Estimating Remaining Useful Life Using Actuarial Methods

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ABSTRACT

In many instances, condition monitoring equipment has not been installed on machinery. Yet, operators still need guidance as to when to perform maintenance that is better than what is offered by the equipment manufacturers. For these systems, running hours, counts, or some other measure of usage may be available. This data, along with failure rate data, can provide an expected time to failure, and the estimated remaining useful life. The failure rate (even small sample size) is used to estimate the shape and scale parameters for the Weibull distribution. Then the conditional expectation of the truncated survival function of the Weibull is used to estimate the time to failure. This is an actuarial technique to solve the conditional survival function problem of: given that the equipment has survived to time x, what is that probability of the equipment surviving to time $x + y$. The inverse cumulative distribution of the truncated survival function can then be used to estimate the remaining useful life, that is: a time when the conditional likelihood of failure is small, such as 10%. The 90% confidence of the shape and scale parameters is then used to give a bound on the remaining useful life. This method is then tested on a real world bearing dataset.

1. INTRODUCTION

There is for many industrial operators, a point where the business conditions force them into reducing costs. When evaluating cost saving measures that impact productivity, condition monitoring is clearing one methodology that is attractive. For most operators, unscheduled maintenance events impact profitability. Unscheduled maintenance events can be impacted through design, condition monitoring, or a combination of both. Design efforts usually take time, and are costly to retrofit into existing platforms. This leaves condition monitoring techniques as one of the most cost effective means to reduce unscheduled maintenance, thus improving productivity and profitability.

There are a number of condition monitoring techniques that an operator can explore, including vibration, acoustic emissions, and oil particle/condition monitoring. These methods give the operator an indication of the damage/state of the machine under monitoring. Other methods, such as usage tracking, can be used as well. These “open loop” (indirect) methods, while not as powerful as the direct (closed loop) measure of damage, such as vibration, can still bring value to the operator in planning a maintenance event.

Consider the operators current maintenance paradigm. The equipment, such as a drilling machine, is designed with an operating life of 20 years. This life does not take into account a number of externalities, such as:

- Oil contamination
- Oil level low/oil starvation
- Unanticipated loads
- Variations in material quality
- Improper maintenance

When the operator experiences an unscheduled maintenance event, for large equipment in remote locations, the down time and loss of productivity can be costly.

However, many large and critical manufacturing equipment have programmable logic control (PLC) units that can record torque, rpm, current, and at the very least, operating hours. This level of detail, in addition to historic failure data, can be used to develop a prognostics health management (PHM) program. This in turns helps advice the operator when to perform maintenance, and reduces the chance of an unscheduled maintenance event. For an operator initiating a condition monitoring (CM) program, this is a powerful and low cost mean to approach this.

Actuarial science is a discipline within mathematics where statistical methods are applied to estimate future contingent events. The discipline has evolved with great pace during the last 30 years due to the development of modern computers, and is an indispensable tool in industries such as insurance, finance and healthcare. Although the discipline is well developed within the listed industries, the techniques are not widespread within the community of engineering.

In this paper, using historic data and PLC operating hours, an estimate of the remaining useful life (RUL) is calculated using a conditional survival function. This is an actuarial procedure to generate a 90% probability of surviving to some future time. This is tied to a Health Index (HI) concept. The HI is a single measure of the total health of the component or equipment under consideration. A HI of zero indicates that the component is "new", i.e. operates perfectly according to specifications. A HI of one indicates that the component operates at the boundary of its specification envelope, in which the operator is advised to perform maintenance.

The RUL is the time from the current HI to an HI of 1, i.e. the estimated time from current time until the time when the component operates outside designated specification envelope. The HI itself is a combination of condition indicators (CIs) fused together, where each CI is a certain measure or statistic chosen to detect faults with minimum false alarm rate. Thus, the CIs should be able to differentiate between different faults and healthy state with maximum confidence.

While this HI concept has been proposed for gear health monitoring (Bechhoefer and He, 2012), the mapping of a conditioned probability of survival to an HI is a new concept.

