Robust Kalman Filter with Recursive Measurement Noise Covariance Estimation Against Measurement Faults

Chingiz Hajiyev

Department of Aeronautical Engineering, Istanbul Technical University
Istanbul, Maslak, 34469 Turkiye
cingiz@itu.edu.tr

ABSTRACT

A new innovation-based recursive measurement noise covariance estimation method is proposed. The presented algorithm is used for Kalman filter tuning, as a result, the robust Kalman filter (RKF) against measurement malfunctions is derived. The proposed innovation-based RKF with recursive estimation of measurement noise covariance is applied for the model of Unmanned Aerial Vehicle (UAV) dynamics. Algorithms are examined for two types of measurement fault scenarios; constant bias at measurements (additive sensor faults) and measurement noise increments (multiplicative sensor faults). The simulation results show that the proposed RKF can accurately estimate UAV dynamics in real time in the presence of various types of sensor faults. Estimation accuracies of the proposed RKF and conventional KF are investigated and compared. In all investigated sensor fault sceneries, the Root Mean Square (RMS) errors of the proposed RKF estimates are lower. The conventional KF gives inaccurate estimation results in the presence of sensor faults.

1. INTRODUCTION

The Kalman Filter (KF) can be used to estimate the states of an Unmanned Aerial Vehicle (UAV). That is the preferred method because it is crucial to exactly know the characteristics, such as velocity, altitude, attitude, etc. Successful aircraft control can be attained when these UAV states are attained without any issues. However, that procedure is contingent on how accurate the measurements are. The filter produces erroneous findings and diverges over time if the measurements are unreliable due to any type of sensor faults. Due to the significance of obtaining fault tolerance in the design of a UAV flight control system, filters should be constructed robustly to overcome such issues.

The Kalman filter method of state estimation is very sensitive to any faults in the measurement system. Changes at the measurement channels considerably reduce the performance of the estimating systems if the state of operation of the measurement system does not match the models employed in the synthesis of the filter. The possible errors can be recovered using adaptive Kalman filters.

A variety of alternative strategies can be used to make the Kalman filter flexible and hence insensitive to a priori measurements or system uncertainties. Multiple-model-based adaptive estimation (MMAE) (White et al., 1998; Maybeck, 1999), innovation-based adaptive estimation (IAE) (Mehra, 1970; Salychez, 1994; Hajiyev and Soken, 2021), and residual-based adaptive estimation (RAE) (Hajiyev and Soken, 2021; Mohamed and Schwarz, 1999) are important approaches for tackling the adaptive Kalman filtering problem. The measurement and/or process noise covariance matrices are immediately adjusted based on changes to the innovation or residual sequences.

The MMAE approach can only be utilized in specific situations because it calls for a number of parallel Kalman filters, and the faults should be known. IAE and RAE methods must use the innovation vectors or residual vectors of *m* epochs in the moving window to estimate the covariance matrices. The number, kind, and distribution of the measurements for all epochs inside a window must be consistent for IAE and RAE estimators. If not, neither the innovation nor the residual vectors can be used to estimate the covariance matrices of the measurement noises.

Another idea is to multiply the noise covariance matrix by a time-dependent variable to scale it. This algorithm is called adaptive fading Kalman filter (AFKF). One approach to creating such an algorithm is to multiply the process or measurement noise covariance matrices by a single adaptive factor (Ding et al., 2007; Jwo and Weng, 2008; Hajiyev, 2007; Hajiyev and Soken, 2012). The AFKF technique can be used if there is a fault in the measurement system, and

https://doi.org/10.36001/IJPHM.2025.v16i1.4204

the filter's insensitivity to present measurement faults can be ensured by multiplying the measurement noise scale factor by the measurement noise covariance matrix. As a result, by applying an adjustment to the filter gain, the filter's accurate estimating behavior will be protected from being affected by inaccurate measurements (Hajiyev, 2007).

An adaptation method based oan the multiple fading factors is provided in (Hajiyev and Soken, 2012; Geng and Wang, 2008; Hajiyev and Soken, 2013; Soken et al., 2014). The main justification for employing several fading factors is the variation in the impacts of the measurement noise covariance change on the estimation performance of each estimated state. It is important to carefully consider how changing the measurement noise covariance will affect each state, especially for complex multivariable systems, and to use a matrix made of multiple fading factors rather than a single factor (so that the adaptation is weighted differently for each state).

The measurement covariance matrix can be modified with the help of the fuzzy inference system (Sabzevari and Chatraei, 2021). The results showed that the proposed adaptive fuzzy extended Kalman filter is robust against disturbances and outliers. Although adaptive Kalman filter algorithms based on fuzzy logic work well in some situations, they are knowledge-based systems that function with linguistic variables and cannot be widely applied to critical systems like aircraft flight control systems since they are human experience-based.

The paper (Zhang et al., 2021) presents a two-step robust adaptive filtering technique, which consists of two filtering steps: the first stage merely detects anomalous observations without taking into account the kinematic model information. Based on the first phase's filtering findings, the second step detects additional kinematic model disturbances and performs adaptive processing.

The sequence orthogonal approach is used to create a preliminary robust correlation Kalman filter (RCKF) in (Chen et al., 2022). To increase its efficiency even more, a higher-order sigma variant of the RCKF is created using a new sigma point generating algorithm. This improved filter can collect the third and fourth central moment information from the system posteriori probability density function.

This study (Wang, et al., 2023) proposes an adaptive Kalman filtering algorithm based on maximum likelihood estimation. It employs a window adaptive selection function and a weight function to adjust the innovation covariance at the k^{th} moment, resulting in a superior measurement noise covariance.

Because the noise estimator cannot be expressed in a recursive form and each previous state vector must be smoothed by the most recent measurements at each point in time, the algorithms in the studies mentioned cannot be used to directly estimate the measurement noise covariance in practical operations. In practical applications, especially in UAV dynamics, the indirect estimation of the measurement noise covariance under measurement faults leads to complex expressions, increases the estimation time and computational load, and may introduce singularity. This will be of vital importance for low-cost small UAVs where computational capabilities are limited.

In this study, a robust Kalman filter with a recursive measurement noise covariance estimator is proposed and applied for the state estimation process of an UAV platform. The results of the proposed robust and conventional Kalman filter algorithms are compared for different types of measurement faults and recommendations about their utilization are given.

