

# Remaining Useful Life Estimation Based on Detection of Explosive Changes: Analysis of Bearing Vibration

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## ABSTRACT

The monitoring of condition variables for maintenance purposes is a growing trend amongst researchers and practitioners where decisions are based on degradation levels. The two approaches in Condition-Based Maintenance (CBM) are diagnosing the level of degradation (diagnostics) or predicting when a certain level of degradation will be reached (prognostics). Using diagnostics determines when it is necessary to perform maintenance, but it rarely allows for estimation of future degradation. In the second case, prognostics does allow for degradation and failure prediction, however, its major drawback lies in when to perform the analysis, and exactly what information should be used for predictions. This encumbrance is due to previous studies that have shown that degradation variable could undergo a change that misleads these calculations. This paper addresses the issue of identifying explosive changes in condition variables, using Control Charts, to determine when to perform a new model fitting in order to obtain more accurate Remaining Useful Life (RUL) estimations. The diagnostic-prognostic methodology allows for discarding pre-change observations to avoid contamination in condition prediction. In addition the performance of the integration methodology is compared against adaptive autoregressive (AR) models. Results show that using only the observations acquired after the out-of-control signal produces more accurate RUL estimations.

## 1. INTRODUCTION AND BACKGROUND

Over time different definitions of diagnostics and prognostics have appeared in maintenance literature (see Sikorska,

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Hodkiewucz and Ma (2011)), however in terms of equipment failure, the great majority of these definitions agree in determining that diagnostics reacts to a failure, while prognostics aims to predict it, and both are based on the condition evolution of the equipment (Sikorska et al., 2011). However, to avoid possible confusions to the reader, in this study, the term failure refers to a level of deterioration that is not desirable for the user due to concerns regarding safety, risk, economical costs, production quality, etc (see Moubray (1992)).

Studies conducted in Condition-Based Maintenance (CBM), have helped in explaining that the condition variable potentially could present a point in time that changes the degradation behavior of the sample under study. For example, Chen and Tsui (2013), and Son, Zhang, Sankavaram and Zhou (2015) acknowledge the existence of a change in bearing degradation that ultimately causes a dramatic increase in vibration levels and variability. Lim and Mba (2014) present what they describe as the typical paths followed by bearing degradation features, with all of them presenting changes in the condition variable. Moreover, degradation can be altered by external sources. Tobon-Mejia, Medjaher, and Zerhouni (2012) showed that even maintenance actions change data behavior. A number of public datasets show examples where this situation is in fact common in bearing degradation. Figure 1 presents bearing degradation, measured in g-forces (g), from the 2012 IEEE Data Challenge (Nectoux, Gouriveau, Medjaher, Ramasso, Chebel-Morello, Zerhouni, and Varnier, 2012), where it is possible to see that at the end of life, the condition variable suffers an exponential increment in its value, from which it does not recover, reaching the failure threshold, and leading to declare the corresponding failure. This scenario, defined in this study as *explosive change*, differentiates itself from other increments in condition variable (such as the one observed around observation 1400 in Figure 1), in

that it leads to declaring an equipment failure, meaning, the equipment degradation is permanent and can only be fixed by performing a maintenance task. It is worth noting that by explosive change the authors refer only to the behavior of the monitored variable, not the underlying physical or chemical phenomenon producing the variable increment, meaning that, if failure in different equipment or degradation process produce similar condition variable patterns, it does not necessarily mean that the degradation process is similar on the equipment.

If practitioners follow the prognostics approach, the presence of these changes might produce misleading estimations of time to failure, a concern previously expressed in the literature (Barraza-Barraza, 2015; Barraza-Barraza, Tercero-Gómez, Beruvides, and Limón-Robles, 2017), suggesting that the identification of the change point in degradation, might improve time-to-failure estimation by fitting a new model.

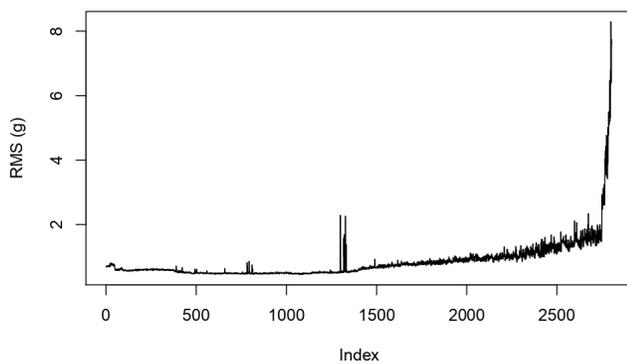


Figure 1. Presence of change in bearing degradation.

Using diagnostics tools that determine when the condition change occurred could lead to the development of more accurate RUL estimation, if information preceding it is disregarded. Visually, the change point is easily detected on the graph; however, a considerable number of after-change observations are required in order to identify its occurrence. The use of control charts (CCs) simplifies this task by allowing the detection of changes within a few "after-change" observations.

The use of Control Charts is a common practice and has previously been used in maintenance for the purpose of detecting degradation changes. Barraza-Barraza, Tercero-Gómez, Limón-Robles, and Beruvides (2016) developed a literature review on the use of CCs for condition monitoring, focused on the type of CC, the level of degradation states declared by the CC, and the costs considered in the model. The review in this study focuses on the type of maintenance action triggered by the CC. Table 1 presents the studies reviewed on statistical monitoring of condition degradation. The columns in Table 1 specify if the maintenance policy considers inspection, preventive or corrective maintenance action. Maintenance ac-

tions are classified according to the approach they take towards failure (see Pintelon and Parodi-Herz (2008)) to avoid confusion, as some authors use replacement (J. Wu & Makis, 2008) or repair (S. Wu & Wang, 2011) as precautionary maintenance. The corrective maintenance action is marked for the cases where the study uses this action as a parameter for CC design.

In addition to these studies, Xie, Goh, and Ranjan (2002) and Quintana, Pisani, and Casal (2015) used a *t*-CC to monitor time between failures to ensure they are occurring at an acceptable rate. As the reader can observe, the literature to date has focused on the area of diagnostics, with preventive/corrective maintenance actions triggered by the alarm in the CCs.

The previous research focus has not included the option of estimating (predicting) when the equipment could possibly fail. Sikorska et al. (2011) and Lei, Li, Guo, Li, Yan and Lin (2018) developed systematic reviews on methodologies for RUL estimation, highlighting their advantages and disadvantages. From these reviews, it can be seen that in terms of data and knowledge requirement, physics-based methodologies require expert knowledge on failure methodology and a degradation model based on physics laws, restricting the area of application of these methodologies; on the other hand, RUL estimation relying on artificial intelligence or neural networks techniques, require the availability of large, run-to-failure datasets; in the same path, some statistical and stochastic methodologies also require historical datasets. However, a variety of studies have explained that run-to-failure, high-quality, training datasets are barely available for CBM purposes (Sikorska et al., 2011; Barraza-Barraza et al., 2017; Lei et al., 2018).

Therefore, there is a gap in the CBM body of knowledge that demands attention, summarized as the necessity of RUL estimation with a methodology that allows the identification of changes in condition variable that might be indicator of an impending failure or the need of a new model fitting, without requiring either run-to-failure or historical data. This research explores a novel sequential approach that integrates first the diagnostics properties of CCs followed by the predicting features of prognostics, through modelling degradation with Autoregressive (AR) models, avoiding requiring historical data. The methodological approach discards pre-change observations to fit a new model with after-change observations only, in order to minimize contamination in the prediction process that ultimately could lead to a more accurate RUL estimation. A dataset from NASA was used as case study to show the feasibility of the proposed approach in practice. Also, extensive Monte Carlo simulations based on real measurements were executed to assess performance. Results are compared with traditional methods found in (Barraza-Barraza et al., 2017; Escobet et al., 2012).

