Lifetime models for remaining useful life estimation with randomly distributed failure thresholds

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ABSTRACT

In order to predict in advance and with the smallest possible uncertainty when a component needs to be fixed or replaced, lifetime models are developed based on the information of the component deterioration trend and its failure threshold to estimate the stochastic distribution of the hitting time (the first time the deterioration exceeds the failure threshold) and the remaining useful life. A primary issue is how to effectively handle the uncertainties related to the component deterioration trend and failure threshold. This problem is here investigated considering a non-stationary gamma process to model the component deterioration and a gamma-distributed failure threshold. Two lifetime models are proposed for comparison on an application concerning deterioration of choke valves used in offshore oil platforms.

1. INTRODUCTION

The capability of predicting when maintenance actions are required is a primary issue for every industry and bears the advantages of enhancing operational safety and maximizing plant reliability. In this respect, to estimate in advance and with an acceptable level of uncertainty the component remaining useful life, one can either define a failure time probability based on the failure times records of a large number of similar components, or exploit the information on the component deterioration trend during operation (Nystad, 2008; Gola & Nystad, 2011a). The latter approach is less conservative and allows tailoring maintenance planning to the specific case and, as a consequence, maximizing the usage of the component.

In practice, lifetime estimation models (van Noortwijk, 2009; Lu & Meeker, 1993) are devised to combine the knowledge of the past deterioration trend and the current degradation state with the failure threshold and to estimate the hitting time (Abdel-Hameed, 1975; Frenk & Nicolai, 2007) and the remaining useful life (van Noortwijk, 2009; Rausand & Høyland, 2004).

The uncertainty associated to the deterioration trend is here modelled by a non-stationary gamma process (Gola & Nystad, 2011a; van Noortwijk, 2009). A gamma process is a stochastic process with independent, non-negative gamma-distributed increments and represents a valuable option to model monotonic processes, i.e. with gradual damage monotonically accumulating over time in a sequence of increments such as wear, fatigue, erosion/corrosion, crack growth, erosion, creep and swell.

The specification of the failure threshold is a critical issue (Nystad, 2008; van Noortwijk, 2009). In fact, using a deterministic threshold is problematic since the same component can fail at different degradation levels. Typically, an unbiased estimate of the threshold mean value, or a conservative lower-bound threshold estimate are supplied. Nevertheless, if the threshold value is set too high, the risk of actual component failure will increase. On the contrary, a conservative low threshold value reduces the risk of failure, but increases the failure probability to a point in which the component can be prematurely put off operation. For some applications, e.g. cable aging due to thermal and mechanic damage (Fantoni & Nordlund, 2009), the designer may not know with certainty what explicit level of degradation causes a failure. If threshold failure data are scarce an alternative source of information are engineers with expertise within the relevant field. Such experts can provide useful information about the threshold probability distribution in form of best estimates of percentiles.

This problem is here tackled by considering the threshold as a random variable with a gamma probability distribution. A likelihood function can then be established based on the expert judgment in terms of percentiles (Welte & Eggen, 2008).

A practical application concerning erosion in choke valves used in the oil and gas industry is considered (Gola & Nystad, 2011a; Bringedal, Hovda, Ujang, With & Kjørrefjord, 2010) and two lifetime models for estimating the remaining useful life are proposed and compared.

2. THE HITTING TIME AND REMAINING USEFUL LIFE

Since the failure threshold variability does not depend on the temporal uncertainty associated to the deterioration trend but only on the historical failure records of the component, it is reasonable to assume that the threshold distribution is independent from the deterioration distribution (Abdel-Hameed, 1975).

In this view, the cumulative density function of the hitting time is defined in Abdel-Hameed (1975) and can be written for each time $t \geq 0$ as:

$$H_Y(t) = \Pr\left(X(t) \geq Y\right)$$

$$= \int_{x=0}^{\infty} \int_{y=0}^{x} f_{X(t)}(x) f_Y(y) dy dx \qquad (1)$$

$$= \int_{x=0}^{\infty} F_Y(x) f_{X(t)}(x) dx$$

where $f_{X(t)}$ is the probability density function (pdf) of the deterioration trend $X(t) \geq 0$, f_Y is the pdf and F_Y is the cumulative density function (cdf) of the failure threshold $Y \geq 0$ (satisfying $F_Y(0) = 0$).