The paper is presented as follows. An optimal Kalman filter for UAV state vector estimation is described in Section 2. Section 3 investigates the influence of sensor biases and measurement noise increment faults on Kalman filter innovation. In Section 4, a new recursive measurement noise covariance estimation approach is proposed for tuning the Kalman filter. In this section, a robust Kalman filter (RKF) against sensor faults is derived. In Section 5, the proposed RKF with recursive measurement noise covariance estimation algorithm is applied to the UAV dynamics model, and the performance of the proposed filter is tested through simulation for the UAV platform state estimation process. The conclusion and results are briefly summarized in the final section.

2. PRELIMINARIES

Consider the linear dynamic system represented by the state equation

$$x(j+1) = Ax(j) + Bu(j) + Gw(j) \tag{1}$$

and measurement equation

$$z(j) = H(j)x(j) + V(j), \tag{2}$$

where $x(j)$ is the system state; A is the system transition matrix; B is the control distribution matrix; $u(j)$ is the control input; $w(j)$ is the random system noise; G is the system noise transition matrix; $z(j)$ is the measurement vector; $H(j)$ is the measurement matrix; $V(j)$ is a random measurement noise.

Assume that $w(j)$ and $V(j)$ are Gaussian white noise random vectors. The following formulas provide their average values and covariances

$$\begin{aligned} &E\left[w(j)\right]=0; E\left[V(j)\right]=0; E\left[w(j)w^T(k)\right] \\ &= Q(j)\delta(jk); E\left[V(j)V^T(k)\right] = R(j)\delta(jk). \end{aligned} \tag{3}$$

where $\delta(jk)$ *is* the Kronecker delta symbol. Note that $\{w(j)\}$ and $\{V(j)\}$ are assumed mutually uncorrelated.

The state vector (1) can be estimated via the optimal linear Kalman filter (LKF) (Kalman, 1960). Equations for the estimation value and gain matrix of the LKF respectively are:

$$\hat{x}(j/j) = \hat{x}(j/j-1) + K(j)\Delta(j) \tag{4}$$

$$K(j) = P(j/j-1)H^T(j)P_\Delta^{-1} \tag{5}$$

where $\hat{x}(j/j-1) = A\hat{x}(j-1/j-1) + Bu(j-1)$ is the extrapolation value, $\Delta(j)$ and $P_\Delta(j)$ are the innovation and innovation covariance respectively. The expressions for the $\Delta(j)$ and $P_\Delta(j)$ are:

$$\Delta(j) = z(j) - H(j)\hat{x}(j/j-1) \tag{6}$$

$$P_\Delta(j) = H(j)P(j/j-1)H^T(j) + R(j) \tag{7}$$

Here $P(j/j-1)$ is the covariance matrix of the extrapolation error.

The innovation sequence (6) will be white Gaussian noise with zero-mean and covariance (7) if the system is functioning normally (Mehra, 1970), i.e. $\Delta(j) \sim N\left(0, P_\Delta(j)\right)$. On the other hand, when there are abnormal changes occurring in the system or measurement channels, it can be assumed that the innovation of faulty system $\Delta_f(j) \sim N\left(\mu(j), P_{\Delta_f}(j)\right)$, where either $\mu(j) \neq 0$ or $P_{\Delta_f}(j) \neq P_\Delta(j)$ or both. Note that faults that only result in $\mu(j) \neq 0$ are generally called additive or bias type faults. They can be denoted as $\Delta_f(j) = \Delta(j) + f(j)$ and satisfy $\Delta_f(j) \sim N\left(\mu(j), P_\Delta(j)\right)$, here $E\left[f(j)\right] = \mu(j)$. Those faults that lead to changes in innovation covariance $P_\Delta(j)$ are called multiplicative or noise increment type faults, which can be denoted as $\Delta_f(j) = F(j)\Delta(j)$ with $\Delta_f(j) \sim N\left(0, F(j)P_\Delta(j)F^T(j)\right)$.

3. THE INFLUENCE OF SENSOR FAULTS ON KALMAN FILTER INNOVATION

The statistical properties of the Kalman filter innovation will alter as a result of measurement bias and sensor noise increase type sensor faults. This section examines the impact of these types of sensor faults on the Kalman filter's innovation sequence.

3.1. Influence of Sensor Biases on the Kalman Filter Innovation

Theorem 1: In the event that measurements are processed using LKF (4)–(7) and a measurement bias arises at an iteration step $j = \tau$, then at iteration $j = \tau$, the innovation bias will be equal to the measurement bias.

Proof: At the step $j = \tau$ where measurement error occurs, the extrapolation value is unbiased and can be expressed as

$$\hat{x}(j+1/j) = A\hat{x}(j/j) + Bu(j) + Gw(j) \tag{8}$$

Innovation of Kalman filter can be written in the form:

$$\begin{aligned}\Delta_b(j+1) = z(j+1) + b_z(j+1) - \\ H(j+1)\hat{x}(j+1/j) = \Delta(j+1) + b_z(j+1)\end{aligned} \tag{9}$$

Formula (9) confirms that the Theorem 1 is proven.

Theorem 2: In the event that measurements are processed using LKF (4) – (7) and a measurement bias arises at an iteration step $j = \tau$, then at all $j > \tau$ steps the innovation bias will be equal to the difference between the measurement bias and predicted observation bias.

Proof: At the first step following the bias occurring at iteration $j = \tau$, the extrapolation value can be expressed as

$$\begin{aligned}&\hat{x}_b(j+1/j) = A\hat{x}_b(j/j) + Bu(j) + Gw(j) \\ &= A\hat{x}(j/j) + A\Delta\hat{x}(j/j) + Bu(j) + Gw(j) \\ &= \hat{x}(j+1/j) + \Delta\hat{x}(j+1/j)\end{aligned} \tag{10}$$

where $\Delta\hat{x}(j+1/j) = A\Delta\hat{x}(j/j)$ is the extrapolation value bias.

