A Joint Predictive Maintenance and Spare Parts Provisioning Policy for Multi-component Systems Using RUL Prediction and Importance Measure

Kim-Anh Nguyen[1], Phuc Do[2], and Antoine Grall[3]

[1,3] *Institut Charles Delaunay, Troyes University of Technology, CNRS UMR 6281, Troyes, France*
kim_anh.nguyen@utt.fr
antoine.grall@utt.fr

[2] *Lorraine University, CRAN, CNRS UMR 7039, Nancy, France*
van-phuc.do@univ-lorraine.fr

Abstract

The paper presents a joint predictive maintenance and spare parts provisioning policy for gradually deteriorating multi-component systems with complex structure. The decision-making process related to maintenance, spare parts ordering, as well as inspections scheduling is based on both RUL prediction and structural importance measure. Moreover, economic dependency between components is studied and integrated in decision rules. In addition, the impacts of the system structure on components deterioration process are also investigated. This dependency may have a significant influence on the RUL estimation of components. In order to evaluate the performance of the proposed joint predictive policy, a cost model is used. Finally, a numerical example of a 6-component system is introduced to illustrate the use and the advantages of the proposed joint maintenance and spare parts provisioning policy.

1. Introduction

Maintenance involves preventive and corrective actions carried out to retain a system or restore it to an operating condition. Optimal maintenance strategies aim to provide optimum system reliability/availability and safety performance at lowest possible maintenance costs. In recent years, condition monitoring and prognostic information are new trends being exploited for maintenance optimization. The use of prognostic information is often dedicated to estimate/predict the remaining useful life (RUL) that may be more advantageous for making decisions related to maintenance, spare parts ordering, as well as inspections scheduling. Several joint maintenance and spare parts inventory strategies using RUL prediction have been introduced in the literature. However, they are applicable to a limited class of systems such as mono-component systems (Elwany & Gebraeel, 2008; Boudhar, Dahane, & Rezg, 2013; L. Wang, Chu, & Mao, 2008), series structures systems with identical components (W. Wang, Pecht, & Liu, 2012; Van Horenbeek, Scarf, Cavalcante, & Pintelon, n.d.; L. Wang, Chu, & Mao, 2009; Xie & Wang, 2008). Today, with the development of industrial manufacturing, the structures of systems become more and more complex in inter-connections with a large number of different components. The inter-connections could be a mixture of well-known basic connections. The above problem remains widely open.

To face with this issue, the aim of this paper is to propose a joint predictive maintenance and spare parts provisioning policy for gradually deteriorating multi-component systems with complex structure. The decision-making associated with maintenance, spare parts ordering, and inspections scheduling is based on both components RUL and their corresponding importance measure. In fact, RUL provides the information about the future health of a component, while the structural importance measure gives a structural importance ranking of a component in the system. Both information should be taken into account in spare parts provisioning and maintenance decision-making. Moreover, economic dependency between components is studied and integrated in decision rules. In addition, the impacts of the system structure on components deterioration process are also considered. This may significantly influence on the components RUL estimation (Nguyen, Do Van, & Grall, 2013a, 2013b). In order to evaluate the performance of the proposed policy, a cost model is used. Furthermore, a simulation approach is developed to find the optimal decision values of the system's inspection time, of preventive maintenance and spare part ordering thresholds corresponding to each component.

This paper is organized as follows. Section 2 is devoted to the system description and deterioration modeling. The reliability/RUL prediction of components and their structural importance are described and discussed. Section 3 focuses on the description of inspection, maintenance, spare part ordering operations and related costs. The proposed joint predictive maintenance and spare parts provisioning policy is described in Section 4. In order to evaluate the performance of the proposed joint policy, a cost model is presented in Section 5. Section 6 is devoted to illustrate the use and the advantages of the proposed joint policy for 6-component system with complex structure. Some numerical results are, in addition, discussed here. Finally, the last section presents the conclusions drawn from this work.

2. SYSTEM DESCRIPTION AND PREDICTIVE RELIABILITY CALCULATION

2.1. Deterioration modeling framework

This paper considers a multi-component system whose components are non-identical, inter-connected according to a complex configuration which could be a mixture of several common basic connections (i.e. connection in series, in parallel, in k-out-of-n), and deteriorate gradually as shown in Fig. 1. To study such systems, the concepts of minimal cut sets, critical and non-critical components should be introduced. The definitions are given as follows:

1. A Minimal cut set (MCS) is a minimal set of components for which when all components of the set are failed, the system is then failed (Rausand & Høyland, 2004);
2. A component is said to be "critical" if a failure of the component, while the other components being in functioning state, lead to a failure of the system and "non-critical" otherwise.

Additionally, in order to model the deterioration of each component i ($i = 1, 2, \dots, N$) of the system, the following general assumptions are considered:

1. The deterioration level of the component i at time t can be measured and described by a scalar random variable X_t^i. Without any maintenance operation on the component i, the deterioration trajectory, $(X_t^i)_{t\geq 0}$, is a stochastic process and increases monotonically over time;
2. The initial deterioration level, X_0^i, is equal to zero, then the component i is considered as new one. The higher X_t^i, the closer the component i to failure. The component i is considered to be failed if X_t^i exceeds a predefined critical threshold D^i and its failure time is then expressed by $T_f^i = \inf\{t \in \mathbb{R}^+ | X_t^i \geq D^i\}$. The D^i can be seen as a deterioration level which must not be exceeded for economical or security reasons;
3. The deterioration increments considered between any two consecutive instants, Δx^i, are supposed to be stationary, nonnegative, and statistically independent.

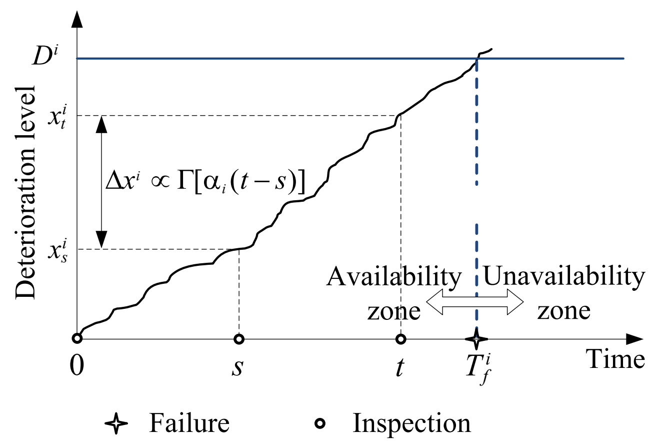

Figure 1. Description of the deterioration process $(X_t^i)_{t\geq 0}$ for component i without maintenance activities.

