On discrete-time observer structures for fault estimation

Dusan Krokavec, Anna Filasová, and Pavol Liščinský

Technical University of Kosice, Kosice, 042 00, Slovakia
dusan.krokavec@tuke.sk, anna.filasova@tuke.sk, pavol.liscinsky@tuke.sk

ABSTRACT

In the paper there is proposed a modified technique for system faults estimation in linear dynamic systems, which gives the possibility simultaneously estimate the system state and faults. Using a discrete-time observer form, the considered faults are assumed to be additive, thereby the principle can be applied for a broader class of time-varying fault signals. An enhanced algorithm is provided to verify stability of the observer with improved performance of fault estimation. Exploiting the given procedure the proposed technique allows to obtain signals that can be further used for thresholds setting in the fault residual scheme. The approach utilizes the measurable output vector variables and the design conditions are based on linear matrix inequality technique.

1. INTRODUCTION

Operating conditions in modern engineering systems are still exposed to possibility of system failure. Sensors and actuators failures or other system components malfunctions, can drastically change the system behavior. Fault tolerant control allows a strategy to improve reliability of the whole system and so many techniques have been proposed especially for sensor and actuator failures. In some cases, fault estimation strategies only are needed to carry on controlling the faulty system and so many sophisticated modifications have been developed, e.g., sliding mode observers, neural network based approaches and adaptive observer technique. The recent bibliographical reviews (Ding 2013, Mahmoud and Xia 2014) give the actual state of the art in this field.

Since the observer-based approach is limited in sensor fault estimation, the principles based on adaptive observers are frequently used to estimate actuator faults for the linear time-invariant systems without external disturbance. The structure makes estimation of actuator faults and the system states simultaneously by integrating the system output errors (Patton and Klinkhieo 2009). Often a derivative term is added which renders that the adaptive fault estimation algorithm can be treated as a special case of the fast adaptive fault estimation algorithm (Zhang, Jiang and Cocquempot 2008). Based on the adaptive fault estimation observer, the augmented structures were constructed to be applied by using actuator fault estimation (Krokavec and Filasová 2015, Shi and Patton 2014). Evidently, the basic principle was appropriately modified to be applied for linear systems with time delays (Filasová, Gontkovič and Krokavec 2013, Zhang and Jiang 2008), a class of nonlinear systems described by Takagi-Sugeno fuzzy models (Krokavec and Filasová 2013) as well as linear stochastic Markovian jumping systems (He 2014, He and Liu 2012). Some generalizations can be found, e.g., in Zhang, Jiang and Shi 2013.

Dealing with the discrete-time system models, the simultaneous system state and actuator fault estimation is done in such a way that the estimation error converges to zero and at the same time the effect of the disturbance input on the fault estimation error is attenuated (Jiang and Chowdhury 2005, Wang, Rodrigues, Theilliol and Shen 2015). Adapting the approaches presented in Tabatabaeipour and Bak 2014, Witczak, Zegar and Pazera 2016, the proposed design method extends the simultaneous principle for actuator fault estimation. Following examination of the model-based fault estimation schemes, an enhanced algorithm using H_∞ approach are provided. The applied enhanced design conditions give extended frameworks for actuator fault estimation in simultaneous discrete-time observer structures. The approach utilizes the measurable output vector variables, design conditions are based on linear matrix inequality (LMI) technique, giving an effective way to calculate the observer parameters and to minimize unknown disturbance effect by solving the problem within LMI constraints. The estimates can be used to compensate fault effects in fault tolerant control structures.

Throughout the paper the following notation is used: $\boldsymbol{x}^T$, $\boldsymbol{X}^T$ denotes the transpose of the vector $\boldsymbol{x}$ and the matrix $\boldsymbol{X}$, respectively, diag$[\,\cdot\,]$ enters up a block diagonal matrix, for a square matrix $\boldsymbol{X} < 0$ means that $\boldsymbol{X}$ is a symmetric negative definite matrix, the symbol $\boldsymbol{I}_n$ indicates the n-th order unit matrix, $I\!R$ notes the set of real numbers, and $I\!R^n$, $I\!R^{n\times r}$ refer to the set of all n-dimensional real vectors and $n \times r$ real matrices, respectively.

2. PROBLEM FORMULATION

A linear discrete-time multi-input, multi-output (MIMO) system in presence of an unknown fault can be described by the state-space equations in the following form

$$\boldsymbol{q}(i+1) = \boldsymbol{F}\boldsymbol{q}(i) + \boldsymbol{G}\boldsymbol{u}(i) + \boldsymbol{H}\boldsymbol{f}(i) + \boldsymbol{V}\boldsymbol{d}(i) \tag{1}$$

$$\boldsymbol{y}(i) = \boldsymbol{C}\boldsymbol{q}(i) \tag{2}$$

where $\boldsymbol{q}(i) \in I\!R^n$, $\boldsymbol{u}(i) \in I\!R^r$, and $\boldsymbol{y}(i) \in I\!R^m$ are vectors of the system, input and output variables, respectively, $\boldsymbol{f}(i) \in I\!R^s$ is the unknown fault vector, $\boldsymbol{d}(i) \in I\!R^p$ is the unknown disturbance, $\boldsymbol{F} \in I\!R^{n\times n}$ is the system dynamic matrix, $\boldsymbol{H} \in I\!R^{n\times s}$ is the fault input matrix, $\boldsymbol{V} \in I\!R^{n\times p}$ is the disturbance input matrix, and $\boldsymbol{G} \in I\!R^{n\times r}$ and $\boldsymbol{C} \in I\!R^{p\times n}$ are the system input and output matrices.

It is assumed that the couple $(\boldsymbol{A}, \boldsymbol{C})$ is observable, changes of fault vector are bounded, i.e., $|\Delta\boldsymbol{f}(i)| \le \boldsymbol{\Delta}_{fmax} < \infty$,

$$\Delta\boldsymbol{f}(i) = \boldsymbol{f}(i{+}1) - \boldsymbol{f}(i) \tag{3}$$

and the value of $\boldsymbol{f}(i)$ is set to zero until a fault occurs.