The meaning of Eq. (1) is illustrated in Figure 1 using the choke valve case study data (see Section 3). Based on the erosion data for the operational time interval $t \in (0, 280]$ (diamonds in Figure 1), the expected value (solid line) and 5th and 95th percentiles (dashed lines) of the fitted gamma process with assumed power law shape are shown. Notice that the functional shape of the erosion process at timestamp $t = 280$ is convex. The failure threshold is here defined as a gamma distribution, i.e. the hazard zone (red contour plot in the Figure). The probability of failure in the operational time is illustrated by the hitting time pdf (blue line).

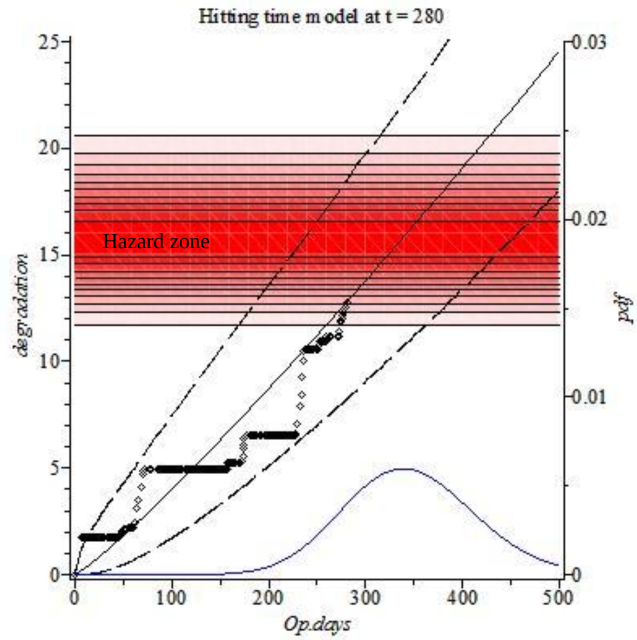

Figure 1. The hitting time probability density function (blue line); fitted gamma process with power-law shape (black solid and dashed lines) and a gamma distributed hazard zone (red).

The remaining useful life at time $t > s \geq 0$ is derived from Eq. (1) (Rausand & Høyland, 2004) and is here calculated by resorting to a state-based approach (Gola & Nystad, 2011b) which accounts for the knowledge of the deterioration state x_s at time $t = s$:

$$RUL(t) = \int_{x_s}^{\infty} \frac{F_Y(x) - F_Y(x_s)}{1 - F_Y(x_s)} f_{X(t)-X(s)}(x - x_s) dx \qquad (2)$$

Recalling that in a time-based perspective the deterioration x is a function of t, the pdf $f_{X(t)-X(s)}(x - x_s)$ represents the probability of having at time t a deterioration increment $x - x_s$ and the term $\left(F_Y(x) - F_Y(x_s)\right) / \left(1 - F_Y(x_s)\right)$ is the left-truncated cdf of the failure threshold providing the probability of having the failure threshold y between the current deterioration state x_s and infinity.

The meaning of Eq. (2) is illustrated in Figure 2. The fitted gamma process is the same with the exception that here there is no uncertainty in the erosion in the operational time interval $t \in (0, 280]$. The expected value (solid line) remains unchanged; the 5th and 95th percentiles (dashed lines) are instead calculated based on the erosion increment $x - x_s$. The left-truncated failure threshold (red contour plot) and the pdf of the RUL (blue line) are finally shown.

