

Direct analysis of non-quadratic phase coupling for detection of linearly modulated signals

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ABSTRACT

The detection of a linearly modulated signal is currently accomplished by applying the Bispectrum. This technique is capable of detecting quadratic phase coupled spectral components, and consequently, can be used in order to reveal a linearly modulated signal presence. However, a linear modulation by itself does not exhibit quadratic phase coupled spectral analysis. Then, the application of the Bispectrum for detecting linearly modulated signals could be unsuccessful. In this paper a general method for detection of linearly modulated signals, which can be applied whether the signals comprise quadratic phase coupled spectral components or not, is proposed. This method is evaluated through numerical simulations and it is applied for detecting a local fault in rolling element bearings. The achieved results are compared with those obtained by the traditional spectral analysis and the Bispectrum, revealing the effectiveness obtained by the application of the proposed method.

1. INTRODUCTION

Second-order signal interactions or transformations analysis is a common issue in Signal Processing (Chaari, Bartelmus, Zimroz, Fakhfakh, and Haddar (2012); Chen & Zuo, 2009). Second order interactions are characterized by Quadratic Phase Coupled (QPC) spectral components, and consequently by linear modulations. Although this paper is focused on linear modulation, the analysis proposed here can be extended to other kind of modulations (e.g., exponential modulations) whenever the phases of the spectral components at both sides around the frequency of the so called carrier signal are phase coupled.

Some published papers have been focused on the problem of detection of phase coupled spectral components, in particular QPC spectral components. In these publications QPC

detection has been applied to different signals; such as biological signals (Venkatakrishnan, Sukanesh, and Sangeetha (2011)), telecommunication signals (Sanaullah, 2013), mechanical vibration signals (Bouillaut & Sidahmed, 2001; Raad & Sidahmed, 2002), etc. In order to detect QPC spectral components, higher-order statistical signal processing, more specifically Bispectrum (Venkatakrishnan et al. (2011); Sanaullah, 2013; Fackrell & McLaughlin, 1995), has been successfully applied. Bispectrum, by definition, detects the quadratic phase coupling among spectral components. However, a linearly modulated signal by itself is not a signal with QPC spectral components necessarily.

This paper proposes a general method for detecting linearly modulated signals based on the analysis of phase coupling among spectral components located at both sides around a center frequency.

The rest of the article is organized as follows. Section 2 presents the well-known Bispectrum's capability of detection of modulated signal once the QPC condition is fulfilled and the problem concerning the application of the Bispectrum on the detection of a linear modulation for a signal without QPC spectral components; in Section 3, the theoretical foundations of the proposed method are explained; Section 4 corroborates these theoretical foundations by numerical simulations; Section 5 presents the results of the application of this method on rolling element bearing fault detection and also a comparison with results obtained by the application of the traditional spectral analysis and the Bispectrum.

2. BISPECTRUM AND THE DETECTION OF LINEARLY SIMULATED SIGNALS

A QPC interaction is produced by a second-order transformation system as follows (Sanaullah, 2013; Gallego, Urdiales, and Ruiz (1999); Seydnejad, 2007):

$$y(t) = ax^2(t) + bx(t) + c, \quad (1)$$

where $x(t)$ and $y(t)$ are the signals at the system input and output, respectively.

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For example, let

$$x(t) = \cos(2\pi f_1 t + \varphi_1) + \cos(2\pi f_2 t + \varphi_2), \quad (2)$$

where φ_1 and φ_2 are independent random variables with uniform probability density function between $-\pi$ and π . Then,

$$\begin{aligned} y(t) = & c + b \cos(2\pi f_1 t + \varphi_1) \\ & + b \cos(2\pi f_2 t + \varphi_2) \\ & + 2a \cos(2\pi f_1 t + \varphi_1) \cos(2\pi f_2 t + \varphi_2) \\ & + a \cos^2(2\pi f_1 t + \varphi_1) \\ & + a \cos^2(2\pi f_2 t + \varphi_2) \end{aligned} \quad (3)$$

As a result of the second-order transformation process, a linearly modulated signal, given by the term $2a \cos(2\pi f_1 t + \varphi_1) \cos(2\pi f_2 t + \varphi_2)$ in Eq. (3), is obtained at the system output. The signal $y(t)$ comprises spectral components at frequencies $0, f_1, f_2, f_1 + f_2, f_1 - f_2, 2f_1$ and $2f_2$. Given that QPC spectral components are defined as spectral components at frequencies f_a, f_b and $f_a + f_b$ with phases equal to φ_a, φ_b and $\varphi_a + \varphi_b$, respectively [3, 4, 5, 6, 7, 9], then it can be stated that $y(t)$ fulfills the QPC condition.

