Remaining Useful Life Estimation of Stochastically Deteriorating Feedback Control Systems with a Random Environment and Impact of Prognostic Result on the Maintenance Process

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ABSTRACT

The objective and originality of this work are twofold. On one hand, it considers the degradation modeling and Remaining Useful Life (RUL) estimation for the closed-loop dynamic systems, which have not been addressed extensively in the literature. On the other hand, the paper examines how the prognosis result impacts the maintenance process. Indeed, due to their natural ageing and/or non desired effects of the operating condition, actuators deal with the loss of effectiveness which is a source of performance degradation of closed-loop system. In this paper, we consider a control system with classical Proportional-Integral-Derivative controller and stochastically deteriorating actuator. It is assumed that the actuators are subject to shocks that occur randomly in time. An integrated model is proposed which jointly describes the states of the controlled process and the actuators degradation. The RUL can be estimated by a probabilistic approach which consists of two steps. First, the system state regarding the available information is estimated online by Particle Filtering method. Then, the RUL of the system is estimated by Monte Carlo simulation. To illustrate the approach and highlight the impact of the prognosis result on the maintenance process, a well-known simulated tank level control system is used. The maintenance decision rule is based on the quantiles of RUL histogram. In order to evaluate the performance of the maintenance policy, a cost model is developed.

1. INTRODUCTION

Respecting the growing demand of safety, reliability and availability of industrial production process, research activity on maintenance modeling has intensively evolved during the last decades. In the context of Condition-Based Maintenance (CBM), system health monitoring information is used to determine its current status and based on this information one can perform maintenance actions to avoid failure (Dieulle, Bérenguer, Grall, & Roussignol, 2003; Van Noortwijk, 2009; Huynh, Barros, & Bérenguer, 2012). However, the CBM approach does not consider specific knowledge about future usage of the system which can be useful information to improve the decision marking (Khoury, Deloux, Grall, & Berenguer, 2013). In this way, a predictive maintenance which combines the prognosis and CBM maintenance seem to be an appropriate approach (Vachtsevanos, Lewis, Roemer, Hess, & Wu, 2006; Do Van, Levrat, Voisin, Iung, et al., 2012).

Generally, prognosis is defined as the prediction of future characteristic of the system such the Remaining Useful Life (RUL) (Si, Wang, Hu, & Zhou, 2011; Sikorska, Hodkiewicz, & Ma, 2011). According to (Jardine, Lin, & Banjevic, 2006) the prognostic approaches can be classified into three main categories: statistical approaches, artificial intelligence approaches and model-based approaches. Many studies are devoted to the RUL estimation of systems, subsystems or components (see reviews by (Peng, Dong, & Zuo, 2010),(Si et al., 2011).

In spite of that, according to the best knowledge of the authors the degradation modeling and RUL estimation process for closed-loop dynamic system such as feedback control system has not been addressed extensively. Indeed, the degradation or wear of components can lead to the gradually decreasing of the control system performance during its operation. One objective of this paper is to propose a probabilistic approach to assess the RUL of feedback control system with stochastically deteriorating actuator within a random environment. The other objective of the paper is to examine the use and the impact of prognostic information on the predictive maintenance decision-making process. In order to deal with the complex interaction between the deterministic behavior of the feedback control system and the stochastic degradation process, a Piecewise Deterministic Markov Process is adopted to describe the whole deteriorating closed-loop system. In this framework, the distribution of the RUL of the system is computed by using a two-step stochastic model-based technique.

The remainder of this paper is organized as follows. Section 2 is devoted to the description of the system characteristics. Section 3 describes the approach for computing the Remaining Useful Lifetime which is relevant to system state estimation using the available condition monitoring information. To illustrate the methodology and also highlight the use of prognostic result in the maintenance process, a specific case study is introduced in Section 4. Some numerical results are also discussed here. Finally, conclusion drawn from this work and possible ways for further studies are given.

2. SYSTEM MODELING AND ASSUMPTIONS

This section is devoted to describe the characteristics of a deteriorating feedback control system whose actuator stochastically degrades through time due to its natural degradation and the impact of the operating condition. The stochastic evolution of set-point which depends on the operating mode is also characterized. No additional sensor is devoted to the monitoring of the actuator degradation, the measurement of controlled output is then used to assess the RUL.

2.1. General structure of a deteriorating feedback control system

Consider a dynamical process which can be described in state-space representation as:

$$\begin{cases} \dot{x}(t) = \mathbf{f}(t, x(t), u(t)) \\ y(t) = \mathbf{h}(t, x(t), u(t)) + \epsilon(t) \end{cases} \quad (1)$$

where $x(t)$ is the state vector of process, $u(t)$ denotes control force acting on the process, $y(t)$ is the measurement of output. Process dynamic function $\mathbf{f}$ and process output function $\mathbf{h}$ can be nonlinear. Here, it is assumed that measurement noises $(\epsilon_t)_{t\in\mathbb{R}_+}$ are independent random variables with a probability density g, not necessarily Gaussian, independent of the process state $(x_t)_{t\in\mathbb{R}_+}$.

The objective of a conventional feedback control system is to maintain the process output $y(t)$ within a desired range defined by a set-point. Such objective can be achieved by the feedback structure with a classical Proportional-Integral-Derivative (PID) controllers which are widely used in industrial applications thanks to their simplicity and performance (Aström & Hägglund, 1995), see Figure 1 for a general scheme of a feedback control system.