2. CONCEPT OF HEALTH AND THE RUL

To simplify presentation and knowledge creation for a user, a uniform meaning across all monitored machines should be developed. The measured CI statistics, e.g. probability density functions (PDFs), will be unique for each component or machine monitored due to different rates, materials, loads, etc. This means that the critical values (thresholds) will be different for each monitored component. By using the HI paradigm, one can normalize the CIs, such that the HI is independent of the component or machine. This facilitates the use of a "stop light" informational system by using nominal (green), warning (yellow) and alarm (red) levels. This paradigm also provides a common nomenclature for the HI, such that:

- The RUL forecasts the time when it is appropriate to do maintenance, not the time until failure.
- The HI ranges from 0 to 1. For vibration based CM, the threshold is set such that the probability of exceeding an HI of 0.5 is the probability of false alarm (PFA). For the conditional probability of survival model, the HI is scaled such that there is only a 0.1 probability of exceeding 1.2. For the conditional model, this was chosen such that the probability of failure at HI 1 is small.
- A warning alert is generated when the HI is greater than or equal to 0.75. Maintenance should be planned by estimating the RUL until the HI is 1.0.
- An alarm alert is generated when the HI is greater than or equal to 1.0. Continued operations could cause collateral damage.

Again, this nomenclature does not define a probability of failure for the component, or that the component fails when the HI is 1.0. Rather, it suggests a change in operator behavior to a proactive maintenance policy: perform maintenance prior to the generations of cascading faults. For example, by performing maintenance on a bearing prior the bearing shedding extensive material, costly gearbox replacement can be avoided.

3. THE WEIBULL PROBABILITY DISTRIBUTION FUNCTION

The Weibull distribution is attributed to Waloddi Weibull in 1951 (Abernaethy, 1996). Extensive research by the U.S. Air Force for fitting of life data suggests that the Weibull analysis is a leading method. Abernethy (1996) reported while working at Pratt & Whitney that the Weibull method worked with extremely small samples: even two or three failures gave good results. This characteristic is important in many industries where the cost of development/applications testing is high. The ability of the Weibull to give relatively good parameter estimates with small sample size, allows this distribution to be used with more advanced techniques, such as failure forecasting and prognostics.

The Weibull PDF is characterized as:

$$f(x,\lambda,k) = {}^{k}/_{\lambda}\left({}^{x}/_{\lambda}\right)^{k-1} e^{-\left({}^{x}/_{\lambda}\right)^{k}},\ x \geq 0 \tag{1}$$

where k is the shape parameter and λ is the scale parameter. It is interesting to note that with $k = 1$, the PDF is exponential, which describes a memoryless process (e.g. where the failure rate is constant over time, or Markovian). For $k = 2$, the PDF is the Rayleigh distribution, which is used extensively in radio frequency/radar models to describe receiver random energy.

While a number of different methods can be used for estimating the parameters k and λ, the maximum likelihood estimator (MLE) (Cohen, 1965) is commonly used, because of its numerical stability.

In general, consider the likelihood of the joint density function of n random sample of $f(x, \lambda, k)$, then

$$L = \prod_{i=1}^{n} f(x,\lambda,k). \tag{2}$$

For a well behaved function, the maximum likelihood of $f(x, \lambda, k)$ is the solution of

$$d\ \ln(L) / d\theta = 0. \tag{3}$$

Differentiating with respects to k and solving:

$$n/k + \sum_{i=1}^{n} \ln x_i \ - 1/\lambda \sum_{i=1}^{n} x_i^k \ln x_i = 0. \tag{5}$$

Differentiating and solving for λ, gives:

$$-n/\lambda + -1/\lambda^2 \sum x_i^k {}_{i=1}^{n} = 0. \tag{6}$$

Solving for k in terms of λ gives:

$$n \sum x_i^k \ln x_i {}_{i=1}^{n} \Big/ \sum x_i^k {}_{i=1}^{n} \ - 1/k\, E[x^k]. \tag{7}$$

Eq. (7) is easily solved for k using the Newton-Raphson method, and then λ is found as $\lambda = E[x^k]$.

3.1. The Conditional Probability of Survival

The Weibull PDF gives the unconditional probability function of the component under analysis. This allows the estimation of the component life from the first moment (e.g. the expected life), and from the second moment, the variance in the life. That said, operators are typically interested in a different question: Given that the component has survived until today, what is the probability that it will survive until tomorrow, or until the next maintenance period. This concept of survival is well established by actuarial models and is the basis for insurance products.