Innovation of Kalman filter is

$$\begin{aligned}&\Delta_b(j+1) = z(j+1) + b_z(j+1) - H(j+1)\hat{x}_b(j+1/j) \\ &= z(j+1) - H(j+1)\hat{x}(j+1/j) + b_z(j+1) - \\ &H(j+1)\Delta\hat{x}(j+1/j) = \Delta(j+1) + \mu(j+1)\end{aligned} \tag{11}$$

where

$$\mu(j+1) = b_z(j+1) - H(j+1)\Delta\hat{x}(j+1/j) \tag{12}$$

is the innovation bias.

The innovation bias is equal to the difference between the measurement bias and predicted observation bias, as may be observed from expression (12), as shown. For all $j > \tau$ steps, this situation applies. As a result, *Theorem 2* is proven. Consequently, measurement bias type sensor faults will cause a bias in the innovation of the Kalman filter.

3.2. Influence of Measurement Noise Increment to the Innovation

Let the measurements are processed by the LKF (4)-(7) and a measurement noise increment occurs at the iteration step $j=\tau$. Measurement noise increment can be simulated by multiplying the measurement noise vector with the diagonal matrix $F(j)$, which diagonal elements meet the following condition: $\sigma_{ii}(j)\ge 1$, $(i=\overline{1,n})$ for $\forall j\ge\tau$. Here n is the dimension of the measurement vector. As it is clear, for the noise increment type sensor fault in the i th measurement channel, the appropriate diagonal element of $F(j)$ will be larger than 1, i.e. $\sigma_{ii}(j)>1$ for $\forall j\ge\tau$ and rest of the measurement channels become 1. Consequently, the diagonal elements of $F(j)$ can be presented in the following form:

$$\sigma_{ii}=\begin{cases}1: \text{ no measurement fault}\\ >1: \text{measurement fault}\end{cases} \tag{13}$$

The measurement model in this case can be written in the form:

$$z(j)=H(j)x(j)+F(j)V(j), \tag{14}$$

where

$$diag\left(F(j)\right)=\begin{cases}\begin{pmatrix}1 & 1 & . & . & . & 1\end{pmatrix}, \quad \text{for } j<\tau\\ \begin{pmatrix}\sigma_{11} & \sigma_{22} & . & . & . & \sigma_{nn}\end{pmatrix} \text{ if } \exists i\in\left(\overline{1,n}\right),\\ \text{where } \sigma_{ii}>1 \text{ for } j\ge\tau\end{cases} \tag{15}$$

Theorem 3: In the event that measurements are processed using LKF (4)–(7) and a measurement noise increment occurs at an iteration $j=\tau$, then at all $j\ge\tau$ steps the measurement noise increment leads to increment in the innovation covariance (7).

Proof. The innovation covariance at the iteration steps $j\ge\tau$ can be expressed as

$$P_{\Delta_{ni}}(j)=H(j)P(j/j-1)H^T(j)+F(j)R(j)F^T(j) \tag{16}$$

The innovation covariance increment is

$$\Delta P_{\Delta_{ni}}(j)=F(j)R(j)F^T(j)-R(j) \tag{17}$$

Since the matrices $F(j)$ and $R(j)$ are assumed to be diagonal, the expression (17) can be rewritten in the following form:

$$\Delta P_{\Delta_{ni}}(j)=\left[F(j)\right]^2R(j)-R(j)=\left\{\left[F(j)\right]^2-I\right\}R(j) \tag{18}$$

where I is the $n\times n$ identity matrix. Because $R(j)$ and $F(j)$ are positive definite diagonal matrices and $F(j)$ has diagonal elements $\sigma_{ii}(j)\ge 1$, $(i=\overline{1,s})$ for $\forall\ j\ge\tau$, then the matrix $\left\{\left[F(j)\right]^2-I\right\}R(j)$ is also positive definite. Since the innovation covariance increment is a positive definite matrix, the *Theorem 3* is valid.

4. RECURSIVE MEASUREMENT NOISE COVARIANCE ESTIMATOR

The statistical properties of the Kalman filter innovation will change as a result of measurement bias and measurement noise increment. Therefore, the innovation (6) can be chosen as the monitoring statistic for the measurement fault compensation purpose. For the compensation of measurement bias or measurement noise increment, the real and theoretical values of the innovation covariance matrices must be compared.

In the absence of measurement fault in the estimation system, the real innovation covariance $C(j)$ is equal to the theoretical one

$$C(j)=H(j)P(j/j-1)H^T(j)+R(j) \tag{19}$$

The real covariance matrix of $\Delta(j)$ is an average of $\Delta(k)\Delta^T(k)$ within a moving window M

$$C(j)=\frac{1}{M-1}\sum_{k=j-M+1}^{j}\Delta(k)\Delta^T(k) \tag{20}$$

Substituting Eq. (20) into (19) we have

$$\frac{1}{M-1}\sum_{k=j-M+1}^{j}\Delta(k)\Delta^T(k)=H(j)P(j/j-1)H^T(j)+R(j) \tag{21}$$

The real innovation covariance matrix $C(j)$ can be estimated by $\Delta(j)\Delta^T(j)$ at the current epoch in order to avoid the smoothness of the average of $\Delta(j)\Delta^T(j)$ within M epochs, which does not adequately reflect the uncertainty of the model errors at the current step

$$C(j)=\Delta(j)\Delta^T(j) \tag{22}$$

Taking into account (19) and (22), the expressions for the measurement noise covariances for $j+1$ and j iterations can be written in the following form

$$R(j+1)=\Delta(j+1)\Delta^T(j+1)-H(j+1)P(j+1/j)H^T(j+1) \tag{23}$$

$$R(j)=\Delta(j)\Delta^T(j)-H(j)P(j/j-1)H^T(j) \tag{24}$$

Therefore $R(j+1)$ minus $R(j)$ equals:

$$R(j+1) - R(j) = \Delta(j+1)\Delta^T(j+1) - \Delta(j)\Delta^T(j) - H(j+1)P(j+1/j)H^T(j+1) + H(j)P(j/j-1)H^T(j) \quad (25)$$

The equation (25) can be written as

$$R(j+1) = R(j) + \Delta(j+1)\Delta^T(j+1) - \Delta(j)\Delta^T(j) - H(j+1)P(j+1/j)H^T(j+1) + H(j)P(j/j-1)H^T(j) \quad (26)$$

If measurements are linear, than $H(j+1) = H(j)$ and the expression (23) can be written in simple form as

$$R(j+1) = R(j) + \Delta(j+1)\Delta^T(j+1) - \Delta(j)\Delta^T(j) + H\left[P(j/j-1) - P(j+1/j)\right]H^T \quad (27)$$

The resulting expression (27) makes it possible to recursively estimate the measurement noise covariance for the Kalman filter tuning. Below the RKF with recursive estimation of measurement noise covariance is applied for the UAV dynamics model.