The PID controller output $u^c(t)$ is given by:

$$u^c(t) = K_P \left[e(t) + \frac{1}{T_I}\int_0^t e(\tau)\mathrm{d}\tau + T_D \frac{\mathrm{d}e(t)}{\mathrm{d}t} \right] \quad (2)$$

where $e(t)$ is the error signal defined as $e(t) = y^{\mathrm{ref}}(t) - y(t)$ with $y^{\mathrm{ref}}(t)$ the desired set-point (the reference output), K_P is the proportional gain, T_I is the integral time and T_D is the derivative time of the PID controller. The adjustment of these three parameters for an optimal system response is extensively studied in control system design (Aström & Hägglund, 1995).

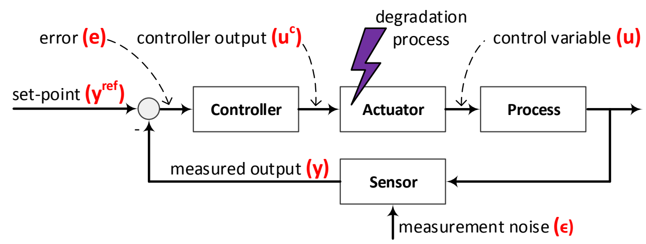


Figure 1. General block diagram of a feedback control system with notations

The output of actuator which is the real control variable acting on the process is defined as a function $\mathbf{g}$ depending on the required value $u^c(t)$ of the controller and on the actual capacity of actuator $C(t)$. $\mathbf{g}$ is a decreasing function w.r.t. $C(t)$:

$$u(t) = \mathbf{g}(u^c(t), C(t)) \quad (3)$$

At the initial stage of working, the actuators operate perfectly, i.e. $C(t) = c_0$ where c_0 is the initial nominal capacity of actuator. In reality, the natural ageing or wear of the parts of the actuator and/or the non desired effects of the operating condition are unavoidable, lead to the decreasing of the actuator's effectiveness $C(t)$ in time and subsequently reduces the control system performance.

2.2. Set-point evolution and operating modes

The evolution of set-point (the mission profile) presents the environmental conditions the system evolves in. According to the demand e.g. of the production process, the desired set-point may change. The random evolution of the set-point is described by a time-homogeneous Markov chain with a finite state space $r_{set} = \{r_1, r_2, \ldots, r_m\}$ describing e.g. the m production phases. Moreover, depending on a operating mode, the transition rate of set-point may be different.

Let $Y^{\mathrm{ref}}(t)$ be the set-point at time t. The evolution of the stochastic process $\{Y_t^{\mathrm{ref}}, t \geq 0\}$ in the operating mode k is expressed by the transition probability matrix P^k with the (i,j)th element equal to:

$$p_{ij}^k(t) = \mathbb{P}(Y_{s+t}^{\mathrm{ref}} = r_j \mid Y_s^{\mathrm{ref}} = r_i) \quad (4)$$

Figure 2 exemplifies the evolution of a set-point which takes value in 2-states space and corresponds to 2 operating modes denoted OM1 and OM2. One can find that the change of set-point occurs more frequently in the operating mode OM2 which is more stressful.

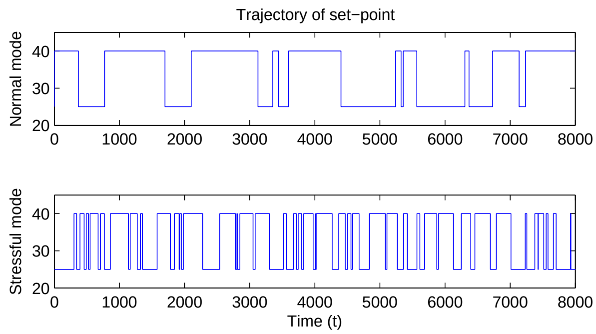


Figure 2. An example of set-point evolution within two operating mode

In practical situation, the set-point and the operating mode is well known at each time and its evolution is easy to identify. In this work, only one set of PID controller parameters is chosen for all known value of set-point r_{set}.

2.3. Actuator degradation behavior

It is assumed that an actuator is subject to shocks that occur randomly through time. Each shock impacts a random quantity of damage to the actuator. Hence, the capacity of the actuator by time t before its failure can be expressed as:

$$C(t) = c_0 - \mathcal{D}(t) \tag{5}$$

where c_0 is the initial capacity of the actuator, $\mathcal{D}(t)$ describes the accumulated deterioration of the actuator at time t (in *capacity* unit)

On the one hand, the actuator is less efficient through time because of its natural degradation. On the other hand, the evolution of set-point also impacts the degradation process of actuator. For example, in a centrifugal pump, an increased demand of pump flow will cause bearing friction and impeller wear to increase at a faster rate.

Natural degradation Due to the natural ageing or wear of the mechanical and/or electrical parts, the actuator capacity decreases through time. At each time ξ_i^{nd} that a shock occurs according to a Poisson process with intensity λ^{nd}, the actuator capacity $C(t)$ decreases a quantity W_i^{nd} which follows a uniform distribution on $[0; \Delta^{nd}]$.

Impact of operating condition As describe in 2.2 the operating conditions which represents the environmental conditions the system evolves in. Their impact on the degradation of the actuator is modeled through another shock process. The shock instant ξ_i^{om} follows a Poisson process with intensity λ^{om} which takes a value corresponding to the actual operating condition OMi. At each time ξ_i^{om} the capacity of the actuator $C(t)$ decreases of a quantity W_i^{om} which follows a uniform distribution on $[0; \Delta^{om}]$. The more frequently the set-point changes in a operating mode OMi, the more frequently damage shock occurs. This is represented by a big value of λ^{om_i}.

Under this modeling assumption, the degradation impacts the actuator only at discrete times. In case where the actuator has a monotone gradual degradation behavior, other processes should be considered e.g. the homogeneous Gamma process (Van Noortwijk, 2009).