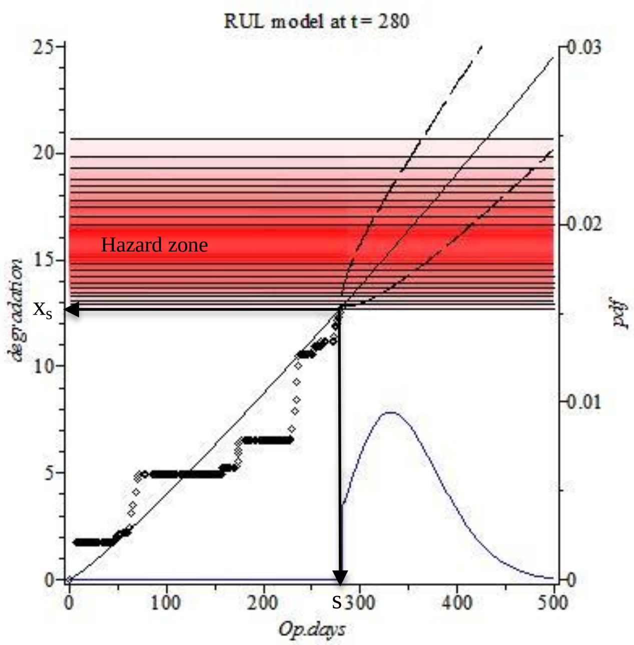

Figure 2. RUL probability density function (blue line); fitted gamma process with power-law shape (black solid and dashed lines); left-truncated gamma distributed hazard zone (red).

Nevertheless, for a distribution without memory (as e.g. the exponential distribution) there is no advantage in left-truncating the cdf of the threshold and therefore the expression of the remaining useful life becomes the same as the left-truncated version of the hitting time.

Notice that since the hitting time model in Eq. (1) considers uncertainty in the whole deterioration trend from $t = 0$ to infinite, the associated uncertainty calculated at $t = s > 0$ is higher than that of the pdf depending only on the prediction from $t = s$ to infinity.

2.1. The deterioration model

The deterioration $X(t)$ is here modelled as a non-stationary gamma process (van Noortwijk, 2009) with a time-dependent pdf written as:

$$f_{X(t)}(x) = \frac{u^{v(t)}}{\Gamma\left(v(t)\right)} x^{v(t)-1} e^{-ux} \qquad (3)$$

where $\Gamma\left(v(t)\right) = \int_0^\infty z^{v(t)-1} e^{-z} dz$ is the gamma function with shape parameter $v(t) > 0$ and scale parameter $u > 0$, $X(0) = 0$ with probability one, the deterioration increment $X(t) - X(s)$ gamma-distributed with shape parameter $v(t) - v(s)$ and scale parameter u for any $t > s \geq 0$ and the stochastic process $\left\{X(t), t \geq 0\right\}$ having independent increments. The shape function $v(t)$ must be non-decreasing, right-continuous and real-valued for $t \geq 0$, with $v(0) = 0$ and $v(\infty) = \infty$. When $v(t)$ is linear the gamma process is stationary and it is non-stationary when $v(t)$ is non-linear.

2.2. The threshold model

Indeed, the hitting time (Eq. 1) and remaining useful life (Eq. 2) models are well suited to handle different types of uncertainties of the failure threshold related for example to the estimate of the initial deterioration (due to imperfect maintenance or production defects), to the manufacturing variability and to the historical measurements.

In this paper, a gamma-distributed failure threshold $Y \sim Ga(y|\alpha, \beta)$ with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$ is considered, with pdf and cdf given for any $y \geq 0$ as:

$$\begin{aligned} f_Y(y) &= \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\beta y} \\ F_Y(y) &= \frac{\gamma(\alpha, \beta y)}{\Gamma(\alpha)}, y \in [0, \infty) \end{aligned} \qquad (4)$$

where $\gamma(\alpha, \beta y) = \int_0^{\beta y} z^{\alpha-1} e^{-z} dz$ is the lower-incomplete gamma function. Notice that the shape parameter α is in this case a time-independent constant.

2.3. Expected deteriorations

In general, the expected deterioration $E(X(t))$ can be linear, concave, convex or any combination of these. As discussed in van Noortwijk (2009), the power law function is a flexible candidate for linear, concave and convex deterioration.

$$E\left(X\left(t\right)\right) = \frac{v(t)}{u} = \frac{ct^b}{u} \qquad (5)$$

In this case, the gamma process is linear and stationary if $b = 1$, non-stationary concave and convex if $b < 1$ and $b > 1$, respectively.

However, the process in Eq. (5) cannot describe a deterioration trend both concave and convex. Given the restrictions on $v(t)$, a candidate process, which describes the expected degradation that is first concave and then convex (i.e., z-shaped) is:

$$E\left(X\left(t\right)\right)=\frac{c\left(\sinh\left(ab\right)+\sinh\left(a\left(t-b\right)\right)\right)}{u} \qquad (6)$$

where the shape parameter $b>0$ is the timestamp of inflection and $a>0$ is related to the size of the derivative in the inflection point.

