Fault Diagnosis in Fuzzy Discrete Event System: Incomplete Models and Learning

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ABSTRACT

Nowadays, determining faults in non-stationary environment and that can deal with the problems of fuzziness impreciseness and subjectivity is a challenging task in complex systems such as nuclear center, or wind turbines, etc. Our objective in this paper is to develop models based on fuzzy finite state automaton with fuzzy variables describing the industrial process in order to detect anomalies in real time and possibly in anticipation. A diagnosis method has for goal to alert actors responsible for managing operations and resources, able to adapt to the emergence of new procedures or improvisation in the case of unexpected situations. The diagnoser module use the outputs events and membership values of each active state of the model as input events.

1. INTRODUCTION

A great number of systems can be naturally viewed as discrete event systems, that why the failure diagnosis problem has been investigated via discrete event system approach. A discrete event system is a dynamic system whose behavior is governed by occurrence of physical events that cause abrupt changes in the state of the system (Liu & Qiu, 2009), (Cassandras & Lafortune, 1999), (Sayed-Mouchaweh & Billaudel, 2012), (Traore, Sayed-Mouchaweh, & Billaudel, 2014). Discrete event system theory, particularly on modeling and fault diagnosis, has been successful employed in many areas such as concurrent monitoring and control of complex system (Cao & Ying, 2005), (Kilic, 2008), (Kwong & Yonge-Mallo, 2011), (Lin & Ying, 2002), (Luo, Li, Sun, & Liu, 2012). Usually, a discrete event system is modeled by Automata (Dzelme-Berzina, 2009), (Mukherjee & Ray, 2014) or Petri Net (Patela

& Joshi, 2013). Automata (or more precisely a finite state automaton) are the prime example of general computational systems over discrete spaces and have a long history both in theory and application (Thomas, 1990), (Moghari, Zahedi, & Ameri, 2011). A finite state automaton is an appropriate tool for modeling systems and applications which can be realized as finite set of states and transition between them depending on some input strings (Doostfatemeh & Kremer, 2004). Thus, the behavior of discrete event system modeled by an automaton is described by the language generated by the automaton. However, sometimes one may need to model systems that cannot be modeled by the current discrete event system modeling methods due to the uncertain and vagueness in the definition of the state and/or transitions. In order to overcome these difficulties, the concepts of the fuzzy states and fuzzy transitions can be used. Every fuzzy transition is associated with a possibility degree, called in the following membership value. This latter can be defined as the possibility of the transition from current (active) state to the next state. The main advantage of fuzzy finite state automaton is that their fuzziness allows them to handle imprecise and uncertain data, which is inherent to real-world phenomena, in the form of fuzzy states and transitions. One of the interesting characteristics of fuzzy automaton is the possibility of several transitions from different current fuzzy states lead to the same next fuzzy state simultaneously, and also the possibility of several transitions from one current fuzzy state lead to the different next fuzzy states simultaneously and consequently several output label can be activated at the same time (Doostfatemeh & Kremer, 2005). For this reason, fuzzy discrete event is very adapted to resolve the ambiguity (or degree of fault) in a fault diagnosis problem especially in the case of multiple faults.

Fault diagnosis in fuzzy discrete event systems is a research area that has received a lot of attention in the recent years and

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has been motivated by the practical need of ensuring the correct and safe functioning of large complex systems (Cabasino & Alessandro Giua, 2010) or multiple collaboration (like crisis situation) (Traore, Chatelet, Soulier, & Gabbar, 2014). Hence, the use of fuzzy finite state automaton in fault diagnosis tasks has gained particular attention in the case of fuzzy discrete event dynamic systems (Gerasimos, 2009; Traore, Chatelet, et al., 2014). Although, most of the approaches proposed in literature for fault diagnosis of fuzzy discrete event systems require a complete and accurate model of the system to be diagnosed. However, the fuzzy discrete event model may have arisen from abstraction and simplification of a continuous time system or through model building from inputoutput data. As such, it may not capture the dynamic behavior of the system completely. Therefore, in this paper, we attempt to develop models based on fuzzy finite state automaton with fuzzy variables describing also the industrial process in order to detect anomalies (eg, critical drifts process that could endanger people) in real time and possibly in anticipation. A diagnosis was developed to alert actors responsible for managing operations and resources, able to adapt to the emergence of new procedures or improvisation in case of unexpected situations.

The diagnoser module use the outputs events and membership values of each active state of the model as its input events. The membership value of the current state is obtained by using the propagation of the membership of the predecessor of the current state. A fault diagnosis approach based on fuzzy automaton is presented in (Rigatos, 2009; Doostfatemeh & Kremer, 2005) and in this paper the propagation (update) approach of the membership of current state is did by using fuzzy roles min, max functions. And a approach based on the conjunctions of the membership of the predecessor of the current state is presented in (Gonzalo & Gracin, 2010). Thus, a propagation approach of the membership based on the sum of the normalized memebreship is proposed in (Wang, Ji, Dong, & Sun, 2013). However all the new approaches proposed in (Rigatos, 2009; Doostfatemeh & Kremer, 2005; Gonzalo & Gracin, 2010; Wang et al., 2013) use only a single membership (weight) for each transition. In our work, we proposed a new approach that use a vector for membership value and a new propagation approach of the membership value is proposed in this paper.

The rest of the paper is organized as follows. In section 2, we present the required background of fuzzy discrete event system and we introduces the concept of single membership value. A new definition of fuzzy state and how the membership value is attributed to each active state are presented in section 3. In section 4, the propagation technique of the membership value is explained. A diagnosis approach using incomplete model is presented in section 5. In section 7 we present the algorithm of our approach.2 An application example is presented in section 7.

2. FUZZY DISCRETE EVENT SYSTEM DECISION MODEL

Fuzzy discrete event system as a generalization of crisp discrete event system may better deal with the problems of fuzziness, impreciseness and subjectivity. In fuzzy discrete event system, the states are fuzzy and every state transition is associated with a possibility degree (*i.e.*, membership value). In this paper, the fuzzy discrete event system theory are applied to develop an innovative model for systems operating in non-stationary environment, specifically for the diagnosis and prognosis (or prediction) to identify and detect possible problems.

A Fuzzy Finite state Automaton (*FFA*) \tilde{G} is a 5-tuple denoted as:

$$\widetilde{G} = \{X, \Sigma, \delta, Y, x_0, F\}$$

where

- $X = \{x_0, \dots, x_n\}$ is the sequence states set
- $\Sigma = {\sigma_0, \cdots, \sigma_m}$ is set of input symbols,
- The fuzzy subset δ : X × Σ × X → [0 1] is a function, called fuzzy transition function. A transition from current state x_i to the next state x_j upon σ_k with the weight ω_{i,j} is denoted as: δ(x_i, σ_k, x_j) = ω_{i,j}.
- $Y = \{y_0, \dots, y_l\}$ is the set non-empty finite set of output,
- $x_0 \in X$ is the set of initial fuzzy states,
- *F* is the (non-empty) set of accepting (or terminating) states.

In the following, we introduce the notion of membership value (mv) associated to each active state. Let the value $\mu^t(x_i)$ be the mv associated to the state active x_i , and $\omega_{i,j}$ represents the weight of transition from state x_i (current state) to state x_j (next state) upon σ_k .