2.4. Piecewise Markov Deterministic Markov Processes

In order to take into account the complex interaction between the stochastic degradation process of actuator and the deterministic behavior of control system, this paper considers the point of view of Piecewise Markov Deterministic Markov Processes (PDMP) which has been first introduced by (Davis, 1993). PDMPs were used to model fatigue growth in (Chiquet, Limnios, & Eid, 2009) and corrosion in (Brandejsky, De Saporta, Dufour, & Elegbede, 2011).

The whole behavior of deteriorating closed-loop system at time t can be resumed by a random variable as:

$$Z_t = \begin{pmatrix} x_t \\ C_t \\ \lambda_t^{om} \\ t \end{pmatrix} \tag{6}$$

with x_t is the physical state variable of controlled process, C_t is the actual capacity variable related to the actuator degradation, λ_t^{om} is a covariate representing the current operating mode of the system and t is the time. The time t is included for the process to be homogeneous in time especially because of the time-varying set-point.

Between two successive shocks reducing the actuator capacity as described by the actuator degradation model, the response of closed-loop system is described by differential equations which combine the process dynamic characteristic and PID controller behavior. Interest readers can refer to (Cocozza-Thivent, 2011; Lorton, Fouladirad, & Grall, 2013) for the detailed definition of a PDMP.

2.5. Condition monitoring model

In this work, no additional sensor is devoted to the monitoring of the actuator degradation. The controlled system output is considered as the only available healthy information. As known that a significant part of the dynamic behavior of the system is shown in the transient period which occurs immediately after a change of set-point, only observations of system output which characterizes the dynamics of deteriorating controlled system is taken in this period (see (Nguyen, Dieulle, & Grall, 2013) for more details of condition monitoring model).

Let introduce the time of prediction $T_{prog} > 0$ which is the time at which the system health can be estimated given all the collected knowledge and a residual lifetime can be derived. If n is the total number of observations until T_{prog}, the observation dates and corresponding system output will be respectively denoted $0 < T_1 < \ldots < T_n \leq T_{prog}$ and $Y_1, Y_2, \ldots, Y_n$ where the observation Y_i is defined from Eq. (1) as:

$$Y_i = \mathbf{h}(T_i, x(T_i), u(T_i)) + \epsilon(T_i) \tag{7}$$

3. RUL ASSESSMENT METHODOLOGY

The Remaining Useful Life at time t RUL_t is defined as the remaining time (from t) before the system can no longer fulfill its requirement anymore:

$$RUL_t = \inf(s \geq t, Z_s \in \mathcal{F}) - t \tag{8}$$

where $\mathcal{F}$ is the failure zone which refers to the set of undesired system states. In the context of the feedback control system, the actual capacity of the actuator has to be greater than a minimal capacity level which relates to the objectives of control system design.

The system state process $(Z_t)_{t\geq 0}$ is a Piecewise Deterministic Markov Process and as shown in (Lorton et al., 2013) the distribution of the RUL of the system conditionally to online available information up to time T_{prog} can be computed by a two-step approach as:

$$\begin{aligned}\mathbb{P}(RUL_{T_{prog}} > s | Y_1 = y_1, \ldots, Y_n = y_n) \\ = \int R_z(s) \mu_{y_1,\ldots,y_n}(dz)\end{aligned} \tag{9}$$

where:

- $\mu_{y_1,\ldots,y_n}(dz)$ is the probability law of the system state at time T_{prog} regarding the available observations $y_1, \ldots, y_n$:

$$\mu_{y_1,\ldots,y_n} = \mathcal{L}(Z_{T_{prog}} | Y_1 = y_1, \ldots, Y_n = y_n) \tag{10}$$

- $R_z(s)$ is the reliability of the system at time s knowing that the initial state value is z:

$$R_z(s) = \mathbb{P}(Z_u \notin \mathcal{F} \quad \forall u \leq s | Z_0 = z) \tag{11}$$

The detail of the approach will be given in the next paragraphs. On one hand, it require the estimation of probability law $\mu_{y_1,\ldots,y_n}(dz)$. On the other hand, it involves the estimation of the conditional reliability knowing $Z_{T_{prog}}$.

3.1. Step 1: Particle Filtering State Estimation

The main task is to estimate the conditional density, $p(z_{T_k}|y_{1:k})$ which represents the probability law of the state at time T_k given the measured value $y_{1:k} = y_1, \ldots, y_k$ of the observation process $Y_{1:k} = \{Y_i, i = 1, \ldots, k\}$ for any $k \leq n$. Let Z_{T_0} be the initial state of the system.

Particle filtering is used here to allow for numerical computation of the filtering density $p(z_{T_k}|y_{1:k})$. The key idea is to approximate the targeted filtering density by a cloud of N_s i.i.d. random samples (particles) $\{z^{(i)}_{T_k}, i = 1, \ldots, N_s\}$ with associated weights $\{w^{(i)}_{T_k}, i = 1, \ldots, N_s\}$, which satisfy $\sum_i w^{(i)}_{T_k} = 1$, so that the target distribution at time T_k can be approximated by

$$p(z_{T_k}|y_{1:k}) \approx \hat{p}(z_{T_k}|y_{1:k}) = \sum_{i=1}^{N_s} w^{(i)}_{T_k} \delta_{z^{(i)}_{T_k}}(\mathrm{d}z_{T_k}) \tag{12}$$

where $\delta_{z^{(i)}_{T_k}}(\mathrm{d}z_{T_k})$ is the Dirac delta mass located in $z^{(i)}_{T_k}$.

