Distributed Adaptive Fault-Tolerant Consensus Control of Multi-Agent Systems with Actuator Faults

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Abstract

This paper presents an adaptive fault-tolerant control (FTC) scheme for leader-follower consensus control of uncertain mobile agents with actuator faults. A local FTC component is designed for each agent in the distributed system by using local measurements and certain information exchanged between neighboring agents. Each local FTC component consists of a fault detection module and a reconfigurable controller module comprised of a baseline controller and an adaptive fault-tolerant controller activated after fault detection. Under certain assumptions, the closed-loop system stability and leader-follower consensus properties of the distributed system are rigorously established. A simulation example is used to illustrate the effectiveness of the FTC method.

1. Introduction

The study of distributed multi-agent systems focuses on the development of control algorithms that enable a team of interconnected agents to accomplish desired team missions. One unique feature of these algorithms is their distributed nature, where each agent takes actions based on information obtained from its local neighbors. This distributed nature has numerous benefits, such as scalability and robustness. The research on distributed multi-agent systems has received increasing attention due to its broad application in numerous areas, such as spacecraft formation flying (Ren & Beard, 2004), smart grid (Pipattanasomporn, Feroze, & Rahman, 2009), and sensor networks (Cortes, Martinez, Karatas, & Bullo, 2004). One key concept in the study of distributed multi-agent systems is to have the team exchange information in order to achieve the desired goal. One typical scenario that was extensively studied is consensus, whose goal is to develop control algorithms such that a team of agents reach agreement on their final states via local interaction. The associated control algorithms are also called consensus algorithms. Albeit simple, the study of consensus provides foundation for the development of more advanced algorithms for more general team missions. For agents with different dynamics, numerous consensus algorithms were developed for single-integrator kinematics (Olfati-Saber & Murray, 2004), double-integrator dynamics (Ren & Atkins, 2007), and general linear dynamics (Li, Duan, Chen, & Huang, 2010), where no model uncertainty was considered. To deal with model uncertainties, new consensus algorithms were developed for single-integrator kinematics (Yu, Chen, & Cao, 2011), double-integrator dynamics (Yu, Chen, Cao, & Kurths, 2010), and general linear dynamics (Li, Ren, Liu, & Fu, 2012).

Since such distributed multi-agent systems are required to operate reliably at all times, despite the possible occurrence of faulty behaviors in some agents, the development of fault diagnosis and accommodation schemes is a crucial step in achieving reliable and safe operations. In the last two decades, significant research activities have been conducted in the design and analysis of fault diagnosis and accommodation schemes (see, for instance, (Blanke, Kinnaert, Lunze, & Staroswiecki, 2006)). Most of these methods utilize a centralized architecture, where the diagnostic module is designed based on a global mathematical model of the overall system and is required to have real-time access to all sensor measurements. Because of limitations of computational resource and communication overhead, such centralized methods are not suitable for large-scale distributed interconnected systems. As a result, in recent years, there has been a significantly increasing research interest in the development of distributed fault diagnosis schemes for multi-agent systems (see, for instance, (Keliris, Polycarpou, & Parisini, 2013; Yan & Edwards, 2008; Ferrari, Parisini, & Polycarpou, 2012; Shames, Teixeira, Sandberg, & Johansson, 2011)).

This paper presents a method for detecting and accommodating actuator faults in a class of distributed nonlinear uncertain multi-agent systems. A fault-tolerant control component is designed for each agent in the distributed system by utilizing local measurements and certain information exchanged between neighboring agents. Each local FTC component consists of two main modules: 1) an online fault detection scheme; and 2) the controller (fault accommodation) module consists of a baseline controller and an adaptive fault-tolerant controller employed after fault detection. Under certain assumptions, the closed-loop system's stability and leader-following consensus properties are established for the baseline controller and adaptive fault-tolerant controller. A simulation example is used to illustrate the effectiveness of the FTC method.

The rest of this paper is organized as follows. Section 2 provides the graph theory notations. Problem formulation for fault-tolerant leader-follower consensus control of multi-agent systems is described in Section 3. The closed-loop system stability and performance before fault occurrence is investigated in Section 4. The distributed fault detection is analyzed in Section 5. The design and analysis of the fault-tolerant control scheme after fault detection is rigorously investigated in Section 6. In Section 7, a simulation example is used to illustrate the effectiveness of the FTC method. Finally, Section 8 provides some concluding remarks.