To estimate faults and the system states simultaneously, the PD state estimator is proposed as

$$\begin{aligned}\boldsymbol{q}_e(i+1) = \boldsymbol{F}\boldsymbol{q}_e(i) + \boldsymbol{G}\boldsymbol{u}(i) + \boldsymbol{H}\boldsymbol{f}_e(t) + \\ +\boldsymbol{J}(\boldsymbol{y}(i) - \boldsymbol{y}_e(i)) + \boldsymbol{L}(\Delta\boldsymbol{y}(i) - \Delta\boldsymbol{y}_e(i))\end{aligned} \tag{4}$$

$$\boldsymbol{y}_e(i) = \boldsymbol{C}\boldsymbol{q}_e(i) \tag{5}$$

where

$$\Delta\boldsymbol{y}(i) = \boldsymbol{y}(i+1) - \boldsymbol{y}(i),\ \Delta\boldsymbol{y}_e(i) = \boldsymbol{y}_e(i+1) - \boldsymbol{y}_e(i) \tag{6}$$

$\boldsymbol{q}_e(i) \in I\!R^n$ is the state observer vector, $\boldsymbol{f}_e(i) \in I\!R^s$ is an estimation of the fault $\boldsymbol{f}(i)$, $\boldsymbol{y}_e(i) \in I\!R^m$ is the vector of estimated output variables, and $\boldsymbol{J}, \boldsymbol{L} \in I\!R^{n\times p}$ are the estimator gain matrices, while $n > m$. Using Eqs. (5), (6), then Eq. (4) can be modified as

$$\begin{aligned}\boldsymbol{q}_e(i+1) = \boldsymbol{F}\boldsymbol{q}_e(i) + \boldsymbol{G}\boldsymbol{u}(i) + \boldsymbol{H}\boldsymbol{f}_e(t) + \\ +(\boldsymbol{J} - \boldsymbol{L})\boldsymbol{C}\boldsymbol{e}_q(i) + \boldsymbol{L}\boldsymbol{C}\boldsymbol{e}_q(i+1)\end{aligned} \tag{7}$$

$$\boldsymbol{e}_q(i) = \boldsymbol{q}(i) - \boldsymbol{q}_e(i), \quad \boldsymbol{e}_y(i) = \boldsymbol{y}(i) - \boldsymbol{y}_e(i) = \boldsymbol{C}\boldsymbol{e}_q(i) \tag{8}$$

The observer is combined with the fault estimation update

$$\Delta\boldsymbol{f}_e(i) = \boldsymbol{M}(\boldsymbol{y}(i) - \boldsymbol{y}_e(i)) + \boldsymbol{N}(\Delta\boldsymbol{y}(i) - \Delta\boldsymbol{y}_e(i)) \tag{9}$$

$$\Delta\boldsymbol{f}_e(i) = \boldsymbol{f}_e(i+1) - \boldsymbol{f}_e(i) \tag{10}$$

and $\boldsymbol{M}, \boldsymbol{N} \in I\!R^{s\times m}$ are the gain matrices.

Analogously, it can be written

$$\Delta\boldsymbol{f}_e(i) = (\boldsymbol{M} - \boldsymbol{N})\boldsymbol{C}\boldsymbol{e}_q(i) + \boldsymbol{N}\boldsymbol{C}\boldsymbol{e}_q(i+1) \tag{11}$$

and, defining $\boldsymbol{e}_f(i) \in I\!R^s$ as follows

$$\boldsymbol{e}_f(i) = \boldsymbol{f}(i) - \boldsymbol{f}_e(i) \tag{12}$$

then

$$\begin{aligned}\boldsymbol{e}_f(i+1) &= \boldsymbol{f}(i+1) - \boldsymbol{f}_e(i+1) = \\ &= \boldsymbol{f}(i+1) - \boldsymbol{f}(i) + \boldsymbol{f}(i) - \boldsymbol{f}_e(i+1) = \\ &= \Delta\boldsymbol{f}(i) + \boldsymbol{f}(i) - \boldsymbol{f}_e(i) - \Delta\boldsymbol{f}_e(i) = \\ &= \boldsymbol{e}_f(i) - (\boldsymbol{M} - \boldsymbol{N})\boldsymbol{C}\boldsymbol{e}_q(i) - \boldsymbol{N}\boldsymbol{C}\boldsymbol{e}_q(i+1) + \Delta\boldsymbol{f}(i)\end{aligned} \tag{13}$$

The task is to design the matrices $\boldsymbol{J}, \boldsymbol{L}, \boldsymbol{M}, \boldsymbol{N}$ in such a way that the observer will be bounded stable and $\boldsymbol{f}_e(i)$ will approximate a varying fault $\boldsymbol{f}(i)$. Actuator faults are represented as an exchange of the matrix $\boldsymbol{H}$ to $\boldsymbol{G}$ for $s = r$.

3. THE OBSERVER-BASED FAULTS ESTIMATION

Lemma 1 *A descriptor form of the PD observer-based discrete-time fault estimator can be constructed as follows*

$$\boldsymbol{E}^\bullet\boldsymbol{e}^\bullet(i{+}1) = \boldsymbol{F}^\bullet\boldsymbol{e}^\bullet(i) + \boldsymbol{V}^\bullet\boldsymbol{d}^\bullet(i) \tag{14}$$

$$\boldsymbol{e}_y(i) = \boldsymbol{C}^\bullet\boldsymbol{e}^\bullet(i) \tag{15}$$

where

$$\boldsymbol{e}^{\bullet T}(i) = \begin{bmatrix} \boldsymbol{e}^{\circ T}(i) & \boldsymbol{e}^{\circ T}(i{+}1) \end{bmatrix},\ \boldsymbol{d}^{\bullet T}(i) = \boldsymbol{d}^{\circ T}(i) \tag{16}$$

$$\boldsymbol{e}^{\circ T}(i) = \begin{bmatrix} \boldsymbol{e}_q^T(i) & \boldsymbol{e}_f^T(i) \end{bmatrix},\ \boldsymbol{d}^{\circ T}(i) = \begin{bmatrix} \boldsymbol{d}^T(i) & \Delta\boldsymbol{f}^T(i) \end{bmatrix} \tag{17}$$

$$\boldsymbol{E}^\bullet = \begin{bmatrix} \boldsymbol{I}^\circ & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},\ \boldsymbol{F}^\bullet = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}^\circ \\ \boldsymbol{F}_e^\circ & -\boldsymbol{W}^\circ \end{bmatrix} \tag{18}$$

$$\boldsymbol{V}^{\bullet T} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{V}^{\circ T} \end{bmatrix},\ \boldsymbol{C}^\bullet = \begin{bmatrix} \boldsymbol{C}^\circ & \boldsymbol{0} \end{bmatrix} \tag{19}$$

$$\boldsymbol{F}_e^\circ = \boldsymbol{F}^\circ - (\boldsymbol{J}^\circ - \boldsymbol{L}^\circ)\boldsymbol{C}^\circ, \quad \boldsymbol{W}^\circ = \boldsymbol{I}^\circ + \boldsymbol{L}^\circ\boldsymbol{C}^\circ \tag{20}$$

$$\boldsymbol{F}^\circ = \begin{bmatrix} \boldsymbol{F} & \boldsymbol{H} \\ \boldsymbol{0} & \boldsymbol{I}_s \end{bmatrix},\ \boldsymbol{V}^\circ = \begin{bmatrix} \boldsymbol{V} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_s \end{bmatrix},\ \boldsymbol{C}^{\circ T} = \begin{bmatrix} \boldsymbol{C}^T \\ \boldsymbol{0} \end{bmatrix} \tag{21}$$

$$\boldsymbol{J}^\circ = \begin{bmatrix} \boldsymbol{J} \\ \boldsymbol{M} \end{bmatrix}, \quad \boldsymbol{L}^\circ = \begin{bmatrix} \boldsymbol{L} \\ \boldsymbol{N} \end{bmatrix}, \quad \boldsymbol{I}^\circ = \begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_s \end{bmatrix} \tag{22}$$