The used particle filter is similar to the Generic Particle Filter in (Arulampalam, Maskell, & Gordon, 2002) with deterministic re-sampling method because it seems to be a computationally cheaper algorithm (Kitagawa, 1996). Indeed, resampling is used to avoid the problem of degeneracy of the algorithm that is, avoiding the situation that all but one of the importance weights are close to zero (Doucet & Johansen, 2009). The algorithm uses the prior distribution $p(z_{T_k}|z^{(i)}_{T_{k-1}})$ based on the simulation of the actuator degradation process and the deterministic behavior of the controlled process which is derived from Eq. (1) to Eq. (5) using a discretized scheme of Eq. (1) and Eq. (2).

Therefore, the real-time state estimation procedure, given the sequence of measurement $y_{1:k}$ can be resumed by the algorithm in Algorithm 1.

3.2. Step 2: RUL estimation

The second step of the presented methodology for the RUL computation requires the estimation of the system reliability starting from the prognostic instant T_{prog} and knowing the approximated pdf of the system state at T_{prog} as given by Eq. (12). Actually, the reliability is computed with the classical Monte Carlo method. The histogram of the RUL is obtained straightforwardly. The mean value or quantiles of the RUL can also be derived. The procedure is illustrated by Algorithm 2.

4. RUL PROGNOSIS AND ITS IMPACT ON MAINTENANCE PROCESS: A CASE STUDY

In the previous section, a methodology to compute the conditional pdf of the RUL of a dynamic system was described. Here, it is illustrated on a well-known feedback control system: a double-tank level control system. A predictive maintenance decision rule which uses the RUL information is also presented which will be compared with an age remplacement strategy.

4.1. Description of the case study

Consider a double-tank level system with cross-sectional area of the first tank S_1 and the second one S_2. Water or other

Algorithm 1 Generic particle filter for system state estimation.

Initialization: $\forall i = 1, \ldots, N_s$.

Draw particle $z_{T_0}^{(i)}$ according to the initial condition of system

Assign corresponding weight $w_{T_0}^{(i)} = \frac{1}{N_s}$

At step k (corresponding to time T_k): Given $\left\{z_{T_{k-1}}^{(i)}, w_{T_{k-1}}^{(i)}\right\}_{i=1}^{N_s}$, do

(a) Importance sampling

Based on the system description (presented in Sections 2), draw particles $\tilde{z}_{T_k}^{(i)} \sim p(z_{T_k}|z_{T_{k-1}}^{(i)})$

(b) Weight update

Based on the likelihoods of the observations y_k collected (Eq. (7)), assign weights $w_{T_k}^{(i)} = w_{T_{k-1}}^{(i)} p(y_k|\tilde{z}_{T_k}^{(i)})$

(c) Weight normalisation

$w_{T_k}^{(i)} = \frac{w_{T_k}^{(i)}}{\sum_{i=1}^{N_s} w_{T_k}^{(i)}}$

(d) Re-sampling decision

If $\hat{N}_{eff} = \frac{1}{\sum_{i=1}^{N_s}(w_{T_k}^{(i)})^2} < N_{thresh}$ then perform deterministic re-sampling: $\left\{\tilde{z}_{T_k}^{(i)}, w_{T_k}^{(i)}\right\}_{i=1}^{N_s} \Rightarrow \left\{z_{T_k}^{(i)}, \frac{1}{N_s}\right\}_{i=1}^{N_s}$

(e) Distribution

$p(z_{T_k}|y_{1:k}) \approx \sum_{i=1}^{N_s} w_{T_k}^{(i)} \delta_{z_{T_k}^{(i)}}(\mathrm{d}z_{T_k})$

Repeat till the prognostic instant T_{prog} is reached

Algorithm 2 RUL estimation.

Given $\left\{z_{T_n}^{(i)}, w_{T_n}^{(i)}\right\}_{i=1}^{N_s}$, N_{depart} number of departure points, N_{traj} number of simulation trajectories for each point

For $j = 1, \ldots, N_{depart}$ **do**

- Generate uniform sample: $u_j \sim U(0,1)$
- Select departure point:

  $z_j^{\text{selected}} = z_{T_n}^{(k)}$ with $\sum_{l=1}^{k-1} w_{T_n}^{(l)} \le u_j < \sum_{l=1}^{k} w_{T_n}^{(l)}$
- **For** $k = 1, \ldots, N_{traj}$ **do**

  Simulate the trajectories according to the system description (presented in Sections 2)

  End

End

Obtain the empirical distribution of RUL

incompressible fluid (i.e. the mass density of fluid ρ is constant) is pumped into the first tank at the top by a pump motor drives. Then, the out flow from the first tank feeds the second tank.

The relation between the inlet flow rate and the pump motor control input u is represented as a first order system (Chen & Chen, 2008):

$$\frac{\mathrm{d}q_{in}}{\mathrm{d}t} = -\frac{1}{\tau_a} q_{in} + \frac{K_a}{\tau_a} u \tag{13}$$

where τ_a is the time constant of pump motor, K_a is the servo amplify gain (with the initial gain $K_{a_{init}}$). The pump saturates at a maximum input u_{max} and it cannot draw water from the tank, so $u \in [0, u_{max}]$.

The fluid leaves out at the bottom of each tank through valves with the flow rates according to the Torricelli rule:

$$q_{j,out} = K_{v_j}\sqrt{2gh_j}, \quad j = 1, 2 \tag{14}$$

where h_j is level of tank j, g is the acceleration of gravity and K_{v_j} is the specified parameter of the valve j.