In this study the deterioration of each component of the system is assumed to be evolved like a homogenous Gamma process, whose characteristic is clearly monotonically increasing. It has been used widely to describe the degradation behaviors in several physical degradation process, e.g. (Grall, Dieulle, Bérenguer, & Roussignol, 2002; Van Noortwijk, 2009). For the Gamma deterioration process, the random increment Δx^i which is considered between two consecutive inspected times, t and s ($t > s$), follows a Gamma probability density function (pdf), $f_{\alpha_i (t-s), \beta_i}(x)$, with shape parameters α_i and scale parameter β_i, with $\alpha_i, \beta_i \in \mathbb{R}^{+*}$:

$$\frac{1}{\Gamma[\alpha_i (t-s)]} \beta_i^{\alpha_i (t-s)} x^{\alpha_i (t-s)-1} e^{-\beta_i x} \mathbb{I}_{\{x\geq 0\}} \quad (1)$$

where: $\Gamma(t) = \int_0^{+\infty} u^{t-1} exp(-u)\, du$ denotes the Euler Gamma function. The parameters α_i and β_i can be estimated from monitored degradation information of the component i. The mean deterioration rate and variance are determined by α_i/β_i and α_i/β_i^2, respectively. Various deterioration behaviors from almost-deterministic to very-chaotic can be modeled by such a stochastic process.

Finally, as mentioned above, if a non-critical component (that is present in the MCS of order greater 1) fails, it does not lead the system to a failure. However, if the component is not replaced as soon as possible, this may be cause to conduct some other components to idle states. More precisely, these components are disabled even if they are not failed. It is also supposed that the degradation level of components being idle state remains unchanged (Nguyen et al., 2013a).

2.2. Predictive reliability calculation

At time t, the reliability $R^i(t)$ of component i is defined as the probability that the component i is in an operational state between times 0 and t:

$$R^i(t) = \mathbb{P}[T_f^i > t] = 1 - \mathbb{P}[T_f^i \leq t] = 1 - \int_0^t f_i(u)du, \quad (2)$$

where: T_f^i is the random variable of time of failure of component i and $f_i(u)$ is its pdf. For the time-based reliability, an item is only considered in two states functioning or failed. Such a consideration only reflects average characteristics of the reliability; it cannot take into account information related to the condition (i.e. deterioration level) of the item during its operating process. Assume now that component i is functioning at time s, let $R^i(t|X_s^i = x_s^i)$ be a conditional reliability of the component i at instant t given its deterioration level at instant $s < t$, $X_s^i = x_s^i$. It can be determined by:

$$\begin{aligned} R^i(t|x_s^i) &= \mathbb{P}[X_t^i < D^i|x_s^i] = 1 - \int_{D^i - x_s^i}^{+\infty} f_{\alpha_i\,(t-s),\beta_i}(x)dx \\ &= 1 - \frac{\Gamma\left[\alpha_i\,(t-s), \beta_i\,(D^i - x_s^i)\right]}{\Gamma[\alpha_i\,(t-s)]}, \end{aligned} \quad (3)$$

where $\Gamma(\alpha, \sigma) = \int_\sigma^{+\infty} x^{\alpha-1}\, exp(-x)\, dx$ is incomplete Gamma function. $R^i(t|x_s^i)$ or $R^i(t|s)$ is also called the predicted reliability and computed at time s. Model parameters (α_i, β_i) can be estimated from complex data, see e.g. (Do Van, Levrat, Voisin, Iung, et al., 2012; Le Son, Fouladirad, Barros, Levrat, & Iung, 2013). The predicted reliability will be used for decision making in maintenance as well as spare part provisioning. Detail description will be presented in Section 4.

2.3. Importance measure

The importance of each component in a multi-component system may be assessed by the measure of structural importance which was proposed by (Birnbaum, 1969). It allows taking into account the topology importance of the logic position of components in a multi-component system to perform various decisions (Nguyen et al., 2013b). The structural importance measure is defined as follows:

Let v_i be a binary variable that describes the state of component i, $(i = 1, \ldots, N)$, such that $v_i = 1$ if the component i is operating and 0 otherwise; and let $\mathbf{v} = (v_1, ..., v_i, ..., v_N)$ be the state vector of the considered system. Then, the system state can be described by a binary/structure function $\Psi(\mathbf{v}) = \Psi(v_1, ..., v_i, ..., v_N)$. Where, $\Psi(\mathbf{v}) = 1$ if the system is operating and $\Psi(\mathbf{v}) = 0$ if the system is in a failed state.

The structural importance measure expresses the relative proportion of the 2^{N-1} possible state vectors which are critical state vectors for component i and is denoted I_B^i. A state vector is considered as critical for component i if for this state vector a change in the value of v_i causes a change of the structure function value. I_B^i is defined for component i as:

$$I_B^i = \frac{\delta_\Psi(i)}{2^{N-1}}, \quad (4)$$

where:

1. $\delta_\Psi(i)$ is the total number of critical state vectors for component i, i.e.

   $$\delta_\Psi(i) = \sum_{(\cdot_i, \mathbf{v})} [\Psi(1_i, \mathbf{v}) - \Psi(0_i, \mathbf{v})]$$

   (hence $1 \leq \delta_\Psi(i) \leq 2^{N-1}$);
2. $(.\,_i, \mathbf{v})$ represents all the possible 2^{N-1} state vectors when the state of component i is fixed and can be either $(1_i, \mathbf{v})$ component i is running or $(0_i, \mathbf{v})$ if it has failed.

In this paper, the structural importance measure is used to make decisions related to maintenance and spare parts ordering. Detail descriptions are presented in Section 4.

3. MAINTENANCE AND SPARE PARTS ORDERING OPERATIONS, AND RELATED COSTS

Inspection operation

In this framework, we assume that a failure of a component is instantaneously revealed by the self-announcing mechanism (e.g. by using smart-sensor) and the deterioration level of working components in system can only be known through periodic inspections at dates $t_k = k\delta t$, with δt is a fixed inter-inspection interval and $k \in \mathbb{N}$. The inspection operation is assumed instantaneous, perfect, non-destructive, and is incurred a cost c_{ins} for each component.

Maintenance operations

Two possible maintenance activities upon each component are preventive replacement (PR) before a failure and corrective replacement (CR) after a failure, which can restore completely the component to "as good as new" state. Both PR and CR activities can be performed at either inspection times or opportunistic maintenance times (i.e. system shutdown times). Also concerning maintenance time, each maintenance activity usually takes a time interval however it is often very small with respect to the time interval between two consecutive inspections. Therefore, in this work the maintenance durations are assumed to be negligible. In some cases, the failed system should be restored as soon as possible. So, some failed components are needed to be replaced immediately. An emergency order with negligible lead-time is then required if spare parts of the failed components are not available. As a result, an emergency ordering cost c_e is incurred for each component in these cases. In other cases, the system should be left in failed state to wait for the arrival of spare parts. Then, a system downtime may appear from the system's failure occurrence time until restored system time. Hence an unavailability cost rate, $c_{d,f}$, is incurred for every unit time when the system elapsed in failed state.