An example of an expected deterioration as in Eq. (6) is the impact of external stress on materials/devices (Mc Pehrson, 2010). The net reaction rate for material/device degradation becomes concave (linear) with low stress and convex with high stress.

2.4. Inference of the model parameters

In practice, the application of the gamma process requires using statistical methods for estimating the parameters from the available measurements. For the gamma process a typical data set consists of inspection times t_i, $i=1,\ldots,n$, where $0=t_0<t_1<t_2<\ldots<t_n$, and the corresponding observations of cumulative amounts of deterioration x_i, $i=1,\ldots,n$, where $0=x_0\le x_1\le x_2\le\ldots\le x_n$. The estimators for the scale parameters u and c for the power law (Eq. 5) and z-shaped (Eq. 6) degradations can be derived by the method of moments or the method of maximum likelihood (van Noortwijk, 2009). The method of moments leads to attractive and simple formulae for the parameters, but it requires knowledge of the shape parameters values of the power law (b) and z-shaped (a,b) degradations which are either given based on experts' opinion (Welte & Eggen, 2008) or can be inferred numerically by least square optimization. On the other hand, the method of maximum likelihood, explained in van Noortwijk (2009), allows estimating directly the shape and scale parameters, at the expenses of larger computational costs. In the application that follows, first least square optimization is used to determine the shape parameters and then the method of moments is applied to calculate the scale parameters (u,c).

Concerning the failure threshold, historical values for a number of similar components can be used to calculate the mean and standard deviation of the failure threshold distribution (Nystad, 2008). For highly reliable components for which failures are rare, one can use few deterioration samples with their associated parameters and Monte Carlo simulations to generate a large number of deterioration paths. Different threshold values randomly selected from a threshold distribution can then be used to estimate the hitting time (Lu & Meeker, 1993). Finally, a source of information is constituted by field experts (Welte & Eggen, 2008). Since the meaning of many probability distribution parameters is rather abstract, experts have usually problems estimating them directly. In fact, experts can provide useful information about the threshold distribution in terms of best estimates (mean, median, mode) or percentiles (e.g. a 10th percentile corresponding to early failures) which can be used to estimate the parameters of two-parametric probability distributions like the gamma distribution.

3. DETERIORATION OF CHOKE VALVES

The application proposed in this paper concerns deterioration of choke valves undergoing erosion (Bringedal et al., 2010; Andrews, Kjørholt & Jøranson, 2005). In offshore oil platforms, choke valves are used on the surface to control the flow of hydrocarbons and protect the equipment from unusual pressure fluctuations. Production experience has shown that choke valves are prone to sand erosion in the disks and in the outlet sleeve (Andrews et al., 2005). The main parameters determining erosion are the impact velocity and the impact angle of the sand grains through the choke discs.

Figure 3. Damage caused by sand erosion. In the picture the original circular holes in the disks have a major wear on the upper side on the left hole and lower side on the right hole.

From the mathematical point of view, the flow characteristic C_V is defined so that the pressure differential Δp across the choke valve is constant and total mass flow rate w through the valve is proportional to the valve flow coefficient C_V which is related to the effective flow cross-section of the valve and therefore depends on the valve opening.

$$w=C_V\sqrt{\frac{\Delta p}{\rho}} \qquad (7)$$

where ρ is the average mixture density. The C_V curve is specific to the valve type and size and for a given valve opening C_V is expected to be constant (Kirmanen, Niemelä, Pyötsiä, Simula, Hauhia & Riihilahti, 2005). The C_V characteristic curve is the baseline for a good as new valve and is often provided by the valve constructors. When erosion occurs, a gradual increase of the effective flow cross-section is observed even at constant pressure drop. Such phenomenon is therefore related to an abnormal increase of the valve flow coefficient with respect to its expected baseline value, hereby denoted as C_V^b. For this reason, for a given valve opening the difference δ_{C_V}

between the actual flow coefficient and its baseline is retained as an indicator of the valve erosion. The difference $\delta_{C_V} = C_V - C_V^b$ is expected to be monotonically increasing throughout the life of the valve, thus reflecting the physical behavior of the erosion process. When δ_{C_V} eventually reaches an established erosion threshold, the valve must be replaced (Gola & Nystad, 2011a)