The cumulative distribution function $F(x, \lambda, k)$, is defined as the integral of $f(x, \lambda, k)$ from 0 to x, or the probability of a component failing between time 0 and x. This allows one to define the probability of surviving to time x as: $1 - F(x, \lambda, k)$. For simplicity, one can assume that the estimates of λ, k, are established.

One can now conceptualize the conditional probability of component survival to time x. This is a subset of the sample space of the random variable X, i.e. those values of X that fail in excess of x. This is the condition survival function, or formally

$$Pr(X > x+n \mid X > x) = S(x+n \mid X > n). \tag{8}$$

This is the probability that the age of failure will exceed $x+n$, given that is does last until x. This is the concept that the probability of survival to $x+n$, given survival to x. From Bayes, one then finds that: $S(x+n \mid X > n) = S(x+n)/S(x)$. In the actuarial sciences, this is called the *lower truncation of the distribution of X*; see (London, 1997).

The more general view of the truncated distribution is to consider the distribution in which X fails between times y and z. This truncated distribution is then given as:

$$S(x|y<X\leq z) = Pr(X>x|y<X\leq z) = Pr(x<X\leq z|y<X\leq z). \tag{9}$$

If this condition probability is multiplied by the probability of obtaining the condition (which is $S(y) - S(z)$), then the unconditional probability for failure between x and z, which is $S(x) - S(z)$, is then

$$S(x|y<X\leq z) = [S(x) - S(z)] / [S(y) - S(z)]. \tag{10}$$

From this, one can derive the expectation of the age of failure, X, of a component known to be functioning at time y. By subtracting y from this expected age of failure, one obtains the expected future life of a component (in the actuarial sciences, this is denoted at the *expectation of life at age y)*. Formally, this is expressed as

$$E[X|X>y] - y. \tag{11}$$

Since $\int_y^\infty f(x|X > y)dx = 1$, the expectation is written as:

$$E[X|X > y] - y = \int_0^\infty t f(t + y|X > y)dt. \tag{12}$$

Note that $f(t+y \mid X > y)$ is the probability distribution function of $(X\text{-}y|X>y)$.

3.2. Some Complications and Deifications

The health concept is based on the idea that the operator does maintenance when it is appropriate. With vibration/oil condition monitoring, there is feedback from measurements that give indications of wear and damage. In the actuarial model, one is given probabilities that relate to failure. The time of estimated failure is not the time when one wants to trigger a maintenance event. Failure causes an unscheduled maintenance event, which leads to higher cost.

This leads to a definitional problem: what should the target condition probability of survival be? While not entirely an ad hoc issue, this is a case where simulation results can be used to evaluate the definition process in a structured way.

- The RUL is the condition probability that, given the component has survived to time y, it will survive until time x.
- The RUL is a conservative value, such that the time x, at which maintenance is performed is such that the reliability of the component is not significantly degraded. For plants such as an off shore oil platforms, it may be difficult to get a replacement component if it fails prior to the planned maintenance event, and downtime is extremely expensive.
- The operator needs a range/confidence in the RUL estimate. The RUL estimate range is taken by the: low probability of failure as the 0.1 estimate of the Weibull parameters, while the high probability of failure is the 0.9 estimate of the Weibull parameters.

- Because one is not interested in the time until failure, but the time until it is appropriate to do maintenance, the expected time of a failure conditioned is given at a probability of exceeding that 83% of that time, with a confidence of 90%.

The expected time of failure is defined as *e*. The last bullet point above thus means that the expected time of failure is the inverse of the lower truncated cumulative distribution function (CDF) at 90%, divided by 1.2. Thus, $RUL=e-t$, where t is the current time.