1. When performing a PR on component i, a PR cost, C_p^i, is incurred and calculated by:
$$C_p^i = c_p^i + c_{ms}, \tag{5}$$
where: c_p^i represents a specific PR cost; c_{ms} is the fixed set-up cost for maintenance, incurred once a time for all PR/CR activities. The set-up cost can be composed by the preparation costs (e.g. rent tools, labors, dissemble, etc.) and the cost of crew traveling. This cost depends on the characteristic of each system. It can be shared when several components are replaced at the same time.
2. Similarly, when performing a CR on component i, a CR cost, C_c^i, is incurred and calculated by:
$$C_c^i = c_c^i + c_{ms} + c_e \mathbb{I}_e^i, \tag{6}$$
where: c_c^i is a specific CR cost. A failure can have disastrous consequences, not only on the incurred cost due to unplanned interventions for example, but also on the environment as well as humans impact, hence it is reasonable to be assumed that $c_c^i > c_p^i$; c_e represents the emergency ordering cost of spare part for a component; $\mathbb{I}_e^i$ is indicator function to indicate that if $\mathbb{I}_e^i = 1$, spare part of component i is emergently purchased and if $\mathbb{I}_e^i = 0$, spare part of component i is not requested.

Spare parts ordering

The spare parts provisioning operation is of continuous time $(S-1, S)$ type of inventory policy which was been applied in several reports in the literature (Moinzadeh & Schmidt, 1991; Armstrong & Atkins, 1996). S is maximum stocking level. The inventory policy is suitable for systems for which demand rate is low but components are expensive (Moinzadeh & Schmidt, 1991). In our work, the studied system consists of N non-identical components which request N independent inventory policies $(S-1, S)$ corresponding to each component. We assume that the maximum number of spare parts is only one. It is either available in stock or present on an outstanding order for each component of the system at any time. This means that the maximum stocking level is $S = 1$ for each component at every time. Under this policy, a possible normal order (upon an inspection cycle) with a lead-time L is regularly placed just after each inspection time t_k for all components of the system. It is assumed that the lead-time L is constant and much lower than the inter-inspection interval. Spare parts of the normal order is delivered at two different dates that are named date1 and date2, respectively. Date1 includes a time interval L after t_k (i.e. at $t_k + L$) and date2 is at the next inspection time t_{k+1}. Let $n_1 \geq 0$ denote the number of spare parts at date1 and $n_2 \geq 0$ denote the number of spare parts at date2, with $n1 + n2 \leq N$. Then, total cost for a normal order per an inspection cycle including set-up cost for placing an order, specific ordering costs, and transportation costs is determined as follows:

$$C_o = \Big[c_{os} + \underbrace{\Big(\sum_{i=1, i\neq j}^{n_1} c_o^i + c_{1,ship}\Big)\mathbb{I}^i_{\{n_1>0\}}}_{\text{delivered at date1}} \tag{7}$$
$$+ \underbrace{\Big(\sum_{j=1, j\neq i}^{n_2} c_o^j + c_{2,ship}\Big)\mathbb{I}^j_{\{n_2>0\}}}_{\text{delivered at date2}}\Big]\mathbb{I}_{\{n_1+n_2>0\}},$$

where:

- c_{os} is the set-up cost for taking an order;
- c_o^i or c_o^j is the specific ordering cost of component i or component j, with $i \neq j$;
- $c_{1,ship}$ and $c_{2,ship}$ are transportation costs corresponding to ordered spare parts delivered at date1 and date2, respectively. Where, $c_{(.),ship}$ is calculated by:
$$c_{(.),ship} = \begin{cases} c_{0,ship} + c_{d,ship}(n_{(.)} - n_0) & \text{if } n_{(.)} > n_0, \\ c_{0,ship} & \text{for otherwise,} \end{cases} \tag{8}$$
where: $(.)$ can be 1 or 2; $c_{0,ship}$ is minimal transportation cost (deterministic cost) for one delivery time; n_0 is the minimal number of spare parts at which a minimal transportation cost $c_{0,ship}$ is incurred; $c_{d,ship}$ is transportation cost per spare part. It is calculated for spare parts only if their package exceeds the minimal number n_0.

In addition, after the ordered spare parts have been delivered, some of them may be utilized immediately for PR and/or CR activities, and the remainder is kept in the stock. For spare parts being in the stock, their deterioration is assumed to remain unchanged, that means they are kept "as-good-as-new". The inventory holding cost for each spare part corresponding to each component per a time unit is $k_h\, c_o^i$, where k_h is inventory holding rate per a spare part per a time unit.

4. JOINT POLICY OF PREDICTIVE MAINTENANCE AND SPARE PARTS PROVISIONING

At each time $t_k = k\,\delta t$ with $k \in \mathbb{N}$, the inspection is made on all functioning components of the system except components which have been selected for PR at the latest inspection time but have not been preventive replaced until the current inspection time. Thank to inspection operations, the deterioration level of each component can be measured. More precisely, for each component i, its the deterioration level at inspection times t_k, $X^i_{t_k} = x^i_{t_k}$, is determined.

The main idea of the proposed joint predictive maintenance and spare parts provisioning policy is to use jointly the structural importance measure and predictive reliability/RUL for selecting the spare parts provisioning and preventive maintenance actions. In fact, at each inspection time the decision rules for component i spare part ordering threshold R_o^i and PR threshold R_p^i are determined based on both structural importance and predictive reliability/RUL of the component. In this way, it is reasonable to be assumed that $R_o^i \geq R_p^i$. Consequently, parameters of the proposed joint policy are needed to be optimized including δt, R_p^i and R_o^i, with $i = 1, ..., N$.

4.1. Maintenance policy

Maintenance activities can only be performed at inspection times (planned maintenance) or when system fails (unplanned maintenance) given that the necessary related spare parts are available. Each planned or unplanned maintenance date is considered as a maintenance opportunity for executing together several preventive and/or corrective maintenance actions. In fact, at each maintenance opportunity, all functioning preventive components which have been selected for PR action and failed components are maintained together if their corresponding spare part (SP) are available. In this way, different maintenance decision rules are proposed for both preventive and corrective maintenance activities.