The valve flow coefficient C_V in a multiphase environment cannot be directly measured, but it can be calculated from the following analytical expression which accounts for the physical parameters involved in the process:

$$C_V = \frac{w_o + w_w + w_g}{N_6 F_p \sqrt{\Delta p}} \sqrt{\frac{f_o}{\rho_o} + \frac{f_w}{\rho_w} + \frac{f_g}{\rho_g J^2}} \qquad (8)$$

where w_o, w_w and w_g are the flow rates of oil, water and gas, f_o, f_w and f_g the corresponding fractions with respect to the total flow rate and ρ_o, ρ_w and ρ_g the corresponding densities. J is the gas expansion factor, F_p is the piping geometry factor and N_6 is a constant equal to 27.3 (Kirmanen, Niemelä, Pyötsiä, Simula, Hauhia & Riihilahti, 2005). The quality of the available data of the physical parameters in Eq. (8) differs because Δp is directly measured, whereas oil, water and gas flow rates are calculated based on daily production rates of other wells of the same field. Improvement of the valve erosion indicator δ_{C_V} based on additional information from well tests carried out throughout the valve life is discussed in Gola and Nystad (2011a). Therefore, in this paper, a single choke valve undergoing erosion is considered and hitting time models and new RUL models based on Eq. (2) are applied to the δ_{C_V} trend obtained in Gola and Nystad (2011a) as a function of the operational days. The valve was opened and checked to be found in a failed state at operational time $t_n = 307$ days.

Expert judgment is here used to define the failure threshold probability distribution (Welte & Eggen, 2008). For a gamma-distributed threshold, the experts provide the best central estimate which, in this case study, is equal to the mean value of the threshold set by the experts ($y = 16$) and they are also asked to assess the boundaries of the interval in which the true value of the threshold falls. A measure of the uncertainty of the expert opinion is the standard deviation of Y. By having the expert claiming that e.g. the true threshold lays between the values 14 and 18 being most likely equal to 16, one can calculate the shape parameter α and the scale parameter β of Eq. (4) from $E(Y) = \alpha/\beta = 16$ and $\sigma_Y(Y) = \sqrt{\alpha/\beta^2} = 2$ which yields $\alpha = 64$ and $\beta = 4$. This hazard zone distribution is shown as a red contour plot in Figure 1. The skewness of a gamma distribution is $2/\sqrt{\alpha} = 0.25$, a value which implies a good fit to the expert's claim.

Figures 4 and 5 show the δ_{C_V} trend and its estimation provided by the power-law (Eq. 5) and z-shaped (Eq. 6) models obtained at different operational days, namely t_n = 100, 200, 250 and 307. Because cumulative amounts of deterioration are measured, the last inspection contains the most information. For the gamma process the expected deterioration at the last inspection time (at time t_n) equals x_n; that is, $E(X(t_n)) = x_n$ (van Noortwijk, 2009). Figures 6 and 7 illustrate the remaining useful life and the associated uncertainty (5th and 95th percentiles) obtained for each $t_n \geq 239$ when using the power-law and z-shaped models, respectively.

Notice that the power-law shape becomes convex only after 250 operational days (Fig. 4), thus leading to an overestimation of the component remaining useful life (Fig. 6) with respect to its theoretical value (red dashed line). On the other hand, using the z-shaped model at 240 operational days one can already identify the final z-shape of the degradation (Fig. 5), with the consequence of obtaining better estimations of the remaining useful life, i.e. closer to its theoretical value and with a reduced uncertainty (Fig. 7).

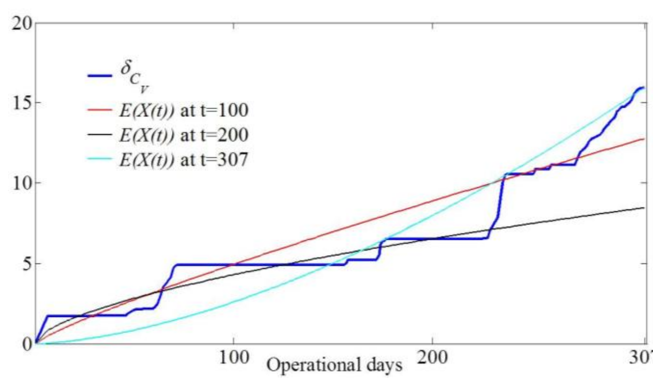

Figure 4. δ_{C_V} trend (thick line) and corresponding estimations provided by the power-law model.