$\boldsymbol{e}^\bullet(i) \in I\!R^{2(n+s)}$, $\boldsymbol{e}^\circ(i) \in I\!R^{n+s}$, $\boldsymbol{d}^\bullet(i), \boldsymbol{d}^\circ(i) \in I\!R^{p+s}$, $\boldsymbol{F}_e^\circ, \boldsymbol{F}^\circ, \boldsymbol{I}^\circ, \boldsymbol{W}^\circ \in I\!R^{(n+s)\times(n+s)}$, $\boldsymbol{V}^\circ \in I\!R^{(n+s)\times(p+s)}$, $\boldsymbol{J}^\circ, \boldsymbol{L}^\circ \in I\!R^{(n+s)\times m}$, $\boldsymbol{C}^\circ \in I\!R^{m\times(n+s)}$, *and* $\boldsymbol{F}^\bullet, \boldsymbol{E}^\bullet \in I\!R^{2(n+s)\times 2(n+s)}$, $\boldsymbol{V}^\bullet \in I\!R^{2(n+s)\times(p+s)}$, $\boldsymbol{C}^\bullet \in I\!R^{m\times 2(n+s)}$, *while* $\Delta\boldsymbol{f}(i)$, $\boldsymbol{e}_q(i)$, $\boldsymbol{e}_f(i)$ *are defined in (3), (8), (12).*

Proof: From the system and observer equations it can see that

$$\begin{aligned}\boldsymbol{e}_q(i+1) &= \boldsymbol{q}(i+1) - \boldsymbol{q}_e(i+1) = \\ &= \boldsymbol{F}\boldsymbol{q}(i) + \boldsymbol{G}\boldsymbol{u}(i) + \boldsymbol{H}\boldsymbol{f}(i) + \boldsymbol{V}\boldsymbol{d}(i) - \\ &\quad -\boldsymbol{F}\boldsymbol{q}_e(i) - \boldsymbol{G}\boldsymbol{u}(i) - \boldsymbol{H}\boldsymbol{f}_e(i) - \\ &\quad -(\boldsymbol{J}-\boldsymbol{L})\boldsymbol{C}\boldsymbol{e}_q(i) - \boldsymbol{L}\boldsymbol{C}\boldsymbol{e}_q(i+1) = \\ &= (\boldsymbol{F} - (\boldsymbol{J}-\boldsymbol{L})\boldsymbol{C})\boldsymbol{e}_q(i) - \boldsymbol{L}\boldsymbol{C}\boldsymbol{e}_q(i+1) + \\ &\quad +\boldsymbol{H}\boldsymbol{e}_f(i) + \boldsymbol{V}\boldsymbol{d}(i)\end{aligned} \tag{23}$$

Combining Eqs. (23), (13) and (8) compactly as follows

$$\begin{aligned}\begin{bmatrix} \boldsymbol{I}_n + \boldsymbol{L}\boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{N}\boldsymbol{C} & \boldsymbol{I}_s \end{bmatrix}\begin{bmatrix} \boldsymbol{e}_q(i{+}1) \\ \boldsymbol{e}_f(i{+}1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{V} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_s \end{bmatrix}\begin{bmatrix} \boldsymbol{d}(i) \\ \Delta\boldsymbol{f}(i) \end{bmatrix} + \\ + \begin{bmatrix} \boldsymbol{F} - (\boldsymbol{J}-\boldsymbol{L})\boldsymbol{C} & \boldsymbol{H} \\ -(\boldsymbol{M}-\boldsymbol{N})\boldsymbol{C} & \boldsymbol{I}_s \end{bmatrix}\begin{bmatrix} \boldsymbol{e}_q(i) \\ \boldsymbol{e}_f(i) \end{bmatrix}\end{aligned} \tag{24}$$

$$\boldsymbol{e}_y(i) = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_q(i) \\ \boldsymbol{e}_f(i) \end{bmatrix} \tag{25}$$

and setting out

$$\begin{bmatrix} \boldsymbol{I}_n + \boldsymbol{L}\boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{N}\boldsymbol{C} & \boldsymbol{I}_s \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_s \end{bmatrix} + \begin{bmatrix} \boldsymbol{L} \\ \boldsymbol{N} \end{bmatrix} \begin{bmatrix} \boldsymbol{C} & \boldsymbol{0} \end{bmatrix} \tag{26}$$

$$\begin{aligned} &\begin{bmatrix} \boldsymbol{F} - (\boldsymbol{J}-\boldsymbol{L})\boldsymbol{C} & \boldsymbol{H} \\ -(\boldsymbol{M}-\boldsymbol{N})\boldsymbol{C} & \boldsymbol{I}_s \end{bmatrix} = \\ &= \begin{bmatrix} \boldsymbol{F} & \boldsymbol{H} \\ \boldsymbol{0} & \boldsymbol{I}_s \end{bmatrix} - \left(\begin{bmatrix} \boldsymbol{J} \\ \boldsymbol{M} \end{bmatrix} - \begin{bmatrix} \boldsymbol{L} \\ \boldsymbol{N} \end{bmatrix} \right) \begin{bmatrix} \boldsymbol{C} & \boldsymbol{0} \end{bmatrix} \end{aligned} \tag{27}$$

then, with the notations (17), (20), (21), (22), Eqs. (24), (25) can be noticed as

$$\boldsymbol{W}^\circ \boldsymbol{e}^\circ(i+1) = \boldsymbol{F}_e^\circ \boldsymbol{e}^\circ(i) + \boldsymbol{V}^\circ \boldsymbol{d}^\circ(i) \tag{28}$$

$$\boldsymbol{e}_y(i) = \boldsymbol{C}^\circ \boldsymbol{e}^\circ(i) \tag{29}$$

Aligning Eq. (28) together with an equality as

$$\boldsymbol{F}_e^\circ \boldsymbol{e}^\circ(i) + \boldsymbol{V}^\circ \boldsymbol{d}^\circ(i) - \boldsymbol{W}^\circ \boldsymbol{e}^\circ(i+1) = \boldsymbol{0} \tag{30}$$

$$\boldsymbol{e}^\circ(i+1) = \boldsymbol{e}^\circ(i+1) \tag{31}$$

the last two equations can be written as

$$\begin{bmatrix} \boldsymbol{I}^\circ & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}^\circ(i+1) \\ \boldsymbol{e}^\circ(i+2) \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}^\circ \\ \boldsymbol{F}_e^\circ & -\boldsymbol{W}^\circ \end{bmatrix} \begin{bmatrix} \boldsymbol{e}^\circ(i) \\ \boldsymbol{e}^\circ(i+1) \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{V}^\circ \end{bmatrix} \boldsymbol{d}^\circ(i) \tag{32}$$

Thus, using the notations (18), (19) then Eq. (32) implies the form (14) and, respecting $\boldsymbol{e}^\bullet(i)$, Eq. (29) can be extended as

$$\boldsymbol{e}_y(i) = \boldsymbol{C}^\circ \boldsymbol{e}^\circ(i) = \begin{bmatrix} \boldsymbol{C}^\circ & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}^\circ(i) \\ \boldsymbol{e}^\circ(i+1) \end{bmatrix} \tag{33}$$

and using the notations (19) then Eq. (33) gives the form (15). This concludes the proof. ■

Using the descriptor approach, the following theorem is proposed to design PD fault observer given by Eqs. (14), (15).