Because there is no closed form solution for the inverse lower truncated CDF, this was calculated numerically via the Newton-Raphson method, such that:

$$S = 1-F \tag{13}$$

where F is the Weibull CDF for the current time x, λ, k. Fe is the Weibull CDF for expected time of failure at time e, λ, k. Further, we define the probability of survival after time e as

$$S(e) = 1 - Fe, \tag{14}$$

and the confidence

$$P = S(e)/S = 0.9 \tag{15}$$

Simulation used to evaluate the performance of this HI paradigm was developed using a shape parameter, $k = 6$, and a mean time to failure of 5.5 years. Then, λ was calculated as:

$$\lambda = {}^{\mu}\!/_{\Gamma\left(1+{}^{1}\!/_{k}\right)} \tag{16}$$

The Weibull random function was then called to simulate the time of failure of 5 components. Then, using these failure times, an estimate of Weibull parameters, a 0.1 and 0.9 confidence of the Weibull was estimated using Cohen's method (Cohen, 1965). Example Matlab© code can be found in the appendix. Figure 1 shows the simulated example of a component that will fail after 4.42 years, where the experiment has run 3.19 years. The RUL is 1.43 years, with a lower limit (e.g. confidence of the RUL) of 0.48 years to 2.28 years.

4. RESULTS AND PERFORMANCE METRICS

Simulation was used to develop the analysis routine and then evaluate the performance of a notional component. Once the analysis engine was tested via simulation, it was applied to a real world fault data set.

4.1. Simulation Results

Simulation is a powerful tool to evaluation the performance of algorithms. Given the expense and time required to study fielded components, the ability to test "what if" conditions requires the establishment of performance metrics to grade the quality of the analysis. Three metrics that were chosen for this study where:

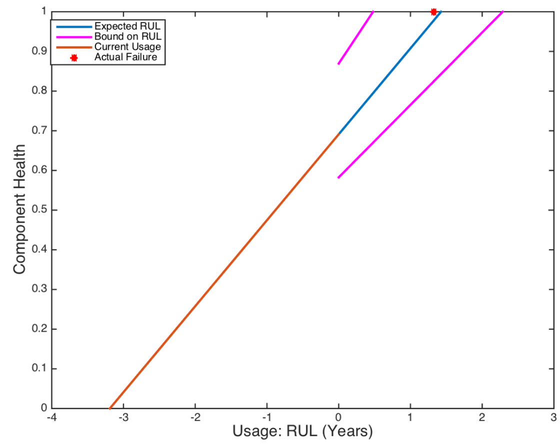

Figure 1 Example of HI and RUL using Condition Survival Function

4.1.1. The Safety Margin

This is the time between the actual time of failure and current time (e.g. when the $HI = 1$). From this, one can estimate the expectation of how much usage of the equipment was lost in replacing prior to failure. Additionally, given that, an opportunity cost can be associated with lost productivity due to a failure, and associated costs can be calculated.

4.1.2. The RUL at the End of the Experiment

The RUL is defined as: the expected life – the current time. The experiment ends when the $HI \geq 1$. Thus, this is a direct measure of the average error in the RUL calculation.

4.1.3. The Future Value of Money at the End of the Experiment.

This cost is calculated on the safety margin, or the time of lost usage on the equipment.

4.2. The Experiment

The experiment was run with 500 trials. For this study, the cost of equipment failure will be taken as $600K per day. The capital cost to replace the failed equipment will be set at $2,000K. The cost of money (for early replacement) is taken as 7%. It is also assumed that the time to replace the failed equipment is 14 days, or $8.4 million in opportunity cost. For this simulation, just 7.6% of the trials failed prior to recommended replacement, of which 0.4% failed within 2 weeks of the RUL estimate (see Figure 2). Thus, the mean opportunity cost of failure in using this model is $638K, or approximately 1 day. Without this replacement model and simply making replacements upon failure, the opportunity cost is, as noted, $8,400K.

Given the distribution of the safety margin (e.g., replacement of the equipment prior to the failure of the equipment, Figure 3), the future value of the money spent on replacing the equipment early is $278K. Because the future value is skewed, the median is somewhat less, at $268K (Figure 4).

