., 2023). The reason why the 1st parity is less sensitive than the 2nd parity may be due to the simplicity of the task, which could introduce the trained PRC some undesirable tolerance to the parameter variation. Moreover, the BER was more sensitive to the change of ω_0 than that of ω_1. This is again the same result as the previous study. It is certain, however, that the parameter sensitivity observed here is highly dependent on the design of PR. Establishing a design theory for PR and task selection to maximize the parameter sensitivity is the major challenge in future study.

5. Conclusion

In this study, we discussed the potential of the concept of PRC-based SHM, in which the target structure itself is used as a computational resource, conceptually a promising way to relieve the heavy computational burden associated with the increase of the sensor density in sensor networks. Particularly in this paper, we examined a specific configuration of PRC, in which the target structure was tailored strongly nonlinear by retrofitting nonlinear attachments, that was trained to learn nonlinear functions. We consider structural damages as changes of the mechanical properties of the PR layer, which allows us to detect the damages as the increase of the prediction error. To demonstrate the effectiveness of the proposed concept, we conducted numerical experiments through a toy problem using a network of interconnected linear oscillators with nonlinear attachments, and performed prediction tasks for parity functions. As a result, it was found that the PRC was able to detect damages in the structure and identify its location.

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Biographies

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Arata Masuda, a professor of mechanical engineering in Kyoto Institute of Technology, received his Ph.D degree in mechanical engineering from Kyoto Institute of Technology in 1999, BS from Kyoto University in 1990. His research has been in the area of smart structures, structural health monitoring, energy harvesting, vibration control, and system identification.