Theorem 1 *The PD fault observer is stable if for a positive scalar $\alpha \in I\!R$, $\alpha \in (0,1)$, there exist symmetric positive definite matrices $\boldsymbol{P}_2^\circ, \boldsymbol{R}_1^\circ, \boldsymbol{R}_2^\circ \in I\!R^{(n+p)\times(n+p)}$, matrices $\boldsymbol{Y}^\circ, \boldsymbol{Z}^\circ \in I\!R^{(n+p)\times m}$ and positive scalars $\gamma_1, \gamma_1 \in I\!R$ such that*

$$\boldsymbol{P}_2^\circ = \boldsymbol{P}_2^{\circ T} > 0, \; \boldsymbol{R}_1^\circ = \boldsymbol{R}_1^{\circ T} > 0, \; \boldsymbol{R}_2^\circ = \boldsymbol{R}_2^{\circ T} > 0 \tag{34}$$

$$\begin{bmatrix} -\boldsymbol{I}^\circ - \alpha \boldsymbol{R}_1^\circ & * & * & * & * & * \\ \boldsymbol{0} & -\alpha \boldsymbol{R}_2^\circ & * & * & * & * \\ \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{\Gamma}^\bullet & * & * & * \\ \boldsymbol{0} & \delta \boldsymbol{I}^\circ & \boldsymbol{0} & -\boldsymbol{I}^\circ & * & * \\ \boldsymbol{\Pi}(4,1) & \boldsymbol{I}^\circ - \boldsymbol{P}_2^\circ - \boldsymbol{Z}^\circ \boldsymbol{C}^\circ & \boldsymbol{P}_2^\circ \boldsymbol{V}^\circ & \boldsymbol{0} & -\boldsymbol{P}_2^\circ & * \\ \boldsymbol{C}^\circ & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{I}_m \end{bmatrix} < 0 \tag{35}$$

$$\boldsymbol{\Pi}(4,1) = \boldsymbol{P}_2^\circ \boldsymbol{F}^\circ - \boldsymbol{Y}^\circ \boldsymbol{C}^\circ + \boldsymbol{Z}^\circ \boldsymbol{C}^\circ \tag{36}$$

where $\boldsymbol{\Gamma}^\bullet$ is a structured LMI matrix variable of the form

$$\boldsymbol{\Gamma}^\bullet = diag \begin{bmatrix} \gamma_1 \boldsymbol{I}_p & \gamma_2 \boldsymbol{I}_s \end{bmatrix} \tag{37}$$

When the above conditions hold, then

$$\boldsymbol{J}^\circ = (\boldsymbol{P}_2^\circ)^{-1} \boldsymbol{Y}^\circ, \quad \boldsymbol{L}^\circ = (\boldsymbol{P}_2^\circ)^{-1} \boldsymbol{Z}^\circ \tag{38}$$

and $\boldsymbol{J}$, $\boldsymbol{L}$ $\boldsymbol{M}$, $\boldsymbol{N}$ can be separated with respect to Eq. (22). Here, $*$ denotes the symmetric item in a symmetric matrix.

Proof: Defining the Lyapunov function

$$\begin{aligned} v(\boldsymbol{e}^\bullet(i)) = \boldsymbol{e}^{\bullet T}(i)(\boldsymbol{E}^{\bullet T} \boldsymbol{P}^\bullet \boldsymbol{E}^\bullet + \boldsymbol{R}^\bullet) \boldsymbol{e}^\bullet(i) + \\ + \sum_{j=0}^{i-1} (\boldsymbol{e}_y^T(j) \boldsymbol{e}_y(j) - \boldsymbol{d}^{\bullet T}(j) \boldsymbol{\Gamma}^\bullet \boldsymbol{d}^\bullet(j)) > 0 \end{aligned} \tag{39}$$

where $\gamma_1, \gamma_2 \in I\!R$ are upper-bounds of square of the H_∞ norms of the generalized disturbance transfer function matrices, then, by taking difference of the function (39), it has to yield for a stable descriptor form of the observer

$$\begin{aligned} \Delta v(\boldsymbol{e}^\bullet(i)) = \boldsymbol{e}_y^T(i) \boldsymbol{e}_y(i) - \boldsymbol{d}^{\bullet T}(i) \boldsymbol{\Gamma}^\bullet \boldsymbol{d}^\bullet(i) - \\ - \boldsymbol{e}^{\bullet T}(i)(\boldsymbol{E}^{\bullet T} \boldsymbol{P}^\bullet \boldsymbol{E}^\bullet + \boldsymbol{R}^\bullet) \boldsymbol{e}^\bullet(i) + \\ + \boldsymbol{e}^{\bullet T}(i+1)(\boldsymbol{E}^{\bullet T} \boldsymbol{P}^\bullet \boldsymbol{E}^\bullet + \boldsymbol{R}^\bullet) \boldsymbol{e}^\bullet(i+1) < 0 \end{aligned} \tag{40}$$

and solving for Eqs. (14), (15) it is obtained

$$\begin{aligned} \Delta v(\boldsymbol{e}^\bullet(i)) = -\boldsymbol{d}^{\bullet T}(i) \boldsymbol{\Gamma}^\bullet \boldsymbol{d}^\bullet(i) + \\ + \boldsymbol{e}^{\bullet T}(i)(-\boldsymbol{E}^{\bullet T} \boldsymbol{P}^\bullet \boldsymbol{E}^\bullet + \boldsymbol{C}^{\bullet T} \boldsymbol{C}^\bullet) \boldsymbol{e}^{\bullet T}(i) + \\ + \boldsymbol{e}^{\bullet T}(i+1) \boldsymbol{E}^{\bullet T} \boldsymbol{P}^\bullet \boldsymbol{E}^\bullet \boldsymbol{e}^\bullet(i+1) + \\ + \begin{bmatrix} \boldsymbol{e}^{\bullet T}(i+1) & \boldsymbol{e}^{\bullet T}(i) \end{bmatrix} \begin{bmatrix} \boldsymbol{R}^\bullet & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{R}^\bullet \end{bmatrix} \begin{bmatrix} \boldsymbol{e}^\bullet(i+1) \\ \boldsymbol{e}^\bullet(i) \end{bmatrix} < 0 \end{aligned} \tag{41}$$