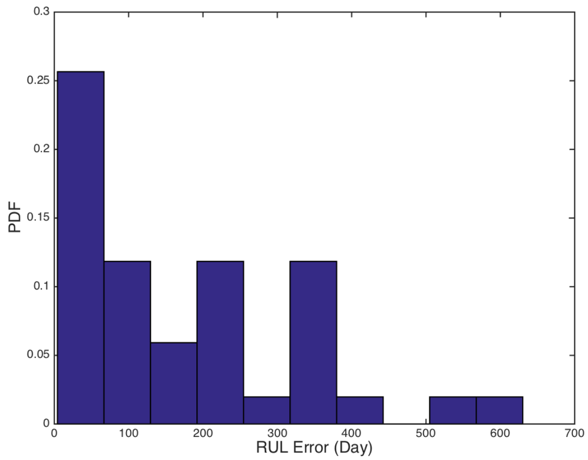

Figure 2 RUL Error based on Failure Occurring Prior to HI of 1

The net benefit of replacement based on this actuarial model is: $8,400K – ($638K + $268K) = $**7,494K**. This is a large cost saving, which can be achieved solely on existing data from in-service failures.

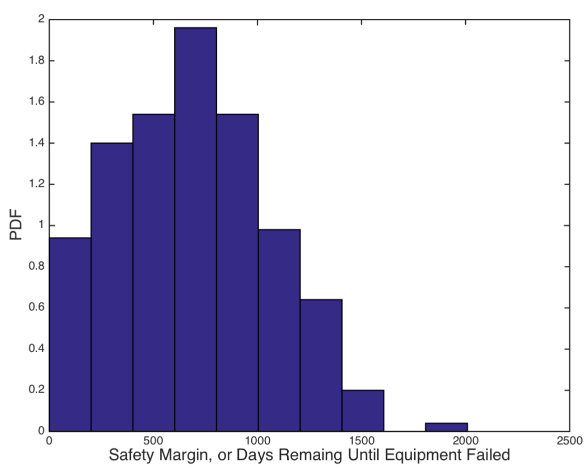

Figure 3 Distribution of Time Until Failure of the Equipment in Days

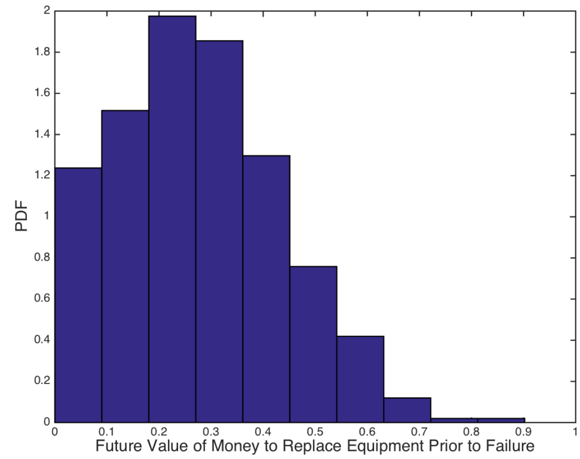

Figure 4 Distribution of Cost of Capital to Replace Equipment Early

4.3. Bearing Fault Data

The time until failure for high speed bearing with an axial crack was generously provided by the National Renewable Energy Laboratory (NREL), Gearbox Reliability Collaborative. The data set consists of bearing with short life (mean age of failure 2.09 years based on 23 examples) and longer life (mean age of failure 4.07 years based on 25 examples).

The experiment was conducted by randomly sampling 6 bearings out of the dataset to estimate the Weibull λ, k parameters. The estimated RUL was compared to the actual life, in a process similar to the simulation study.

Entering assumptions for cost can be defined based on historic industry values. The cost of replacing a high speed bearing "up tower" (i.e. prior to failure and the associated collateral damaged associated with failure) is $50K, and two days of lost production. The cost associated with a "down tower" (i.e. after the failure occurs, during which there is the cost of replacing the gearbox, and a mobilization cost for the crane), is $400K and 30 days of lost production. The lost revenue for a day of power production will be taken as $1K.

4.3.1. Short Life Bearing Replacement Policy

Based on the random draw of the six bearing failure times, a λ of 2.26 and k of 13.15 was calculated from for the Weibull. The expected life the bearing, from the inverse lower truncated CDF at the start of the experiment is 1.58 years. At the time of replacement, the expected value of the days remaining until failure (Figure **5**) was 0.48 years.