With *FFA*, there is the possibility of several transitions from different active states lead to the same next state simultaneously as shown in Figure 1.(a). Thus, the possibility of several transitions from one active state lead to the different next states simultaneously as shown in Figure 1.(b), and consequently several output label can be activated at the same time (Doostfatemeh & Kremer, 2005). Thus, with *FFA* it is possible to have more than one start state. For this reason, fuzzy discrete event system concept is more convenient in the investigation of the problems of multiple faults.

When an input σ_k occurs at time *t*, all active state at this time, are those states to which there is at least one transition on the input event σ_k . Then, the fuzzy set of all active state at time *t* is called active state set at time *t*. The active state set denoted



Figure 1. A example of a part of FFA.

 X_{act} consists of the state and their mv's and given by:

 $\begin{aligned} X_{act}(t) &= \left\{ (x_j, \mu^t(x_j)) | \exists (x_i \in \mathbf{A}_{x_j}, \sigma_k \in \Sigma) \land x_j \in X_{succ}(x_i, \sigma_k) \right\}, \\ \mathbf{A}_{x_j} &= X_{pred}(x_j, t) = \left\{ x_i \mid \exists \sigma'_k \text{ s.t } x_j \in \varphi_1(x_i, \sigma'_k) \text{ and } x_j \in X_{act}(t) \right\}, \\ X_{succ}(x_i, \sigma_k) &= \left\{ x_j \mid x_j \in \varphi_1(x_i, \sigma_k) \right\}, \\ \delta(x_i, \sigma_k, x_j) &= \omega_{i,j}. \end{aligned}$

where φ_1 (and φ_2) is the extension of the transition function $\varphi: X \times \Sigma \rightarrow$ of the crisp discrete event system as presented in (Traore, Sayed-Mouchaweh, & Billaudel, 2014). The function φ_1 gives the state reached from a state $x_i \in X$ and a given input $\sigma_k \in \Sigma$ and φ_2 defines the output sequence from state x_i when the input σ_k occurs.

The relation of φ_1 and φ_2 are given by

$$\varphi_1(x_i, \sigma_k) = \left\{ x_j \mid \exists y_j \text{ such that } (x_j, y_j) \in \varphi(x_i, \sigma_k) \right\}, \varphi_2(x_j, \sigma_k) = \left\{ y_j \mid \exists x_j \text{ such that } (x_j, y_j) \in \varphi(x_i, \sigma_k) \right\},$$

where $y_i \in Y$.

For example in Figure 1.(a) $\varphi_1(x_1, 1) = \varphi_1(x_8, 1) = \varphi_1(x_{13}, 1) = x_3$ and $\varphi_2(x_3, 1) = y_3$.

The state x_i is the state at time t - 1 and x_j is the active state at time t and $\mu^t(x_j)$ is the membership of the state x_j at time t. The state set in $X_{pred}(x_j, t)$ is all predecessors state set of the active state x_j and $X_{succ}(x_j, \sigma_k)$ is all successors set of the state x_j on input symbol σ_k . The successor $X_{succ}(x_j, \sigma_k)$ is the set of all x_j which will be reached via transition function $\varphi_1(x_j, \sigma_k)$.

In the following, all successors set of x_i is denoted by

 $X_{succ}(x_j, \stackrel{all}{\rightarrow})$, when the next states depend to the occurrence of different input events.

We use the same notation for the active state, when the upon entrance is a string $\Gamma \in \Sigma^*$. The active state set of the string Γ is given by:

$$X_{act}(\Gamma) = X_{act}(t_0 + |\Gamma|),$$

where $|\Gamma|$ represent the length of Γ .

For example in Figure 1.(a), at time t_1 , the active state set is

$$X_{act}(t_1) = \{x_1, x_8, x_{13}\} \text{ and}$$

$$X_{succ}(x_1, 1) = \{x_3\},$$

$$X_{succ}(x_8, 1) = \{x_3\} \text{ and}$$

$$X_{succ}(x_{13}, 1) = \{x_3\},$$

At time time t_2 , the active state set is $X_{act}(t_2) = \{x_3\}$ and $X_{pred}(x_3, t_2) = \{x_1, x_8, x_{13}\}$, that mean the state x_3 is forced to take several different mv at time t_2 , when the input 1 is red (occurred). Hence, x_3 is a state with multi-membership, that we will call in the following multi-membership state.

In Figure 1.(b), each $mv \mu^t(x_j)_{j=7,10,14}$ of the state x_j at time t is computed by using the function Ψ_1 , named augmentation transition function. The function Ψ_1 should satisfy the two following axioms.

- 1. $0 \leq \Psi_1(\mu^t(x_i), \delta(x_i, \sigma_k, x_j)) \leq 1$,
- 2. $\Psi_1(0,0) = 0$ and $\Psi_1(1,1) = 1$

To compute $\mu^t(x_j)$, the function Ψ_1 use two parameters: $\mu^{t-1}(x_i)$ at time t-1 and the weight $\omega_{i,j} = \delta(x_i, \sigma_k, x_j)$ of the transition.

Same examples of Ψ_1 are:

Arithmetic Mean

$$-\mu^{t}(x_{j}) = \Psi_{1}(\mu^{t-1}(x_{i}), \delta(x_{i}, \sigma_{k}, x_{j})),$$

= $Mean(\mu^{t-1}(x_{i}), \omega_{i,j}),$
= $\frac{\mu^{t-1}(x_{i}) + \omega_{i,j}}{2},$

Geometric Mean

$$-\mu^{t}(x_{j}) = \Psi_{1}(\mu^{t-1}(x_{i}), \delta(x_{i}, \sigma_{k}, x_{j})),$$

= $GMean(\mu^{t-1}(x_{i}), \omega_{i,j}),$
= $\sqrt{\mu^{t-1}(x_{i}) \times \omega_{i,j}},$

where $\mu^{t-1}(x_i)$ is the *mv* of the corresponding predecessor of x_j .

The *mv* of each active state is used as the level of activation of each active state and the active state can be multi-membership state. However, each active state must be a single *mv* to use the function Ψ_1 . For this reason, the function Ψ_2 is introduced to compute the single *mv* corresponding to the state that was forced to take several *mv* by these predecessors. The single membership value $\mu^t(x_j)$ of each multi-membership state given by:

$$-\mu^{t}(x_{j}) = \Psi_{2}^{m} [\Psi_{1}(\mu^{t-1}(x_{i}), \omega_{i,j})],$$

where m is the number of simultaneous transitions from states

 x_i, \cdots, x_m to state x_j prior to time t.