It is evident that the last item in the inequality (41) corresponds to the difference of the Lyapunov function for a linear discrete-time system with the same state vector as in Eq. (41). If such state vector converges to equilibrium with a convergence rate factor β then $\boldsymbol{e}^\bullet(i+1) \leq \beta \boldsymbol{e}^\bullet(i)$ and

$$\begin{aligned} -(1-\beta^2) \boldsymbol{e}^{\bullet T}(i) \boldsymbol{R}^\bullet \boldsymbol{e}^\bullet(i) = -\alpha \boldsymbol{e}^{\bullet T}(i) \boldsymbol{R}^\bullet \boldsymbol{e}^\bullet(i) < \\ < \begin{bmatrix} \boldsymbol{e}^{\bullet T}(i+1) & \boldsymbol{e}^{\bullet T}(i) \end{bmatrix} \begin{bmatrix} \boldsymbol{R}^\bullet & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{R}^\bullet \end{bmatrix} \begin{bmatrix} \boldsymbol{e}^\bullet(i+1) \\ \boldsymbol{e}^\bullet(i) \end{bmatrix} < 0 \end{aligned} \tag{42}$$

where $\alpha \in (0,1)$. Then, in consequence,

$$\begin{aligned} \Delta v(\boldsymbol{e}^\bullet(i)) < -\boldsymbol{d}^{\bullet T}(i) \boldsymbol{\Gamma}^\bullet \boldsymbol{d}^\bullet(i) + \\ + \boldsymbol{e}^{\bullet T}(i)(-\boldsymbol{E}^{\bullet T} \boldsymbol{P}^\bullet \boldsymbol{E}^\bullet + \boldsymbol{C}^{\bullet T} \boldsymbol{C}^\bullet - \alpha \boldsymbol{R}^\bullet) \boldsymbol{e}^{\bullet T}(i) + \\ + \boldsymbol{e}^{\bullet T}(i+1) \boldsymbol{E}^{\bullet T} \boldsymbol{P}^\bullet \boldsymbol{E}^\bullet \boldsymbol{e}^\bullet(i+1) < 0 \end{aligned} \tag{43}$$

Rewriting the last element in the inequality (43) as follows

$$\begin{aligned} \boldsymbol{e}^{\bullet T}(i+1) \boldsymbol{E}^{\bullet T} \boldsymbol{P}^\bullet \boldsymbol{E}^\bullet \boldsymbol{e}^\bullet(i+1) = \\ = \boldsymbol{e}^{\bullet T}(i) \boldsymbol{F}^{\bullet T} \boldsymbol{P}^\bullet \boldsymbol{F}^\bullet \boldsymbol{e}^\bullet(i) + \boldsymbol{d}^{\bullet T}(i) \boldsymbol{V}^{\bullet T} \boldsymbol{P}^\bullet \boldsymbol{V}^\bullet \boldsymbol{d}^\bullet(i) + \\ + \boldsymbol{e}^{\bullet T}(i) \boldsymbol{F}^{\bullet T} \boldsymbol{P}^\bullet \boldsymbol{V}^\bullet \boldsymbol{d}^\bullet(i) + \boldsymbol{d}^{\bullet T}(i) \boldsymbol{V}^{\bullet T} \boldsymbol{P}^\bullet \boldsymbol{F}_{ce}^\bullet \boldsymbol{e}^\bullet(i) \end{aligned} \tag{44}$$

and defining the composed vector

$$\boldsymbol{e}_c^{\bullet T}(i) = \begin{bmatrix} \boldsymbol{e}^{\bullet T}(i) & \boldsymbol{d}^{\bullet T}(i) \end{bmatrix} \tag{45}$$

the inequality (43) can be written equivalently as

$$\boldsymbol{e}_c^{\bullet T}(i) \boldsymbol{P}_c^\bullet \boldsymbol{e}_c^\bullet(i) < 0 \tag{46}$$

$$\boldsymbol{P}_c^{\bullet}=\begin{bmatrix}\boldsymbol{P}_c^{\bullet}(11) & \boldsymbol{F}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{V}^{\bullet}\\ \boldsymbol{V}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{F}^{\bullet} & -\boldsymbol{\Gamma}^{\bullet}+\boldsymbol{V}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{V}^{\bullet}\end{bmatrix}<0 \tag{47}$$

$$\boldsymbol{P}_c^{\bullet}(11)=\boldsymbol{F}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{F}^{\bullet}+\boldsymbol{C}^{\bullet T}\boldsymbol{C}^{\bullet}-\alpha\boldsymbol{R}^{\bullet}-\boldsymbol{E}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{E}^{\bullet} \tag{48}$$

Separating the inequality (47) as

$$\begin{aligned}\boldsymbol{P}_c^{\bullet}=&\begin{bmatrix}\boldsymbol{C}^{\bullet T}\boldsymbol{C}^{\bullet}-\boldsymbol{F}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{F}^{\bullet}-\boldsymbol{R}^{\bullet} & \boldsymbol{0}\\ \boldsymbol{0} & -\boldsymbol{\Gamma}^{\bullet}\end{bmatrix}+\\ &+\begin{bmatrix}\boldsymbol{F}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{F}^{\bullet} & \boldsymbol{F}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{V}^{\bullet}\\ \boldsymbol{V}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{F}^{\bullet} & \boldsymbol{V}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{V}^{\bullet}\end{bmatrix}\end{aligned} \tag{49}$$

then, since it yields,

$$\begin{bmatrix}\boldsymbol{F}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{F}^{\bullet} & \boldsymbol{F}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{V}^{\bullet}\\ \boldsymbol{V}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{F}^{\bullet} & \boldsymbol{V}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{V}^{\bullet}\end{bmatrix}=\begin{bmatrix}\boldsymbol{F}^{\bullet T}\\ \boldsymbol{V}^{\bullet T}\end{bmatrix}\boldsymbol{P}^{\bullet}\begin{bmatrix}\boldsymbol{F}^{\bullet} & \boldsymbol{V}^{\bullet}\end{bmatrix} \tag{50}$$

the Schur complement property implies that the matrix inequality (47) can be reformulated as

$$\begin{bmatrix}\boldsymbol{C}^{\bullet T}\boldsymbol{C}^{\bullet}-\boldsymbol{E}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{E}^{\bullet}-\alpha\boldsymbol{R}^{\bullet} & \boldsymbol{0} & \boldsymbol{F}^{\bullet T}\boldsymbol{P}^{\bullet}\\ \boldsymbol{0} & -\boldsymbol{\Gamma}^{\bullet} & \boldsymbol{V}^{\bullet T}\boldsymbol{P}^{\bullet}\\ \boldsymbol{P}^{\bullet}\boldsymbol{F}^{\bullet} & \boldsymbol{P}^{\bullet}\boldsymbol{V}^{\bullet} & -\boldsymbol{P}^{\bullet}\end{bmatrix}<0 \tag{51}$$