Bispectrum (third-order spectral cumulant) is a common technique applied for detecting QPC spectral components. The Bispectrum of a signal $z(t)$ can be obtained as follows (Venkatakrisnan et al. (2011); Sanaulah, 2013; Fackrell & McLaughlin (1995); Rivola & White (1998)):

$$B(f_a, f_b) = E\{Z(f_a)Z(f_b)Z^*(f_a + f_b)\} \quad (4)$$

where $Z(f)$ is the Fourier transform of $z(t)$.

Let $z(t)$ denote the signal obtained from Eq. (3) corrupted by a zero mean Gaussian noise, $w(t)$, as

$$z(t) = y(t) + w(t), \quad (5)$$

The Bispectrum of signal $z(t)$ is in theory different from zero only for the bispectral components $(f_a - f_b, f_b)$ and (f_a, f_b) . Such nonzero bispectral components are indicating that certain spectral components in $z(t)$ fulfill the QPC condition, and consequently the signal $z(t)$ comprises a linearly modulated signal.

An important question must be taken into consideration: a linearly modulated signal by itself does not necessarily comply with the QPC condition; then the application of Bispectrum for detecting linearly modulated signal can be unsuccessful.

3. GENERAL METHOD PROPOSED FOR LINEARLY MODULATED SIGNAL DETECTION

In order to detect a linearly modulated signal, the calculation of the phase coupling among spectral components at both sides around a center frequency can be carried out depending on the characteristics of the modulated signal to detect. For example, the following equations calculate the spectral components phase coupling for detecting a linearly modulated signal with unsuppressed carrier, a linearly modulated signal with suppressed carrier, and a linearly modulated signal with only odd spectral components around the carrier frequency, respectively:

$$D_0(f, \alpha)_3 = E\{Z(f - \alpha)Z(f + \alpha) \cdot |Z(f)|e^{-j2 \cdot \text{ang}[Z(f)]}\} \quad (6)$$

$$D_2(f, \alpha)_4 = E\{Z(f - \alpha)Z(f + \alpha) \cdot Z^*(f - 2\alpha)Z^*(f + 2\alpha)\} \quad (7)$$

$$D_3(f, \alpha)_4 = E\{Z(f - \alpha)Z(f + \alpha) \cdot Z^*(f - 3\alpha)Z^*(f + 3\alpha)\} \quad (8)$$

where $E\{\cdot\}$ is the expected value operator, $Z(f)$ is the Fourier transform of the signal under study and “ $\text{ang}[w]$ ” is the angle of the complex number w .

For example, given a linearly modulated signal with unsuppressed carrier at frequency equals to f_c , there will be an $\alpha = f_m$, provided that $f_c + f_m$ and $f_c - f_m$ are the frequencies of two nonzero modulated signal spectral components, such that $D_0(f_c, f_m)_3 \neq 0$. An example of a linearly modulated signal spectrum is shown in Figure 1.

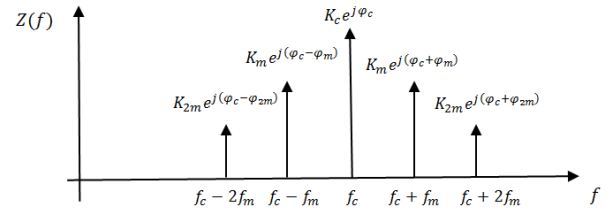


Figure 1. Example of a linearly modulated signal spectrum.

Moreover, the modulation signal detection algorithm is immune to the corrupting noise, then it can be applied under low signal-to-noise rates. For example, given a signal $z(t)$ comprising an amplitude modulation signal, $a(t)$, with unsuppressed carrier at frequency equals to f_c and modulating signal with frequency equal to f_m , plus a random signal (noise), $a_n(t)$, the Eq. (6), at frequencies f_c, f_m can be written as follows:

$$D_0(f_c, f_m)_3 = E\{Z(f_c - f_m)Z(f_c + f_m) \cdot |Z(f_c)|e^{-j2 \cdot \text{ang}[Z(f_c)]}\} \quad (9)$$