If a measurement bias occurs at the iteration step $j = \tau$, and the biased innovation sequence is denoted by $\Delta_b(j)$, then the biased innovation   is defined as,

$$\Delta_b(j) = \Delta(j) \qquad j = 1,2,\ldots\ \tau-1 \quad (28)$$

$$\Delta_b(j+1) = \Delta(j+1) + b_z(j+1) \qquad j = \tau \quad (29)$$

$$\Delta_b(j+1) = \Delta(j+1) + \mu(j+1) \qquad j = \tau+1, \tau+2, \ldots \quad (30)$$

When $j < \tau$, the mathematical expectation of the real innovation covariance matrix (22) can be determined by the following equation

$$E\left[C(j)\right] = P_\Delta(k) = H(j)P(j/j-1)H^T(j) + R(j) \quad (31)$$

In the case of $j > \tau$, in the sample innovation covariance a biased values $\Delta_b(j+1) = \Delta(j+1) + \mu(j+1)$ is used instead of an unbiased value $\Delta(j+1)$, where $\mu(j+1)$ is the innovation bias

$$C_b(j) = \Delta_b(j)\Delta_b^T(j) \quad (32)$$

Remark. Note that the expected value of the innovation $\Delta_b(j)$ in this case is not zero, therefore the formula (32) is not a real covariance. This is the square of innovation. Bias type measurement fault may be converted to the square of innovation and such type of faults can be compensated using the covariance matching techniques.

Statement. For iteration steps $j > \tau$, measurement bias leads to an increase in the mathematical expectation of the square of innovation.

Proof. It is proven in *Theorem 2* that the measurement bias will cause bias in the innovation of the Kalman filter.

The mathematical expectation of the square of innovation (32) for $j > \tau$ can be written as

$$E\left[C_b(j)\right] = E\left[\Delta_b(j)\Delta_b^T(j)\right] = E\left\{\left[\Delta(j) + \mu(j)\right]\left[\Delta(j) + \mu(j)\right]^T\right\} = E\left[\Delta(j)\Delta^T(j) + \Delta(j)\mu^T(j) + \mu(j)\Delta^T(j) + \mu(j)\mu^T(j)\right] \quad (33)$$

Taking into account $E\left[\Delta(j)\right] = 0$, and the absence of correlation between the parameters $\Delta(j)$ and $\mu(j)$, we have

$$E\left[C_b(j)\right] = E\left[C(j)\right] + E\left[\mu(j)\mu^T(j)\right] \quad (34)$$

Expressions (12) and (34) prove the *Statement*. Consequently, the measurement bias will increase the mathematical expectation of the square of innovation.

It can be seen from the *Theorem 2* and the *Statement* above that the measurement bias is transferred to the innovation bias and changes the mathematical expectation of the square of innovation (22). As a result, the measurement bias is transferred to the mathematical expectation of the square of innovation. Thus, the square of innovation can be used to compensate of measurement bias. Therefore, the measurement bias will increase the mathematical expectation of the square of innovation (22). As a result, according to formulas (24) and (27), the expected value of the measurement noise covariance matrix R will generally(17) increase, resulting in a smaller Kalman gain, which will reduce the influence of measurements on the state update process and increase the influence of the mathematical model of the system. As a result, the filter's resilience against the measurement bias fault is ensured, while the degradation of the estimate method induced by the measurement bias fault is avoided.

5. RESULTS OF SIMULATION

Proposed innovation-based adaptive KF algorithm is applied to the UAV platform dynamics model. As the experimental platform, the ZAGI UAV was selected, and Kalman filter applications were carried out while taking into consideration its dynamics and characteristics (Matthews, 2006). The Appendix-B presents a mathematical model of the combined longitudinal and lateral dynamics of the UAV. The LKF (4)–(7) is utilized to estimate the UAV state vector.

Simulations are carried out in 1000 steps over a time frame of 100 seconds with a sampling time of 0.1 seconds. Two different types of measurement fault scenarios—constant bias in measurements and measurement noise increment—are taken into consideration during simulations to test the proposed innovation-based robust Kalman filter with recursive estimate of measurement noise covariance.

5.1. Constant Bias in Measurements

A constant bias term is added to the measurements of pitch angle gyro after the 30th second of simulation

$$z_{\theta}(j) = z_{\theta}(j) + v_{\theta}(j) + 0.5\,,\ (j \geq 300) \qquad (35)$$

The innovation-based RKF with recursive estimation of measurement noise covariance results for the pitch angle in the presence of pitch angle gyro bias are presented in Fig. 1. The findings of the RKF's state estimation are compared to the actual values in the first section of the figure. The estimation error based on the actual values of the UAV states is depicted in the second portion of the picture. The estimation error variance is shown in the final section.

Fig. 1 shows that the proposed innovtion-based RKF with recursive estimation of measurement noise covariance achieves estimation of the states accurately in the presence of bias at the pitch angle gyro. In this case RKF gives sufficiently good estimation results by totally eliminating the estimation error caused by the bias in the pitch angle gyro.

Fig. 2 displays the results of the conventional KF estimation for this case. As can be seen, the conventional KF estimates shift after the 30th second of simulation (after the pitch angle gyroscope fails), and the estimation results are erroneous.