Analogously, writing

$$\begin{bmatrix}\boldsymbol{C}^{\bullet T}\boldsymbol{C}^{\bullet} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}=\begin{bmatrix}\boldsymbol{C}^{\bullet T}\\ \boldsymbol{0}\\ \boldsymbol{0}\end{bmatrix}\boldsymbol{I}_m\begin{bmatrix}\boldsymbol{C}^{\bullet} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix} \tag{52}$$

then, applying the Schur complement property ones more again, the inequality (51) can be configured as

$$\begin{bmatrix}-\boldsymbol{E}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{E}^{\bullet}-\alpha\boldsymbol{R}^{\bullet} & \boldsymbol{0} & \boldsymbol{F}^{\bullet T}\boldsymbol{P}^{\bullet} & \boldsymbol{C}^{\bullet T}\\ \boldsymbol{0} & -\boldsymbol{\Gamma}^{\bullet} & \boldsymbol{V}^{\bullet T}\boldsymbol{P}^{\bullet} & \boldsymbol{0}\\ \boldsymbol{P}^{\bullet}\boldsymbol{F}^{\bullet} & \boldsymbol{P}^{\bullet}\boldsymbol{V}^{\bullet} & -\boldsymbol{P}^{\bullet} & \boldsymbol{0}\\ \boldsymbol{C}^{\bullet} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{I}_m\end{bmatrix}<0 \tag{53}$$

For a symmetric positive definite matrix $\boldsymbol{P}^{\bullet}$ it yields

$$\begin{aligned}&\boldsymbol{E}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{E}^{\bullet}=\\ &=\begin{bmatrix}\boldsymbol{I}^{\circ} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{P}_{11}^{\circ} & \boldsymbol{P}_{12}^{\circ}\\ \boldsymbol{P}_{12}^{\circ T} & \boldsymbol{P}_{22}^{\circ}\end{bmatrix}\begin{bmatrix}\boldsymbol{I}^{\circ} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}=\begin{bmatrix}\boldsymbol{P}_{11}^{\circ} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}\geq 0\end{aligned} \tag{54}$$

and since there are no other constraints justified on the rest block elements of the matrix $\boldsymbol{P}^{\bullet}$, to ensure that the matrix $\boldsymbol{P}^{\bullet}$ is positive definite, it can be set, for a positive $\delta\in I\!R$ and a symmetric positive definite $\boldsymbol{P}_2^{\circ}\in I\!R^{(n+s)\times(n+s)}$

$$\boldsymbol{P}^{\bullet}=\begin{bmatrix}\boldsymbol{I}^{\circ} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{P}_2^{\circ}\end{bmatrix}>0,\quad \boldsymbol{E}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{E}^{\bullet}=\begin{bmatrix}\boldsymbol{I}^{\circ} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix} \tag{55}$$

Thus, in consequence, it can obtain

$$\boldsymbol{P}^{\bullet}\boldsymbol{F}^{\bullet}=\begin{bmatrix}\boldsymbol{I}^{\circ} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{P}_2^{\circ}\end{bmatrix}\begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}^{\circ}\\ \boldsymbol{F}_e^{\circ} & -\boldsymbol{W}^{\circ}\end{bmatrix}=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}^{\circ}\\ \boldsymbol{P}^{\circ}\boldsymbol{F}_e^{\circ} & \boldsymbol{I}^{\circ}-\boldsymbol{P}^{\circ}\boldsymbol{W}^{\circ}\end{bmatrix} \tag{56}$$

$$\boldsymbol{P}^{\bullet}\boldsymbol{V}^{\bullet}=\begin{bmatrix}\boldsymbol{I}^{\circ} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{P}_2^{\circ}\end{bmatrix}\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{V}^{\circ}\end{bmatrix}=\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{P}_2^{\circ}\boldsymbol{V}^{\circ}\end{bmatrix} \tag{57}$$

Substituting the blocks (55)-(56) into the inequality (53) it is

$$\begin{bmatrix}-\boldsymbol{I}^{\circ}-\alpha\boldsymbol{R}_1^{\circ} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{F}_e^{\circ T}\boldsymbol{P}_2^{\circ} & \boldsymbol{C}^{\circ T}\\ \boldsymbol{0} & -\alpha\boldsymbol{R}_2^{\circ} & \boldsymbol{0} & \boldsymbol{I}^{\circ} & \boldsymbol{I}^{\circ}-\boldsymbol{W}^{\circ T}\boldsymbol{P}_2^{\circ} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{\Gamma}^{\bullet} & \boldsymbol{0} & \boldsymbol{V}^{\circ T}\boldsymbol{P}_2^{\circ} & \boldsymbol{0}\\ \boldsymbol{0} & \delta\boldsymbol{I}^{\circ} & \boldsymbol{0} & -\boldsymbol{I}^{\circ} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{P}_2^{\circ}\boldsymbol{F}_e^{\circ} & \delta\boldsymbol{I}^{\circ}-\boldsymbol{P}_2^{\circ}\boldsymbol{W}^{\circ} & \boldsymbol{P}^{\circ}\boldsymbol{V}^{\circ} & \boldsymbol{0} & -\boldsymbol{P}_2^{\circ} & \boldsymbol{0}\\ \boldsymbol{C}^{\circ} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{I}_m\end{bmatrix}<0 \tag{58}$$

where

$$\boldsymbol{R}^{\bullet}=\mathrm{diag}\begin{bmatrix}\boldsymbol{R}_1^{\circ} & \boldsymbol{R}_2^{\circ}\end{bmatrix}>0 \tag{59}$$

Since the products $\boldsymbol{P}^{\circ}\boldsymbol{F}_e^{\circ},\ \boldsymbol{P}^{\circ}\boldsymbol{W}^{\circ}$ can be written as

$$\boldsymbol{P}_2^{\circ}\boldsymbol{F}_e^{\circ}=\boldsymbol{P}_2^{\circ}(\boldsymbol{F}^{\circ}-(\boldsymbol{J}^{\circ}-\boldsymbol{L}^{\circ})\boldsymbol{C}^{\circ})=\boldsymbol{P}_2^{\circ}\boldsymbol{F}^{\circ}-\boldsymbol{Y}^{\circ}\boldsymbol{C}^{\circ}+\boldsymbol{Z}^{\circ}\boldsymbol{C}^{\circ} \tag{60}$$

$$\boldsymbol{P}_2^{\circ}\boldsymbol{W}^{\circ}=\boldsymbol{P}_2^{\circ}(\boldsymbol{I}^{\circ}+\boldsymbol{L}^{\circ}\boldsymbol{C}^{\circ})=\boldsymbol{P}_2^{\circ}+\boldsymbol{Z}^{\circ}\boldsymbol{C}^{\circ} \tag{61}$$

where

$$\boldsymbol{P}_2^{\circ}\boldsymbol{J}^{\circ}=\boldsymbol{Y}^{\circ},\quad \boldsymbol{P}_2^{\circ}\boldsymbol{L}^{\circ}=\boldsymbol{Z}^{\circ} \tag{62}$$

then (58) implies (35). This concludes the proof. ■

The above design procedure is convex H_∞ optimization problems with a set of LMI constraints, which can be solved efficiently using available LMI solvers.