2. GRAPH THEORY NOTATIONS

A directed graph $\mathcal{G}$ is a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, \cdots, v_P\}$ is a set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges, and P is the number of nodes. An edge is an ordered pair of distinct nodes (v_j, v_i) meaning that the ith node can receive information from the jth node. For an edge (v_j, v_i), node v_j is called the parent node, node v_i the child node, and v_j is a neighbor of v_i. An undirected graph can be considered as a special case of a directed graph where $(v_i, v_j) \in \mathcal{E}$ implies $(v_j, v_i) \in \mathcal{E}$ for any $v_i, v_j \in \mathcal{V}$. A directed graph contains a directed spanning tree if there exists a node called the root such that the node has directed paths to all other nodes in the graph.

The set of neighbors of node v_i is denoted by $N_i = \{j : (v_j, v_i) \in \mathcal{E}\}$. The weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \Re^{P\times P}$ associated with the directed graph $\mathcal{G}$ is defined by $a_{ii} = 0$, $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The topology of an intercommunication graph $\mathcal{G}$ is said to be fixed, if each node has a fixed neighbor set and a_{ij} is fixed. It is clear that for undirected graphs $a_{ij} = a_{ji}$. The Laplacian matrix $L = [l_{ij}] \in \Re^{P\times P}$ is defined as $l_{ii} = \sum_{j\in N_i} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. Both $\mathcal{A}$ and L are symmetric for undirected graphs and L is positive semidefinite.

3. PROBLEM FORMULATION

3.1. Distributed Multi-Agent System Model

Consider a set of M agents with the dynamics of the ith agent, $i = 1, \cdots, M$, being described by the following dynamics

$$\begin{aligned} \dot{x}_i &= \phi_i(x_i) + u_i(y_i, y_J) + \eta_i(x_i, t) \\ &\quad + \beta_i(t - T_i) f_i(u_i(y_i, y_J)) \\ y_i &= x_i + d_i \, , \end{aligned} \tag{1}$$

where $x_i \in \Re^n$, $u_i \in \Re^n$, and $y_i \in \Re^n$ are the state vector, input vector, and output vector of the ith agent, respectively. Additionally, y_J contains the output variables of neighboring agents that directly communicate with agent i, including the time-varying leader to be tracked (i.e., y^r) as agent number $M + 1$, i.e., $J = \{j : j \in N_i\}$, $\phi_i : \Re^n \mapsto \Re^n$, $\eta_i : \Re^n \times \Re^+ \mapsto \Re^n$ and $f_i : \Re^n \mapsto \Re^n$ are smooth vector fields, and $d_i \in \Re^n$ is a possibly time-varying vector. The model given by

$$\begin{aligned} \dot{x}_i &= \phi_i(x_i) + u_i \\ y_i &= x_i \end{aligned} \tag{2}$$

represents the *known nominal* dynamics of the ith agent with ϕ_i being the known nonlinearity, while the healthy system is described by

$$\begin{aligned} \dot{x}_i &= \phi_i(x_i) + u_i + \eta_i(x_i, t) \\ y_i &= x_i + d_i \, . \end{aligned} \tag{3}$$

The difference between the nominal model Eq. (2) and the actual (healthy) system dynamics Eq. (3) is due to vector fields η_i and d_i representing the modeling uncertainty in the state dynamics and output measurement of the ith agent, respectively.

The term $\beta_i(t - T_i) f_i(u_i)$ denotes the changes in the dynamics of ith agent due to the occurrence of an actuator fault. Specifically, $\beta_i(t - T_i)$ represents the time profile of a fault which occurs at some unknown time T_i, and $f_i(u_i) = [\,\theta_{i1}u_{i1}, \cdots, \theta_{in}u_{in}\,]^T$ is an actuator fault function representing partial loss of effectiveness of the actuators, where the fault parameter $\theta_{ip} \in (-1, 0]$, $p = 1, \cdots, n$, characterizes the unknown magnitude of the actuator fault. In this paper, the time profile function $\beta_i(\cdot)$ is assumed to be a step function (i.e., $\beta_i(t - T_i) = 0$ if $t < T_i$, and $\beta_i(t - T_i) = 1$ if $t \geq T_i$). The system model (1) allows the occurrence of faults in multiple agents but it is assumed there is only a single fault in each agent at any time.