The structure of this paper is as follows: Section 2 explains

Table 1. Maintenance actions in CC applications to maintenance

Reference	Inspection	Precautionary Maintenance	Corrective Maintenance
J. Wu and Makis (2008)	X	X	X
Wu and Wang (2011)		X	X
Bouslah, Gharbi and Pellerin (2015)		X	X
Tagaras (1988)	X	X	
Tambe and Kulkarni (2015)		X	X
Chan and Wu (2009)	X	X	
Yin, Zhang, Zhu, Deng and He (2015)		X	X
Ben-Daya and Rahim (2000)		X	
Cassady, Bowden, Liew and Pohl (2000)		X	
Yeung, Cassady and Schneider (2007)		X	
Panagiotidou and Tagaras (2010)		X	
Mehraffrooz and Noorossana (2011)		X	
Ho and Quinino (2012)			X
Panagiotidou and Nenes (2009)		X	
Liu, Yu, Ma and Tu (2013)	X	X	X
Xiang (2013)		X	X
Wang (2012)	X	X	X
Zhou and Zhu (2008)		X	X
Wang and Zhang (2008)	X		
Panagiotidou and Tagaras (2012)		X	
Ivy and Nembhard (2005)			X
Linderman, McKone-Sweet and Anderson (2005)		X	X

the methodology proposed in this study. Section 3 presents the adaptive model used for performance comparison. Section 4 presents the development of a simulation model, whose analysis results, using the proposed methodology, are presented in Section 5. Section 6 presents an analysis of real-life degradation of bearings. Finally, a discussion and conclusions drawn are provided in Section 7.

## 2. DIAGNOSTICS-PROGNOSTICS INTEGRATION METHODOLOGY

The methodology presented in this study assumes that i) the maintenance practitioner is concerned with equipment that presents a single, failure mode; ii) there is a single, primary, measurable variable related directly or indirectly to the failure mode; iii) the variable related to the failure mode can be monitored; iv) there is no evidence or information about the equipment operating conditions; v) there is no historical, run-to-failure data available; vi) there is enough knowledge to determine an acceptable failure threshold directly related to the condition variable and well-established according to the needs of the practitioner; vii) the practitioner is interested in predicting condition evolution and RUL estimation.

The proposed method uses an Exponentially Weighted Moving Average for Stationary Process (EWMAS) CC for condition monitoring followed by a RUL estimation, where data is assumed to follow an AR(p) structure. Section 2.1 describes data assumptions required for condition monitoring and RUL estimation. Section 2.2 presents the EWMAS CC

used for monitoring and imminent failure detection. Section 2.3 introduces a state-space AR(p) model with calculations of expected RUL and minimum expected RUL to perform forecasts. Finally, Section 2.4 provides a summary algorithm for the implementation process of the proposed sequential monitoring-estimation scheme.

It is worth noting that, individually, each method, EWMAS and RUL estimation, have been previously proposed. However, their sequential application, and, specially, the effect of using control charts to improve RUL estimation and how it compares with traditional approaches in the presence of explosive changes have not been previously addressed in the literature.

### 2.1. The AR(p) Degradation Model

A number of studies (Nectoux et al., 2012; Barraza-Barraza, 2015; Xie et al., 2002; Escobet et al., 2012) have shown that AR structures are useful to model degradation variables, as they acknowledge the presence of autocorrelation among observations, and do not require historical failure data for parameter estimation. This study assumes that the degradation process can be modeled using the AR structure

$$y_t = \begin{cases} \mu_0 + \phi_{01}y_{t-1} + \phi_{02}y_{t-2} + \dots + \phi_{0p}y_{t-p} + \epsilon_t & \text{if } 1 \leq t \leq \tau \\ \mu_1 + \phi_{11}y_{t-1} + \phi_{12}y_{t-2} + \dots + \phi_{1p}y_{t-p} + \epsilon_t & \text{if } \tau < t \end{cases} \quad (1)$$

where  $y_t$  is the condition measurement at time  $t$ ,  $\mu_0, \phi_{01}, \phi_{02}, \dots, \phi_{0p}$  are the pre-change unknown parameters,  $\mu_1, \phi_{11}, \phi_{12}, \dots, \phi_{1p}$  stand for the after-change unknown parameters, and  $\epsilon_t \sim N(0, \sigma_e^2), t = 1, 2, \dots$ . To avoid confusion with notation from the different areas that come together in this work, the authors decided to present all random variables and constants in lower case letters. For selection of the most appropriate autoregressive order, and validation of residual assumptions, refer to the work of Box, Jenkins, Reinsel and Ljung (2015). In the methodology proposed here, parameters of equation (1) are estimated using the commonly known Ordinary Least Squares (OLS) algorithm (Box et al., 2015; Searle, 2012).

It is assumed that the monitored equipment presents a known, single failure mode; that there is a univariate, available-to-measure condition variable linked to this failure mode; that a known, acceptable and well-established failure threshold exists in terms of the condition variable, determined after a failure analysis or experience from the practitioner; that the degradation process suffers one change in the equipment lifetime; and that the variance for residuals,  $\sigma_e^2$ , does not change through the in-control stage of the process.

Readers might be concerned with the fact that time series models, such as the one assumed in this study, were developed for stationary processes, and the degradation data might not follow this assumption; however, this should not be an issue since practical applications can use transformations such as differencing, log transformations, etc., to achieve stationarity (Box et al., 2015; Croux et al., 2011, Kirchgässner, Wolters, & Hassler, 2012).

## 2.2. EWMAST Control Charts

The use of an AR structure for modeling the degradation process of bearings raises the concern of auto-correlated data when applying control charts since most of them are based on the assumption of independence. Different studies have addressed the issue of auto-correlated data in control chart applications which has taken the form of reviews and suggestions on how to deal with it (Woodall & Montgomery, 2014; Peñabaena Niebles et al., 2013; Prajapati & Singh, 2012). Given that Time Series provide the ability to model different levels of correlation, Alwan and Roberts (1988) suggested modeling the observations in conjunction with them and, in addition, to monitor the fitting errors with a standard control chart, provided that these errors are independent, identically distributed normal random variables. However, this approach not only requires knowledge about the model, but also can have a poor change-detection performance (Capizzi & Masarotto, 2007).

To overcome this issue, Zhang (1998) adapted the Exponentially Weighted Moving Average (EWMA) CC, named the

EWMAST CC, whose monitoring statistic is

$$z_t = \lambda y_t + (1 - \lambda)z_{t-1}, t = 1, 2, \dots \quad (2)$$

where  $y_t$  is the observed stationary process, such that, in control  $E[y_t] = \mu, \forall t$ , and the autocovariance function  $R(k) = E[(y_t - \mu)(y_{t+k} - \mu)]$  depends only on the lag  $k$  (Capizzi & Masarotto, 2007);  $0 \leq \lambda \leq 1$  is known as the smoothing parameter; and  $z_0 = \mu$ . According to Zhang (1998); Capizzi & Masarotto (2007),  $z_t$  is asymptotically stationary, and  $E[z_t] = \mu$  is constant. The EWMAST CC incorporates estimations of the data autocorrelation structure into calculations for the control limit. Considering the variance for  $z_t$  (Perry & Pignatiello Jr, 2010), is

$$\sigma_{z_t}^2 \simeq \left[ \frac{\lambda}{2 - \lambda} \right] \sigma_y^2 \times \left\{ 1 - (1 - \lambda)^{2t} + 2 \sum_{k=1}^{t-1} \rho(k)(1 - \lambda)^k \times \left[ 1 - (1 - \lambda)^{2(t-k)} \right] \right\} \quad (3)$$

where  $\sigma_y^2 = R(0)$  is the variance for  $y_t$ ;  $\rho(k) = \frac{R(k)}{R(0)}$ ,  $k = 0, 1, 2, \dots$  stands for the  $k$ -th autocorrelation of  $y_t$  (for more detail, the reader can refer to Zhang (1998); Capizzi & Masarotto (2007) and Perry & Pignatiello Jr (2010).