Analyzing Eq. (10), it is evident that $\Delta\boldsymbol{f}(i)$ acts on the observer dynamics as an extension of unknown disturbance.

4. ILLUSTRATIVE EXAMPLE

In the example, there is considered the state-space representation of the system (1), (2), where

$$\boldsymbol{F}=\begin{bmatrix}1.0620 & -0.0045 & 0.2381 & -0.1961\\ -0.0220 & 0.8444 & -0.0021 & 0.0261\\ 0.0370 & 0.1546 & 0.7762 & 0.1967\\ 0.0011 & 0.1547 & 0.0454 & 0.9267\end{bmatrix}$$

$$\boldsymbol{G}=\boldsymbol{H}=\begin{bmatrix}0.0003 & -0.0156\\ 0.2095 & 0.0001\\ 0.0630 & -0.1109\\ 0.0630 & -0.0030\end{bmatrix},\ \boldsymbol{V}=\begin{bmatrix}-0.0380\\ 0.0696\\ 0.2317\\ 0.0000\end{bmatrix}$$

$$\boldsymbol{C}=\begin{bmatrix}4 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix},\quad \sigma_d^2=0.1$$

while the sampling period is $T_s=0.04s$.

Solving (34), (35) with respect to the LMI matrix variables $\boldsymbol{P}$, $\boldsymbol{H}$, and $\boldsymbol{Y}$ using Self-Dual-Minimization (SeDuMi) package for MATLAB®, the estimator parameter design problem is solved as feasible and the estimator gain matrix are computed for the tuning parameter value $\alpha=0.2$ as follows

$$\boldsymbol{J}=\begin{bmatrix}0.0454 & -0.1916\\ -0.0032 & 0.0473\\ -0.0019 & 0.2044\\ 0.0025 & 0.0177\end{bmatrix},\ \boldsymbol{L}=\begin{bmatrix}-0.2189 & 0.0023\\ 0.0018 & 0.0214\\ -0.0494 & 0.0078\\ -0.0001 & -0.9145\end{bmatrix}$$

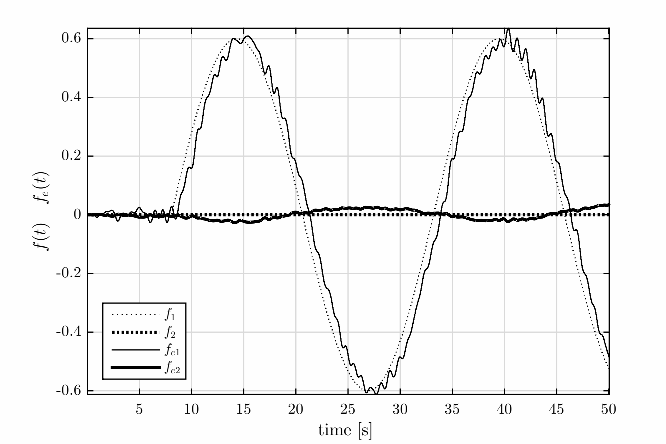

Figure 1. The first actuator fault and its estimation

$$\boldsymbol{M} = \begin{bmatrix} 0.0009 & 0.0087 \\ -0.0026 & -0.0006 \end{bmatrix}, \ \boldsymbol{N} = \begin{bmatrix} 0.0003 & 0.0098 \\ -0.0024 & -0.0009 \end{bmatrix}$$

while

$$\gamma_1 = 14.5643, \quad \gamma_2 = 17.2154$$

These parameters guaranty the stable actuator fault observer, where the observer system matrix eigenvalues spectrum is

$$\rho((\boldsymbol{I}_n + \boldsymbol{L}\boldsymbol{C})^{-1}(\boldsymbol{F} - (\boldsymbol{J} - \boldsymbol{L})\boldsymbol{C})) = \{-0.0671, \ -0.0091, \ 0.7833 \pm 0.0561\,\mathrm{i}\}$$

while the extended observer system matrix eigenvalue spectrum is

$$\rho((\boldsymbol{I}^{\circ} + \boldsymbol{L}^{\circ}\boldsymbol{C}^{\circ})^{-1}(\boldsymbol{F}^{\circ} - (\boldsymbol{J}^{\circ} - \boldsymbol{L}^{\circ})\boldsymbol{C}^{\circ})) = \{-0.0673, -0.0092, \ 0.8033 \pm 0.0426\,\mathrm{i}, \ 0.9556, \ 0.9916\}$$

The tuning parameter $\alpha = 0.2$ is set experimentally, adapting the observer to the system dynamics and the maximal value of the fault signal amplitude.

For simulation purposes, the equilibrium of the system was stabilized by the controller

$$\boldsymbol{u}(i) = -\boldsymbol{K}\boldsymbol{q}(i)$$

where, exploiting the pole-placement method which offers the possibility to design the linear state controller for the linear systems, the control law gain matrix is computed, using MATLAB® function $place(\,\cdot\,)$ with the prescribed eigenvalues spectrum of the closed-loop system matrix $\rho(\boldsymbol{F} - \boldsymbol{G}\boldsymbol{K}) = \{0.2, \, 0.3, \, 0.4, \, 0.5\}$, as follows

$$\boldsymbol{K} = \begin{bmatrix} -0.9147 & 2.5091 & 0.2236 & 7.9481 \\ -17.6130 & 0.1020 & -8.2106 & 5.1556 \end{bmatrix}$$

The simulation results with the initial conditions

$$\boldsymbol{q}(0) = [0.2, \, 0.0, \, 0.0, \, 0.0]^T, \ \boldsymbol{q}_e(0) = \Delta\boldsymbol{q}_e(0) = \boldsymbol{0}$$

are given in Fig. 1 and Fig. 2, where Fig. 1 presents the fault signal, as well as its estimation, reflecting a single actuator fault at the first actuator.

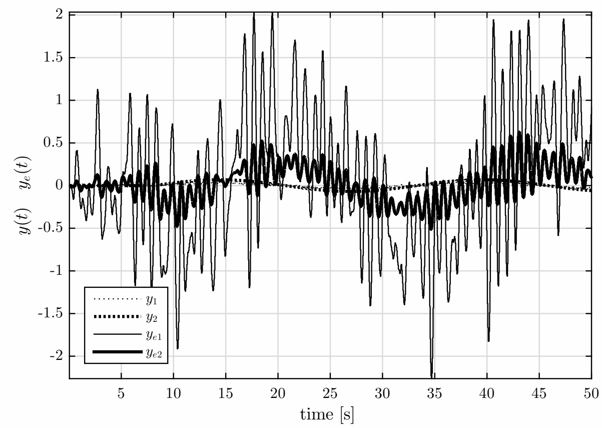

Figure 2. Outputs of the system and their estimations

The sinusoidal actuator fault is modeled as

$$f(i+1) = \begin{cases} 0\,; & i < 200 \\ 0.6\sin(x(i-200))\,; & k \geq 200 \end{cases}$$

an starts at the time instant $t_f = 200\,T_s = 8s$. Note, the fault considered in simulation doesn't cause closed-loop system instability.