Maintenance decisions at t_k

Each inspection time $t_k = k\,\delta t$ (with $k = 1, 2...$) the inspection and maintenance decisions for each component i ($i = 1, ..., N$) are the following:

- If component i has already failed, it is correctively replaced if its SP is available;
- If component i is functioning, an inspection operation is firstly carried out, i.e. the deterioration level of the component is measured, $x^i_{t_k}$. Secondly, the predictive reliability of the component i $R^i(t_{k+1}|x^i_{t_k})$ is estimated (see again subsection 2.2). The main idea to build preventive maintenance decision rules for component i is to jointly consider its structural importance and predictive reliability. To this end, a fixed PR thresholds, R_p^i ($0 < R_p^i \leq 1$), is introduced as follows:

$$R_p^i = K_p\, I_B^i, \quad \text{with} \quad 0 < K_p \leq \frac{1}{\min\limits_{i=1,...,N}(I_B^i)}, \qquad (9)$$

  The coefficient K_p is the same for all components. I_B^i is the importance measure of component i and is calculated by Eq. (4).

  Finally, the preventive maintenance decision rules is the following:

  - If $R^i(t_{k+1}|x^i_{t_k}) \leq R_p^i$, then component i is selected for preventively replacement action. It is immediately replaced if its spare part is available otherwise the component will be replaced at a maintenance opportunity when its SP is available;
  - If $R^i(t_{k+1}|x^i_{t_k}) > R_p^i$, no maintenance action is carried out on component i.

Maintenance decisions between (t_k, t_{k+1})

This is concerned with unplanned maintenance which could occur randomly between (t_k, t_{k+1}) (i.e. the system fails). If the failure of component i does not lead the system to failed state, then no corrective maintenance action on the failed component i is carried out and the decisions related to this component will be placed at the next maintenance opportunity. Otherwise, if the failure of component i leads the system to failed state, then the decision rules of the system restoration are the following:

- If the component i is critical one, and
  - if its spare part is available, a corrective replacement is immediately carried together then the system is immediately restored;
  - if the spare part of i is present on an outstanding order, then the system is left in failed state and will be restored as soon as when the ordered spare part of i is delivered;
  - if the spare part of i neither available nor present on an outstanding order, then an emergency order is placed for the spare part of i. The system will be restored right away the arrival of this spare part;
- If the component i is non-critical one, and
  - if there is at least one spare part of a MCS that contains the component i which is available, then the system is immediately restored;
  - if there is not any spare part of the MCS (that contains the component i) available; but if at least one spare part of this MCS is present on an outstanding order, then the system is left in failed state and will be restored as soon as possible when the ordered spare parts is delivered.
  - if there is not any spare part of the MCS available or present on an outstanding order, then the spare part of i is emergently ordered. The system will be restored immediately the arrival of this spare part.

4.2. Spare parts provisioning policy

At every time, it is assumed that the maximum number of spare parts is only one which is either available in stock or presenting on an outstanding order for each component of the system. By inspection operations, a normal order is placed right away after the time t_k for the $(k + 1)$-th inspection cycle, in which a spare part if any of a component can only be delivered at either date1 or date2. The delivery is illustrated in Figure 2.

At time t_k, spare parts ordering rules are as follows:

1. If component i has failed and if its spare part is not available, spare part of i will be delivered at date1;

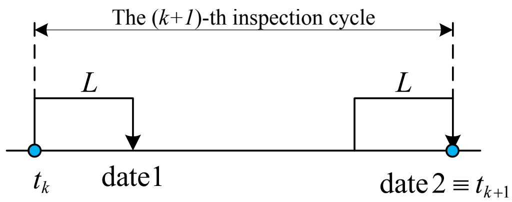

Figure 2. Illustration of delivery of a normal order.

2. If operating component i, but $R^i(t_{k+1}|x^i_{t_k}) \leq R^i_p$ and its spare part is not available, then spare part of i will also be delivered at date1;

3. If the predictive reliability $R^i(t_{k+1}|x^i_{t_k})$ is higher the PR threshold R^i_p but lower or equal the ordering threshold R^i_o, then spare part of i will be delivered at date2. The ordering threshold introduced here is formulated as:

$$R^i_o = K_o\, I^i_B, \quad \text{with} \quad 0 < K_o \leq \frac{1}{\min\limits_{i=1,\dots,N}(I^i_B)}, \tag{10}$$

Each I^i_B only depends on the system configuration and remains unchanged with time. Therefore, the optimal PR threshold R^{i*}_p and the optimal ordering threshold R^{i*}_o for each component i can be determined from the global optimal coefficients K^*_p and K^*_o, respectively. K_p, K_o and δt are the decision parameters of the proposed joint predictive policy which have to be optimized. For this purpose, a cost model is proposed to evaluate the performance of the joint policy based on the long-term mean cost rate criteria. It is presented in the next section.

5. PERFORMANCE EVALUATION OF PROPOSED JOINT POLICY

Accumulative total cost until time t of whole system includes costs of CR and PR (including set-up costs of maintenance), inspection costs, downtime costs, spare parts ordering costs (including set-up costs of the purchase, transportation costs), and inventory holding costs:

$$C_T(t) = \underbrace{C_{corr}(t) + C_{prev}(t) + C_{ins}(t)}_{C_M(t)\text{: costs related to maintenance}} + \underbrace{C_{downtime}(t) + C_o(t) + C_{hold}(t)}_{C_I(t)\text{: costs related to inventory}} \tag{11}$$

To assess the performance of the proposed joint policy, the long-term expected average costs of maintenance and inventory per unit time is considered. It is defined as:

$$C^{\infty}_T(K_p, K_o, \delta t) = \lim_{t\to\infty} \frac{E[C_M(t)] + E[C_I(t)]}{t} \tag{12}$$

If t is large enough, Eq. (12) can be rewritten as follows:

$$C^{\infty}_T(K_p, K_o, \delta t) \simeq \frac{E[C_M(N_m\,\delta t)] + E[C_I(N_m\,\delta t)]}{\textit{\#Operating time of system}}, \tag{13}$$

where: *#Operating time of system* = $N_m\,\delta t$ - *#Total downtime of system*, N_m is the number of inspection times in $[0, t]$ of whole system.