The spectral components in $z(t)$ at frequencies $f_c - f_m$, $f_c + f_m$ and f_c correspond with spectral components of modulation signal and noise, then:

$$D_0(f_c, f_m)_3 = E\left\{ \begin{aligned} & [A_{-m}e^{j(\varphi_c - \varphi_m)} + A_{n_{-m}}e^{j\theta_{n_{-m}}}] \cdot \\ & [A_{+m}e^{j(\varphi_c + \varphi_m)} + A_{n_{+m}}e^{j\theta_{n_{+m}}}] \cdot \\ & [A_c e^{-j2\varphi_c} + A_{n_c}e^{j\theta_{n_c}}] \end{aligned} \right\} \quad (10)$$

where A_{-m} and $\varphi_c - \varphi_m$, A_{+m} and $\varphi_c + \varphi_m$, A_c and φ_c , are the amplitudes and phases of spectral components of the modulation signal at frequencies $f_c - f_m$, $f_c + f_m$ and f_c , respectively, and $A_{n_{-m}}$ and $\theta_{n_{-m}}$, $A_{n_{+m}}$ and $\theta_{n_{+m}}$, A_{n_c} and θ_{n_c} , are the amplitudes and phases of spectral components of the random signal at frequencies $f_c - f_m$, $f_c + f_m$ and f_c , as well.

The development of Eq. (10) results in

$$D_0(f_c, f_m)_3 = E\{A_{-m}A_{+m}A_c\} \quad (11)$$

the significant magnitude of which will be indicating the modulation signal presence. Furthermore, Eq. (11) reveals the independent nature of the algorithm with respect to the corrupting noise magnitude.

4. NUMERICAL SIMULATIONS

In order to evaluate the proposed method, 1000 realizations of a discrete linearly modulated signal with non-quadratic phase coupled spectral components, corrupted by a zero mean gaussian noise with variance equal to 4, are generated by using the software Matlab®, according to the following equation:

$$\begin{aligned} y[nT_s] = & \cos(2\pi f_1 nT_s + \varphi_1) \\ & + \cos(2\pi f_2 nT_s + \varphi_2) \\ & + \cos(2\pi f_1 nT_s + \varphi_3) \cos(2\pi f_2 nT_s + \varphi_2) \\ & + w(nT_s) \end{aligned} \quad (7)$$

where $T_s = 1$ ms, $f_1 = 10$ Hz, $f_2 = 200$ Hz, and φ_1 , φ_2 , and φ_3 are independent discrete random variables with uniform probabilistic density function between $-\pi$ and π . It should be noted that a linear modulation signal is produced but the spectral components are not quadratic phase coupled.

The Bispectrum of $y[nT_s]$ does not reveal any information about the linearly modulated signal. This can be observed in Figure 2. The magnitude of the bispectral components at frequencies $(f_1, f_2) = (190, 10)$ and $(f_1, f_2) = (200, 10)$, calculated for the simulation signal, are shown in

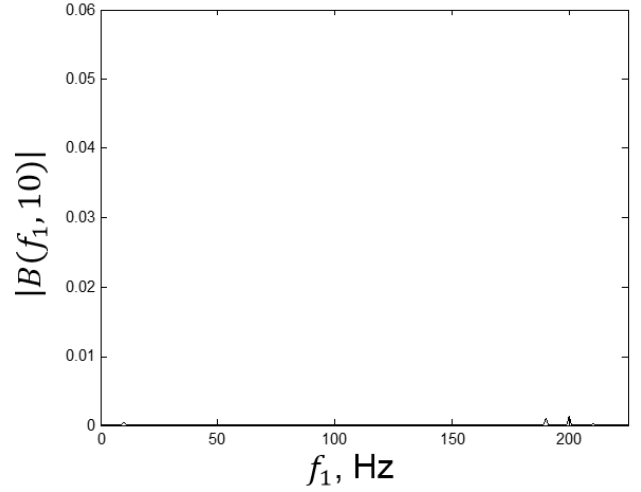


Figure 3.

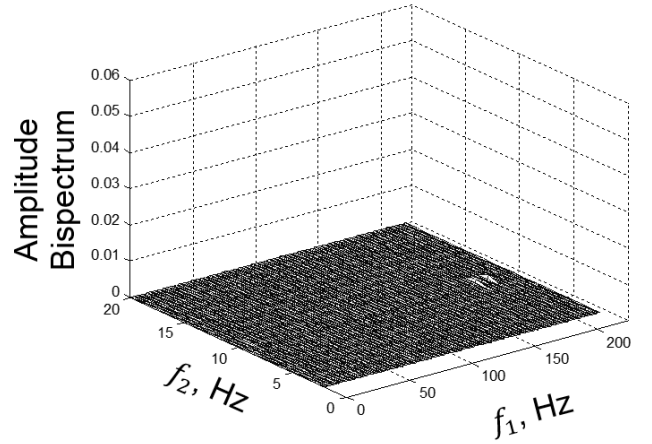


Figure 2. Amplitude Bispectrum of a linearly modulated signal with non-quadratic phase coupled spectral components.