The net present value of replacing the bearing early was: $53.7K, including lost revenue for the maintenance. This compares to a cost of 430K for the current practice. The net benefit of actuarial model is: $430K - $53.7K = $376.3K

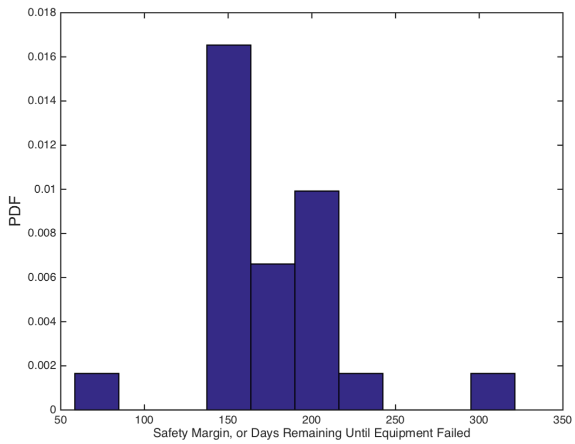

Figure 5 Short Bearing Life Margin Of Safety PDF

4.3.2. Long Life Bearing Replacement Policy

For the long bearing life experiment, the λ was 4.27 and k of 9.5, giving a life the bearing, from the inverse lower truncated CDF at the start of the experiment of 2.81 years. At the end of the experiment, the expected lost years of usage (i.e. the margin of safety) was 1.1142 years (Figure **6**)

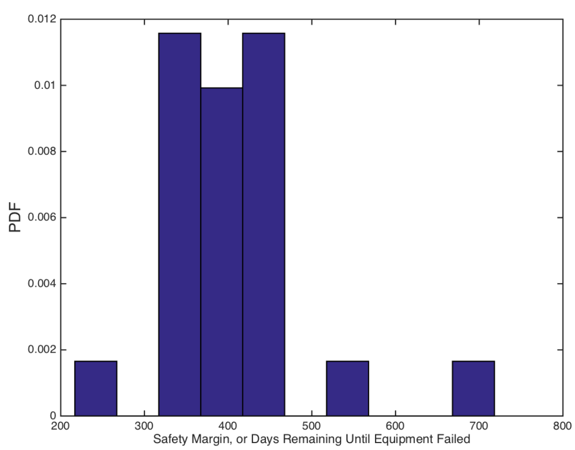

Figure 6 Long Bearing Life Margin of Safety PDF

The mean cost of this policy was $57.9K. Given a similar cost structure for replacing the bearing when its failed, the net benefit of the method was $372.1K

5. CONCLUSION

Actuarial methods provide a means to examine condition probabilities of survival. This in turn can be used to modify existing maintenance practices to replace equipment when it is appropriate, versus when the equipment has failed. From a human factors perspective, the usage and current health of the equipment is present to the operator in such a way that maintenance is planned when the health is at 0.75, and performed when the component/equipment is at an HI of 1 or greater.

This conservative process insures that the probability of unscheduled maintenance is small. For the simulated data, this results in a significant opportunity cost saving relative to the current “run to failure” model. Even taking into account the cost of money, and the rare cases in which this model fails, this paradigm resulted in a 90% cost saving over traditional maintenance models ($7.5 million savings on an $8.4 million dollar estimated cost).

For the real world bearing data, this resulted in a cost reduction of 6:1 (e.g. ~$55K cost of replacing early, vs. a $430K cost when failed). It can be argued that, perhaps the failed bearing does not require a “down tower” repair. Even under these circumstances, its likely that the cost benefit is at least the lost revenue due to lost production, or 30K.

While this modeled used time, reduction in system variance may be improved by using other metrics of usage, such as power hours or some other more direct measure of load or wear on the equipment.

The use of simulation of the actuarial method would allow optimization and minimization of opportunity costs. This could be achieved by adjusting the expected life to HI mapping. This model is based on certain assumptions in the cost of money, the opportunity cost due to lost productivity, and the cost of the equipment. These clearly can be argued and updated as needed.

ACKNOWLEDGEMENT

We would like to thank Dr. Shaugnwen Sheng, and Dr. Jonathan Keller, of the National Renewable Energy Laboratory, the Gearbox Reliability Consortium, for there help on this paper.