To develop a cost model for evaluating the policy performance, the additional following notations will be used in this section:

$\mathbb{I}_{\{x^i_t \geq D^i\}}$ indicates whether component i is failed at time t before any decision is made.
If $x^i_t \geq D^i$, $\mathbb{I}_{\{x^i_t \geq D^i\}} = 1$: failed;
if $x^i_t < D^i$, $\mathbb{I}_{\{x^i_t \geq D^i\}} = 0$: functioning;

$\mathbb{I}^i_{PS}(t)$ indicates whether component i satisfies PR condition ($0 < R^i(t+\delta t|x^i_t) \leq R^i_p$) at time t.
$\mathbb{I}^i_{PS}(t) = 1$: satisfying; $\mathbb{I}^i_{PS}(t) = 0$: otherwise;

$\mathbb{I}^i_{CR}(t)$ indicates whether component i is correctively replaced at time t. $\mathbb{I}^i_{CR}(t) = 1$: replaced;
$\mathbb{I}^i_{CR}(t) = 0$: otherwise;

$\mathbb{I}_{ER}(t)$ indicates whether there is a component that must be made emergency CR at time t.
$\mathbb{I}_{ER}(t) = 1$: emergency CR;
$\mathbb{I}_{ER}(t) = 0$: no emergency CR;

$\mathbb{I}^i_{stock}(t)$ indicates whether spare part of i is avaiable in stock at time t. $\mathbb{I}^i_{stock}(t) = 1$: available;
$\mathbb{I}^i_{stock}(t) = 0$: unavailable;

$\mathbb{I}^i_{outstd}(t)$ indicates whether spare part of i is present on an outstanding order at time t. $\mathbb{I}^i_{outstd}(t) = 1$: present; $\mathbb{I}^i_{outstd}(t) = 0$: not present;

$\mathbb{I}^i_{OD1,k}$ indicates whether, in k-th cycle, a purchase decision for spare part of i with date1 is placed.
$\mathbb{I}^i_{OD1,k} = 1$: ordered; $\mathbb{I}^i_{OD1,k} = 0$: not ordered;

$\mathbb{I}^i_{OD2,k}$ indicates whether, in k-th cycle, a purchase decision for spare part of i with date2 is placed.
$\mathbb{I}^i_{OD2,k} = 1$: ordered; $\mathbb{I}^i_{OT1,k} = 0$: not ordered.

Inspection cost $C_{ins}(t)$
At each time t_k, the inspection is made on all functioning components of the system except for components satisfying PR condition at latest inspection time but for which any replacement action has been carried out until the current inspection time. The total inspection cost over the time span t is formulated:

$$C_{ins} = c_{ins} \sum_{i=1}^{N} \sum_{k=1}^{N_m} \mathbb{I}^i_{ins}(t_k), \tag{14}$$

where, $\mathbb{I}^i_{ins}(t_k)$ indicates whether an inspection action on the component i should be implemented at time t_k. $\mathbb{I}^i_{ins}(t_k) = 1$ means that an inspection is needed and $\mathbb{I}^i_{ins}(t_k) = 0$ otherwise. $\mathbb{I}^i_{ins}(t_k)$ is defined as follows:

$$\mathbb{I}^i_{ins}(t_k) = \begin{cases} 0 & \text{either } \mathbb{I}_{\{x^i_{t_k} \geq D^i\}} = 1 \text{ or } \Big(\mathbb{I}^i_{PS}(t_{k-1}) = 1 \\ & \text{but any replacement action (PR or CR)} \\ & \text{has been carried out until the current} \\ & \text{inspection time } t_k\Big); \\ 1 & \text{otherwise.} \end{cases}$$

Corrective and preventive replacement costs
$C_{corr}(t) + C_{prev}(t)$
Between two inter-inspection times (the inspection time is not including), an intervention to restore the system if and only if the system has been failed. Therefore, total replacement cost can be separated into a replacement cost at inspection times and a replacement cost outside of inspection times (opportunistic maintenance times). Let $M \in \mathbb{N}$ denote the total number of intervention times in order to restore the system from failed state without inspection times. And let t_m represent the m-th intervention time on the system ($t_m \neq t_k$). If $M \neq 0$, the total replacement cost of the system during its mission is formulated as follows:

$$\sum_{m=1}^{M} \Big(\sum_{i=1,i\neq j}^{N} c^i_p \mathbb{I}^i_{PR}(t_m) + \sum_{j=1,j\neq i}^{N} c^j_c \mathbb{I}^j_{CR}(t_m) + c_{ms} + c_e \mathbb{I}_{ER}(t_m)\Big)$$
$$+ \sum_{k=1}^{N_m} \Big(\sum_{i=1,i\neq j}^{N} c^i_p \mathbb{I}^i_{PR}(t_k) + \sum_{j=1,j\neq i}^{N} c^j_c \mathbb{I}^j_{CR}(t_k) + c_{ms}\Big) \tag{15}$$

where:

- At inspection time t_k, $\mathbb{I}^i_{PR}(t_k)$ and $\mathbb{I}^j_{CR}(t_k)$ are defined as follows:

$$\mathbb{I}^i_{PR}(t_k) = \begin{cases} 1 & \text{if } \Big(\mathbb{I}_{\{x^i_{t_k} \geq D^i\}} = 0 \text{ and } \mathbb{I}^i_{ins}(t_k) = 1 \\ & \text{and } \mathbb{I}^i_{PS}(t_k) = 1 \text{ and } \mathbb{I}^i_{stock}(t_k) = 1\Big), \\ & \text{or } \Big(\mathbb{I}_{\{x^i_{t_k} \geq D^i\}} = 0 \text{ and } \mathbb{I}^i_{ins}(t_k) = 0 \\ & \text{and } \mathbb{I}^i_{stock}(t_k) = 1\Big), \\ 0 & \text{otherwise;} \end{cases}$$

$$\mathbb{I}^j_{CR}(t_k) = \begin{cases} 1 & \text{if } \mathbb{I}_{\{x^i_{t_k} \geq D^i\}} = 1 \text{ and } \mathbb{I}^i_{stock}(t_k) = 1 \\ 0 & \text{otherwise.} \end{cases}$$

- At time $t_m \neq t_k$, if it is assumed that the failure occurrence of components of the system are not simultaneous, the failed system is restored if there is at least one necessary spare part for CR action (i.e. the spare part can be either available in stock or bought emergently or the ordered spare part has just been delivered). The system is failed due to:
  (i) a critical component i. Then $\mathbb{I}_{ER}(t_m)$ is defined:

$$\mathbb{I}_{ER}(t_m) = \begin{cases} 1 & \text{if its spare part } i \text{ is not available} \\ & \text{and outstanding in any current order,} \\ 0 & \text{otherwise.} \end{cases}$$

  (ii) a non-critical component i. Then $\mathbb{I}_{ER}(t_m)$ is defined:

$$\mathbb{I}_{ER}(t_m) = \begin{cases} 1 & \text{if there is neither any spare part of} \\ & \text{components in MCS that contains the} \\ & \text{component } i \text{ is available nor} \\ & \text{presenting on outstanding order,} \\ 0 & \text{otherwise.} \end{cases}$$

  In addition, other failed components and functioning components that satisfied PR condition at latest inspection time but for which any replacement has been made until the current time (t_m) are also opportunistically replaced at this instant. Therefore, the indicators $\mathbb{I}^i_{PR}(t_m)$ and $\mathbb{I}^j_{CR}(t_m)$ are determined as follows:

$$\mathbb{I}^i_{PR}(t_m) = \begin{cases} 1 & \text{if } \Big(\mathbb{I}_{\{x^i_{t_m} \geq D^i\}} = 0 \text{ and} \\ & \mathbb{I}^i_{PS}(t_k < t_m) = 1 \text{ and} \\ & \mathbb{I}^i_{PR}(t_k < t_m) = 0 \text{ and} \\ & \mathbb{I}^i_{stock}(t_m) = 1\Big), \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathbb{I}^j_{CR}(t_m) = \begin{cases} 1 & \text{if } \mathbb{I}_{\{x^j_{t_m} \geq L^j\}} = 1 \text{ and } \mathbb{I}^j_{stock}(t_m) = 1, \\ 0 & \text{otherwise;} \end{cases}$$

Note that after each preventive replacement of the component i at $t = t_k$ or $t = t_m$, $\mathbb{I}^i_{PS}(t)$ should always be reset zero.