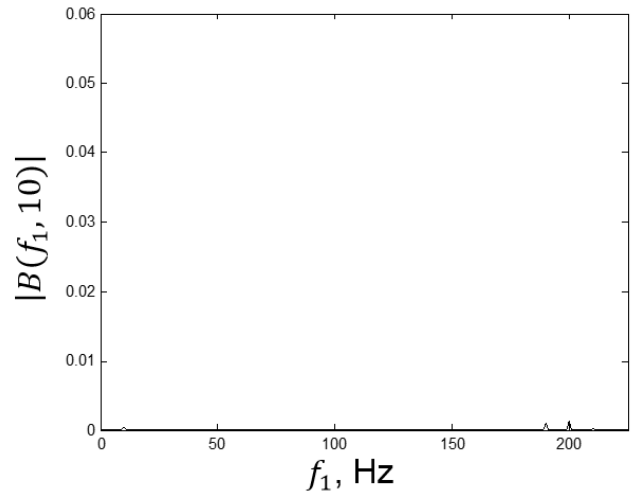


Figure 3. Sketch of function $|B(f_1, 10)|$ for a linearly

modulated signal with non-quadratic phase coupled spectral components.

As expected, it can be observed in

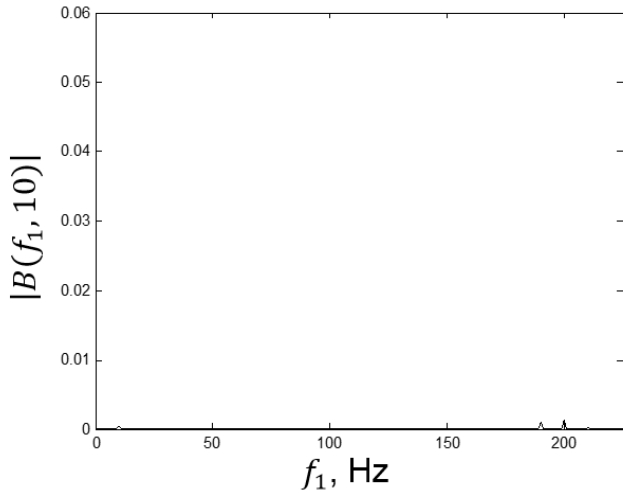


Figure 3 that the application of the Bispectrum for detecting a linearly modulated signal is not effective.

If $D_0(f, \alpha)_3$ is computed for the simulation signal, a nonzero value at $(f, \alpha) = (200, 10)$, indicating that a modulated signal is present, is obtained (see

(200,10) can be observed in

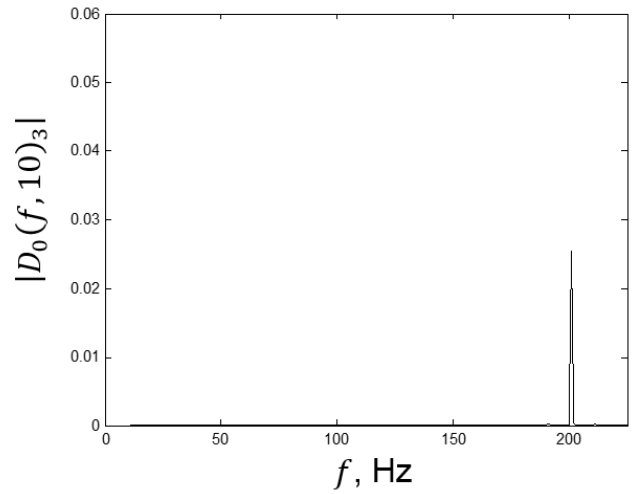


Figure 5, where the magnitude of the function $D_0(f, \alpha)_3$, for $\alpha = 10$ Hz, is plotted.