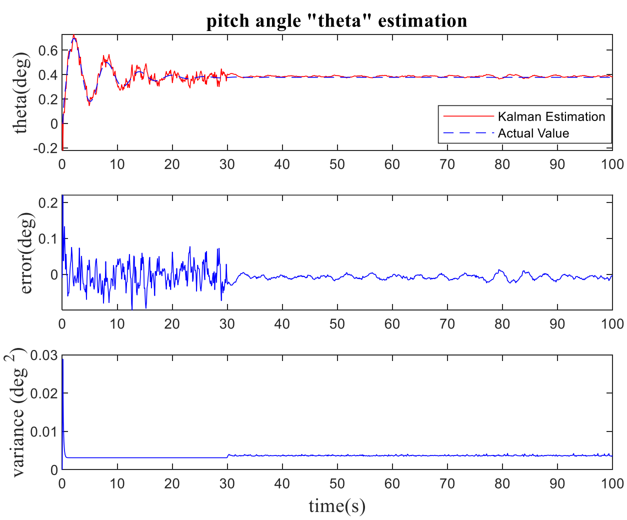

Figure 1. RKF pitch angle estimation results in the presence of bias at the pitch angle gyro

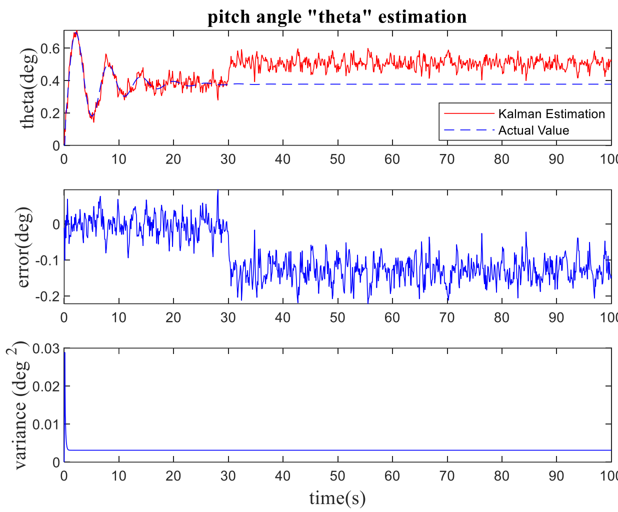

Figure 2. The conventional KF's pitch angle estimation results in the presence of bias at the pitch angle gyro

Forward velocity estimation results via RKF with recursive estimation of measurement noise covariance and conventional KF in the presence of bias at the pitch angle gyro are given in Fig.3 and Fig.4 respectively. The presented graphs show that conventional KF estimates after the 30th second of simulation are biased (see Fig. 4), but the proposed RKF with recursive estimation of measurement noise covariance gives fairly accurate estimation results throughout the entire estimation interval (Fig. 3).

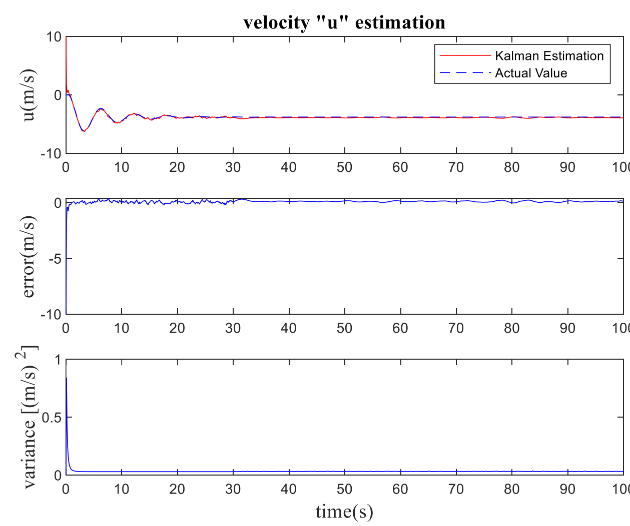

Figure 3. RKF forward velocity estimation results in the presence of bias at the pitch angle gyro

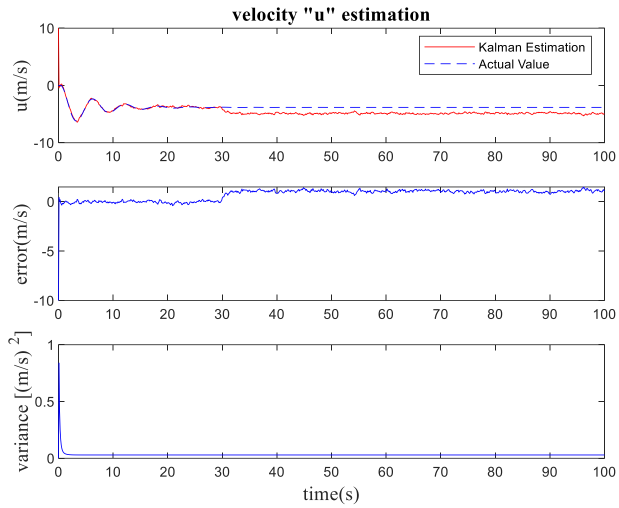

Figure 4.The conventional KF's forward velocity estimation results in the presence of bias at the pitch angle gyro

Similar results were obtained for other state parameters.

5.2. Measurement Noise Increment

In the second measurement malfunction scenario, the measurement fault is defined as the pitch angle gyro measurement noise's standard deviation multiplied by a constant term after the 30th second

$$z_{\theta}(j) = z_{\theta}(j) + v_{\theta}(j) \times 3 \, , \, (j \geq 300) \, . \qquad (36)$$

The proposed innovation-based RKF with recursive estimation of measurement noise covariance results for this case are presented in Fig.5.

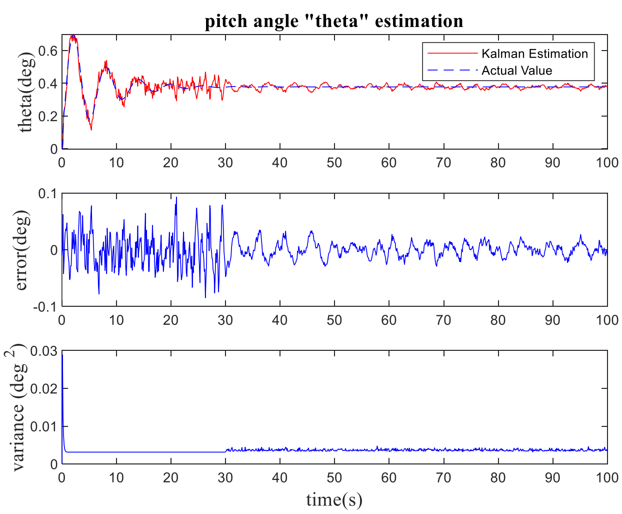

Figure 5. RKF pitch angle estimation results in the presence of noise increment at the pitch angle gyro measurements

As seen from the graphs presented in Fig.5, the proposed innovtion-based RKF with recursive estimation of measurement noise covariance gives sufficiently good estimation results in the presence of measurement noise increment at the pitch angle gyro.