Downtime cost $C_{downtime}(t)$
It is assumed that lead-time for emergency orders is negligible. Thus, in the k-th inspection cycle, the downtime of system is equal zero during from $(t_{k-1} + L)^+$ to $(t_k - L)^-$. The downtime of system can only occur in the period from t^+_{k-1} to $t_{k-1} + L$ and in the period from $t_k - L$ to t_k. Thus the downtime of the system in the k-th inspection cycle is determined as:

$$C_{downtime}(t) = c_{d,f} \sum_{k=1}^{N_m} (t_{ef1,k} + t_{ef2,k}) \tag{16}$$

where $t_{ef1,k}$ is the time elapsed by the system in the failed state in the period from t^+_{k-1} to $t_{k-1} + L$, and $t_{ef2,k}$ is the time elapsed by the system in the failed state in the period from $t_k - L$ to t_k in the k-th inspection cycle.

Ordering cost $C_o(t)$

$$\sum_{k=1}^{N_m} \Big[\Big(\sum_{i=1,i\neq j}^{n_1} c^i_o \mathbb{I}^i_{OD1,k} + c_{1,ship}\Big)\mathbb{I}_{\{n_1>0,k\}} \tag{17}$$
$$+ \Big(\sum_{j=1,j\neq i}^{n_2} c^j_o \mathbb{I}^j_{OD2,k} + c_{2,ship}\Big)\mathbb{I}_{\{n_2>0,k\}} + c_{os}\Big]\mathbb{I}_{\{n_1+n_2>0,k\}}$$

where, $\mathbb{I}^i_{OD1,k}$ and $\mathbb{I}^j_{OD2,k}$ can be defined as follows:

$$\mathbb{I}^i_{OD1,k} = \begin{cases} 1 & \text{if } \left(\mathbb{I}_{\{x^i_{t_{k-1}} \geq D^i\}} = 1 \text{ or } \mathbb{I}^i_{PS}(t_{k-1}) = 1\right) \\ & \text{and } \mathbb{I}^i_{stock}(t) = 0, \\ 0 & \text{for otherwise;} \end{cases}$$

$$\mathbb{I}^j_{OD2,k} = \begin{cases} 1 & \text{if } R^j_p < R^j(t_k|x^i_{k-1}) \leq R^j_o \text{ and } \mathbb{I}^j_{stock}(t) = 0, \\ 0 & \text{for otherwise;} \end{cases}$$

$c_{1,ship}$ and $c_{2,ship}$ are the transportation costs are calculated as Eq. (8).

Spare parts holding cost $C_{hold}(t)$

$$C_{hold}(t) = \sum_{i=1}^{N} c^i_0\, k_h \sum_{z_i=1}^{Z_i} t^i_{hold,z_i} \tag{18}$$

where:

- Z_i total number of spare parts of component i that are used to replace (preventively and correctively) during the system's mission;
- t^i_{hold,z_i} holding time interval of z_i-th spare part. It is determined by:

$$t^i_{hold,z_i} = t^i_{output,z_i} - t^i_{input,z_i} \tag{19}$$

  where, t^i_{output,z_i} and t^i_{input,z_i} are instants when z_i-th spare part is stocked and taken away from the inventory, respectively.

Determining optimal solutions of proposed joint policy
The optimal solution of the joint policy $(K_p, K_o, \delta t)$ can be obtained by minimizing the expected global average cost per unit of time of whole system $C^{*,\infty}_T(K_p, K_o, \delta t)$ i.e.:

$$\begin{aligned} C^{\infty}_T(K^*_p, K^*_o, \delta t^*) &= \min_{K_p, K_o, \delta t} C^{\infty}_T(K_p, K_o, \delta t) \\ \text{subject to: } & 0 \leq L < \delta t, \\ & K_p \in \left(0, \frac{1}{\min\limits_{i=1,\ldots,N}\left(I^i_B\right)}\right], \\ & K_o \in \left(0, \frac{1}{\min\limits_{i=1,\ldots,N}\left(I^i_B\right)}\right], \\ & K_o \geq K_p. \end{aligned} \tag{20}$$

The numerical calculation can be done by Monte Carlo simulation. The optimal PR thresholds R^{i*}_p and the optimal ordering thresholds R^{i*}_o corresponding to each system component is directly derived from the optimal value K^*_p and K^*_o, respectively.

6. NUMERICAL EXAMPLE

The main aim of this section is to validate and to analyze the performance of the proposed joint policy of maintenance and spare parts provisioning. For this end, a study is performed on a 6-component system whose the degradation evolution of each component is assumed to be a gamma process. The system structure is shown in Figure 3.

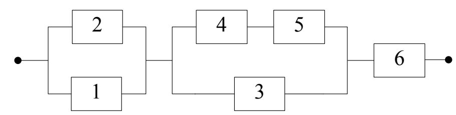

Figure 3. Reliability block diagram of the system consist of six components.

The parameters related to all components such as deterioration parameters, prefixed failure thresholds, ordering costs, preventive and corrective replacement costs, and importance measures are listed in Table 1. The parameters related to the system are inspection cost $c_{ins} = 3$, set-up cost of maintenance operation $c_{ms} = 30$, downtime cost rate $c_{d,f} = 30$, emergency ordering cost $c_e = 100$, set-up cost for placing an order $c_{os} = 3$, minimal transportation cost for a delivery time $c_{0,ship} = 30$, transportation cost for one spare part $c_{d,ship} = 5$, minimal number of spare parts of an order $n_0 = 2$, inventory holding rate per a spare part per time unit $k_h = 0.004$, and lead-time $L = 10$ time units. The components of the system are s-independent and their parameters have been arbitrarily chosen for the purpose of the numerical study.

Table 1. Parameters for each component.