Remark 1: The distributed multi-agent system model given by Eq. (1) is a nonlinear generalization of the single integrator dynamics considered in literature (for instance, (Ren & Beard, 2008)). In this paper, in order to investigate the fault-tolerance and robustness properties, the fault function $f_i(u_i)$ and modeling uncertainties η_i and d_i are included in the system model.

The objective of this paper is to develop a robust distributed fault diagnosis and fault-tolerant leader-following consensus control scheme for the class of distributed multi-agent systems described by Eq. (1). The following assumptions are made throughout the paper:

Assumption 1. Each component of the modeling uncertainties, represented by $\eta_i(x_i, t)$ and d_i in Eq. (1) and also the rate of change of the measurement uncertainty represented by $\dot{d}_i$, has a known upper bound, i.e., $\forall p = 1, \cdots, n$, $\forall x_i \in \Re^n$, $\forall u_i \in \Re^n$, and $\forall y_i \in \Re^n$,

$$|\eta_{ip}(x_i, t)| \leq \bar{\eta}_{ip}(y_i, t) \tag{4}$$

$$|d_{ip}| \leq \bar{d}_{ip} \tag{5}$$

$$|\dot{d}_{ip}| \leq \Xi_{ip} \, , \tag{6}$$

where the the bounding function $\bar{\eta}_{ip}$, $\bar{d}_{ip}$, and Ξ_{ip} are known and uniformly bounded.

Assumption 2. The nonlinear term $\phi_i(x_i)$ in Eq. (1) satisfies a Lipschitz condition (Rajamani, 1998): $\forall x_i,\, y_i \in \Re^n$,

$$|\phi_{ip}(x_{ip}) - \phi_{ip}(y_{ip})| \leq \sigma_{ip}|x_{ip} - y_{ip}| \, , \tag{7}$$

where σ_{ip} is a known Lipschitz constant.

Assumption 3. The communication topology among followers is undirected and the leader has directed paths to all followers.

Assumption 1 characterizes the class of modeling uncertainty under consideration. The bound on the modeling uncertainty is needed in order to distinguish between the effects of faults and modeling uncertainty during the fault diagnosis process (Emami-Naeini, Akhter, & Rock, 1988). Assumption 2 provides Lipschitz condition on the nominal nonlinearity $\phi_i(x_i)$ in Eq. (1), which is needed for FDI and FTC designs. Assumption 3 is needed to ensure that the information exchange among agents is sufficient for the team to achieve the desired team goal.

3.2. Fault-Tolerant Control Structure

In this paper, we investigate the FTC problem of leader-following consensus. Specifically, the objective is to develop distributed robust FTC algorithms to guarantee that each agent's output converges to the time-varying reference output of the leader even in the presence of modeling uncertainty and actuator fault.

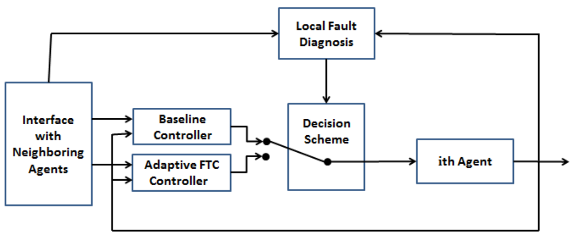

Figure 1. Distributed FTC architecture for the ith agent

The distributed FTC architecture considered is shown in Figure 1. First of all, we define two important time–instants: T_i is the fault occurrence time; $T_d > T_i$ is the time–instant when a fault is detected; The structure of the fault-tolerant controller for the ith agent takes on the following general form (Zhang, Parisini, & Polycarpou, 2004):

$$\begin{aligned} \dot{\omega}_i &= \begin{cases} g_0(\omega_i, y_i, y_J, t), & \text{for} \quad t < T_d \\ g_D(\omega_i, y_i, y_J, t), & \text{for} \quad t \geq T_d \end{cases} \\ u_i &= \begin{cases} h_0(\omega_i, y_i, y_J, t), & \text{for} \quad t < T_d \\ h_D(\omega_i, y_i, y_J, t), & \text{for} \quad t \geq T_d \end{cases} \end{aligned} \tag{8}$$

where ω_i is the state vector of the distributed controller; g_0, g_D and h_0, h_D are nonlinear functions to be designed according to the following qualitative objectives:

1. In a fault free mode of operation, a baseline controller guarantees the output of ith agent $y_i(t)$ should track the leader's time-varying output y^r, even in the presence of plant modeling uncertainty.
2. If an actuator fault is detected, the baseline controller is reconfigured to compensate for the effect of the fault. This new controller should guarantee the boundedness of system signals and leader-following consensus, even in the presence of fault.

4. BASELINE CONTROLLER DESIGN

In this section, we design the baseline controller and investigate the closed-loop system stability and performance before fault occurrence. The dynamics of the agents before fault occurrence (i.e., for $0 \leq t < T_i$) is given by Eq. (3). Without loss of generality, let the leader be agent number $M+1$ with a time-varying reference output (i.e., $y_{M+1} = y^r$). The baseline controller for the ith agent can be designed as:

$$\begin{aligned} u_{ip} &= -(\bar{\eta}_{ip} + \sigma_{ip}\bar{d}_{ip} + \Xi_{ip} + \kappa_i) sgn\Big(\sum_{j \in N_i} k_{ij}\tilde{y}_{ij}\Big) \\ &\quad - \sum_{j \in N_i} (k_{ij}\tilde{y}_{ij}) - \phi_{ip}(y_{ip}), \end{aligned} \tag{9}$$

where u_{ip} and y_{ip} are the pth component of the input and output vectors of the ith agent, respectively, $p = 1, \cdots, n$, $i = 1, \cdots, M$, $\tilde{y}_{ij} \triangleq y_{ip} - y_{jp}$, κ_p is a positive bound on $|\dot{y}^r_p|$ (i.e., $\kappa_p \geq |\dot{y}^r_p|$), $sgn(\cdot)$ is the sign function, N_i is the set of neighboring agents that directly communicate with the ith agent including the leader, and k_{ij}, for $j \in N_i$, are positive constants. Notice that $k_{im} = 0$, for $m \notin N_i$.

Note that, by adding a leader, the topology graph of the system has a spanning tree with the leader as its root. First, we need the following Lemmas:

Lemma 1. *(Ren & Beard, 2008) The Laplacian matrix $L \in \Re^{P \times P}$ of a directed graph $\mathcal{G}$ has at least one 0 eigenvalue with $\mathbf{1}_P$ as its right eigenvector, where $\mathbf{1}_P$ is a $P \times 1$ column vector of ones, and all nonzero eigenvalues of L have positive real parts. 0 is a simple eigenvalue of L if and only if the directed graph $\mathcal{G}$ has a spanning tree.*

Lemma 2. *Consider a connected graph $\mathcal{G}$ with the leader as the $(M+1)$th node. The matrix*

$$\bar{\mathcal{L}} \triangleq \Psi\mathcal{L} + \mathcal{L}^T\Psi \tag{10}$$

is positive semidefinite and has a simple zero eigenvalue with $\mathbf{1}_{M+1}$ as its right eigenvector, where $\Psi \in \Re^{(M+1)\times(M+1)}$ is the Laplacian matrix of the graph as if the communication between leader and followers is undirected, and $\mathcal{L} \in \Re^{(M+1)\times(M+1)}$ is the Laplacian matrix of the graph with a directed leader.

Proof. The proof of the above Lemma can be found in (Khalili, Zhang, Polycarpou, Parisini, & Cao, 2015). □

Remark 2: It is worth noting that the Laplacian matrix Ψ for the undirected graph is only considered for the purpose of controller performance analysis. The actual distributed control topology is directed, since the leader is only sending the data and does not receive any data from other agents.

The following result characterizes the stability and leader-following performance properties of the controlled system before fault occurrence.