Fig. 6 displays the results of the conventional KF estimation for the pitch angle. As seen, the accuracy of conventional KF estimates deteriorates after the 30th second of simulation (after the pitch angle gyroscope fails).

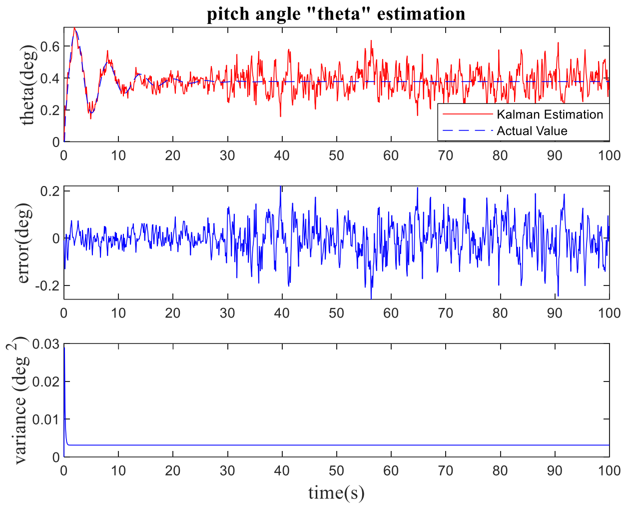

Figure 6. The conventional KF's pitch angle estimation results in the presence of noise increment at the pitch angle gyro measurements

Forward velocity estimation results via RKF with recursive estimation of measurement noise covariance and conventional KF in the case of noise increment at the pitch angle gyro measurements are shown in Fig.7 and Fig.8 respectively.

The presented graphs show that the accuracy of conventional KF estimates increases after the 30th second of simulation (See Fig. 8), however, the proposed RKF with recursive estimation of measurement noise covariance gives fairly good estimation results over the entire estimation interval (Fig. 7).

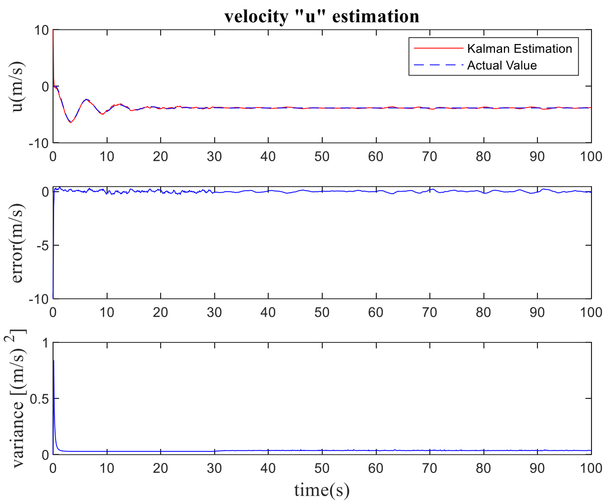

Figure 7. RKF forward velocity estimation results in the presence of noise increment at the pitch angle gyro measurements

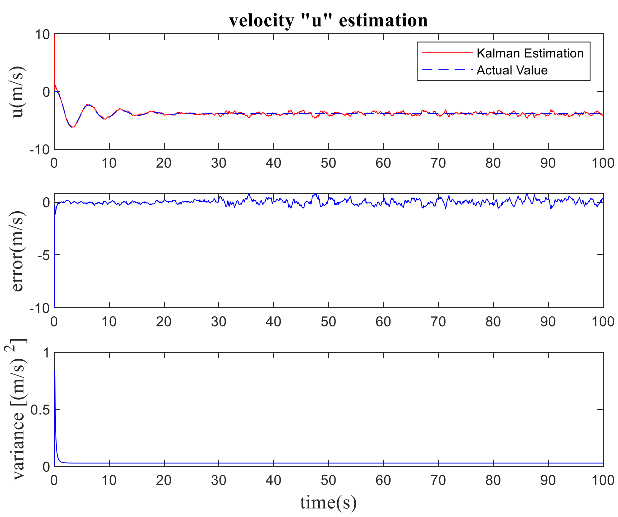

Figure 8. The conventional KF's forward velocity estimation results in the presence of noise increment at the pitch angle gyro measurements

In all investigated sensor fault scenarios, the proposed recursive RKF estimation results outperform those of the conventional KF.

5.3. RMS Errors of the RKF and Conventional KF

Root Mean Square (RMS) errors of the innovation-based RKF with recursive estimation of measurement noise covariance and conventional KF estimates in the presence of pitch angle gyro bias are presented in Table 1. As can be seen from the results presented in Table 1, the proposed RKF is superior for both longitudinal and lateral parameters in the presence of pitch angle gyro bias. RMS errors of conventional KF are sufficiently greater than the RMS errors of the proposed robust filter.

RMS errors of the proposed RKF and conventional KF estimates in the presence of measurement noise increment at the pitch angle gyro are presented in Table 2.

Method	RKF	Conv.KF
Δu	0.0795	0.5309
Δw	0.0176	0.1738
Δq	0.0137	0.0651
$\Delta \theta$	0.0105	0.1357
Δh	0.1074	0.3068
$\Delta \beta$	0.0098	0.0286
Δp	0.0059	0.0386
Δr	0.0091	0.0285
$\Delta \phi$	0.0124	0.0475

Table 1. RMS errors of the proposed RKF and conventional KF estimates in the presence of pitch angle gyro bias

Method	RKF	Conv.KF
Δu	0.0811	0.1782
Δw	0.0172	0.0847
Δq	0.0133	0.0648
$\Delta \theta$	0.0129	0.0749
Δh	0.0869	0.1523
$\Delta \beta$	0.0108	0.0289
Δp	0.0076	0.0413
Δr	0.0104	0.0284
$\Delta \phi$	0.0150	0.0497

Table 2. RMS errors of the proposed RKF and conventional KF estimates in the presence of measurement noise increment at the pitch angle gyro

The presented in Table 2 results show that, the RKF gives better results for both longitudinal and lateral parameters in the presence of measurement noise increment at the pitch angle gyro. The RMSE results of conventional KF are worst compared to the robust filter.