The function Ψ_2 should satisfy the minimum requirements following axioms:

1.
$$0 \leq \Psi_{i=1}^{W} [\Psi_{1}(\mu^{t-1}(x_{i}), \omega_{i,j})] \leq 1,$$

2.
$$\Psi(\phi) = 0,$$

3.
$$\Psi_{2}^{W} [\Psi_{1}(\mu^{t-1}(x_{i}), \omega_{i,j})] = \mathbf{v}, \text{ if } \forall (\Psi_{1}(\mu^{t-1}(x_{i}), \omega_{i,j}) = \mathbf{v}).$$

Same examples of Ψ_2 are:

• Maximum multi-membership resolution

$$- \mu^{t}(x_{j}) = \max_{i=1 \text{ to } m} [\Psi_{1}(\mu^{t-1}(x_{i}), \omega_{i,j})],$$

• Arithmetic mean multi-membership resolution

$$- \mu^{t}(x_{j}) = \frac{\left[\sum_{i=1}^{m} \Psi_{1}(\mu^{t-1}(x_{i}), \omega_{i,j})\right]}{m}$$

The problems of diagnosis via fuzzy discrete event system in non-stationary environment can be deal with to imprecise or uncertain factors. For that, we introduce a new definition of the state x_i to deal with to imprecise and/or uncertain factors. In the following, the state x_i is defined by a fuzzy state \tilde{x}_i that can take into account the imprecise or uncertain factors. The definition of the fuzzy state \tilde{x}_i is presented in the next section.

3. MODELING METHOD DUE TO THE VAGUENESS IN NON-STATIONARY ENVIRONMENT

To take into account to the fuzziness and impreciseness, the new definition of the fuzzy state is defined as:

$$\widetilde{x}_i = \left\{ (s_1, \mu_{\widetilde{x}}^t(s_1)), \cdots, (s_p, \mu_{\widetilde{x}}^t(s_p)) \right\} \text{ and } s_p \in S,$$

S is the linguistic value set (label) represented as fuzzy subset of the respective universes of discourse ω . The linguistic values are associated to the fuzzy state $\tilde{x}_i \in X$. Figure 2 shown an example of this association:

 $S = \{s_1, s_2, s_3\}.$



Figure 2. Fuzzy variable

When the entered input prior at time *t* has been σ_k , all active states at this time, are those states to which there is at least

one transition on the input event σ_k (Doostfatemeh & Kremer, 2005). The new definition of active state X_{act} is given by:

$$X_{act}(t) = \left\{ (\widetilde{x}_j, V_{\widetilde{x}_j}^t) \mid \exists (\widetilde{x}_i \in \mathbf{A}_i, \sigma_k \in \Sigma) \land \widetilde{x}_j \in \mathbf{B}_j \right\} \text{ and,}$$

$$V_{\widetilde{x}_i}^t = \left[\mu_{\widetilde{x}_i}^t(s_1) \cdots \mu_{\widetilde{x}_i}^t(s_k) \cdots \mu_{\widetilde{x}_i}^t(s_p) \right],$$

$$X_{act}(t) = \left\{ \left(\widetilde{x}_j, \left[\mu_{\widetilde{x}_i}^t(s_1) \cdots \mu_{\widetilde{x}_i}^t(s_k) \cdots \mu_{\widetilde{x}_i}^t(s_p) \right] \right) \right\},$$
with $\mathbf{A}_i = X_{pred}(\widetilde{x}_j, t)$ and $\mathbf{B}_j = X_{succ}(\widetilde{x}_i, \sigma_k)$ and
$$\left\{ \begin{array}{l} X_{pred}(\widetilde{x}_j, t) = \left\{ \widetilde{x}_i \mid \exists \sigma_k \text{ s.t } \widetilde{x}_j \in \varphi_1(\widetilde{x}_i, \sigma_k) \And \widetilde{x}_j \in X_{act}(t) \right\}, \\ X_{succ}(\widetilde{x}_i, \sigma_k) = \left\{ \widetilde{x}_j \mid \widetilde{x}_j \in \varphi_1(\widetilde{x}_i, \sigma_k) \right\}, \end{array} \right\}$$

where $\delta(\tilde{x}_i, \sigma_k, \tilde{x}_j) = \omega_{i,j}^{\top}$ (vector) and $\tilde{x}_i, \tilde{x}_j \in X$ and $\sigma_k \in \Sigma$. In the following, the membership degree associated to the state \tilde{x}_i at time *t* is denoted as:

$$\mu_{\widetilde{x}_i}^t(s_k)$$

4. PROPAGATION APPROACH OF THE MEMBERSHIP VAL-UES AFTER OCCURRENCE OF A EVENT

To compute the membership degree, we will use the length of the predecessors of each single active state \tilde{x}_j to know if the current state has been forced to take multi-membership or to take only a single membership value. If the length of the predecessors of the active state x_j , *i.e.*, $|X_{pred}(\tilde{x}_j,t)|$, at time *t* is "0" or "1" that mean the active state is not forced to take multi-membership. Hence each $mv \mu_{\tilde{x}_j}^t(s_k)$ of the active state x_j is estimated by the following relation:

$$\begin{cases} \mu_{\widetilde{x}_{j}}^{t}(s_{k}) = \max_{l=1 \ to \ |S|} \left(R_{k,l}^{s_{l}}(\widetilde{x}_{i}) \right), \ s_{k} \in S \ \text{and}, \\ R_{k,l}^{s_{l}}(\widetilde{x}_{i}) = \min \left(\mu_{\widetilde{x}_{i}}^{t-1}(s_{l}), \mu_{\omega_{i,j}}^{t}(s_{k}) \right) \ \text{and}, \\ \boldsymbol{\omega}_{i,j} = \left[\mu_{\omega_{i,j}}^{t}(s_{1}) \cdots \mu_{\omega_{i,j}}^{t}(s_{p}) \right]^{\top}, \end{cases}$$

$$U_{x_{j}}^{t+1} = \begin{pmatrix} R_{1,1}^{s_{1}}(\widetilde{x}_{i}) & R_{1,2}^{s_{2}}(\widetilde{x}_{i}) & \cdots & R_{1,p}^{s_{p}}(\widetilde{x}_{i}) \\ R_{2,1}^{s_{1}}(\widetilde{x}_{i}) & \ddots & \vdots \\ \vdots & \ddots & R_{p-1,p}^{s_{p}}(\widetilde{x}_{i}) \\ R_{p,1}^{s_{1}}(\widetilde{x}_{i}) & \cdots & R_{p,p-1}^{s_{p-1}}(\widetilde{x}_{i}) & R_{p,p}^{s_{p}}(\widetilde{x}_{i}) \end{pmatrix}$$

where |S| is the length of *S* and $R_{k,l}^{s_l}(\tilde{x}_i)$ is the propagation function for the linguistic variable s_l (k = 1 to |S|) for the state \tilde{x}_i .

In Figure 3.(b) we can see how the $mv \mu_{\tilde{x}_i}^t(s_k)$ is estimated

(computed) by using the $mv \ \mu_{\tilde{x}_i}^{t-1}(s_k)$ of the predecessor of the state \tilde{x}_j and the propagation function and the fuzzy vector

(weight)
$$\omega_{i,j} = \left[\mu_{\omega_{i,j}}^{t+1}(s_1) \cdots \mu_{\omega_{i,j}}^{t+1}(s_k) \cdots \mu_{\omega_{i,j}}^{t+1}(s_p) \right]$$
.