Using Equation (3) and properties from statistic  $z_t$  in Equation (2), the EWMAST control limits are defined as

$$H_U = \mu + L\sigma_{z_t} \quad (4)$$

$$H_L = \mu - L\sigma_{z_t} \quad (5)$$

with  $L$  a constant, commonly selected in a way that for a given  $\lambda$ , the in-control average run length ( $ARL_0$ ) of the CC is equal to a desired value. For guidelines to select  $\lambda$  values, refer to Zhang (1998); Capizzi & Masarotto (2007) and Montgomery (2009).

## 2.3. RUL Estimation

After the EWMAST CC detects an out-of-control observation, RUL estimations take place. To simplify calculations presented in this section, previous studies (Barraza-Barraza, 2015; Barraza-Barraza et al., 2017) relied on a variation of the State-Space formulation for an AR(p), explained as:

$$y(t+1) = \mathbf{C}[\mathbf{A}y(t) + \mathbf{K}\epsilon(t+1)], \quad (6)$$

where

$$A_{(p+1) \times (p+1)} = \begin{bmatrix} 1 & \phi_1 & \phi_2 & \dots & \phi_{p-1} & \phi_p \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix},$$

$$\mathbf{y}(t)_{(p+1) \times 1} = \begin{bmatrix} \mu \\ y_t \\ y_{t-1} \\ \vdots \\ y_{t-p+1} \end{bmatrix}, \mathbf{K}_{(p+1) \times 1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\mathbf{C}'_{(p+1) \times 1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Note that Matrix  $A$  in (7) is composed by the parameters of the AR model, the vector  $\mathbf{y}(t)$  contains the last  $p$  condition measurements, and vector  $C$  is used to retrieve the condition prediction, making  $y(t+1)$  a scalar. Derived from the State-Space notation, the expected condition prediction at the prediction horizon  $h$ , given the information obtained up to time  $t$ , is

$$\hat{y}(t+h|t) = CA^h \mathbf{y}(t) \quad (8)$$

Barraza-Barraza et al. (2017) provided, in their research, formulations to estimate expected RUL ( $\widehat{RUL}_E$ ), and the minimum expected RUL ( $\widehat{RUL}_{min}$ ), given a known failure threshold  $B$ . The  $\widehat{RUL}_E$  estimation is

$$\widehat{RUL}_E = \min\{h : \hat{y}(t+h|t) \geq B\} \quad (9)$$

$\widehat{RUL}_{min}$  depends on the assumption of the Gaussian distribution for the estimation error (Barraza-Barraza et al., 2017; Escobet et al., 2012), which allows the estimation of the upper confidence interval for the  $h$ -step-ahead condition forecast, defined as

$$\hat{y}_{1-\gamma}(t+h|t) = \hat{y}(t+h|t) + z_\gamma \times \hat{\sigma}(t+h|t) \quad (10)$$

where  $\gamma$  is the uncertainty level in condition forecasting, selected according to the practitioner's experience;  $z_\gamma$  is the upper  $\gamma$  quantile for the standard normal distribution, and  $\hat{\sigma}^2(t+h|t)$  is the forecast standard deviation. In their study, Barraza-Barraza et al. (2017) calculate the forecast standard deviation and forecast variance as

$$\hat{\sigma}(t+h|t) = \sqrt{\hat{V}(t+h|t)} \quad (11)$$

$$\hat{V}(t+h|t) = \hat{\sigma}_\epsilon^2(t) \times C \left( \sum_{i=1}^h A^{i-1} K K' (A^{i-1})' \right) \quad (12)$$

respectively, where  $\hat{\sigma}_\epsilon^2(t) = \sum_{i=1}^t \frac{\epsilon_i^2}{t-(p+1)}$  is the residual estimated variance at time  $t$ , and  $A$ ,  $K$  and  $C$  already defined in (7), for derivation of these formulas review Barraza-Barraza (2015); Barraza-Barraza et al. (2017). Using Equation (10), the minimum expected RUL is

$$\widehat{RUL}_{min} = \min\{h : \hat{y}_{1-\gamma}(t+h|t) \geq B\} \quad (13)$$

Starting at time  $t > p+1$ ,  $\widehat{RUL}_E$  and  $\widehat{RUL}_{min}$  are calculated following Algorithm 1.

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#### Algorithm 1. RUL Estimation

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- (1) Establish  $h = 0$
  - (2) While  $\hat{y}(t+h|t) \leq B$ 
    - a. Make  $h = h + 1$
    - b. Calculate new forecast  $\hat{y}(t+h|t)$  with equation (6)
    - c. Forecast upper condition interval  $\hat{y}_{1-\gamma}(t+h|t)$  with equation (10)
    - d. Obtain  $\widehat{RUL}_E$  and  $\widehat{RUL}_{min}$  with equations (9) and (13) respectively
- 

## 2.4. General algorithm for Diagnostics-Prognostics Methodology

The methodology proposed in this research is based on the assumption that the degradation process can be modeled with an AR(p) model. It is stipulated that a change in the condition variable can be detected using the EWMAST CC. Upon detection of a change, pre-change observations are discarded, and only after-change measurements are used for fitting a new AR(p) model that is then used for condition prediction and RUL estimation. Algorithm 2 presents the general steps for this methodology.

By means of the definition of *explosive change*, if a structural deformation occurs to the bearing, generating this type of change, the methodology proposed in this study requires continuing with after-change monitoring and discarding pre-change information, precisely since the mentioned structural deformation causes the previous model not to appropriately represent the condition-variable behavior anymore. Therefore, the goal of this methodology is to identify this change and update the model in order to have a statistical tool that provides more useful information for RUL estimation and maintenance-decision making.

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**Algorithm 2.** Diagnostics-Prognostics Methodology (AR-EWMAST)
 

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- (1) Define  $\lambda$  and  $L$  from equations (2) and (4-5) to achieve a desired ARL.
  - (2) Set  $t = 0$  and  $z_0 = \mu_0$ .
  - (3) While  $H_L \leq z_t \leq H_U$ .
    - a. Set  $t = t + 1$
    - b. Obtain a new observation  $y_t$
    - c. Calculate  $z_t$  from equation (2)
    - d. Calculate  $\sigma_{z_t}^2$  from equation (3)
    - e. Update  $H_U$  and  $H_L$  from equations (4-5)
  - (5) Set  $\hat{\tau} = t$
  - (6) Obtain new observations until  $t \geq \hat{\tau} + p + 1$ .
  - (7) Fit new  $AR(p)$  with  $y_t, y_{t+1}, \dots$  where  $t \geq \hat{\tau} + 1$ .
  - (8) Estimate RUL using Algorithm 1.
- 

This methodology, unlike the ones described at section 1, especially artificial intelligence and machine learning algorithms, does not require a training data set (large or small), nor a great computational effort for model fitting. Additionally, the monitoring process and its change detection is easier to interpret and link to the physical degradation. On their study, Ran et al. (2019) mention that, although machine learning methods can deal with high dimensionality on data, they easily over-fit the model and obtain poor prediction accuracy, a situation that the proposed methodology overcomes by continuously adapting the model parameters to the condition variable behavior.