Using the mass balance equation, the process can be described by following equations:

$$\begin{cases} \dfrac{\mathrm{d}h_1(t)}{\mathrm{d}t} = \dfrac{1}{S_1} q_{in} - \dfrac{K_{v1}}{S_1}\sqrt{2gh_1(t)} \\ \dfrac{\mathrm{d}h_2(t)}{\mathrm{d}t} = \dfrac{K_{v1}}{S_1}\sqrt{2gh_1(t)} - \dfrac{K_{v2}}{S_2}\sqrt{2gh_2(t)} \end{cases} \quad (15)$$

The water level of tank 2 is measured by a level measurement sensor and controlled by adjusting the pump motor control input which is calculated by a PID controller. The overall tank level control system is shown in Figure 3.

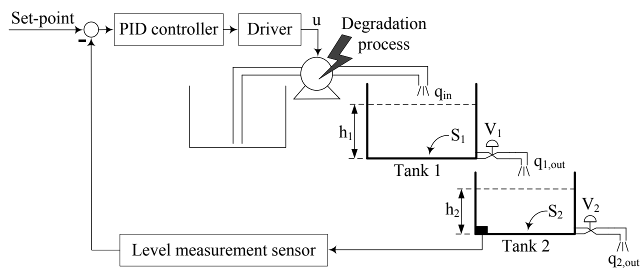


Figure 3. A double-tank level control system

Degradation process Due to degradation of the pump, its capacity $C(t) = K_a(t) = K_{a_{init}} - \mathcal{D}(t)$ stochastically decreases according to the presented model in Section 2.3. To have simple and comprehensible case study, we suppose that the set-point admits only two values r_1 and r_2 with $r_1 < r_2$.

It is assumed that the system evolves in a two-states operating mode: the normal mode (OM1) and the stressful mode (OM2). At each T_{change} time duration the operating mode can change. The evolution of operating mode is described by a Markov chain as represented as Figure 4 where set-point changes more frequently in OM2.

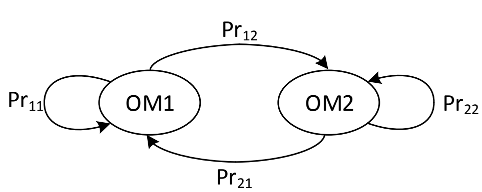


Figure 4. Operating mode Markov chain

The sojourn times in the different values of system set-point are characterized by a continuous-time Markov chain whose the transition rate matrix corresponding to the operating mode OMi is:

$$P_i = \begin{pmatrix} -\alpha_i & \alpha_i \\ \alpha_i & -\alpha_i \end{pmatrix} \quad (16)$$

where the parameters α_i describe transition rates of set-point of the operation mode OMi. Set-point changes more frequently in mode OM2 so $\alpha_2 > \alpha_1$.

Failure zone of the system According to Eq. (13) and Eq. (15), the steady states are obtained at instant t_{ss} if

$$u(t_{ss}) = \frac{S_1}{S_2}\frac{K_{v_2}}{K_a(t_{ss})}\sqrt{2gh_2(t_{ss})} \quad (17)$$

Since $u(t_{ss}) \leq u_{max}$ then

$$K_a(t_{ss}) \geq \frac{S_1}{S_2}\frac{K_{v_2}}{u_{max}}\sqrt{2gh_2(t_{ss})}$$

that means the actual capacity of the actuator must be greater than a minimal capacity defined in the control system design phase. In this case of study, this accepted value is defined as:

$$K_{a_{\min}} = \frac{S_1}{S_2}\frac{K_{v_2}}{u_{max}}\sqrt{2g\max_i r_i} = \frac{S_1}{S_2}\frac{K_{v_2}}{u_{max}}\sqrt{2gr_2} \quad (18)$$

Thus, the RUL of the system is the remaining time before the process Z enters in the failure zone which is defined as:

$$K_a(t) \leq K_{a_{\min}} \quad (19)$$

Under all these considerations, the behavior of water tank level control system can be summed up using the process $Z = (Z_t)_{t\in\mathbb{R}_+}$, where Z_t is given by:

$$Z_t = (K_a(t), h_1(t), h_2(t), \lambda^{om}(t), t) \quad (20)$$

The current state of the system at time t is then a five-component vector Z_t, which includes the current capacity of the pump, the water levels of two tanks, the current operating mode and the current time t.

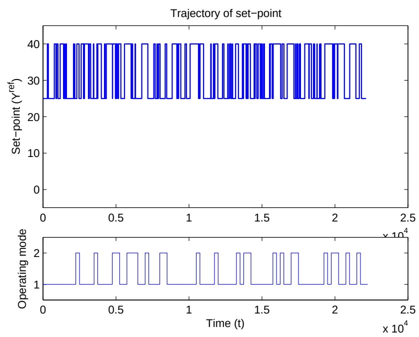
4.2. Numerical illustrations

Numerical values for double-tank level control system are summed up in Table 1.

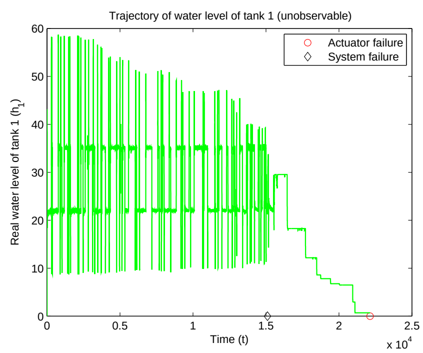
Figure 5 represents one trajectory of the process Z until the failure of system. The evolution of set-point with successive change of set-point values is illustrated in Figure 5(a). The water level of tank 1 and tank 2 $h_1(t)$ and $h_2(t)$ are reflected in Figure 5(b) and Figure 5(c). Figure 5(d) shows real (unobservable) value of actuator capacity $K_a(t)$.