Example 1 in Figure 3.(a):

• Suppose at time *t* the active state is:

$$\begin{cases} X_{act}(t) = \left\{ \left(\widetilde{x}_j, V_{\widetilde{x}_j}^t \right) \right\}, \\ V_{\widetilde{x}_j}^t = \left[\mu_{\widetilde{x}_j}^t(s_1) \cdots \mu_{\widetilde{x}_j}^t(s_k) \cdots \mu_{\widetilde{x}_j}^t(s_p) \right], \end{cases}$$

• Predecessor of \tilde{x}_i

$$\begin{cases} X_{pred}(\tilde{x}_{j},t) = \{\tilde{x}_{i}\}, \\ V_{\tilde{x}_{i}}^{t-1} = \left[\mu_{\tilde{x}_{i}}^{t-1}(s_{1})\cdots\mu_{\tilde{x}_{i}}^{t-1}(s_{k})\cdots\mu_{\tilde{x}_{i}}^{t-1}(s_{p})\right], \\ X_{act}(t-1) = \left\{(\tilde{x}_{i},V_{\tilde{x}_{i}}^{t-1})\right\}, \end{cases}$$

However, if the active state \tilde{x}_j is forced,

i.e $|X_{pred}(\tilde{x}_j,t)| > 1$, to take several membership value, then the $mv \ \mu_{\tilde{x}_j}^t(s_k)$ of the state \tilde{x}_j at time *t* is computed by using a new function $\Psi_{s_k}^{\tilde{x}_j}$, named augmentation transition function. But the function $\Psi_{s_k}^{\tilde{x}_j}$ should satisfy the two following axioms.

1.
$$0 \leq \Psi_{s_{\tau}}^{\tilde{x}_{j}} \left(\mu_{\tilde{x}_{i}}^{\tau}(s_{\tau}), \cdots, \mu_{\tilde{x}_{i'}}^{\tau}(s_{\tau}) \right) \leq 1,$$

2.
$$\Psi_{s_{k}}^{\tilde{x}_{j}}(0, \cdots, 0) = 0 \text{ and } \Psi_{s_{k}}^{\tilde{x}_{j}}(1, \cdots, 1) = 1,$$

where \tilde{x}_i and $\tilde{x}_{i'}$ are the predecessors of \tilde{x}_j .

For instance, the function $\Psi_{s_k}^{\tilde{\chi}_j}$ can be:

• Arithmetic mean multi-membership resolution

$$\mu_{\widetilde{x}_{j}}^{t}(s_{\tau}) = \Psi_{s_{k}}^{\widetilde{x}_{j}}\left(\mu_{\widetilde{x}_{i}}^{\tau}(s_{\tau}), \cdots, \mu_{\widetilde{x}_{i'}}^{\tau}(s_{\tau})\right),$$
$$= Mean\left(\mu_{\widetilde{x}_{i}}^{\tau}(s_{\tau}), \cdots, \mu_{\widetilde{x}_{i'}}^{\tau}(s_{\tau})\right)$$

example in Figure 5

$$\begin{split} \mu_{\widetilde{x}_{j}}^{t}(s_{\tau}) &= \Psi_{s_{k}}^{\widetilde{x}_{j}}\left(\mu_{\widetilde{x}_{i}}^{\tau}(s_{\tau}), \mu_{\widetilde{x}_{i'}}^{\tau}(s_{\tau})\right), \\ \mu_{\widetilde{x}_{j}}^{\tau}(s_{\tau}) &= \max_{l=1 \ to \ |S|}\left(R_{\tau,l}^{s_{l}}(\widetilde{x}_{i})\right), \\ R_{\tau,l}^{s_{l}}(\widetilde{x}_{i}) &= \min\left(\mu_{\widetilde{x}_{i}}^{t-1}(s_{l}), \mu_{\omega_{i,j}}^{t}(s_{\tau})\right), \end{split}$$

Maximum multi-membership resolution

$$\begin{split} \mu_{\widetilde{x}_{j}}^{t}(s_{\tau}) &= \Psi_{s_{k}}^{\widetilde{x}_{j}}\left(\mu_{\widetilde{x}_{i}}^{\tau}(s_{\tau}), \cdots, \mu_{\widetilde{x}_{i'}}^{\tau}(s_{\tau})\right), \\ &= Max\left(\mu_{\widetilde{x}_{i}}^{\tau}(s_{\tau}), \cdots, \mu_{\widetilde{x}_{i'}}^{\tau}(s_{\tau})\right), \end{split}$$

Example 2, in Figure 4, we assume that, at time t - 1 the active states are \tilde{x}_i and $\tilde{x}_{i'}$, *i.e.*, $X_{act}(t-1) = {\tilde{x}_i, \tilde{x}_{i'}}$ and we



Figure 3. Membership assignment: Estimation of $mv \mu_{\tilde{x}_j}^t(s_k)$ for the next state \tilde{x}_j at time *t*.

suppose the next input is " σ_r ", then the active state at time *t* is x_j , *i.e.*, $X_{act}(t) = {\tilde{x}_j}$. Hence, the active state \tilde{x}_j at time *t* is forced to take several membership.



Figure 4. Example of *FFA* with an active state that forced to take multi-membership.

$$\begin{cases} \widetilde{x}_i = \left\{ \left(s_1, \mu_{\widetilde{x}_i}^{t-1}(s_1) \right), \cdots, \left(s_p, \mu_{\widetilde{x}_i}^{t-1}(s_p) \right) \right\}, \\ \widetilde{x}_{i'} = \left\{ \left(s_1, \mu_{\widetilde{x}_{i'}}^{t-1}(s_1) \right), \cdots, \left(s_p, \mu_{\widetilde{x}_{i'}}^{t-1}(s_p) \right) \right\}. \end{cases}$$

then, at time *t*, the states \tilde{x}_i and $\tilde{x}_{i'}$ are the predecessors of \tilde{x}_j . In Figure 5, we can see the propagation of *mv* of the states \tilde{x}_i and $\tilde{x}_{i'}$ for the estimation of the active state's *mv* that has been forced to take multi-membership (*i.e.*, state \tilde{x}_j) as shown in Figure 4. The active state at time *t* is given by:

$$\widetilde{x}_j = \left\{ \left(s_1, \mu_{\widetilde{x}_j}^t(s_1) \right), \cdots, \left(s_p, \mu_{\widetilde{x}_j}^t(s_p) \right) \right\}.$$