In all investigated sensor fault scenarios, the proposed RKF gives better estimation results then the conventional KF.

6. CONCLUSION

This study proposes a novel recursive method for estimating measurement noise covariance for Kalman filter tuning. Based on the presented innovation approach to recursively estimate the measurement noise covariance, a robust Kalman filter against sensor faults is presented. The sensor fault compensation in this filter is accomplished with a simple change to the conventional KF.

The proposed RKF with recursive estimation of measurement noise covariance is applied for the UAV dynamics model. Two alternative scenarios of measurement error are evaluated on algorithms; constant bias at measurements (additive sensor faults) and measurement noise increments (multiplicative sensor faults). The simulation results show that the proposed innovation-based RKF with recursive estimation of measurement noise covariance can accurately estimate the UAV dynamics in real time in the presence of various types of sensor faults.

Estimation accuracies of the proposed RKF and conventional KF are compared. In all investigated sensor fault sceneries, the estimation accuracy of the proposed RKF is superior. The conventional KF gives inaccurate estimation results in the presence of sensor faults.

The innovation-based RKF with recursive estimation of measurement noise covariance can be recommended as the reliable UAV state estimator in the flight control system in the presence of sensor faults.

Further research will be aimed at developing RKF with recursive estimation of process noise covariance and its application to the UAV dynamics model.

REFERENCES

Chen, X., Cao, L., Guo, P., and Xiao, B. (2022). A higher-order robust correlation Kalman filter for satellite attitude estimation. *ISA Transactions*, vol. 124, 2022, pp. 326–337.

Ding, W., Wang, J., Rizos, C., and Kinlyside, D. (2007). Improving adaptive Kalman estimation in GPS/INS integration. *The Journal of Navigation*, vol. 60, no.3, pp. 517–529.

Geng, Y., and Wang, J. (2008). Adaptive estimation of multiple fading factors in Kalman filter for navigation applications. *GPS Solutions*, vol. 12, no.4, pp. 273–279.

Hajiyev, C. (2007). Adaptive filtration algorithm with the filter-gain correction applied to integrated INS/Radar altimeter. *Proc. Inst. Mech. Eng., Part G*, vol. 221, no.5, pp. 847–885.

Hajiyev, C., and Soken, H.E. (2012). Robust estimation of UAV dynamics in the presence of measurement faults. *Journal of Aerospace Engıneerıng*, vol.25, pp. 80-89.

Hajiyev, C., and Soken, H E. (2013). Robust Adaptive Kalman Filter for estimation of UAV dynamics in the presence of sensor/actuator faults. *Aerospace Science and Technology*, vol. 28, pp. 376–383.

Hajiyev, C., & Soken, H.E. (2021). *Fault Tolerant Attitude Estimation for Small Satellites.* Boca Raton, FL, USA and Abingdon, Oxon, England: Taylor & Francis Group, LLC, CRC Press,

Jwo, D.J., and Weng, T.P. (2008). An adaptive sensor fusion method with applications in integrated navigation. *The Journal of Navigation*, vol. 61, no.4, pp. 705–721.

Kalman, R.E. (1960). A new approach to linear filtering and prediction problems. *ASME Journal of Basic Engineering*, vol. 82, pp. 35-45.

Matthews, J.S. (2006). Adaptive Control of Micro Air Vehicles, M.Sc. Thesis, Department of Electrical and Computer Engineering, Brigham Young University, Utah, USA.

Maybeck, P.S. (1999). Multiple model adaptive algorithms for detecting and compensating sensor and actuator/surface failures in aircraft flight control systems. *International Journal of Robust and Nonlinear Control*, vol.9, no.14, pp. 1051-1070.

Mehra, R.K. (1970). On the identification of variance and adaptive Kalman filtering. *IEEE Transactions on Automatic Control*, vol.15, no.2, pp. 175-184.

Mohamed, A.H., and Schwarz, K.P. (1999). Adaptive Kalman filter for INS/GPS. *Journal of Geodesy,* vol.73, no.4, pp.193-203.

Sabzevari, D., and Chatraei, A. (2021). INS/GPS sensor fusion based on adaptive fuzzy EKF with sensitivity to disturbances. *IET Radar, Sonar& Navigation*, vol. 15, pp. 1535-1549.

Salychez, O.S. (1994). Special studies in dynamic estimation procedures with case studies in inertial surveying. *ENGO 699.26 lecture notes*, Department of Geomatics Engineering, University of Calgary, Canada.

Soken, H.E., Hajiyev, C., and Sakai, S. (2014). Robust Kalman filtering for small satellite attitude estimation in the presence of measurement faults. *European Journal of Control,* vol. 20, pp. 64-72.

Yechout, T. R., Morris, S.L., Bossert, D.E., and Hallgren, W.F. (2003). *Introduction to Aircraft Flight Mechanics: Performance, Static Stability, Dynamic Stability, and Classical Feedback Control,* AIAA Education Series, Virginia, USA.

Wang, L., Cheng, J., Qi, B., Cheng, S., Chen, S. (2023). An adaptive Kalman filtering algorithm based on maximum likelihood estimation. *Measurement Science and Technology,* vol. 34**,** paper 115114. https://doi.org/10.1088/1361-6501/ace9ef

White, N.A., Maybeck, P.S. and DeVilbiss, S.L. (1998). MMAE detection of interference/jamming and spoofing in a DGPS-aided inertial system. *IEEE Transactions on Aerospace and Electronic Systems*, vol. 34, no.4, pp. 1208-1217.

Zhang, Q., Zhao, Lu., and Zhao, Lo. (2021). A Two-step robust adaptive filtering algorithm for GNSS kinematic precise point positioning. *Chinese Journal of Aeronautics*, vol. 34, Iss. 10, pp. 210-219.

DOI:10.1016/j.cja.2020.10.033

APPENDIX-A

RKF With Recursive Measurement Noise Covariance Estimation

The proposed RKF with recursive measurement noise covariance estimation includes the following operations:

1) One step further prediction of the value (determination of the extrapolation value)

$$\hat{x}(j+1/j) = A\hat{x}(j/j) + Bu(j) \qquad \text{(A-1)}$$

2) Determination of the difference of the measurement and the extrapolation value (innovation sequence)

$$\Delta(j+1) = z(j+1) - H\hat{x}(j+1/j) \qquad \text{(A-2)}$$

3) Calculation of the covariance matrix of extrapolation error

$$P(j+1/j) = AP(j/j)A^T + BD_u(j)B^T + GQ(j)G^T \qquad \text{(A-3)}$$

where $D_u(j)$ is the covariance matrix of control inputs.