### 3. ALTERNATIVE ADAPTIVE AND NON-ADAPTIVE AR MODELS FOR RUL ESTIMATION

This section presents two methods against which the proposed methodology is comparable: adaptive and non-adaptive approaches for multi-step-forecast models, summarized in Algorithm 3 and Algorithm 4, respectively. These methods use the same AR structure as presented in equation (1), with variations on the algorithm used for parameter estimation and adaptation to parameter change.

#### 3.1. Adaptive AR model for RUL Estimation

This model uses Recursive Least Square (RLS) estimators to adapt over possible changes that the condition variable might suffer through the degradation process. Young Young (2011) explained that RLS are useful in providing information about the presence of non-stationarity in the process and the possible nature of the variations in the parameters. The RLS algorithm has been applied previously for degradation prognostics (Barraza-Barraza et al., 2017; Escobet et al., 2012) and showed a good performance in condition prediction when dealing with trended data. The RLS algorithm follows the

next equations:

$$\epsilon_t = \frac{y_t - \hat{\phi}'_{t-1} \mathbf{y}_{t-1}}{1 + \mathbf{y}'_{t-1} F_{t-1} \mathbf{y}_{t-1}} \quad (14)$$

$$\hat{\phi}_t = \hat{\phi}_{t-1} + F_{t-1} \mathbf{y}_{t-1} \epsilon_t, \quad (15)$$

$$F_t = \frac{1}{\lambda_1} \left[ F_{t-1} - \frac{F_{t-1} \mathbf{y}_{t-1} \mathbf{y}'_{t-1} F_{t-1}}{\lambda_2 + \mathbf{y}'_{t-1} F_{t-1} \mathbf{y}_{t-1}} \right] \quad (16)$$

where  $\hat{\phi}_t = [\mu, \phi_1, \phi_2, \dots, \phi_p]$  is the parameter estimated vector at time  $t$ , and the observation vector at time  $t$  is  $\mathbf{y}_t = [1, y_{t-1}, y_{t-2}, \dots, y_{t-p}]$ ,  $F$  is the adaptation or gain matrix, restricted to  $F_0 > 0$ ;  $\lambda_1$  and  $\lambda_2$  are called the forgetting factor parameters restricted to  $0 < \lambda_1 \leq 1, 0 < \lambda_2 \leq 2$ . For guidelines on value selection for these parameters refer to Young (2011), Landau & Zito (2006) and Ljung & Söderström (1983), that also provide deeper detail for this algorithm and development of these equations. If there is prior information about  $\hat{\phi}$  and  $F$ , that information is used as the initial conditions for  $F_0$  and  $\hat{\phi}_0$ , otherwise they could set as  $F_0 = \mathbf{I}$ ,  $\hat{\phi}_0 = \mathbf{0}$  (more detail in Ljung & Söderström (1983)). Algorithm 3 presents the adaptive AR model steps, that will be referred to as *AR-RLS* in the rest of the document.

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**Algorithm 3.** Adaptive AR model (AR-RLS)
 

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- (1) Define the AR order  $p$  using at least  $k \geq p + 1$  observations
  - (2) For each new observation obtained after  $k$  do
    - a. Update recursive parameters using equations (14)-(16)
    - b. If RUL estimation is desired follow Algorithm 1, using  $\hat{\phi}_t$  as the parameters for matrix  $A$  in equation (7)
- 

#### 3.2. Non-adaptive AR model for RUL estimation

This model estimates the AR parameters through the OLS algorithm, differing from the diagnostics-prognostics methodology proposed in this study on the parameter updating process. This model, updates matrix  $A$  in equation (7) when a new observation is available. Algorithm 4 presents the steps followed in this model. Throughout the rest of this study, this model will be known as *AR-OLS*.

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**Algorithm 4.** Non-Adaptive AR model (AR-OLS)
 

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- (1) Define the AR order  $p$  using at least  $k \geq p + 1$  observations
  - (2) For each new observation obtained after  $k$  do
    - a. Update parameters using OLS and 1 to  $k$
    - b. If RUL estimation is desired follow Algorithm 2
-

#### 4. ANALYSIS OF BEARING DEGRADATION

Bearing degradation has been widely discussed in maintenance literature due to the extensive use of bearings in equipment with rotary parts (see Tandon & Choudhury (1999), and Jammu & Kankar (2011)). Bearing degradation presents a particular characteristic that complicates its study, which is related to its variability in degradation paths and failure times. Different experiments in bearing degradation (Nectoux et al., 2012; Wang & Zhang, 2008; NASA, 2008) have shown that even under the same operating conditions, bearings fail at different times, different vibration levels, and present different failure modes. This study uses the IEEE Data Challenge Dataset (Nectoux et al., 2012) as a starting point to develop simulation analysis.

A particular degradation behavior is simulated in the model presented in this research in order to evaluate the performance of the proposed methodology. It is necessary to run a simulation model due to the small available quantity of bearings in public datasets that also suffer a single explosive change in their degradation. This restriction should not be considered as an uncommon scenario in bearing degradation, as the three public datasets present this behavior in at least one of their bearings. In addition, there is no simulation model that properly characterizes bearing degradation in all its varied complexities. However, this simulation model imitates the particular behavior of a specific type of degradation in bearings; with the caveat that the model structure and parameters, the failure threshold, and any other parameter used in this simulation are by no means an intended characterization of general bearing degradation.

The following subsections describe the simulation model and the treatment given to some public datasets found on bearing degradation. These public datasets were used to test the methodology not only under simulated conditions, but also for real life scenarios.

##### 4.1. Simulating Bearing Degradation

The simulated model was obtained from an analysis performed on the 2012 IEEE Data Challenge Dataset (Barraza-Barraza, 2015). An AR (2) was fitted to bearing 1 of group 1 from this dataset (Figure 1), since this bearing presented one sustained and explosive change in its condition variable. Upon completion of this fitting, parameters were adjusted to obtain a behavior similar to Bearing 6 in Figure 2. The final simulation model is

$$y_t = \begin{cases} 0 + \phi_{1,0}y_{t-1} + \phi_{2,0}y_{t-2} + \epsilon_t, & 1 \leq t \leq \tau \\ \delta + \phi_{1,1}y_{t-1} + \phi_{2,1}y_{t-2} + \epsilon_t, & \tau < t, \end{cases} \quad (17)$$

where  $\epsilon_t$  are independents and normally distributed,  $\delta = [0.10, 0.75]$ ,  $\phi_{1,0} = 0.2$ ,  $\phi_{2,0} = 0.1$  and  $\phi_{1,1} = 0.7$ ,  $\phi_{2,1} = 0.4$ .

Two different change-point scenarios were established, with

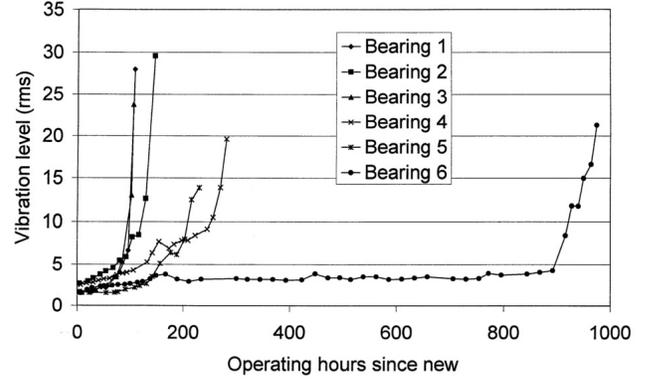


Figure 2. Bearing Degradation Behavior

the change occurring either at  $\tau = 1000$  or at  $\tau = 2000$ . The simulation model presented earlier is based on two distinct databases, each one with different time units. Wang's dataset (2002) uses operating hours, as time unit, while 2012 IEEE Dataset (Nectoux et al., 2012) uses sample number as indexing variable. This simulation uses  $t$  as a unitless-indexing variable, since selecting the unit time does not affect the methodology performance, as long as samples are acquired at equi-distant-time points.