input σ_r

$$\begin{pmatrix} \mathbf{y}_{i_{1}}^{\mathbf{s}}(\mathbf{s}_{1}) \lor \langle \mathbf{R}_{1,1}^{\mathbf{s}}(\mathbf{x}_{1}) \rangle \\ \mu_{\omega_{i,j}}^{\mathbf{t}}(\mathbf{s}_{1}) \lor \langle \mathbf{R}_{1,k}^{\mathbf{s}}(\mathbf{x}_{1}) \rangle \\ \mu_{\omega_{i,j}}^{\mathbf{t}}(\mathbf{s}_{1}) \lor \langle \mathbf{R}_{1,k}^{\mathbf{s}}(\mathbf{x}_{1}) \rangle \\ \mu_{\omega_{i,j}}^{\mathbf{t}}(\mathbf{s}_{1}) \lor \langle \mathbf{R}_{p,1}^{\mathbf{s}}(\mathbf{x}_{1}) \rangle \\ \vdots & \vdots \\ \mu_{\omega_{i,j}}^{\mathbf{t}}(\mathbf{s}_{1}) \lor \langle \mathbf{R}_{p,1}^{\mathbf{s}}(\mathbf{x}_{1}) \rangle \\ \mu_{\omega_{i,j}}^{\mathbf{t}}(\mathbf{s}_{1}) \lor \langle \mathbf{R}_{p,k}^{\mathbf{s}}(\mathbf{x}_{1}) \rangle \\ \mu_{\omega_{i,j}}^{\mathbf{t}}(\mathbf{s}_{1}) \lor \langle \mathbf{R}_{p,k}^{\mathbf{s}}(\mathbf{x}_{1}) \rangle \\ \mu_{\omega_{i,j}}^{\mathbf{t}}(\mathbf{s}_{1}) \lor \langle \mathbf{R}_{p,k}^{\mathbf{s}}(\mathbf{x}_{1}) \rangle \\ \mu_{\omega_{i,j}}^{\mathbf{t}}(\mathbf{s}_{1}) \lor \langle \mathbf{R}_{1,k}^{\mathbf{s}}(\mathbf{x}_{1}) \rangle \\ \vdots & \vdots \\ \mu_{\omega_{i,j}}^{\mathbf{t}}(\mathbf{s}_{1}) \lor \langle \mathbf{R}_{1,k}^{\mathbf{s}}(\mathbf{x}_{1}) \rangle \\ \vdots & \vdots \\ \vdots \\ \mu_{\omega_{i,j}}^{\mathbf{t}}(\mathbf{s}_{1}) \lor \langle \mathbf{R}_{1,k}^{\mathbf{s}}(\mathbf{x}_{1}) \rangle \\ \mu_{\omega_{i,j}}^{\mathbf{t}}(\mathbf{s}_{1}) \lor \langle \mathbf{R}_{1,k}^{\mathbf{s}}(\mathbf{x}_{1}) \rangle \\ \mu_{\omega_{i,j}}^{\mathbf{t}}(\mathbf{s}_{1}) \lor \langle \mathbf{R}_{1,k}^{\mathbf{t}}(\mathbf{x}_{1}) \rangle$$

Figure 5. Membership assignment, *i.e.*, the *mv* of a state that has been forced to take multi-membership

The active state at time t is \tilde{x}_j , then the active output at this time is y_j .

The active output set is the fuzzy set of all active output (*i.e.*, output labels together with their mv's) at time t denoted as $Y_{act}(t)$, is called the active output set at time t and $Y_{act}(t)$ is given by:

$$Y_{act}(t) = \left\{ \left(y_j, V_{\tilde{x}_j}^t \right) \right\}$$
 and $Y_{act}(\Gamma) = Y_{act}(t_0 + |\Gamma|),$

where $V_{\tilde{x}_j}^t$ is the grade membership of the state \tilde{x}_j at time *t*. Our diagnosis approach use the fuzzy set of output events of the model as input events of the diagnoser module and the diagnoser output are membership degrees of fault related to the faulty component of the system. This is accomplished by using the defuzzification.

5. DIAGNOSIS USING INCOMPLETE MODEL

The research on the fault diagnosis problem for such system with fuzziness is interesting and an important challenge. Furthermore, the transition from one state to another is also vague. The goal of the diagnoser for fuzzy discrete system is to detect and identify the occurrence of a specific behavior of the system.

In this paper, a standard diagnoser is a Finite State Automata (*FSA*) built to detect the occurrence of a specific behavior of \tilde{G} .

Let $\widetilde{G} = \{X, \Sigma, \delta, Y, \widetilde{x}_0, F\}$ be the fuzzy discrete event model for a dynamical system that we want supervise. The set $Y = \left\{ \left(y_0, V_{\widetilde{x}_0}^{t_0}\right), \cdots, \left(y_j, V_{\widetilde{x}_j}^{t}\right) \right\}$ is the fuzzy output of \widetilde{G} .



Figure 6. Diagnoser using output model constituted of a fuzzy set as input events.

The standard diagnoser that we use here, is a *FSA* that takes the output sequence $\Delta_{\widetilde{G}} = y_1 y_2 \cdots$ of \widetilde{G} as its input as shown in Figure 6, with λ : the operating condition of the system that can be normal or abnormal mode.

The diagnoser $D_{\widetilde{G}}$ of the model G is given by:

$$D_{\widetilde{G}} = (Z, Y, \zeta, \lambda, \widetilde{z}_0, \Omega),$$

with

• Z is the set of standard diagnoser state,

- *Y* is the set of standard diagnoser input, we recall, *Y* is the output of model *G*,
- λ is the set of standard diagnoser output,
- $\zeta: Z \times Y \times \to Z \times \lambda$ is the standard diagnoser state transition function,
- \tilde{z}_0 is the start state set of the standard diagnoser,
- $\Omega \in Z$ is the (non-empty) set of terminal states

Let ζ_1 and ζ_2 be the two projections of ζ of $D_{\widetilde{G}}$, with ζ_1 and ζ_2 are given by

$$\begin{cases} \zeta_1(\widetilde{z}_k, y_{k+1}) = \{\widetilde{z}_{k+1} \mid \exists \lambda_i \land (\widetilde{z}_{k+1}, \lambda_i) \in \zeta(\widetilde{z}_k, y_{k+1})\}, \\ \zeta_2(\widetilde{z}_k, y_{k+1}) = \{\lambda_i \mid \exists \widetilde{z}_{k+1} \land (\widetilde{z}_{k+1}, \lambda_i) \in \zeta(\widetilde{z}_k, y_{k+1})\}. \end{cases}$$

with $\lambda_i = \lambda(\tilde{z}_{k+1})$ and $\tilde{z}_k \subseteq Z$ is the estimate state of $D_{\tilde{G}}$ at time *k*.

The diagnoser state transition is given by

$$\begin{cases} (\tilde{z}_{k+1}, \lambda(\tilde{z}_{k+1})) = \zeta(\tilde{z}_k, y_{k+1}), \\ \lambda(\tilde{z}_{k+1}) = \zeta_2(\tilde{z}_k, y_{k+1}), \\ \tilde{z}_{k+1} = \zeta_1(\tilde{z}_k, y_{k+1}), \\ = X_{succ}(\tilde{z}_k, \stackrel{all}{\to}) \cap \zeta_1(\tilde{z}_k, y_{k+1}), \end{cases}$$

If the properties of the system are not sufficiently known, a learning diagnoser is used for the fault diagnosis of the system. A learning diagnoser is a standard diagnosis that tolerates missing information (*i.e.*, transitions and states) about the system to be diagnosed. The learning diagnoser must be able to learn the true model of the system \tilde{G} , when missing information about the system are presented.

Let $D_{\widetilde{G}_n}$ be the nominal diagnoser created from the nominal model \widetilde{G}_n of the system. Suppose that we have the nominal model \widetilde{G}_n and the output sequence $(y_0y_1y_2\cdots)$. Then, \widetilde{G}_n is consistent with the output sequence if

 $\widetilde{z}_{k+1} = X_{succ}(\widetilde{z}_k, \stackrel{all}{\rightarrow}) \cap \zeta_1(\widetilde{z}_k, y_{k+1}) \neq \emptyset \ \forall k \ge 1.$ Otherwise, \widetilde{G}_n is inconsistent with the output sequence. When \widetilde{G}_n is inconsistent with the output sequence, then

 $X_{succ}(\tilde{z}_k, \stackrel{all}{\rightarrow}) \cap \zeta_1(\tilde{z}_k, y_{k+1}) = \emptyset$ for some *k*, causing the diagnoser to fail.