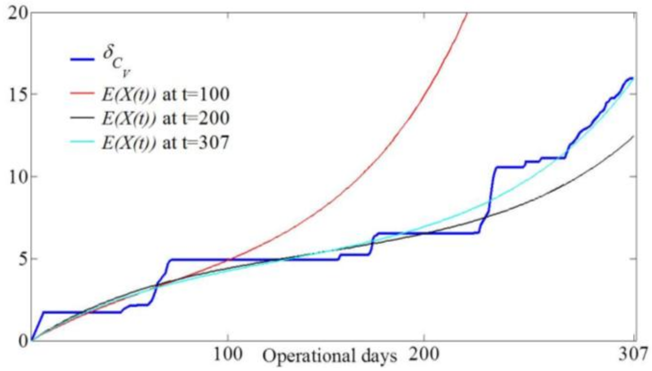

Figure 5. δ_{C_V} trend (thick line) and corresponding estimations provided by the z-shaped model.

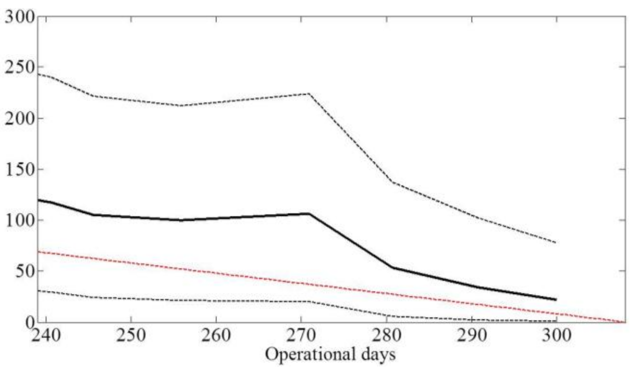

Figure 6. Remaining useful life estimation with the power-law model (95% confident interval) and theoretical value (red dashed line).

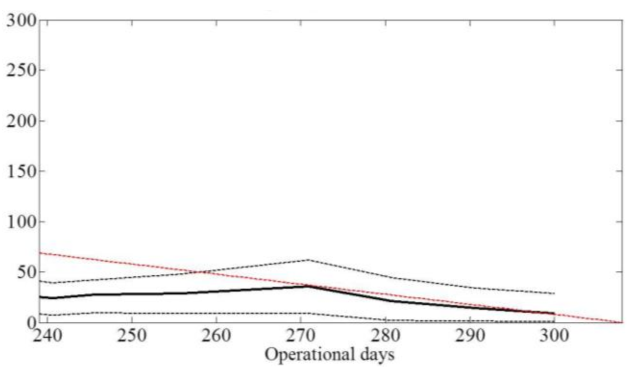

Figure 7. Remaining useful life estimation with the z-shaped model (95% confident interval) and theoretical value (red dashed line).

4. Conclusion

This paper has investigated the problem of estimating the remaining useful life of components using stochastic lifetime models and considering randomly-distributed failure thresholds. In particular, gamma processes with power-law and z-shaped shape functions (i.e. first concave, then convex) have been proposed to predict the deterioration; a gamma distribution has been considered to model the failure threshold for it is frequently used as a probability model in life testing and it is a flexible distribution for modeling the uncertainty in experts' opinions. The failure threshold distribution is also known to contain only positive real values, i.e. $x \in [0, \infty)$.

A case study of erosion of choke valves used in offshore oil platforms has been considered and the results of the expected deterioration calculation and the remaining useful life estimation given by the power-law and z-shaped models have been compared. An a priori knowledge of the overall shape of the deterioration is valuable. In this respect, with some efforts the shape of the expected erosion can be assumed beforehand by some engineering expertise. However, the model is general and can be applied also to other cases where the distribution of the parameters for a maintenance model must be estimated.

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