On Determination of the Non-periodic Preventive Maintenance Scheduling with the Failure Rate Threshold for Repairable System

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ABSTRACT

Determination of a preventive maintenance scheduling is regarded as a key part in manufacturing system to maintain the equipment in good condition. In practice, many preventive maintenance policies is used in manufacturing system to reduce the unexpected failures and increase sustainability of system. In this paper, the failure rate of system is used as a condition variable and a decision variable, and preventive maintenance policy is then developed for minimizing the expected maintenance cost rate. The imperfect preventive maintenance activities of the proposed models are modeled via the arithmetic reduction model which uses the age reduction factor or the hazard reduction factor. The results of the numerical example shows that the model based on age reduction can not only extend the system lifetime but also reduce the expected maintenance cost rate although the preventive maintenance cost is significantly higher than those of the model based on hazard reduction.

1. INTRODUCTION

For extending functioning life of manufacturing systems, it is important to determine the condition variable of maintenance activities in planning preventive maintenance scheduling. In practice, the preventive maintenance policy focused on high reliability level of system may result in high maintenance cost because of the frequent preventive maintenance (PM) activities. On the contrary, insufficient PM activities cannot extend functioning life for system due to excessive system breakdowns. However, it is difficult to determine an optimal level of system reliability because of numerous uncertainties such as the state of system, the level of system deterioration, maintenance costs, and repair effects. Especially, the determination of how to deal with repair effects is important when establishing a maintenance model because the system hazard intensity is changed according to the repair effects.

The maintenance activities can be generally divided into corrective maintenance (CM) activities and preventive maintenance (PM) activities. CM activities are unscheduled, and are immediately performed to restore a system after any breakdowns. CM activities are commonly modeled via minimal repairs. After CM activities, the system is restored to a state of "As bad as old". PM activities are usually scheduled, and are performed to reduce the likelihood of system breakdowns. PM activities are assumed as perfect or imperfect. In perfect PM case, after PM activities, the system is restored to a state of "As good as new". In imperfect PM case, the system is restored to a "better than as old" state; this is widely used because of the fact that it can represent the uncertainty and the deterioration process.

To represent the effectiveness of imperfect maintenance, several models have been proposed. One is the (p,q) model, in which minimal repair is performed with probability p and perfect repair is performed with probability (1-p) (Nakagawa, 1979). Another model is the virtual age model proposed by (Kijima, 1989; Kijima & Sumita, 1986), in which system is restored to a state of "better than as old" by age reduction factors. Other model is the hybrid model composed of age reduction factor and hazard increasing factor (D. Lin, Zuo, & Yam, 2001). The other model is the ARI and the ARA model, in which ARI model is based on the hazard reduction factors and ARA model is based on the age reduction factor (Doyen & Gaudoin, 2004). In this study, the analysis is conducted for how each reduction factors affects the proposed model based on the ARI model and the ARA model.

Ever since the early work of (Barlow & Hunter, 1960), various works for maintenance policies have been conducted, and has been divided into two types according to maintenance criteria (Ahmad & Kamaruddin, 2012). Time-based maintenance policies aim to determine the scheduling of PM for minimizing maintenance cost, and maintenance activities are periodically performed. However, it is hard for periodic PM to prevent the frequently occurring breakdowns in the late period of system life because of severe system degradation (Huang, Chen, & Ho, 2013). Condition-based preventive maintenance policies aim to determine the scheduling of

PM minimizing maintenance cost, and maintenance activities are performed when condition variable such as reliability and hazard intensity reaches a certain value. In research on condition-based preventive maintenance policies, age reduction factor has been widely used to represent the effectiveness of imperfect maintenance (Liao, Pan, & Xi, 2010; Z. L. Lin & Huang, 2010; Z.-L. Lin, Huang, & Fang, 2015). However, it is not realistic to consider only the age reduction factor because it cannot reflect the case where the efficiency of imperfect maintenance is worse. Therefore, it is necessary to conduct the analysis for how each reduction factor to represent the effectiveness of imperfect maintenance affect the system restoration. Finally, in this study, a condition-based PM policy based on failure rate threshold is proposed, in which the arithmetic reduction model is used for representing the effectiveness of imperfect maintenance

This paper is organized as follows. Section 2 explains basic assumptions, and derives a condition-based PM model. Section 3 provides a numerical example based on two cases. Finally, Section 4 discusses the conclusion.