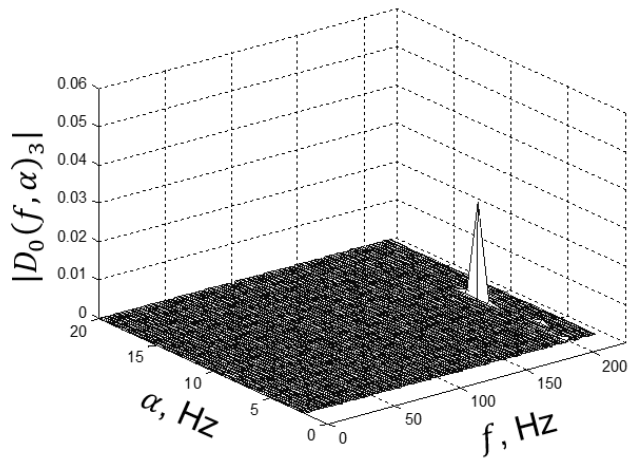


Figure 4). The amplitude of the component at $(f, \alpha) =$

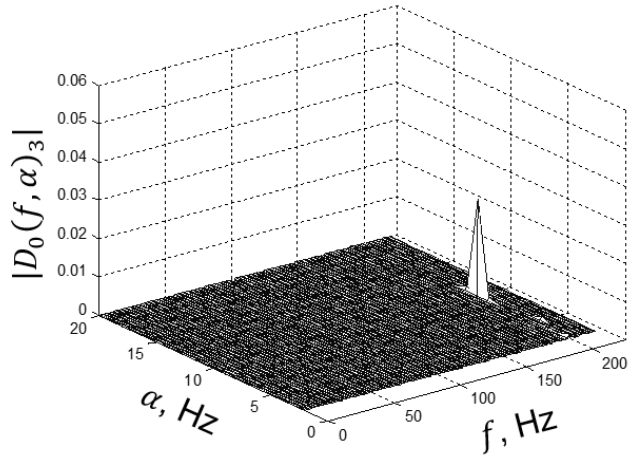


Figure 4. Sketch of $|D_0(f, \alpha)_3|$ for a linearly modulated signal with non-quadratic phase coupled spectral components.

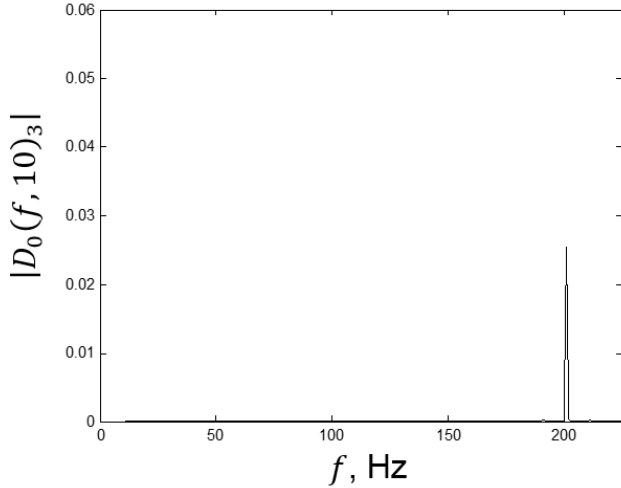


Figure 5. Sketch of function $|D_0(f, 10)_3|$ for a linearly modulated signal with non-quadratic phase coupled spectral components.

5. DETECTION OF A VIBRATION MODULATED SIGNAL PRODUCED BY A ROLLING ELEMENT BEARING WITH A LOCAL FAULT

When a rolling element bearing has a local fault produces a vibration with an amplitude modulation waveform (Seydnejad, 2007). This signal does not satisfy the QPC condition necessarily. The modulating signal is periodic (with main frequency known as “fault characteristic frequency”, f_{ca}), it depends on the physical characteristics of the bearing components, and it is proportional to the rotating frequency of the shaft supported by the bearing (Seydnejad, 2007). The detection of this fault is performed by identifying the modulated signal spectral components, spaced in the fault characteristic frequency that arise around the frequencies associated to the natural frequencies of the mechanical system (Randall & Antoni (2011); Hernandez & Caveda (2008)). The accurate identification of such modulated signal can be affected either by spectral components of vibrations from sources unrelated to the bearing fault mechanism or the background noise. Under these conditions, the application of the proposed method represents a suitable tool.

In order to test the effectiveness of the application of the proposed method on bearing fault detection, $D_0(f, \alpha)_3$ is employed for analyzing a vibration signal experimentally obtained from a rolling element bearing with a local fault. $D_0(f, \alpha)_3$ is chosen because the modulated signal has a significant spectral component magnitude at the carrier frequency. The Spectrum and the Bispectrum are also calculated for comparison purposes.