Perry and Pignatiello (2010) used changes larger than 1.00 for  $\delta$ , however, they are too big for the scenario studied in this research, consequently, the proposed ones here are more conservative. Perry and Pignatiello's (2010) change-point estimation assumes a stationary behavior before and after the change, the latter being a condition not fulfilled in the scenario under study in this research, which is why their methodology is not used in this case. The failure threshold was defined by averaging the 17 standardized failure thresholds of the IEEE Dataset, meaning,

$$B = \frac{1}{17} \sum_{i=1}^{17} \frac{RMS_{N,i}}{SD_{50,i}}, \quad i = 1, 2, \dots, 17, \quad (18)$$

where  $RMS_{N,i}$  stands for the last Root Mean Square (RMS) observation and  $SD_{50,i}$  represents the standard deviation from the first 50 observations, both on the  $i$ th bearing. The result for this operation is 135.2313, rounded to 135. The first 10 observations from each simulation were discarded in order to work with a steady-state behavior in the simulation. Figure 3 shows the change sizes and change point cases used to simulate scenarios of degradation; for each scenario, 1000 degradation simulations were generated.

##### 4.2. RUL Estimation Points

In their study, Barraza-Barraza et al. (2017) established the bearing forecasting times or RUL estimation points based on the percentage of degradation found. Nonetheless, this approach is not suitable for the scenario addressed in this study

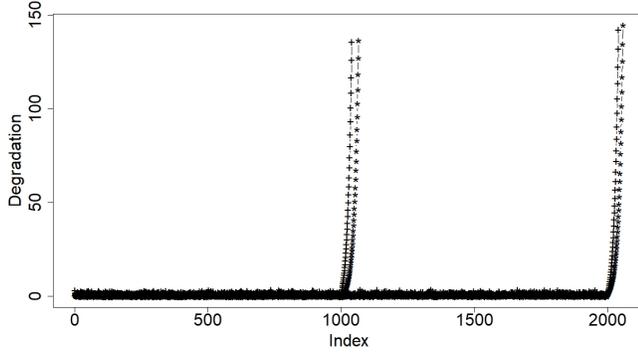


Figure 3. Simulation Scenarios

due to the triggering maintenance actions of the control charts. The forecasting times were selected in terms of CC alarm ( $\hat{\tau}_{CC}$ ) and an observation window large enough to fit a new AR(p), meaning that RUL estimations occur at times  $\hat{\tau}_{CC} + w(p + 1)$ , where, for this study,  $w = 3, 9, 12$  when  $\tau = 1000$  and  $w = 3, 6, 9$  if  $\tau = 2000$ . The values for  $w$  were selected in accordance to the length of the different simulated degradations. The idea of successive RUL estimation is based on the principle of variable monitoring intervals applied to diagnostics methodologies (Gardner, 2006).

### 4.3. Performance Indicators

Bias and Mean Absolute Deviation (MAD) statistics serve as indicators to evaluate the performance of RUL estimations. Table 2 shows the formula to estimate these statistics, where  $N$  is the number of simulations with RUL estimation under each simulated scenario. To simplify notation, in this table,  $\widehat{RUL}_{E,i}$  stands for the estimated RUL for simulation  $i$ , estimated with equation (9), whereas  $RUL_i$  indicates the true RUL value for simulation  $i$ .

Table 2. Statistics for Performance Evaluation

Performance Indicator	Formula
Bias	$\frac{1}{N} \sum_{i=1}^N (\widehat{RUL}_{E,i} - RUL_i)$
MAD	$\frac{1}{N} \sum_{i=1}^N  \widehat{RUL}_{E,i} - RUL_i $

## 5. RESULTS OF BEARING DEGRADATION ANALYSIS

### 5.1. Analysis of simulated bearing degradation

Table 3 presents the parameters used in the analysis of the simulated dataset, along with a reference to the formula each parameter appears on. Table 4 contains parameters whose values depend on the time a change occurred ( $\tau$ ).  $L$  is retrieved from the `spc` package in R, when  $\mu_0 = 0$  and  $ARL_0$  according to Table 4. In Statistical Control Process literature, values of  $0.05 \leq \lambda \leq 0.25$  are commonly used (Mont-

gomery, 2009; Qiu, 2013). According to RLS literature (Laudau & Zito, 2006) and previous studies (Barraza-Barraza et al., 2017), the selected values for  $\lambda_1$  and  $\lambda_2$  provide a larger forgetting factor for previous history.

Table 3. Common parameters used in simulation analysis

Parameter	Value	Equation	Parameter	Value	Equation
$p$	2	(1)	$\lambda$	0.25	(2)
$\lambda_1$	0.99	(16)	$\lambda_2$	2	(16)

Table 4. Specific parameters for different change points

Parameter	Reference Equation	$\tau = 1000$	$\tau = 2000$
$ARL_0$	(4-5)	1000	2000
$L$	(4-5)	3.126072	3.336692

Figure 4 shows the bias distribution in change estimation for each scenario in the simulation. The x-axis arranges boxplots according to the change size on  $\delta$ , while the colors represent the change point ( $\tau$ ). It is worth noting that this study assumes no false alarms appear as it creates misleading bias over RUL estimation performance measurements. Independently of the value for  $\tau$ , the EWMAST CC detected faster a large change ( $\delta = 0.75$ ), than the small one ( $\delta = 0.25$ ).

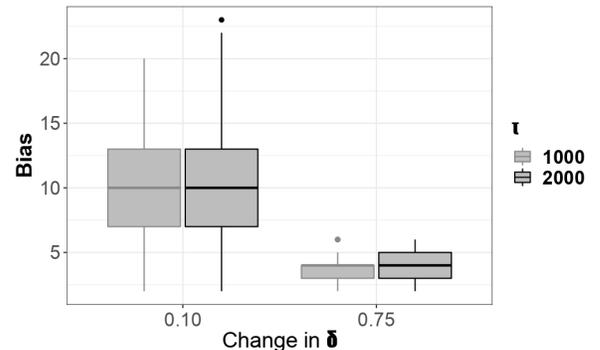


Figure 4. Change point estimation by change size and simulation length

After the EWMAST CC raises an alarm at time  $\hat{\tau}$ , the next step is to predict RUL. The results presented in this section corresponds to each scenario and model, and in terms of bias and MAD, Tables 5 and 6 respectively, along with their standard deviation. These tables contain cases where methods *AR-RLS* and *AR-OLS* do not report results, and it could be due to a combination of three situations: a) failure occurred before the point  $\hat{\tau} + w \times (p + 1)$ , a possibility if  $\tau = 2000$  and  $\delta = 0.75$ ; b) extremely large condition prediction, meaning that when  $h > 10000$ , the prediction horizon in equation

(8), the process stops as it surpasses the true RUL by a large amount; c) the algorithm predicts a decrement on degradation, and the prediction process has to stop as it would not reach the failure threshold. Although the three methods could present these situations, the *AR-RLS* and *AR-OLS* are more affected by them, as they do not present results in various scenarios. In one particular case, the *AR-RLS* method produced only one useful RUL estimation (not falling in the three cases presented in this paragraph), and therefore, a *NA* appears in standard deviation for Bias.

For graphical description, Figure 5 presents a comparison of bias distribution for a given scenario. This first approach to the results show that the *AR-EWMAST* methodology proposed in this study performs a more accurate RUL estimation compared against the other methods. Tables 5 and 6 confirm that these results are consistent throughout all the scenarios, with *AR-EWMAST* developing a smaller bias and MAD. In addition, this method is also more precise, with smaller standard deviations for these performance indicators.