Let σ_{new} be a new event detected and not found in Σ of \widetilde{G}_n , then the new set of input events of \widetilde{G}_n is given by

$$\Sigma_{new} = \Sigma \cup \{\sigma_{new}\}.$$

A transition $x_i \xrightarrow{\sigma_{new}/\omega_{i,j}} x_j$ is ordered pair of state denoting a transition from the state x_i to the state x_j . Let φ' be the extend

function transition of φ_{new} of the system \widetilde{G}_n such that

$$\varphi_{new}(x_i, \sigma_i) = \begin{cases} x_j \text{ if } \sigma_i = \sigma_{new} & \& \\ \varphi_1(x_i, \sigma_i) \text{ otherwise,} \end{cases} \begin{cases} \Sigma : \Sigma \leftarrow \sigma_{new}, \\ \text{and} \\ X : X \leftarrow x_j \text{ if } x_j \notin X, \end{cases}$$

Let be a dynamic model \widetilde{G}' of \widetilde{G}_n defines as

$$\widetilde{G}' = extend(\widetilde{G}_n, X', \Pi) = (X \cup X', \Sigma \cup \Pi, Y, \varphi_{new}, \widetilde{x}_0),$$

and \widetilde{G}' is called the extension of \widetilde{G}_n by X' and Π , with X' is the set containing all new states and Π is the set containing all new transitions found. The weight $\omega_{i,j}$ for the new transition detected is estimated by the expert of the system. The set transition Π is empty, if the model \widetilde{G} of the system is consistent with the output sequence.

The output of a fuzzy system should be defuzzified in an appropriate way to be usable by the environment.

6. ALGORITHM OF A LEARNING DIAGNOSER

The algorithm presented in (Algorithm 1) is a learning algorithm that allows to add new transitions and/or states. All the new transition and state are validated by an expert of the system. The newly proposed diagnoser approach in the algorithm allows us to deal with the problem of failure diagnosis for fuzzy discrete event system, which many better deal with the problem of fuzziness, impreciseness and uncertainness in fault diagnosis. Before the presentation of the algorithm we are going to give the definition of each step.

Let σ_k be the event red by the system at time *t*, then

$$\begin{cases} \widetilde{x}_j = \boldsymbol{\varphi}_1(\widetilde{x}_i, \boldsymbol{\sigma}_k), \\ y_j = y_{k+1} \\ = \boldsymbol{\varphi}_2(\widetilde{x}_i, \boldsymbol{\sigma}_k), \end{cases}$$

From these two above relations $X_{succ}(\tilde{x}_i, \sigma_{k+1})$ and $X_{pred}(\tilde{x}_j, t)$ are computed by:

$$\begin{cases} X_{succ}(\widetilde{x}_i, \sigma_{k+1}) = \{ \forall \, \widetilde{x}_s \in X \mid \widetilde{x}_s \in \varphi_1(\widetilde{x}_i, \sigma_{k+1}) \} \\ X_{pred}(\widetilde{x}_j, t) = \{ \forall \, \widetilde{x}_i \in X \mid \widetilde{x}_j \in \varphi_1(\widetilde{x}_i, \sigma_k) \} \end{cases}$$

If the intersection $X_{succ}(\tilde{x}_i, \sigma_k) \cap \zeta_1(z_k, y_{k+1})$ is not empty, that means, in the model the transition from state \tilde{x}_i to \tilde{x}_j . Otherwise the diagnoser detect a new transition and/or state when the intersection $X_{succ}(\tilde{x}_i, \sigma_k) \cap \zeta_1(z_k, y_{k+1})$ is empty. The definition of $X_{succ}(\tilde{x}_i, \sigma_k)$ and $\zeta_1(z_k, y_{k+1})$ are given respectively in section 2 and 5. When we are in the case where

 $X_{succ}(\widetilde{x}_i, \sigma_k) \cap \zeta_1(z_k, y_{k+1}) \neq \emptyset$, the number of the states in $X_{pred}(\widetilde{x}_j, t)$, *i.e.*, $X_{act}(t-1)$ is used to estimate the membership degree of the active states as explained in section 4. The

complexity of this algorithm is that we can have a explosion of the transitions and/or state if the nominal model is not well estimated (build).

7. APPLICATION TO CRISIS MANAGEMENT

Crisis management is a special type of collaboration involving several different groups and actors. The challenge is how to handle the coordination and interactions between these different involved groups and actors during the crisis management and to detect abnormalities (e.g., critical process deviations, evolution towards dangerous or blocked situations, etc.) on-line or to predict the evolution of the current situation towards a dangerous or critical state.

During the crisis management, the capacity to take fast and efficient decisions is a very important challenge for a better exit of crisis. Because, the context and characteristics of crisis such as extent of actors and roles, the management becomes more difficult in order to take efficient decisions, but also to exchange information or to coordinate different groups involved. The difficult to take a decision can be also due to random factors, such as stress, emotional impact, road conditions, weather conditions, etc. For this reason, it is important to integrate these factors in the model of crisis management for decision-making. The FFA presented above is used to takes into account the stress of the actors involved in the crisis management.

7.1. FFA model of crisis management

Here, we propose a model (no generic model) applied on the team SAMU¹ from Hospital of Troyes in France, during TEAN² exercise.

The team of SAMU is composed of the following actors:

- Rear Base ³ (*RB*): Operations Coordination,
- Communication Center (*CC*): collecting information and sharing with *RB*,
- First Team: first intervention, sending the first evaluation (result) about the crisis to the *CC*,
- Advanced Medical Post (*AMP*): Intervention and evacuation of victims, sending the complete evaluation to the *CC*.

The FSA of the TEAN exercise is shown in Figure 7.

The discrete event model showed in Figure 7 for *TEAN* exercise, allows one hand to monitor the communication and coordination between various groups involved in crisis management, and also to supervise some specific behaviors that

while input is σ_k and active state time t - 1 is x_i **do** | read symbol σ_k ;