A vibration signal produced by a rolling element bearing with an incipient local fault in the bearing outer race ($f_{ca} = 98$ Hz), supporting a shaft rotating at 1500 RPM (25 Hz), is sampled at 50 kHz and analyzed.

A portion of the amplitude spectrum of the vibration produced by the incipient local fault in the bearing outer race is shown in

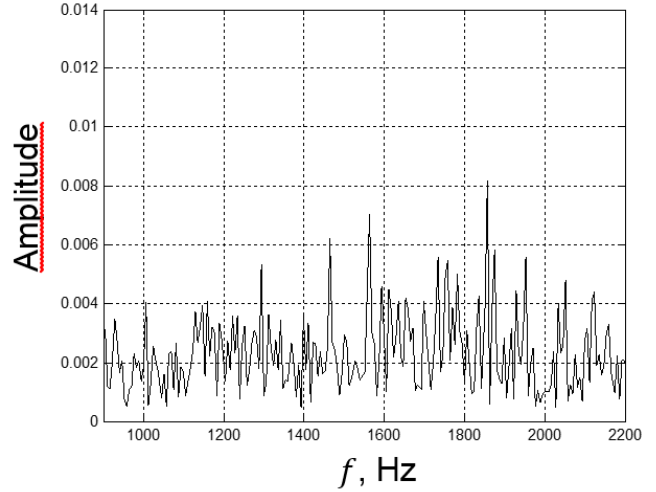


Figure 6. This section of the spectrum shows one of the spectral bands at which some natural frequencies of the mechanical system are excited. As it can be shown in

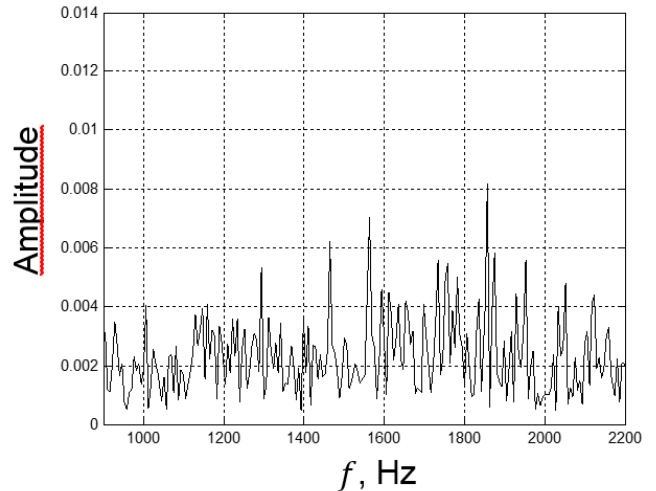


Figure 6, the local fault is so weak that the associated spectral pattern (peaks spaced in $f_{ca} = 98$ Hz) is not possible to be distinguished.

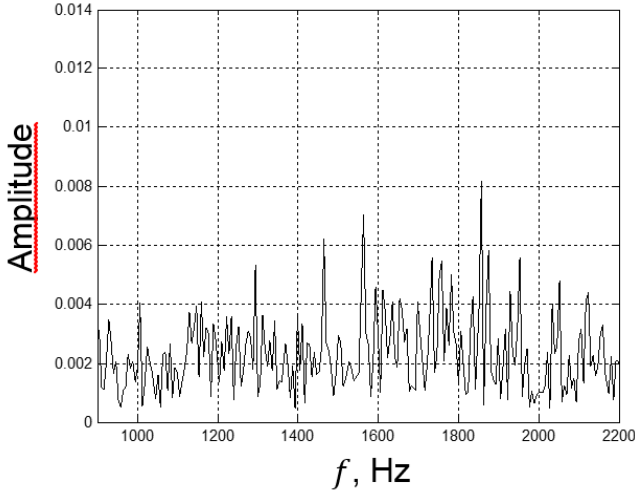


Figure 6. Amplitude spectrum of the vibration produced by an incipient local fault in a bearing outer race ($f_{ca} = 98$ Hz).