4) Calculation of the measurement noise covariances for $j+1$ iteration

$$\begin{aligned} R(j+1) &= R(j) + \Delta(j+1)\Delta^T(j+1) - \Delta(j)\Delta^T(j) \\ &+ H\left[P(j/j-1) - P(j+1/j)\right]H^T \end{aligned} \qquad \text{(A-4)}$$

5) Determining the innovation covariance

$$P_\Delta(j+1) = HP(j+1/j)H^T + R(j+1) \qquad \text{(A-5)}$$

6) Calculation of the Kalman gain

$$K(j+1) = P(j+1/j)H^T + P_\Delta^{-1}(j+1) \qquad \text{(A-6)}$$

7) Determining the estimated value

$$\hat{x}(j+1/j+1) = \hat{x}(j+1/j) + K(j+1)\Delta(j+1) \qquad \text{(A-7)}$$

8) Calculation of the covariance matrix of estimation error

$$P(j+1/j+1 = P(j+1/j) - K(j+1)HP(j+1/j) \qquad \text{(A-8)}$$

9) Storage of values $R(j+1)$, $\hat{x}(j/j)$, $P(j+1/j+1)$ and repetition of the loop.

APPENDIX-B

Mathematical Model of the UAV Flight Dynamics

The dynamic characteristic of an UAV must be known in order to build a Kalman filter for the state estimation. In general, equation derivation process for an UAV may be examined in two steps; derivation of the rigid body equations of motion and the linearization (Yechout et al., 2003).

In general, the equations are considered in two phases; longitudinal and lateral. These nonlinear equations can be linearized by using the small perturbation theory. Hereafter, the term $\Delta(.)$ is used for representing the perturbed state. Consequently, the linearized longitudinal equations of motion of UAV in the state space form is,

$$x^{lon}(j+1) = A^{lon}x^{lon}(j) + B^{lon}u^{lon}(j) \qquad \text{(B-1)}$$

where

$$x^{lon}(j) = \begin{bmatrix}\Delta u & \Delta w & \Delta q & \Delta\theta & \Delta h\end{bmatrix}_j^T;$$

$$A^{lon} = \begin{bmatrix} X_u & X_w & 0 & -g & 0 \\ Z_u & Z_w & u_0 & 0 & 0 \\ M_u + M_{\dot{w}}Z_u & M_w + M_{\dot{w}}Z_w & M_q + M_w u_0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix};$$

$$B^{lon} = \begin{bmatrix} X_{\delta e} & X_{\delta T} \\ Z_{\delta e} & Z_{\delta T} \\ M_{\delta e} + M_{\dot{w}}Z_{\delta e} & M_{\delta T} + M_{\dot{w}}Z_{\delta T} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; u^{lon}(j) = \begin{bmatrix} \Delta\delta_e \\ \Delta\delta_T \end{bmatrix}_j. \qquad \text{(B-2)}$$

and the linearized lateral equations of motion of UAV in the state space form is,

$$x^{lat}(j+1) = A^{lat}x^{lat}(j) + B^{lat}u^{lat}(j) \qquad \text{(B-3)}$$

where

$$x^{lat}(j) = \begin{bmatrix} \Delta\beta & \Delta p & \Delta r & \Delta\phi \end{bmatrix}_j^T;$$

$$A^{lat} = \begin{bmatrix} \dfrac{Y_\beta}{u_0} & \dfrac{Y_p}{u_0} & -\dfrac{u_0 - Y_r}{u_0} & \dfrac{g\cos(\theta_0)}{u_0} \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix};$$

$$B^{lat} = \begin{bmatrix} 0 & \dfrac{Y_{\delta r}}{u_0} \\ L_{\delta a} & L_{\delta r} \\ N_{\delta a} & N_{\delta r} \\ 0 & 0 \end{bmatrix}; u^{lat}(j) = \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix}_j . \tag{B-4}$$

Here, Δu , Δw are the velocity components $\Delta p, \Delta q, \Delta r$ are the angular rates, $\Delta\delta_e$, $\Delta\delta_a$ and $\Delta\delta_r$ are the elevator, aileron and the rudder deflections, $\Delta\delta_T$ is the change in the thrust, $\Delta\theta$ is the pitch angle about y axis, $\Delta\phi$ is the roll angle about x axis, $\Delta\beta$ is the sideslip angle, Δh is the height, θ_0 and u_0 are the values of related terms in the steady state flight, g is the gravity constant and X_u, X_w, $X_{\delta e}$, $X_{\delta T}$, Z_u, Z_w, $Z_{\delta e}$, $Z_{\delta T}$, M_u, M_w, M_q, $M_{\dot{w}}$, Y_r, Y_p, Y_β, $Y_{\delta r}$, L_β, L_p, L_r, $L_{\delta a}$, $L_{\delta r}$, $M_{\delta e}$, $M_{\delta T}$, $N_{\delta a}$, $N_{\delta r}$, N_β, N_p, N_r are the stability derivatives, their values can be found in (Matthews, 2006).

Integrating the longitudinal and lateral equations of UAV results in the equations as (Hajiyev and Soken, 2012)

$$\begin{aligned} x(j+1) &= Ax(j) + Bu(j) + Gw(j) \\ z(j+1) &= Hx(j+1) + V(j+1) \end{aligned} \tag{B-5}$$

where

$$x(j) = \begin{bmatrix} \Delta u & \Delta w & \Delta q & \Delta\theta & \Delta h & \Delta\beta & \Delta p & \Delta r & \Delta\phi \end{bmatrix}_j^T$$

$$u(j) = \begin{bmatrix} \Delta\delta_e & \Delta\delta_T & \Delta\delta_a & \Delta\delta_r \end{bmatrix}_j^T$$

$$A = \begin{bmatrix} A^{lon} & 0 \\ 0 & A^{lat} \end{bmatrix}, B = \begin{bmatrix} B^{lon} & 0 \\ 0 & B^{lat} \end{bmatrix}$$