As depicted in Figure 5, the actuator fails completely (i.e. $K_a = 0$) at 22129.4 time units, but the failure of system here is 15102.6 time units. One can find that after the system failure instant the water level of tank 2 (the controlled variable) cannot track the evolution of desired set-point.

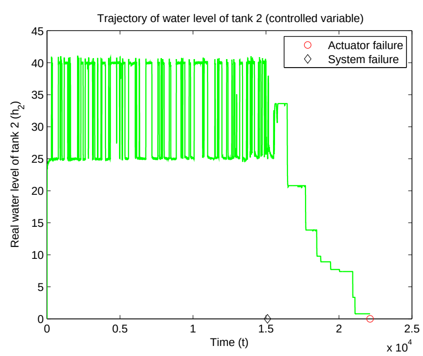
The only available health information of the system is the noisy observations of the water level of the tank 2 which are recorded during the transient periods whenever the set-point changes. For instance, let consider the prognostic time $T_{prog} = 8875.2$ time units i.e. at 64th change instant of the



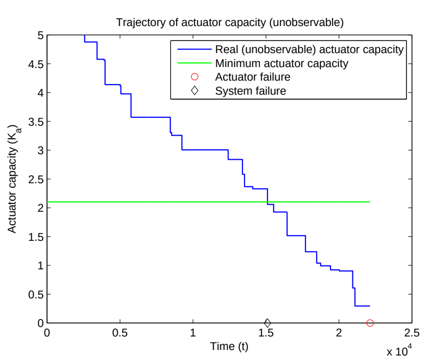
(a)



(b)



(c)



(d)

Figure 5. A trajectory of the water tank level control system until failure of actuator: (a) Set-point, (b) Water level of tank 1, (c) Water level of tank 2 and (d) Actuator capacity

Table 1. Double-tank model

Physical parameters		
$S_1 = 25$	$K_{v_1} = 8$	$\tau_a = 1$
$S_2 = 20$	$K_{v_2} = 6$	$g = 9.82$
$u_{max} = 100$	$\sigma = 0.05$	
PID controller parameters		
$K_P = 12.9896$	$T_I = 99.8432$	$T_D = 2.3727$
Initial condition: t = 0		
$h_1(0) = 0$	$h_2(0) = 0$	$K_{a_{init}} = 5.0$
Natural degradation		
$\lambda^{nd} = 10^{-3}$	$\Delta^{nd} = 0.5$	
Operating mode evolution		
$\Pr_{11} = 0.75$	$\Pr_{12} = 0.25$	$T_{change} = 250$
$\Pr_{21} = 0.75$	$\Pr_{22} = 0.25$	
Varying set-point		
$\alpha_1 = 0.006$	$r_1 = 25$	$\lambda_1^{om} = 5.10^{-4}$
$\alpha_2 = 0.01$	$r_2 = 40$	$\lambda_2^{om} = 10^{-3}$
$\Delta^{om} = 0.3$		

set-point, this health information is shown in Figure 6.

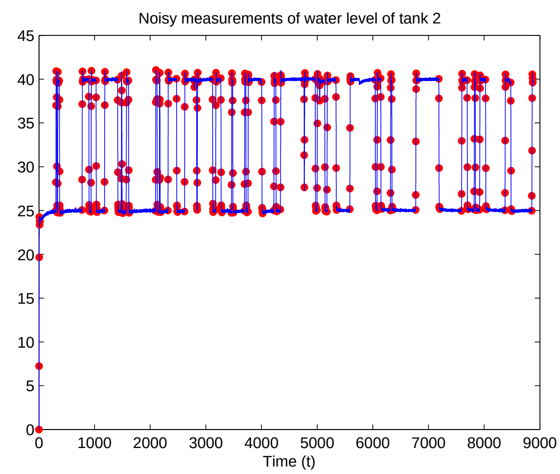
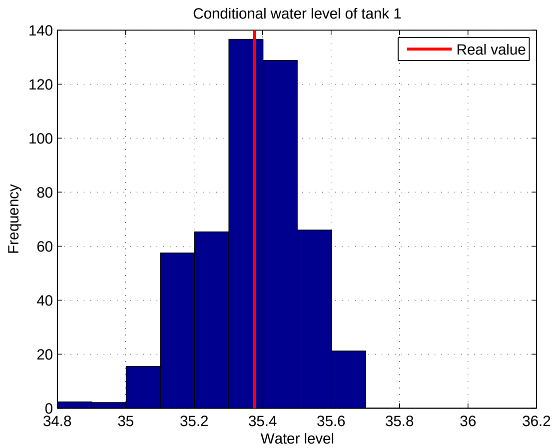


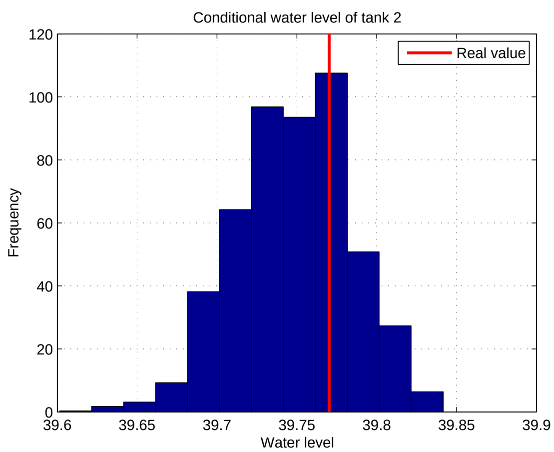
Figure 6. Noisy observations of water level of tank 2

The first step of the method is to compute the conditional state of the system knowing the noisy measurement of h_2 until the prognostic time T_{prog}. Approximations of the pdfs are represented in Figure 7(a) for the water level of tank 1, Figure 7(b) for the water level of tank 2 and Figure 7(c) for the actuator capacity with $N_s = 500$ particles.