The Amplitude Bispectrum of the vibration produced by the incipient local fault in the bearing outer race is shown in Figure 7a and the Amplitude Bispectrum contour is shown in Figure 7b. As it can be seen in Figure 7b, the modulated signal is not detected by the Bispectrum and the fault characteristic frequency, in this case $f_{ca} = 98$ Hz, is not identified either. The representation of the magnitude of the Bispectrum along de frequency $f_2 = 98$ Hz (see Figure 7c) shows clearly the unsuccessful application of the Bispectrum for detecting the modulated signal.

$D_0(f, \alpha)_3$ is calculated for the vibration produced by the same incipient local fault in the bearing outer race. The absolute value of the computed $D_0(f, \alpha)_3$ and its corresponding contour are shown in Figure 8a and Figure 8b, respectively. As it can be seen in Figure 8a and Figure 8b, nonzero components at $(f, k \cdot 98)$, $k = 1, 2, \dots$, are obtained, which indicates that several phase coupled spectral components, spaced in 98 Hz, have been excited. Therefore, it is possible to assure that the modulation has been detected and that the fault characteristic frequency has been identified. The sketch of the magnitude of the function $D_0(f, \alpha)_3$ along the frequency $\alpha = 98$ Hz is shown in Figure 8c, where the achieved effectiveness is better revealed in comparison with that obtained by the Bispectrum (see Figure 7c).

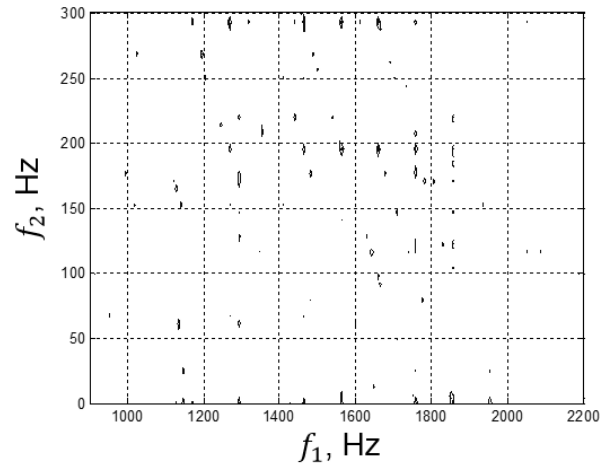
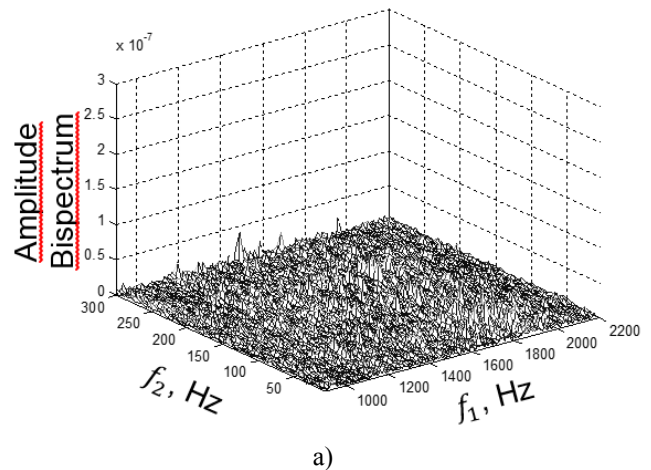
6. CONCLUSION

A method for detecting linearly modulated signals has been proposed. It was demonstrated that this method is suitable to be applied for detecting the distinct phase coupling among the modulated signal spectral components, whether such a

coupling is quadratic or not. This method could also be applied for detecting other kind of modulations (e.g., exponential modulations) since spectral components at both sides of the carrier frequency are found to be phase coupled. The proposed method for detecting a linearly modulated signal is based on the phase relationships among spectral components around a center frequency.

This method has been experimentally evaluated, and its applicability has been verified by detecting a local fault in rolling element bearings.

It has been demonstrated through the numerical and experimental works that better results can be achieved by applying the proposed method, in comparison with the use of the conventional spectral analysis and the Bispectrum analysis. In fact, the Bispectrum analysis is one of the currently most used technique for detecting phase coupled spectral components.