Comp.	α_i	β_i	D^i	c^i_o	c^i_p	c^i_c	I^i_B
1	0.8	1.25	40	120	36	96	0.15625
2	1.3	1.8	38	120	36	96	0.15625
3	0.8	1.5	45	180	54	144	0.28125
4	0.6	0.9	42	150	45	120	0.09375
5	0.7	0.8	39	150	45	120	0.09375
6	0.5	1.3	50	250	75	200	0.46875

6.1. Experimental results

The PR thresholds are used to determine components that should be preventively replaced and the ordering thresholds are to determine components that should be ordered to prepare available spare parts for next preventive replacements. In this proposed joint model, an order is placed just after inspection time t_k with the two possible delivery dates, where date2 is to prepare available spare parts for PRs at t_{k+1} while date1 (that is earlier than date2) is to replenish as soon as possible spare parts if PRs and/or CRs cannot be performed at t_k on corresponding components due to the unavailability of spare parts. The lack is partly due to the uncertainty in the RUL prediction. Clearly the spare parts are required at date1 to reduce the system's breakdown and the emergency ordering costs, but on the other hand they may make the inventory holding costs increase. Hence, it is necessary to choose carefully the appropriate decision parameters of K_p, K_o, and δt in order to balance these costs.

According to a set of all given parameters, in order to find the optimum decision parameters (i.e. K_p^*, K_o^*, and δt^*), the expected global average cost rate C_T^∞ is evaluated with different values of K_p, K_o, and δt by using Eq. (11) and Eq. (13)-(20). The obtained minimum global average cost rate is 20.129 with three corresponding decision parameters: $K_p^* = 1.51$, $K_o^* = 3.63$, and $\delta t^* = 45$, i.e. $C_T^{*,\infty}(1.51, 3.63, 45) = 20.129$. The optimal PR thresholds corresponding to each component are inferred from the K_p^* by using Eq. (9), such that: $R_p^{1*} = R_p^{2*} = 0.24$, $R_p^{3*} = 0.42$, $R_p^{4*} = R_p^{5*} = 0.14$, and $R_p^{6*} = 0.71$. Similarly, the optimal ordering thresholds corresponding to each component are inferred from the K_o^* by using Eq. (10), such as: $R_o^{1*} = R_o^{2*} = 0.56$, $R_o^{3*} = 1$, $R_o^{4*} = R_o^{5*} = 0.34$, and $R_o^{6*} = 1$. The results show that the PR threshold of the critical component is much higher than that of the non-critical components. The same conclusion is drawn for the ordering thresholds. The ordering threshold of each component is much higher than its PR threshold. It is also noted in this case that the optimal ordering threshold of component 3 and component 6 are equal to one, this means that the spare part of the two components must be regularly replenished at each inspection date t_k.

Figure 4 shows the cost surface considering at the $\delta t^* = 45$ as a function of the PR coefficient K_p and the ordering coefficient K_o. The surface is clearly convex.

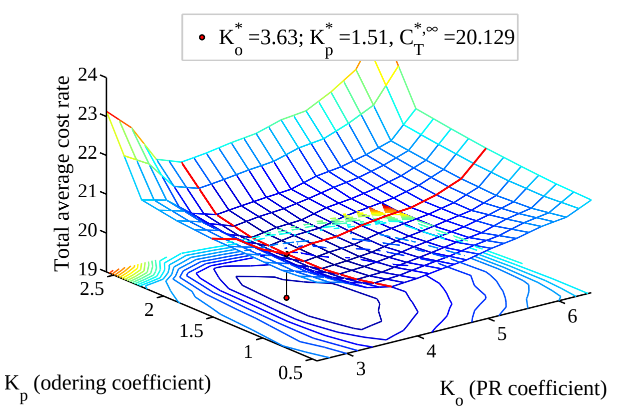

Figure 4. Cost surface of the proposed joint policy as a function of the K_p and the K_o considering at $\delta t^* = 45$.

6.2. Comparison of the joint and separate optimized approach

Considering the benefits form the proposed joint model under the jointly optimized approach, a comparison with a traditional maintenance model and a traditional provisioning model, which are separately optimized, is performed. Under the separately optimized approach, the expected mean cost rate of the maintenance model, C_M^∞, depends only on the inspection cost, the preventive and corrective replacement costs, and the set-up cost; while the expected mean cost rate of the inventory model, C_I^∞, depends solely on the downtime cost, the costs related to spare parts ordering, and the inventory holding cost. Figure 5 shows the average cost rate as a function of the inter-inspection time interval for the joint and separate optimization with the same given parameters.

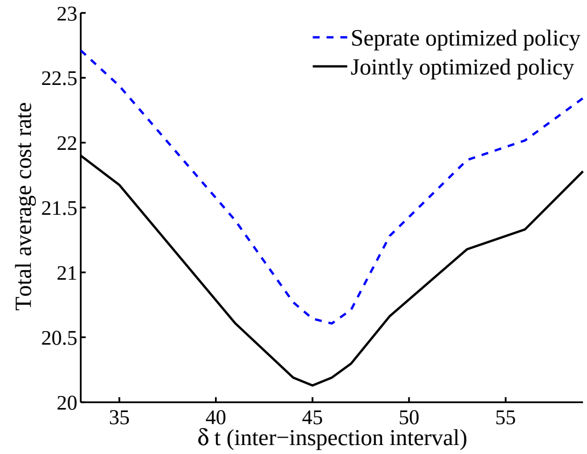

Figure 5. Relation of the total average cost rate between the joint and separate optimization.

The obtained results clearly show that the cost of jointly optimized policy, in most situations, is lower than that of their separately optimized counterparts (i.e. the sum of C_M^∞ and C_I^∞). This is because all cost parameters associated to the maintenance and the inventory are simultaneously considered in the joint model, hence achieving more appropriate values for the decision variables.

The relative cost difference between the two approaches varies from 2% to 3.5%, and of course, this difference is dependent upon the input parameters of the system. In the next paragraph, the influences of some main parameters such as the lead-time, the holding cost, and the set-up cost on the proposed joint policy are studied.

6.3. Sensibility analysis

To investigate the influences of the lead-time on the total average cost of the proposed policy, the numerical experiments are carried out for the different values of the lead-time. Figure 6 exhibits the optimum values of $C_T^{*,\infty}$ increase when the lead-time increases from 1 to 21 time units (the other given parameters remains unchanged).