From the simulation results in Fig. 1 it can be observed that an acceptable difference exists between the signal reflecting the single actuator fault and its estimation.

To demonstrate the fault observer functionality in noisy environment, the measurable noise system outputs and their estimates are presented Fig. 2. It is evident that the deterministic variables are deeply embedded. Especially from this point of view the results are presented for unforced system mode.

5. CONCLUDING REMARKS

The presented fault estimation method for linear discrete-time systems provides useful and easily implementable structure in process of fault estimation. The principally LMI design conditions novelty, based on a descriptor model of the fault PD estimator error exploitation, requires no iterative procedures and guarantees the asymptotic stability of proposed PD observer structure. The tuning parameter α has to be set interactively, reflecting the real observer state convergence rate limit, and incorrect values of this parameter would results in unstable or noisy response of the fault estimation signals. A simulation example, subject to given type of failure, demonstrates the effectiveness of the proposed form of the fault estimation design technique.

ACKNOWLEDGMENT

The work presented in the paper was supported by VEGA, the Grant Agency of the Ministry of Education and the Academy of Science of Slovak Republic, under Grant No. 1/0348/14. This support is very gratefully acknowledged.

REFERENCES

Ding, S. (2013) *Model-based fault diagnosis techniques. Design schemes, algorithms, and tools*, Berlin, Springer-Verlag.

Filasová, A., Gontkovič, D. & Krokavec, D. (2013). Observer-based fault estimation for linear systems with distributed time delay. *Archives of control sciences, 23*(2), 169-186.

He, S. (2014). Fault estimation for T-S fuzzy Markovian jumping systems based on the adaptive observer. *International journal of control, automation and systems, 12*(5), 977-985.

He, S. & Liu, F. (2012). Adaptive observer-based fault estimation for stochastic Markovian jumping systems. *Abstract and applied analysis, 2012*, Article ID 176419, 11p, doi 10.1155/2012/176419.

Jiang, B. & Chowdhury, F.N. (2005). Fault estimation and accommodation for linear MIMO discrete-time systems. *IEEE transactions on control systems technology, 13*(3), 493-499.

Krokavec, D. & Filasová, A. (2013). Actuator faults reconstruction using reduced-order fuzzy observer structures. In *Proceedings of the 12th EUCA European control conference ECC'13*, Zurich, Switzerland, 4299-4304.

Krokavec, D. & Filasová, A. (2014). H_∞ control of discrete-time stochastic state-multiplicative systems constrained in state by equality constraints. *Procedings of 19th IFAC world congres*, Cape Town, South Africa, 8699-8704.

Krokavec, D. & Filasová, A. (2015). Design of PD observer-based fault estimator for a class of Takagi-Sugeno descriptor systems. *IFAC-PapersOnLine, 48*(11), 754-759.

Mahmoud, M.S. & Xia, Y. (2014). *Analysis and synthesis of fault-tolerant control systems*, Chichester, John Wiley & Sons.

Patton, R.J. & Klinkhieo, S. (2009). Actuator fault estimation and compensation based on an augmented state observer approach. In *Proceedings of joint 48th IEEE conference on decision and control and 28th Chinese control conference*, Shanghai, China, 8482-8487.

Shi, F. & Patton, R.J. (2014). Simultaneous state and fault estimation for descriptor systems using an augmented PD observer. In *Procedings of 19th IFAC world congres*, Cape Town, South Africa, 8006-8011.

Tabatabaeipour, S.M. & Bak, T. (2014). Robust observer-based fault estimation and accommodation of discrete-time piecewise linear systems. *Journal of the Franklin institute, 35*(1), 277-295.

Wang, Z., Rodrigues, M., Theilliol, D. & Shen, Y. (2015). Fault estimation filter design for discrete-time descriptor systems. *IET control theory & applications, 9*(10), 1587-1594.

Witczak, M., Zegar, D. & Pazera, M. (2016). Design of an adaptive sensor and actuator fault estimation scheme with a quadratic boundedness approach. In *Advanced and intelligent computations in diagnosis and control*, Z. Kowalczuk (Ed.), Cham, Springer, 49-59.

Zhang, K. & Jiang, B. (2008). Analysis and design of adaptive fault estimation for time-varying delay systems. In *Proceedings of the 27th Chinese control conference*, Kunming, China, 38-42.

Zhang, K., Jiang, B. & Cocquempot, V. (2008). Adaptive observer-based fast fault estimation. *International journal on control automation and systems, 6*(3), 320-326.

Zhang, K. & Jiang, B. & Shi, P. (2013). *Observer-based fault estimation and accomodation for dynamic systems*, Berlin, Springer-Verlag.

BIOGRAPHIES

Dušan Krokavec received M.Sc. degree in automatic control in 1967 and Ph.D. degree in technical cybernetics in 1982 from the Faculty of Electrical Engineering, Slovak University of Technology in Bratislava, Slovakia. In 1984 he was promoted Associated Professor from the Technical University in Košice, Slovakia, and in 1999 he was appointed Full Professor in automation and control. Since 1971 he is with the Department of Cybernetics and Artificial Intelligence, Faculty of Electrical Engineering and Informatics, Technical University of Košice. In the long term, he specializes in stochastic processes in dynamic systems, digital control systems and in dynamic system fault diagnosis. He is a member of the IFAC Technical Committees TC 1.4. Stochastic Systems as well as TC 6.4. Fault Detection, Supervision & Safety of Technical Processes - SAFEPROCESS.

Anna Filasová graduated in technical cybernetics and received M.Sc. degree in 1975, and Ph.D. degree in 1993, both from the Faculty of Electrical Engineering and Informatics, Technical University of Košice, Slovakia. In 1999 she was appointed Associated Professor from the Technical University in Košice in technical cybernetics. She is with the Department of Cybernetics and Artificial Intelligence, Faculty of Electrical Engineering and Informatics, Technical University of Košice. Her main research interests are in robust and decentralized control and fault-tolerant control systems.

Pavol Liščinský received the B.E. degree in cybernetic in 2010 and the M.Sc. degree in cybernetics and information control systems in 2012, both from the Department of Cybernetics and Artificial Intelligence, Technical University of Košice, Slovakia. He is currently working towards the Ph.D. degree in intelligent systems at the Department of Cybernetics and Artificial Intelligence. His main research interests include fault estimation, control reconfiguration and cascade control systems.