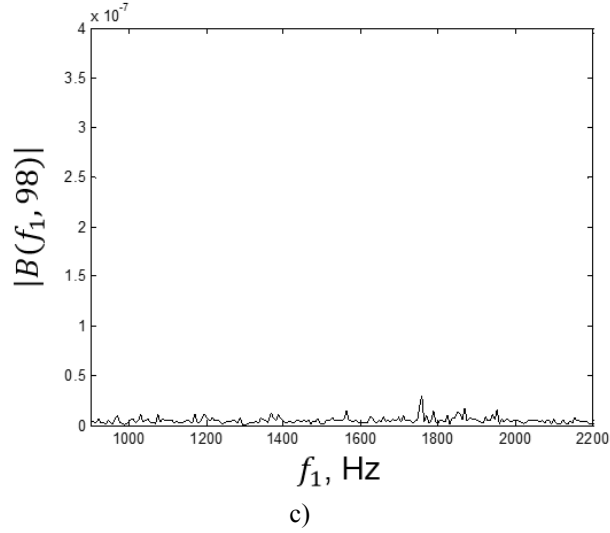


Figure 7. Bispectrum of the vibration produced by an incipient local fault in a bearing outer race ($f_{ca} = 98$ Hz).
 a) Amplitude Bispectrum. b) Amplitude Bispectrum contour. c) Amplitude Bispectrum along the frequency $f_2 = 98$ Hz.

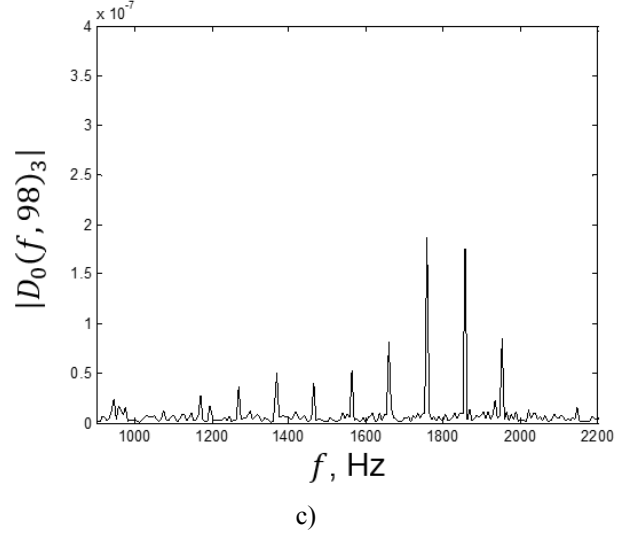
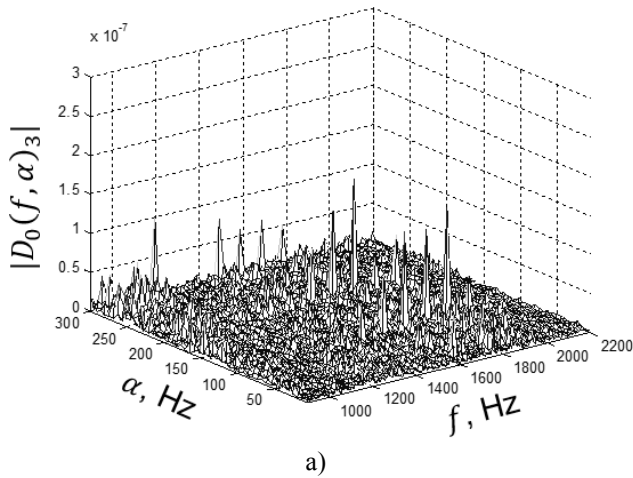
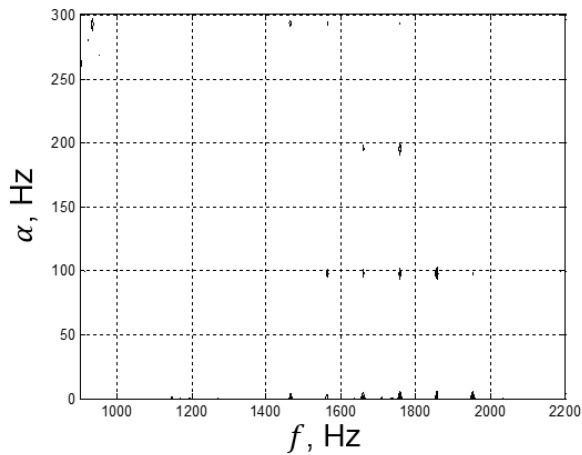


Figure 8. $D_0(f, \alpha)_3$ for the vibration produced by an incipient local fault in a bearing outer race ($f_{ca} = 98$ Hz).
 a) Sketch of $|D_0(f, \alpha)_3|$. b) Contour of the magnitude of $D_0(f, \alpha)_3$. c) Sketch of $|D_0(f, \alpha)_3|$ along the frequency $\alpha = 98$ Hz.



a)



b)

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