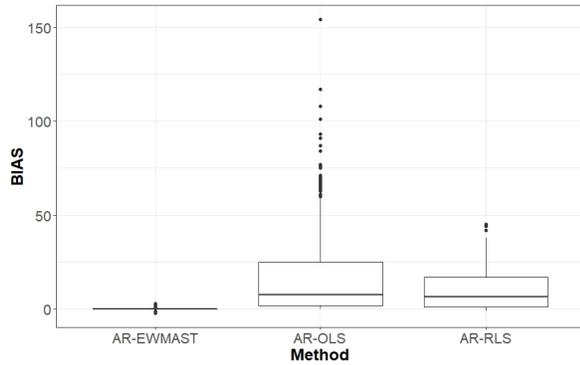


Figure 5. Results comparison among three methods

The results show that Bias and MAD decrease as the after-change observations become more available as presented in Figure 6, where prediction from the three methods are compared when they occur at the same point. This graphs help to exemplify how *AR-RLS* and *AR-OLS* predict a slow increment in degradation when few after-change observations are available, meaning that the pre-change information that both models are still carrying has a negative effect on their prediction. The availability of out-of-control observations help these models improve their performance in degradation prediction, as the middle and bottom graphs in Figure 6 show.

## 5.2. Analysis of Real Bearing Degradation. The IMS Dataset

In order to test the methodology with real-degradation data, the IMS Bearing Dataset is used. This is a dataset available for public download, consisting on three experiments over four bearings, each experiment. The four bearings were installed on a shaft that rotated at constant speed (2000 RPM)

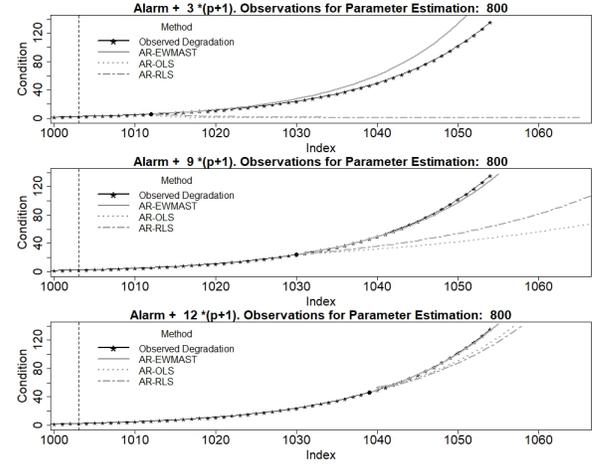


Figure 6. A random path over different RUL estimation approaches

and constant radial load of 6000 lbs, until the bearings exceeded its designed lifetime of 100 million revolutions (for more detail refer to Lee, Qiu, Yu and Lin (2007)). Each file in the dataset contained raw vibrations that, for the purpose of this study, were transformed to obtain the RMS for each sample time with the formula in Equation (19). For experiment 1, the dataset provides vibration measurements from two channels for each bearing; therefore, global RMS is obtained with Equation (20). After a visual analysis of the RMS plots for all bearings, only bearing 2 was selected due to the presence of a degradation change at the end of life.

$$RMS_{v/h,t} = \sqrt{\frac{1}{20480} \sum_{i=1}^{20480} acc_{v/h,t}^2} \quad (19)$$

$$Global\ RMS_{G,t} = \sqrt{RMS_{Ch1,t}^2 + RMS_{Ch2,t}^2} \quad (20)$$

$i = 1, 2, \dots, 2156$

Figure 7 presents the RMS for this bearing 2 from IMS Bearing Dataset. A visual inspection in Figure 7 shows a clear degradation change around observation 160 that could be due to an alteration in the load or an issue in the DAQ measurement system<sup>1</sup>. However, since the alteration occurred at an early life state, discarding the first 170 observations to minimize its effect does not affect data analysis.

The bearing degradation was smoothed using Holt's exponential smoothing for trended data (Gardner, 2006), with equations

$$S_t = \alpha RMS_t + (1 - \alpha) (S_{t-1} + T_{t-1}) \quad (21)$$

$$T_t = \varphi (S_t - S_{t-1}) + (1 - \varphi) T_{t-1} \quad (22)$$

<sup>1</sup>According to an email exchange with professors in charge of the experiment

Table 5. Bias and standard deviation (SD) results in RUL estimation

Length	Change	Window	Method	Bias	SD	Length	Change	Window	Method	Bias	SD
1000	0.10	3	AR-EWMAST	8.95	33.46	2000	0.10	3	AR-EWMAST	9.4	41.14
			AR-OLS	-	-				AR-OLS	-	-
			AR-RLS	-	-				AR-RLS	9.00	NA
		9	AR-EWMAST	0.00	0.77			6	AR-EWMAST	0.07	0.77
			AR-OLS	28.71	14.67				AR-OLS	438.79	880.28
			AR-RLS	17.60	5.30				AR-RLS	-	-
	12	AR-EWMAST	-0.01	0.37	9		AR-EWMAST	0.00	0.36		
		AR-OLS	1.85	0.69			AR-OLS	4.42	1.28		
		AR-RLS	3.14	0.86			AR-RLS	-	-		
	0.75	3	AR-EWMAST	0.41	3.20		0.75	3	AR-EWMAST	0.37	2.86
			AR-OLS	-	-				AR-OLS	-	-
			AR-RLS	43.64	46.83				AR-RLS	512.79	864.54
		9	AR-EWMAST	0.01	0.57			6	AR-EWMAST	0	0.55
			AR-OLS	2.58	0.98				AR-OLS	14.78	3.89
			AR-RLS	1.32	0.87				AR-RLS	4.78	1.09
	12	AR-EWMAST	0.00	0.27	9		AR-EWMAST	0.00	0.25		
		AR-OLS	-0.31	0.46			AR-OLS	0.29	0.45		
		AR-RLS	-0.23	0.42			AR-RLS	0.37	0.48		

Table 6. MAD results in RUL estimation

Length	Change	Window	Method	MAD	Length	Change	Window	Method	MAD
1000	0.10	3	AR-EWMAST	16.08	2000	0.10	3	AR-EWMAST	16.8
			AR-OLS	-				AR-OLS	-
			AR-RLS	-				AR-RLS	9
		9	AR-EWMAST	0.51			6	AR-EWMAST	0.52
			AR-OLS	28.71				AR-OLS	438.79
			AR-RLS	17.6				AR-RLS	-
	12	AR-EWMAST	0.14	9		AR-EWMAST	0.13		
		AR-OLS	1.85			AR-OLS	4.42		
		AR-RLS	3.14			AR-RLS	-		
	0.75	3	AR-EWMAST	2.29		0.75	3	AR-EWMAST	2.2
			AR-OLS	-				AR-OLS	-
			AR-RLS	43.64				AR-RLS	512.79
		9	AR-EWMAST	0.31			6	AR-EWMAST	0.3
			AR-OLS	2.58				AR-OLS	14.78
			AR-RLS	1.34				AR-RLS	4.78
	12	AR-EWMAST	0.07	9		AR-EWMAST	0.06		
		AR-OLS	0.31			AR-OLS	0.29		
		AR-RLS	0.23			AR-RLS	0.37		

where  $RMS_t$  is  $RMS$  at time  $t$ ;  $S_t$  is the smoothed level of  $RMS_t$ ;  $T_t$  is the smoothed additive trend at time  $t$ ;  $\alpha, 0 \leq \alpha \leq 1$  is the smoothing parameter for  $RMS$ ; and  $\varphi, 0 \leq \varphi \leq 1$ , the smoothing parameter for trend.