The results obtained from the sensibility analysis show that when the lead-time increases, it leads to decrease the optimal inter-inspection time interval and increase the optimal PR thresholds as well as the optimal ordering thresholds. This means that: the system should be inspected more frequently, the components need to be preventively maintained earlier

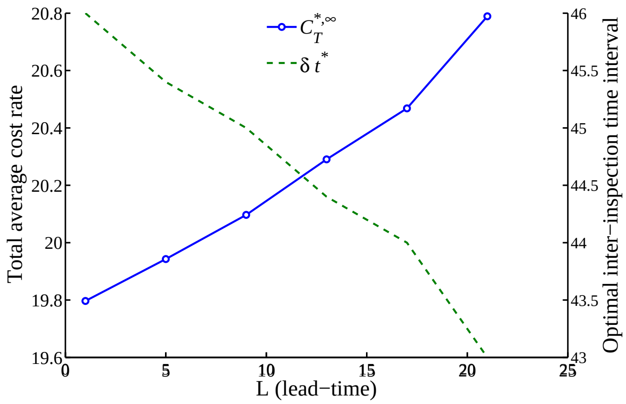

Figure 6. Influence of the lead-time on the proposed joint policy.

(compared to their lifetime), and the spare parts also need to be ordered earlier so as to prevent a failure of components which may lead to the system failure.

The effect of the inventory holding rate is shown in Table 2, where the total average cost increases significantly as the increase of the inventory holding rate.

Table 2. Optimal results with given inventory holding rates.

k_h	δt^*	K_p^*	K_o^*	$C_T^{*,\infty}$
0.0	46	1.69	3.81	19.39
0.004	45	1.51	3.63	20.12
0.008	45	1.51	3.45	21.46
0.012	44	1.33	3.45	22.53
0.016	44	1.33	3.10	23.52
0.020	43	1.33	3.10	24.62

Herein, when the inventory holding rate k_h is varying from 0 to 0.02 with increments of 0.004, the optimal ordering thresholds (as well as the optimal PR thresholds) decrease to reduce the inventory levels. Besides, δt^* also decreases. This shows the system should be inspected more frequently in order to reduce the risk due to the decrease of the inventory levels.

Table 3 shows the influence of the set-up cost of the maintenance c_{ms} on the total average cost when the c_{ms} varies from 0 to 100 cost units.

It is surprising that the higher the set-up cost is the higher the total average cost is. When the c_{ms} increases, the optimal PR thresholds as well as the optimal ordering thresholds increase, which indicates that the components of the system tends to be preventively maintained earlier. At the same time the optimal inspection cycle decreases slightly. Consequently, there are more selected components in a group for the PR activities in order to save set-up cost.

Table 3. Optimal results with given maintenance set-up costs.

c_{ms}	δt^*	K_p^*	K_o^*	$C_T^{*,\infty}$
0	46	1.33	2.92	19.55
20	45	1.33	3.28	20.21
40	45	1.51	3.63	20.68
60	45	1.69	3.81	21.03
80	45	1.86	3.81	21.59
100	44	1.86	3.98	22.16

7. CONCLUSION

In this paper, a joint predictive maintenance and spare parts provisioning policy for multi-component systems with complex inter-connections is proposed. Predictive reliability/RUL of components and their structural importance measure are jointly used and integrated in maintenance and spare parts decision-makings. Moreover, both economic and structural dependencies are investigated and considered in the proposed policy. This allows a better modeling of multi-component system. In addition, to evaluate the performance of the proposed joint predictive policy, a cost model is used. Finally, Monte-Carlo simulation approach is implemented in order to final the optimal decision parameters. The numerical results show that the proposed joint policy is more appropriate than when considering maintenance policy and spare parts provisioning one separately. The joint combination of predictive reliability and structural importance measure can provides a powerful tool for decision-making on maintenance et spare parts provisioning.

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NOMENCLATURE

N	number of components of the system
i	index for components, with $i = 1, 2, ..., N$
$X_t^i = x_t^i$	deterioration level of component i measured at time t
$(X_t^i)_{t\geq 0}$	stochastic process describing the deterioration of component i over time t
α_i, β_i	shape and scale parameters of Gamma distribution for component i
c_p^i	specific preventive cost for component i
c_c^i	specific corrective cost at failure for component i (generally $c_c^i > c_p^i$)
c_{ins}	inspection cost for each component
c_{ms}	set-up cost for a maintenance operation
$c_{d,f}$	loss cost per time unit incurred by the system in the failed state due to shortage of spare parts
c_{os}	set-up cost for placing an order and independent of the ordered quantities of spare parts

c_o^i	spare part ordering cost for component i
c_e	emergency ordering cost for one spare part
$c_{0,ship}$	minimal transportation cost for a delivery
$c_{d,ship}$	transportation cost per a spare part
n_0	minimal number of spare parts of an order at which a cost $c_{0,ship}$ is incurred
k_h	inventory holding rate per a spare per time unit
δt	inter-inspection time interval (inspection cycle)
L	lead-time for a regular order, $L > 0$
t_k	k-th inspection time, $t_k = k\,\delta t$ and $k \in \mathbb{N}$
$R^i(t\|s)$	predictive reliability of component i at time t given that component i has survived for time s
I_B^i	structural importance measure of component i
K_p	preventive replacement coefficient
K_o	spare parts ordering coefficient
R_p^i	PR threshold defined for component i
R_o^i	ordering threshold defined for component i
$C_T(t)$	cumulative total cost at time t
$C_M(t)$	cumulative maintenance cost at time t
$C_I(t)$	cumulative inventory cost at time t
N_m	number of inspection times of the whole system within $[0, t]$
C_T^∞	long-term expected total average cost rate
C_M^∞	long-term expected maintenance average cost rate
C_I^∞	long-term expected inventory average cost rate

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BIOGRAPHIES

Kim-Anh Nguyen holds a Bachelor's degree in Electrical Engineering from the Danang University of Science & Technology (in 2004), and a Science Master degree in Control Engineering and Automation from the University of Danang (in 2009), Danang, Vietnam. He is currently pursuing his doctorate in System Safety and Optimization at the University of Technology of Troyes,

Troyes, France. He is a member of Vietnam Automation Association since 2009. He obtained four national awards in the filed of Control Engineering and Automation. His current researches are interesting in stochastic modeling of systems deterioration, prognostic and health management techniques, maintenance and inventory decision-making, optimization and evaluation.

Phuc DO is currently associate professor at Lorraine University, Research Centre for Automatic Control (CRAN CNRS UMR 7039), France. He received his Ph.D. in Systems Optimisation and Dependability in 2008 from Troyes University of Technology (France) where he held an assistant professor position from 2009 to 2011. His research interests include stochastic modeling of systems deterioration, optimization of maintenance policies (condition-based maintenance, prognostics for maintenance decision-making, opportunistic and dynamic grouping maintenance), reliability importance measures and their related applications.

Antoine Grall is Professor at the Troyes University of Technology and currently head of the Statistics, Operations Research and Numerical Simulation Department. He is member of Charles Delaunay Institute (Systems Modeling and Dependability Group). His current research interests include maintenance policies evaluation and optimization, stochastic degradation modeling for prognosis, and modeling for quantitative analysis of systems dependability.