2. METHODOLOGY

2.1. Assumptions

1. Every PM activity is performed when the system failure rate reaches a certain threshold value.
2. The deterioration process of the system is assumed as the non-homogeneous Poisson process (NHPP), and can be modeled by the Weibull power-law intensity given as $h(t) = \alpha\beta t^{\beta-1}$ $t \geq 0$ is the operation time, $\alpha > 0$ is the scale parameter, and $\beta > 1$ is the deterioration parameter.
3. When breakdown occur before PM activities, the minimal repairs are performed.
4. Every PM activity is imperfect, and the system is restored to a state of "better than old but worse than new" after PM activity. The reduction factors of age and hazard are used as the effectiveness of imperfect maintenance denoted as ρ_i for $i = 1, 2, ..., N-1$.
5. (*N*-1) imperfect PM activities are performed during system lifetime, and the replacement of the system is conducted at the Nth PM activity. In addition, the repair time for both minimal repair and PM is assumed to be negligible.
6. The minimal repair cost, the PM cost, and the replacement cost are assumed to be a constant.

2.2. Imperfect Maintenance Models

In the present work, imperfect PM activities are modeled via the arithmetic reduction model of (Doyen & Gaudoin, 2004). The arithmetic reduction model is appropriate to express the effectiveness of imperfect PM because of the assumption that the hazard intensity function is reduced after PM activity. Moreover, it is composed of two models; one is the ARI model which uses the hazard reduction factor. In the ARI model, after PM activity, the hazard intensity is reduced via reduction factor ρ_i for $i = 1, 2, ..., N-1$. The hazard reduction process after the *i*th imperfect PM activity can be modeled as

$$H_i = \rho_i \left(H_{i-1} + h(T_i) - h(T_{i-1}) \right), \tag{1}$$

for $i = 1, 2, ..., N-1$, and H_{i-1} is similar to the virtual age process proposed by (Kijima, 1989; Kijima & Sumita, 1986). The hazard intensity function after the *i*th imperfect PM activity can be derived as

$$\begin{aligned} h_{i+1}(t) &= h(t) - (h(T_i) - H_i) \\ &= h(t) - \sum_{j=1}^{i} h(T_j)(1-\rho_j) \left\{ \prod_{k=1}^{i} \rho_k \Big/ \prod_{k=1}^{j} \rho_k \right\}. \end{aligned} \tag{2}$$

The other one is the ARA model which uses the age reduction factor. In the ARA model, after PM activity, the elapsed time of hazard intensity is reduced via reduction factor ρ_i for $i = 1, 2, ..., N-1$. The age reduction process after the *i*th imperfect PM activity can be modeled as

$$V_i = \rho_i \left(V_{i-1} + T_i - T_{i-1} \right), \tag{3}$$

for $i = 1, 2, ..., N-1$, and V_{i-1} follows the virtual age process proposed by (Kijima, 1989; Kijima & Sumita, 1986). The hazard intensity function based on the age reduction factors can be derived as

$$\begin{aligned} h_{i+1}(t) &= h(t - (T_i - V_i)) \\ &= h\left(t - \sum_{j=1}^{i} T_j (1-\rho_j) \left\{ \prod_{k=1}^{i} \rho_k \Big/ \prod_{k=1}^{j} \rho_k \right\} \right). \end{aligned} \tag{4}$$

2.3. Condition-based Preventive Maintenance Model with Failure rate Threshold

The number of system breakdowns until the Nth PM activity can be calculated via the definition of NHPP and can be derived as

$$Mr = \int_0^{T_1} h_1(t)dt + \cdots + \int_{T_{N-1}}^{T_N} h_N(t)dt. \tag{5}$$

The time point of each PM activity is given as the operation time of the system. The *i*th PM interval can be calculated as

$$x_i = T_i - T_{i-1}. \tag{6}$$

The system is replaced at the Nth PM activity, and hence the system lifetime can be determined as

$$T_o = T_N. \tag{7}$$

In this study, every PM activity is performed when the failure rate threshold reaches a certain value. The failure rate threshold of the proposed model is not predetermined, and is derived via the expected maintenance cost function. The constraints of the proposed model based on failure rate threshold can be given as

$$h_{i+1}(t) = \theta \tag{8}$$

, for $T_i \le h_{i+1}(t) \le T_{i+1}$. Hence, the expected maintenance cost per unit time can be derived as

$$C(N,\ \theta) = \frac{MrC_m + (N-1)C_p + C_r}{T_o}. \tag{9}$$

The decision variable is an optimal frequency of scheduled PM actions and an optimal failure rate threshold, in that the expected maintenance cost per unit time can be minimized. Using the assumptions stated in section 2, the optimization problem of the proposed model can be formulated as

$$\begin{aligned} &\text{minimize} \quad C(N,\theta) = \frac{MrC_m + (N-1)C_p + C_r}{T_N} \\ &\text{subject to} \quad h_i(t) = \theta^*,\ i = 1,2,..,N. \end{aligned} \tag{10}$$