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BIOGRAPHIES

Eric Bechhoefer is the president of GPMS, Inc., a company focused on the development of low cost condition monitoring systems. Dr. Bechhoefer is the author of over 100+ juried papers on condition monitoring and prognostics health management, and holds 23 patents in the field of CBM.

Rune Schlanbusch received his MSc degree in Space Technology from Narvik University College (NUC), Norway in 2007, and PhD degree in Engineering Cybernetics from NTNU, Norway in 2012. He currently holds positions as senior researcher at Teknova, Norway and II associate professor at Department of Technology, NUC.

Tor Inge Waag is a Technical Advisor at MHWirth AS and holds a part time position as senior researcher at Teknova AS in Norway. He has received both his MSc and PhD degree in Physics from NTNU, Norway. His primary work focus is at present on condition monitoring, signal processing, and prognostics health management.

APPENDIX

```
function [safety,error,fHI] = getRUL (lam,k)

safety = zeros(100,1);
error = zeros(100,1);
fHI = zeros(100,1);
for j = 1:500,

  if nargin == 0,
    mv = 5.5;
    k = 6;
    lam = mv/gamma(1+1/k);

    sampe = wblrnd(lam,k,1,5);
    actual = wblrnd(lam,k);
    [parmhat, parmbnd] = wblfit(sampe,.1);

    k = parmhat(2);
    lam = parmhat(1);

    khi = parmbnd(1,2);
    lamhi =  parmbnd(1,1);
    klw = parmbnd(2,2);
    lamlw = parmbnd(2,1);
    pr = 0;
  end

  y = linspace(0,actual,100);

  rul = zeros(1,100);
  rulHi = rul;
  rulLw = rul;
  life = rul;
  hi = rul;
  hlw = rul;
  hhi = rul;

  for i = 1:100
    crt = y(i);


    e = invLowTrunCDF(crt,lam,k,.1)/1.2;
    elw =  invLowTrunCDF(crt,lamlw,klw,.1)/1.2;
    ehi =  invLowTrunCDF(crt,lamhi,khi,.1)/1.2;

    rul(i) = e-crt;
    rulHi(i) = ehi-crt;
    rulLw(i) = elw-crt;
    life(i) = e;
    hi(i) = crt/e;
    if hi(i) > 1,
      break;
    end
    hlw(i) = crt/elw;
    hhi(i) = crt/ehi;
    if pr == 1,
      plot([0    rul(i)],[hi(i)    1],[0    rulLw(i)],[hlw(i)    1],'m',y(1:i)-
crt,hi(1:i),actual-crt,1,'r*',[0 rulHi(i)],[hhi(i) 1],'m','LineWidth',2)
      axis([-8 5 0 1])
      xlabel('Usage: RUL (Years)','FontSize',14)
      ylabel('Component Health','FontSize',14)
      legend('Expected  RUL','Bound  on  RUL','Current  Usage','Actual
Failure')
      pause(.1)
    end
  end
  safety(j) = actual - crt;
  error(j) = rul(i);
  fHI(j) = hi(i);
  if hi(i) < 1,
    disp(['Failed  prior  to  repair:  rul  =  '  num2str(rul(i))  ',  HI  '
num2str(hi(i))])
  end

end

figure(1)
hist(safety);

title('Safety Factor')
figure(2)
hist(error)
title('Error in RUL')
figure(3)
hist(fHI)
title('Final HI')

function n = invLowTrunCDF(x,lam,k,p)

global kl;
global laml;
global pTarget;
global xl;

small = 1 -1e-5;
upper = wblinv(small,lam,k);
kl = k;
laml = lam;
xl = x;

pTarget = 1-p;

n = fminbnd('setLowTurnCDF',x,upper);


function x = setLowTurnCDF(val)
global kl;
global laml;
global pTarget;
global xl;

p = lowTrunCDF(xl,val,laml,kl);

x = (p-pTarget)^2;

function p = lowTrunCDF(x,n,lam,k)

F = wblcdf(x,lam,k);

S = 1-F;
```

```
Sn = 1-wblcdf(n,lam,k);

p = Sn./S;

p(p<0) = 0;
```