Table 7 contains the parameters used for the analysis of bearing 2. In order to design the EWMAST CC parameters, the first 500 observations (considered as a sufficient number of measurements for a good parameter estimation) were used to check stationarity and parameter estimation. After applying the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) and Augmented Dickey-Fuller (ADF) tests, the data resulted in a no stationary time series, therefore, differencing of order one was required. With this differencing, the data was transformed into a stationary process.

The parameter  $\lambda$  was selected after testing the EWMAST CC

Table 7. Parameters used in practical example

Parameter	Value	Equation	Parameter	Value	Equation
$p$	1	(1)	$\lambda$	0.25	(2)
$ARL_0$	1000	(4-5)	$L$	3.14	(4-5)
$\lambda_1$	0.99	(16)	$\lambda_2$	2	(16)
$\alpha$	0.1	(21)	$\varphi$	0.5	(22)

performance with  $\lambda = 0.05, 0.1, 0.25, 0.3$ , and finding that alarm was triggered at equal monitoring times. The `spc` package in `R` gave the value for  $L$  in Table 7.  $\lambda_1$  and  $\lambda_2$  took the same values as in the analysis of simulated degradation.

Figure 8 presents the monitoring process in three graphs: the upper graph shows the RMS differenced data in order to acquire stationarity. The center graph presents the  $z_t$  CC statis-

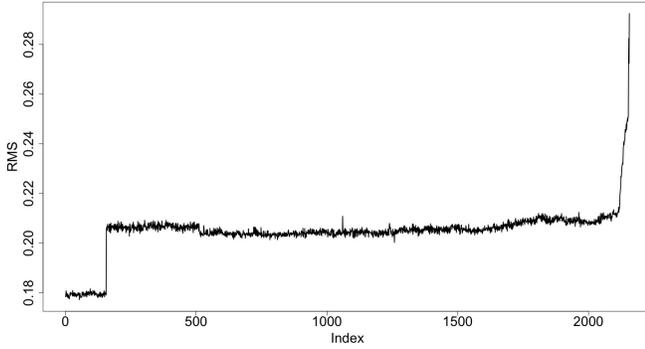


Figure 7. Global RMS for Bearing 2 from IMS Bearing Dataset

tic, considering that the first 500 observations were used for process mean estimation and CC set up; this graph also shows the upper control limit. Finally, the lower graph presents the original data and the time where the alarm appears (vertical dashed line). This figure shows the transformations the data suffered in order to fulfill the CC requirements of stationarity in the in-control state, and the change detected by the CC at observation 1920 in original data.

After the CC detected a change in the condition, prediction takes place. Figure 9 presents a comparison of prediction performance for the three methodologies: *AR-EMWAST*, *AR-RLS* and *AR-OLS*. In this case, the reader can notice how the condition predicted through the *AR-OLS* methodology follows an almost horizontal path, due to the pre-change information its parameters still carry. The *AR-RLS*, develops a steeper slope in its prediction, showing that, although its parameters are adjusting to the new direction that the degradation is taking, this adjustment is not fast enough, contrary to what the *AR-EMWAST* methodology is doing. The method that forgets all pre-change information is performing better in predicting the condition path of the degradation.

RUL estimation took place at every after-change point to analyze the behavior of the three methods as more after-change information became available. Figure 10 presents the evolution of RUL estimation performed by the three methods (interrupted lines), and the real RUL (solid line). In this graph, the goal is to lie as closest as possible to the solid line, as that would represent a smaller bias in RUL estimation. As the figure shows, both *AR-RLS* and *AR-OLS* over estimate the RUL, with an adequate prediction only at the end of life, around observation 1950; it can be seen that the *AR-EMWAST* gives closer estimations to the actual RUL, with a small under estimation at early estimation stages, and over estimation at the end of life, showing that forgetting all pre-change observation is a better option in RUL estimation, than carrying pre-change information or adapting estimators to the degradation evolu-

tion.

### 5.3. Remarks on AR-RLS predictions

In the first attempts at RUL estimation for the simulated dataset, the *AR-RLS* method presented issues with the degradation path it predicted, due to the amount of observations used for parameter estimation ( $k$  in Algorithm 3). Figure 11 presents an example of these issues, for a scenario where  $\tau = 1000$  and  $w = 3$ . In the upper graph of this figure, the starting parameters were estimated using 500 observations, the reader can observe the wave developed in degradation prediction, due to a bad parameter estimation and adaptation. Predictions in the middle and bottom graphs used 800 and 1000 observations, respectively, and in this case *AR-RLS* followed the same path as the *AR-OLS* method. Given these circumstances, all predictions with *AR-RLS* used 800 observations for initial parameter estimation, since 1000 was too close to a change and there seemed to be no significant difference in the predicted path.

### 5.4. Discussion

The results showed that, for both datasets, the differences in performance can be explained in terms of the capabilities each model has to forget the pre-change information, which misleads condition prediction (Figure 9 illustrates this phenomenon). The *AR-OLS* model performs bad RUL estimations due to the pre-change history it still carries in its parameters; the adaptive feature of the *AR-RLS* model allows it to progressively forget the pre-change information, however, this forgetting process takes its time, and therefore, RUL estimations become more accurate and precise as more after-change information becomes available. The proposed methodology (*AR-EMWAST*), on the other hand, is capable of estimating RUL with accuracy and precision at early after-change stages (few observations after the CC triggered an alarm).

The difference among Bias and MAD for different  $\tau$  in Tables 5 and 6 provides evidence of the effect of pre-change information as a misleading factor in RUL estimations. For *AR-OLS* when  $\tau = 1000$ , Bias in RUL estimations is relatively small, in comparison with the scenario when  $\tau = 2000$ . For *AR-RLS*, is more dramatic, as the scenario for  $\tau = 2000$  did not provide useful RUL estimations to calculate Bias and MAD. It is worth noting that the amount of pre-change information does not affect *AR-EMWAST* results, as it uses only after-change observations.

More evidence on the effect that pre-change information has in RUL estimation is provided by the decrement in bias and MAD as more after-change information becomes available. When  $w = 3$ , both *AR-RLS* and *AR-OLS* do not provide useful RUL estimations. The *AR-EMWAST* in these cases, provided accurate RUL estimations even with few after-change observations.