 $\widetilde{x}_i = \boldsymbol{\varphi}_1(\widetilde{x}_i, \boldsymbol{\sigma}_k);$ $y_j = y_{k+1} = \boldsymbol{\varphi}_2(\widetilde{x}_i, \boldsymbol{\sigma}_k);$ $X_{succ}(\widetilde{x}_i, \sigma_{k+1}) = \{ \forall \ \widetilde{x}_s \in X \mid \widetilde{x}_s \in \varphi_1(\widetilde{x}_i, \sigma_{k+1}) \} ;$ **if** \widetilde{x}_i is the start state, i.e \widetilde{x}_0 **then** $X_{pred}(\widetilde{x}_i,t) = \emptyset;$ else $X_{pred}(\widetilde{x}_i, t) = \{ \forall \, \widetilde{x}_i \in X \mid \widetilde{x}_i \in \varphi_1(\widetilde{x}_i, \sigma_k) \};$ end if $(X_{succ}(\widetilde{x}_i, \sigma_k) \cap \zeta_1(z_k, y_{k+1}) \neq \emptyset)$ then if $(|X_{pred}(\widetilde{x}_j, t)| = \emptyset)$ then $|X_{act} = \widetilde{x}_0;$ $X_{succ}(\widetilde{x}_0, a_k) = \left\{ \forall \, \widetilde{x}_s \in X \mid \widetilde{x}_s \in \varphi_1(\widetilde{x}_j, a_k) \right\};$ else if $(|X_{pred}(\widetilde{x}_j,t)| = 1)$ then $\mu_{\widetilde{x}_i}^t(s_k) = \max_{l=1 \ to \ |S|} (r_{lk}(\widetilde{x}_i))$ and $r_{l,k}(\widetilde{x}_i) = \min\left(\mu_{\widetilde{x}_i}^{t-1}(s_l), \mu_{\omega_{i,j}}^t(s_k)\right);$ $V_{\widetilde{x}_{i}}^{t} = \left[\mu_{\widetilde{x}_{i}}^{t}(s_{1}) \cdots \mu_{\widetilde{x}_{i}}^{t}(s_{p}) \right];$ $X_{act} = \left\{ \left(\widetilde{x}_j, V_{\widetilde{x}_j}^t \right) \right\} \text{ and } \\ X_{succ}(\widetilde{x}_j, a_k) = \left\{ \forall \ \widetilde{x}_s \in X \mid \widetilde{x}_s \in \varphi_1(\widetilde{x}_j, a_k) \right\};$ else active state have been forced to take different several mv; $\boldsymbol{\mu}_{\widetilde{x}_{i}}^{t+1}(s_{\tau}) = \Psi_{s_{k}}^{\widetilde{x}_{j}}\left(\boldsymbol{\mu}_{\widetilde{x}_{i}}^{\tau}(s_{\tau}), \cdots, \boldsymbol{\mu}_{\widetilde{x}_{i'}}^{\tau}(s_{\tau})\right);$ $\begin{aligned} X_{act} &= \left\{ \left(\widetilde{x}_j, V^t(\widetilde{x}_j) \right\} ; \\ X_{succ}(\widetilde{x}_j, a_k) &= \left\{ \forall \, \widetilde{x}_s \in X \mid \widetilde{x}_s \in \varphi_1(\widetilde{x}_j, a_k) \right\} ; \end{aligned}$ end Diagnoser method; the active diagnoser state is: $z_k \in D_k$; else go to inconsistency; detection of new transition and/or state; $X_{succ}(\widetilde{x}_i, \sigma_k) \cap \zeta_1(z_k, y_{k+1}) = \emptyset;$ we suppose for all new transition; $\delta(\widetilde{x}_i, \sigma_k, \widetilde{x}_i) = 0;$ if $(\tilde{x}_i \in X \& \sigma_k \in \Sigma)$ then *new transition between* \tilde{x}_i (*past state*) to \tilde{x}_i (active state); else if $\widetilde{x}_i \in X \& \sigma_k \notin \Sigma$ then update Σ ; $\Sigma \leftarrow \sigma_k$; else update X and Σ ; $X \leftarrow \widetilde{x}_i$; $\Sigma \leftarrow \sigma_k$; end end end



¹SAMU is Service emergency medical assistance.

²TEAN is the name of the exercise.

³Other word, Rear Base is decision makers



Figure 7. A example of modelisation of a scenario of crisis with finite state automaton and the weight corresponds to the stress of actors involved.

are critical situations. Thus, the factor's stress of the actors involved is estimated for decision-making.

Consider the *FFA* in Figure 7 with several transition overlaps and several output labels. It is specified as:

$$\widetilde{G}_n = (X, \Sigma, \delta, Y, \widetilde{x}_0, F),$$

The dashed line in Figure 7, between states 6 and 7 represents a critical event. The occurrence of event "f" bring the system in a critical mode corresponding to state \tilde{x}_7 and $\omega_{i,j}$ is the stress of actors involved in crisis management.

In this example:

$$\begin{split} X &= \{\widetilde{x}_0, \widetilde{x}_1, \cdots, \widetilde{x}_7\}, \text{ the set of fuzzy states,} \\ \Sigma &= \{a, b, c, d, e, f, g, h\}, \text{ set of input symbols,} \\ Y &= \{y_1, y_2, y_3, y_4, y_6, y_7\}, \text{ set of output events,} \\ \widetilde{x}_0 &= \left\{ (L, \mu_{\widetilde{x}_0}^{t_0}(L)), (M, \mu_{\widetilde{x}_0}^{t_0}(M)), (H, \mu_{\widetilde{x}_0}^{t_0}(H)) \right\}, \text{ starting state,} \\ S &= \{\text{Low}(L), \text{ Moderate}(M), \text{ High}(H)\}, \text{ Stress of actors,} \end{split}$$

$$\lambda(\widetilde{x}_i) = \begin{cases} F_1, & \text{if } i=7, \\ N, & otherwise. \end{cases}$$

and the definitions of the states \tilde{x}_i , output y_i and the events are given respectively in tables 1, 2 and 3.

States	Definition
<i>x</i> ₀	No crisis
<i>x</i> ₁	Onset Crisis
<i>x</i> ₂	Communication center (CC)
<i>x</i> ₃	Police men
<i>x</i> ₄	Emergency department
<i>x</i> ₅	Advanced Medical Post (AMP)
<i>x</i> ₆	Accident area
<i>x</i> ₇	The model is unpredictable for this crisis situation

Table 1. List and definition of states.

Output labels	Definition
<i>y</i> ₀	No coming call
<i>y</i> 1	Accident is happen
<i>y</i> ₂	Information arrived to CC
<i>y</i> ₃	Information arrived to police office
У4	Preparation of the Intervention Team
У5	Preparation of the AMP agent
У6	New Actors arrived in the accident area
<i>y</i> ₇	uncontrolled situations (conditions)

Table 2. List and definition of outputs.

events	Definition
a	A call from (or about) a accident
b	Sending Team to the accident site
С	Sending information to CC and police office
d	Sending information to Emergency
е	Sending the first evaluation to CC
h	Sending final evaluation to CC
f	End of crisis management without success
g	End of crisis management with success

Table 3. List and definition of the transitions (events).

we suppose at the beginning $\mu_{\tilde{x}_0}^{t_0}(L) = 0.5$, $\mu_{\tilde{x}_0}^{t_0}(M) = 0.65$, $\mu_{\tilde{x}_0}^{t_0}(H) = 0.9$, that mean the stress level estimated at time t_0 associated to $\tilde{x}_0 = \{(L, 0.5), (M, 0.65), (H, 0.9)\}$ may be "Low" with possibility 0.5, and "Medium" with possibility 0.65, and "High" with possibility 0.9. Thus, all the other *mv* are computed by using approaches presented in section 2.