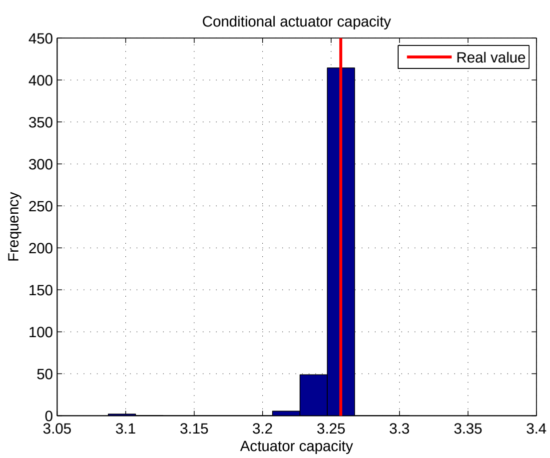
The last step of the method is to compute the distribution of the RUL of the system starting at T_{prog} knowing the approximated pdf of the system state at T_{prog}. The RUL distribution has been obtained by Monte Carlo simulation with 2500 trajectories describing the system evolution from its state at the prognostic time until its failure. The resulting RUL is depicted in Figure 8.



(a) Conditional water level of tank 1



(b) Conditional water level of tank 2



(c) Conditional actuator capacity parameter

Figure 7. Conditional distribution of the system state at time $T_{prog} = 15046.8$ time units given the noisy measurements of h_2 for $N_s = 500$ particles

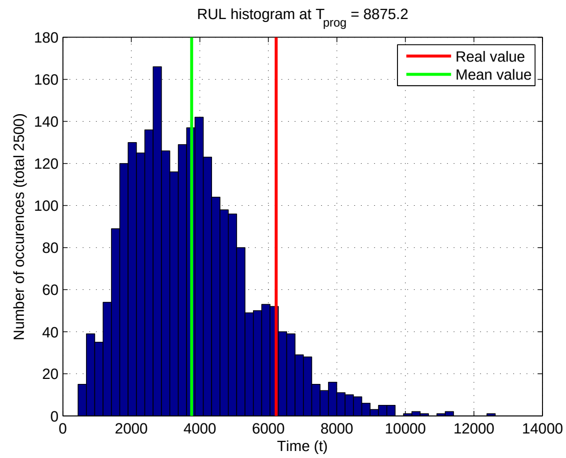


Figure 8. Remaining Useful Lifetime of the water tank level control system at time $T_{prog} = 8875.2$ time units

4.3. Maintenance strategies

To show how the prognosis information can be incorporated in maintenance decision-making, this section will compare a predictive maintenance which uses the on-line available information and an age based remplacement strategy. A cost model which is the long-run expected maintenance cost rate including the unavailability cost is developed in order to evaluated the performance of these maintenance strategies.

Predictive maintenance In this paragraph, a predictive maintenance policy is considered. Under this maintenance strategy, the system is replaced upon failure (corrective replacement action) or at a specified maintenance date which is calculated using the RUL information (preventive maintenance action). Both maintenance actions put the system back in as-good-as-new state, the interventions take negligible times and their costs are fixed. It is assumed that the replacement actions can only be performed at the opportunities (the instants of possible changes of operating mode, i.e. each time duration T_{change}). Therefore, there are a system inactivity after the stoppage of the system and an additional cost is incurred by the time d_i from the stoppage until the next replacement at a cost rate C_d which may correspond to production loss per unit of time.

The preventive maintenance date is updated through the working time of the system. Indeed, at each change of the setpoint, the associated RULs and the next maintenance time can be re-computed using the previously described methodology with the new arrival condition information. At each prognostic time T_{prog} the maintenance date which is the RUL of the system with a given failure probability η can be written using Eq. (9) as:

$$RUL(T_{prog}, \eta) \quad = \quad \sup\{\nu : \mathbb{P}(RUL_{T_{prog}} < \nu | Y_1 = y_1, \dots, Y_n = y_n) \leq \eta\} \quad (21)$$

where η is a decision parameter to be optimized. For a trade-off between the result accuracy and time computation, 500 particles and 2500 trajectories for RUL computation are chosen.

To assess the performance of the maintenance policy, a widely used criterion which is the expected maintenance cost per unit over an infinite time span is considered

$$C^{\infty}_{Pred}(\eta) = \lim_{t\to\infty} \frac{C_{Pred}(t,\eta)}{t} \tag{22}$$

where $C_{Pred}(t,\eta)$ is the cumulative maintenance cost at time t can be described as:

$$C_{Pred}(t,\eta) = \sum_{i=1}^{N_p(t)} C_p + \sum_{j=1}^{N_c(t)} C_c + C_d.d(t) \tag{23}$$

where $N_p(t)$, $N_c(t)$ are respectively the number of preventive maintenance and of corrective replacement in $[0,t]$; $d(t)$ is the total inactivity time of the system in $[0,t]$.

This cost criterion is then evaluated by stochastic Monte Carlo simulation. The optimal value of decision parameters η is obtained by miminizing the expected cost rate, i.e.,

$$C^{\infty}_{Pred}(\eta^*) = \min_{\eta}\{C^{\infty}_{Pred}(\eta), 0 < \eta < 1\} \tag{24}$$

Table 2. Maintenance costs

C_c	C_p	C_d
200	150	5

With the maintenance costs summarized in Table 2, the optimal values of $\eta = 0.45$ with the cost rate $C^{\infty}_{Pred}(\eta^*) = 0.08989$ (see Figure 9).