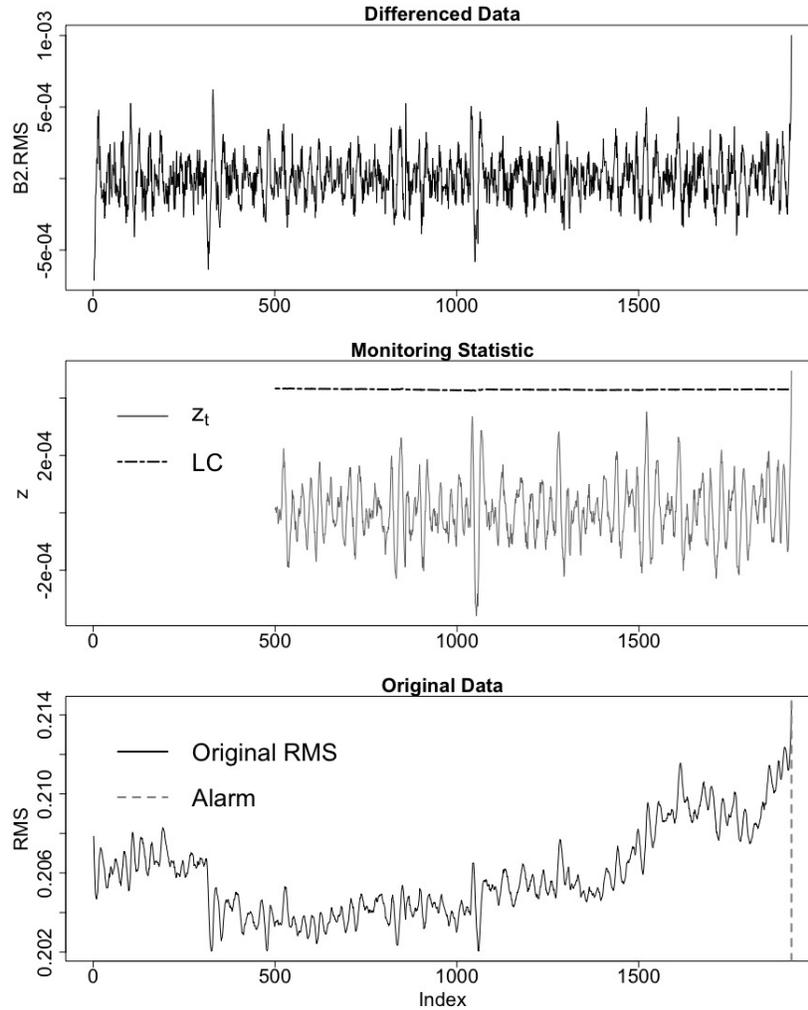


Figure 8. Graphical description of monitoring of IMS Bearing 2

## 6. CONCLUSIONS

This study developed a new diagnostics-prognostics methodology for RUL estimation based on detection of explosive changes and using only after-change observations for model fitting. The proposed methodology *AR-EWMAST* monitors the condition variable using an EWMAST CC, and discards all pre-change observation once the CC triggers an alarm, with the remaining observations, an AR(p) model is adjusted and condition prediction, leading to RUL estimation, is performed. Two other models found in CBM literature serve as comparison methodologies in this study: *AR-RLS* and *AR-OLS*. *AR-OLS* updates parameters using all available observations through OLS, while *AR-RLS* iteratively adapts the parameters with the RLS algorithm. The study used two datasets in order to test the performance of these methodologies, a simulation of bearing degradation, and a real-life degradation dataset.

The three methods compared in this study present different degrees of a forgetting feature in fitting a model for RUL estimation. Starting with the *AR-OLS*, that uses all available information, causing its predictions to be inaccurate, and imprecise. The adaptiveness of the *AR-RLS* model, allows it to perform more accurate RUL estimations than the *AR-OLS*, however, this model presents the disadvantage of requiring a large amount of in-control observations in order for it to develop an acceptable degradation prediction. While the proposed diagnostics-prognostics methodology, the *AR-EWMAST*, by not acknowledging any pre-change information, avoids contamination in its model fitting process and, in consequence, estimates RUL more accurately and precisely than the other two methods.

The *AR-EWMAST* methodology provided better RUL estimations than the models it was compared to in both datasets; its feature of using only observations obtained after the EWMAST CC alarm is responsible for its performance. The ap-

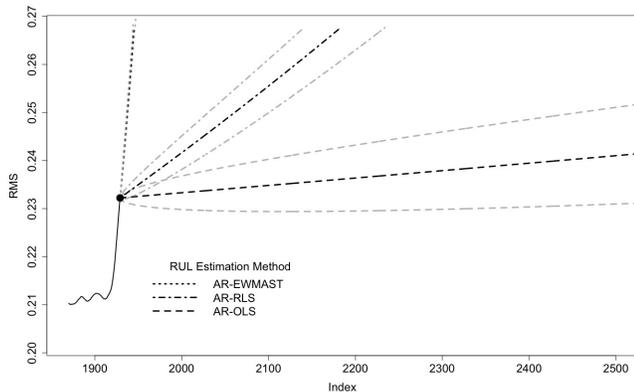


Figure 9. Prediction paths of RUL estimation methods

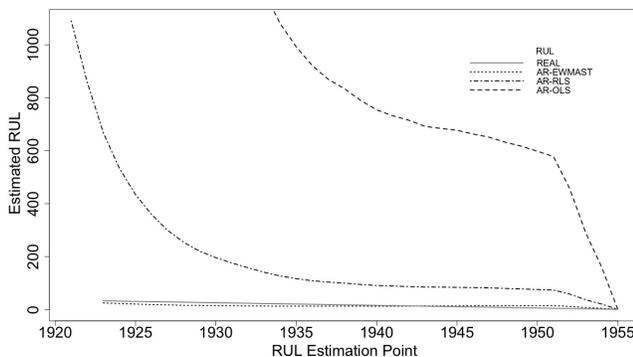


Figure 10. RUL prediction evolution over different prediction methods

plication of the EWMAST provides the advantage of not requiring fitting a model in the in-control state of degradation, especially since this pre-change model might not be the same as in the out-of-control phase.

Future research might evaluate if this methodology could be improved by working with scenarios where 1) the degradation also suffers a change in condition variance; 2) the change does not occur at the end of life; 3) there is more than one change, or even, when there is a smaller change after the explosion in degradation condition. Exploring different time series models, which considers either AR or Moving Average residuals, is also left to future research.

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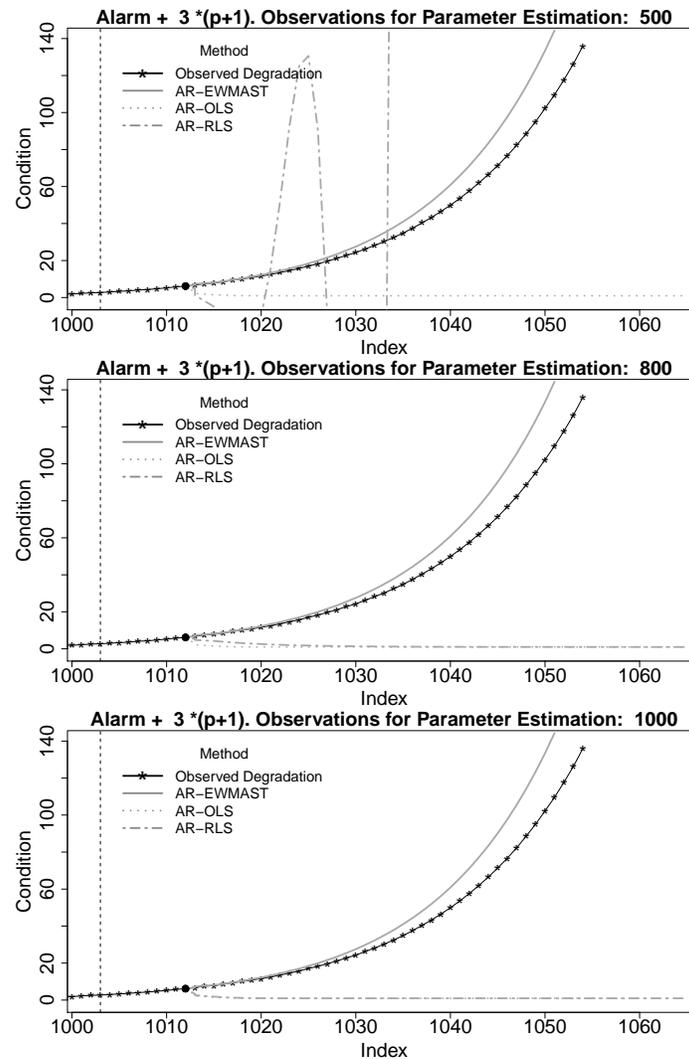


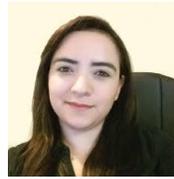
Figure 11. Behavior of AR-RLS prediction path over different initial samples sizes

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