Assuming that \widetilde{G}_n starts operating at time t_0 and the next three input are "**a**" respectively (one at a time), active states and their mv's at each time step are as follows.

at time
$$t_0$$

 $X_{act}(t_0) = \{(\widetilde{x}_0, V^{t_0}(\widetilde{x}_0))\}$ with
 $V_{\widetilde{x}_0}^{t_0} = \begin{bmatrix} \mu_{\widetilde{x}_0}^{t_0}(L) & \mu_{\widetilde{x}_0}^{t_0}(M) & \mu_{\widetilde{x}_0}^{t_0}(H) \end{bmatrix},$
 $\begin{cases} X_{succ}(\widetilde{x}_0, a) = \widetilde{x}_1, \\ X_{succ}(\widetilde{x}_0, \stackrel{all}{\rightarrow}) = \{\widetilde{x}_1\}. \end{cases}$
(1)

 $X_{succ}(\widetilde{x}_j, \stackrel{all}{\rightarrow})$ is the set of all successors of state \widetilde{x}_j ,

at time
$$t_1$$
, input is "a"
 $X_{act}(t_1) = \left\{ \left(\tilde{x}_1, V_{\tilde{x}_1}^{t_1} \right) \right\},$
 $Y_{act}(t_1) = \left\{ \left(y_1, V_{\tilde{z}_1}^{t_1} \right) \right\},$ and here $V_{\tilde{z}_1}^{t_1} = V_{\tilde{x}_1}^{t_1}$
 $X_{pred}(\tilde{x}_1, t_1) = \tilde{x}_0$, and $|X_{pred}(\tilde{x}_1, t_1)|$ is the number of pre-
decessors of active state \tilde{x}_1 . $|X_{pred}(\tilde{x}_j, t)| = 1$, then, the
active state \tilde{x}_1 is not forced to take multi-membership.
Hence, the *mv* of the active state \tilde{x}_1 is computed by:

$$\begin{pmatrix} \mu_{\widetilde{x}_{1}}^{t_{1}}(s_{k}) = \max_{l=1 \ to \ 3} \left(r_{lk}(\widetilde{x}_{i}) \right), \ s_{1} = L, \ s_{2} = M, \ s_{3} = H \text{ and}, \\ r_{l,k}(\widetilde{x}_{i}) = \min \left(\mu_{\widetilde{x}_{i}}^{t}(s_{l}), \mu^{t+1}(s_{k}) \right) \& \mu^{t}(s_{j}) \in \omega_{i,j}, \text{ and} \\ \omega_{0,1} = [0.75 \ 0.3 \ 0.2]^{\top}, \text{ stress factor at time } t_{1}$$



Figure 8. Propagation of membership value of the state \tilde{x}_0

After the propagation of $V_{\tilde{x}_0}^{t_0}$ shown in figure 8, we get $\tilde{x}_1 = \{(L, 0.75), (M, 0.3), (H, 0.2)\}$

 $X_{succ}(\widetilde{x}_1,c) = \{\widetilde{x}_2,\widetilde{x}_3\},\$

7.2. Diagnoser model of TEAN exercise

The standard diagnoser for the fuzzy discrete event system of crisis management model illustrated in Figure 7 is shown in Figure 9, with $\tilde{z}_0 = {\tilde{x}_0}$. Each state of the diagnoser $D_{\tilde{G}_n}$, shown as a rounded box in Figure 9, is a set of states of the system. An output symbol corresponding to the operating condition of the system is associated with each diagnoser state. For example, to see the importance of having a complete model for the diagnoser, we suppose at time t_1 the output sequence " y_0y_1'' (see Figure 7) is observed, then the state estimate is $\tilde{z}_1 = {\tilde{x}_1}$ and the operating condition from \tilde{z}_0 is $\lambda(\tilde{z}_1) = N$. The successors of state estimate \tilde{z}_1 is: $Z_{succ}(\tilde{z}_1) = {\tilde{z}_2, \tilde{z}_3} = {\tilde{x}_2, \tilde{x}_3}$. If the next output symbol y_{t+1} is anything other than y_2 and y_3 , we get

$$Z_{succ}(\widetilde{z}_1) = X_{succ}(\widetilde{z}_1, \overset{all}{\rightarrow}) \cap \zeta_1(\widetilde{z}_1, y_{t+1}) = \emptyset,$$

that means the observation generated after y_k is inconsistent with the model dynamic and the diagnoser cannot proceed. When the output sequence is inconsistent with the system's model, then we have to revise the model of \tilde{G}_n by adding new state(s) and/or new transition(s) respectively in X and Σ , that we believe are missing in the nominal model. This situation may be interpreted as a normal or abnormal situation, because we add new states and/or transitions. Detecting and adding new states and/or transitions in X and/or in Σ of G is called learning diagnoser.



Figure 9. Diagnoser of fuzzy discrete event system model shown in Figure 7.

8. CONCLUSION AND PERSPECTIVES

In this paper, we have presented the definition of a fuzzy discrete event system and we presented the main advantage of fuzzy automaton, to handle imprecise and uncertain data in non-stationary environment. Our approach uses the fuzzy set of output events of the model as the input events of the diagnoser. We have formalized the construction of the learning diagnoser based on evolving fuzzy finite state automaton that are used to perform fuzzy diagnosis. In particular, we have proposed a learning diagnoser approach based on evolving fuzzy finite state automaton that allows to add new transitions and states. The newly proposed diagnoser approach allows us to deal with the problem of failure diagnosis for fuzzy discrete event system, which many better deal with the problem of fuzziness, impreciseness and uncertainness in the failure diagnosis. The approach presented in this paper has been to real case of crisis management.

Future research will focus on the development of fault diagnosis by using fuzzy finite automaton, that takes more than one random factor. Thus, in our future work we will consider the diagnoser with partial observability about the input events.

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Dr. Moussa Traore is a Postdoctoral Researcher at the University of Waterloo (ON), Canada. He is working under the supervision of Prof. Sebastian Fischmeister within the Real-Time Embedded Software Group at the Department of Electrical and Computer Engineering of the University of Waterloo. Prior to joining the Real-Time Embedded Software Group, he was a Postdoctoral Researcher at the University of Technology of Troyes, France in 2014. Before that, he was a Postdoctoral Researcher at the University of Reims, France in 2013. He obtained his Ph.D degree in Computer science, Control and Signal processing from the Department of Automatic Control and Computer Science (Informatique Automatique IA), during a collaborative framework between the Ecole Nationale Suprieure des Mines de Douai and the University Lille 1, France. He holds a Master of Science, Optimization and safety of functioning Systems, from the University of Technology of Troyes, France in 2006 and a Bachelor's degree in Electrical Engineering from the Faculty of Sciences and Technology of Nouakchott, Mauritania in 2004. His research area is Diagnostic (Anomaly detection) and Prognostic of dynamic systems operating in nonstationary environments, for continuous and discrete systems. His research work allowed him to publish several journal papers and international conference papers in the area of anomaly detection, prognosis and predictive maintenance. He served as International reviewer for several International Conferences. Dr. Hossam A. Gabbar Dr. Hossam A. Gabbar is Associate Professor in the Faculty of Energy Systems and Nuclear Science, and cross appointed in the Faculty of Engineering and Applied Science, University of Ontario Institute of Technology (UOIT). He obtained his Ph.D. degree (Safety Engineering) from Okayama University (Japan), while his undergrad degree (B.Sc.) is in the area of automatic control from Alexandria University, Egypt. He is specialized in safety and control engineering where he worked in process control and safety in research and industrial projects in Japan and Canada. Since 2004, he was tenured Associate Professor in the Division of Industrial Innovation Sciences at Okayama University, Japan. And from 2001, he joined Tokyo Institute of Technology and Japan Chemical Innovative Institute (JCII), where he participated in national projects related to advanced distributed control and safety design and operation synthesis for green energy and production systems. He developed new methods for automated control recipe synthesis and verification, safety design, and quantitative and qualitative fault simulation.

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