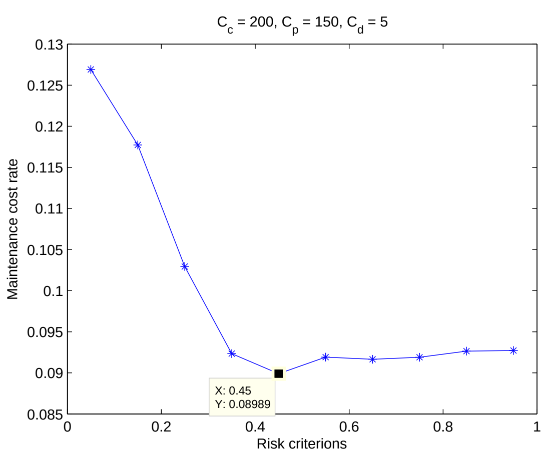


Figure 9. Long run expected maintenance cost per unit of time

Age-based remplacement strategy Like previously described predictive maintenance strategy, the maintenance actions are also executed only at the opportunities. The different point is that the system is preventively replaced at a specified date which does not change through the working time of the system. This specified date t_{Prev} is the parameter to be optimized.

Figure 10 illustrates the evolution of the system degradation behavior and the maintenance policy.

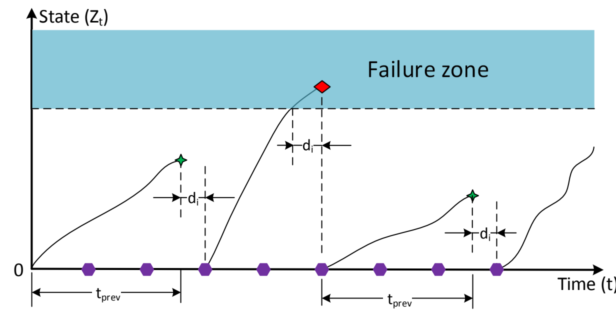


Figure 10. Illustration of considered systematic maintenance

The cumulative maintenance cost at time t in this strategy is:

$$C_{Prev}(t,t_{Prev}) = \sum_{i=1}^{N_p(t)} C_p + \sum_{j=1}^{N_c(t)} C_c + C_d.d(t) \tag{25}$$

where $N_p(t)$, $N_c(t)$ are respectively the number of preventive maintenance and of corrective replacement in $[0,t]$; $d(t)$ is the total inactivity time of the system in $[0,t]$.

The long run expected maintenance cost per unit of time is:

$$C^{\infty}_{Prev}(t_{Prev}) = \lim_{t\to\infty} \frac{C_{Prev}(t,t_{Prev})}{t} \tag{26}$$

This cost criterion is then evaluated by stochastic Monte Carlo simulation. The optimal value of preventive replacement age t^*_{Prev} is obtained by minimizing the expected cost rate, i.e.,

$$C^{\infty}_{Prev}(t^*_{Prev}) = \min_{t_{Prev}}\{C^{\infty}_{Prev}(t_{Prev}), t_{Prev} > 0\} \tag{27}$$

As represented in Figure 11, the optimal values of $t^*_{Prev} = 4750$ with the cost rate $C^{\infty}_{Prev}(t^*_{Prev}) = 0.03722$.

On the considered case study, the opportunist age-based remplacement policy and the predictive one efficiencies are very close to each other. This shows the effect of maintenance opportunities in the structure of the decision rule. Indeed, as represented in Figure 11, the age-based strategy can easily take into account the effects of maintenance opportunities. The local optima on the expected cost rate are coincide with the opportunities dates which lead to the cancel the inactivity cost. On the other hand, the predictive maintenance does

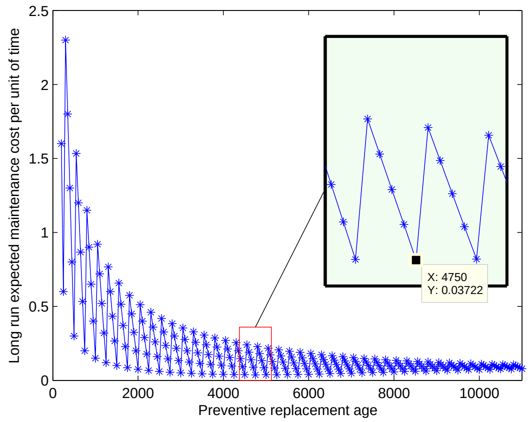


Figure 11. Evolution of long run expected maintenance cost per unit of time

not take directly into account the existence of maintenance opportunities and the decision rule is not well suited. As the cost of inactivity per unit of time is very high compared to the unit replacement cost the predictive maintenance cost is slightly higher tant the age-based one.

5. CONCLUSION

The present paper proposes a modeling framework using PDMP that shows the ability to combine the deterministic behavior of a feedback control system with the stochastic degradation process for the actuator. On the one hand, the actuator is less efficient through time because of natural degradation process. On the other hand, the set-point level impacts also the degradation process of actuator. Particle filtering technique is used to estimate on-line the state of considered system regarding only the noisy observations of closed system output. By using a methodology based on the assumption of Markov property, the Remaining Useful Lifetime can be deduced with Monte Carlo simulation. A simulated double-tank level control system was used as a case study to illustrate the efficiency of the proposed approach and the use of the prognostic information in order to optimize the decision-making process. A predictive maintenance whose the decision rules use the RUL estimation is compared with an age-based remplacement strategy. The long run expected maintenance cost per time unit is then used to assess the performance of two strategies. The results show the useful of RUL information on maintenance decision-marking process. However, the impact of maintenance opportunities should be taken into account in the structure of predictive decision rule.

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