The optimal failure rate threshold can be determined as

$$\theta^*(N) = \alpha\beta\left(\frac{(N-1)C_p + C_r}{C_m S(N)\alpha(\beta-1)}\right)^{\frac{\beta-1}{\beta}} \tag{11}$$

by solving the first-order partial derivatives of $C(N,\theta)$ with respect to θ to zero and $S(N)$ is changed with the reduction factors. The time point of the *i*th PM activity based on the age reduction factor can be derived as

$$T_i = \left(i - \sum_{k=1}^{i-1}\rho_i\right)T_1. \tag{12}$$

The time point of the ith PM activity based on the hazard reduction factor can be derived as

$$T_i = \left(i - \sum_{k=1}^{i-1}\rho_i\right)^{1/(\beta-1)} T_1. \tag{13}$$

The first PM interval can be given as

$$T_1 = \left(\theta^*(N^*)/\alpha\beta\right)^{1/(\beta-1)}. \tag{14}$$

The corresponding optimal solutions can be obtained by the following algorithm:

Step 1: Solve $dC(N,\theta)/d\theta = 0$ with respect to R_{th}.
Step 2: Set $N = 1$ and calculate the failure rate threshold $\theta^*(N=1)$, and substituting both $N=1$ and $\theta^*(N=1)$ into $C(N,\ \theta^*(N))$; then, go to step 3.
Step 3: Let $N = N+1$. Calculate the failure rate threshold $\theta^*(N+1)$, and substituting both $N+1$ and $\theta^*(N+1)$ into $C(N+1,\ \theta^*(N+1))$; then, go to step 4.
Step 4: If $C(N,\ \theta^*(N)) < C(N+1,\ \theta^*(N+1))$ then attain the corresponding optimal solution, and stop the process, and go to step 5; else, go to step 3.

3. NUMERICAL EXAMPLE

In this section, an optimal condition-based PM maintenance strategy based on the failure rate threshold is designed considering the two cases, and a numerical example was conducted based on two cases.

Case 1: In this case, every PM activity is performed by the talented mechanic and hence the PM cost is significantly high. Every PM activity is immediately performed when the failure rate threshold reaches an optimal value. After PM activity, the system is restored by the age reduction factor.

Case 2: The un-talented mechanic is employed and hence the PM cost is significantly low. After PM activity, the system is restored by the hazard reduction factor. The basic assumptions of maintenance policy are the same as Case 1. It is should be noted that the level of system restoration is significantly lower than those of Case 1.

The parameter of the power-law intensity were assumed to be $\alpha = 1.8$ and $\beta = 2.6$. The reduction factors were given as $\rho_i = (i+1)/(2i+1)$ (El-Ferik & Ben-Daya, 2006), which increased as the number of PM activities increased. The costs for maintenance activities were given as $C_m = 2$, $C_r = 3$, $C_p = 1$ for Case 1, $C_p = 0.5$ for Case 2. The result is summarized in Table 1.

Table 1. The optimal solutions for each case

	Case 1	Case 2
N^*	6	3
$\theta^*(N^*)$	2.0088	2.3227
$C(N^*,\ \theta^*(N^*))$	5.5990	6.0388
T_o	2.5000	1.0764

From Table 1, the results of Case 1 were significantly better than those of Case 2 even though the PM cost was set higher. It shows that the system can be more restored when the age reduction factor is used as the reduction factor for maintenance activity under the power-law process. Moreover, the optimal failure rate threshold of Case 1 was lower than those of Case 2. The system lifetime and the expected maintenance cost rate of Case 1 were longer than those of Case 2.

4. CONCLUSION

Generally, the assumptions that the effectiveness of PM activity is imperfect, is a realistic because it can reflect the situation of real world. This study provides an analysis for the effectiveness of imperfect PM activity. The result of the analysis shows that the age reduction factor can more restore the system than the hazard reduction factor. Decision-makers can be able to determine how to model the reduction factor to a given system with the analysis result. If the given system is expensive, the use of age reduction factor is a reasonable because of the fact that the more sustainability of system operation can be guaranteed when the age reduction factor is used. Moreover, the proposed model provides the optimal failure-rate threshold. The optimal failure rate threshold can guarantee the minimum expected maintenance cost rate.

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