Theorem 1. *In the absence of faults, the baseline controller described by Eq. (9) has the following properties:*

1. *The leader-follower consensus is achieved asymptotically with a time-varying reference state, i.e. $y_i - y^r \to 0$ as $t \to \infty$;*
2. *All states are bounded, and $x_i - x_j \to d_j - d_i$ as $t \to \infty$.*

Proof. We know from Eq. (1) that $\dot{y}_{ip} = \dot{x}_{ip} + \dot{d}_{ip}$, for $p = 1, \cdots, n$. Based on Eqs. (9) and (3), the closed-loop system dynamics are given by

$$\begin{aligned} \dot{y}_{ip} &= \phi_{ip}(x_{ip}) - \phi_{ip}(y_{ip}) - \sum_{j \in N_i} k_{ij}\tilde{y}_{ij} + \eta_{ip}(x_i, t) + \dot{d}_{ip} \\ &\quad -(\bar{\eta}_{ip} + \sigma_{ip}\bar{d}_{ip} + \Xi_{ip} + \kappa_p) sgn\Big(\sum_{j \in N_i} k_{ij}\tilde{y}_{ij}\Big). \end{aligned} \tag{11}$$

We can represent the collective output dynamics as

$$\dot{y}^p = -\mathcal{L}y^p + \tilde{\phi}^p + \zeta^p - \bar{\zeta}^p + \dot{d}^p, \tag{12}$$

where $y^p \in \Re^{M+1}$ is comprised of the pth output component of the $M+1$ agents, including the leader as the $(M+1)$th agent, i.e., $y^p = [\, y_{1p},\, y_{2p},\, \cdots,\, y_{Mp},\, y_p^r \,]^T$, the terms $\zeta^p \in \Re^{M+1}$, $\bar{\zeta}^p \in \Re^{M+1}$, and $\tilde{\phi}^p \in \Re^{M+1}$ are defined as

$$\zeta^p \triangleq [\eta_{1p}, \quad \cdots, \quad \eta_{Mp}, \quad 0]^T , \tag{13}$$

$$\bar{\zeta}^p \triangleq \left[\bar{\zeta}_{1p}, \quad \cdots, \quad \bar{\zeta}_{Mp}, \quad 0\right]^T , \tag{14}$$

$$\tilde{\phi}^p \triangleq \left[\tilde{\phi}_{1p}, \quad \cdots, \quad \tilde{\phi}_{Mp}, \quad 0\right]^T , \tag{15}$$

$$\dot{d}^p \triangleq \left[\dot{d}_{1p}, \quad \cdots, \quad \dot{d}_{Mp}, \quad 0\right]^T , \tag{16}$$

where $\bar{\zeta}_{ip} \triangleq (\bar{\eta}_{ip}+\sigma_{ip}\bar{d}_{ip}+\Xi_{ip}+\kappa_p)sgn\Big(\sum_{j\in N_M} k_{ij}\tilde{y}_{ij}\Big)$, and $\tilde{\phi}_{ip} \triangleq \phi_{ip}(x_{ip}) - \phi_{ip}(y_{ip})$, $i = 1, \cdots, M$. We consider the following Lyapunov function candidate:

$$V_p = y^{pT}\Psi y^p = \frac{1}{2}\sum_{i=1}^{M}\sum_{j\in N_i} k_{ij}(y_{ip}-y_{jp})^2 + \frac{1}{2}\sum_{i=1}^{M} k_{i(M+1)}(y_{ip}-y_{(M+1)p})^2 , \tag{17}$$

where Ψ is defined in Lemma 2, and $y_{(M+1)p}$ is the pth component of the leader's constant output y^r. Then, the time derivative of the Lyapunov function Eq. (17) along the solution of Eq. (12) is given by

$$\dot{V}_p = -y^{pT}\bar{\mathcal{L}}\, y^p + 2\dot{y}_p^r \sum_{i=1}^{M} k_{i(M+1)}(y_p^r - y_{ip}) + 2y^{pT}\Psi(\tilde{\phi}^p + \zeta^p - \bar{\zeta}^p + \dot{d}^p) , \tag{18}$$

where $\bar{\mathcal{L}}$ is defined in (10). Based on Eq. (13), and noticing that $\eta_{(M+1)p}$ is zero, we have

$$y^{pT}\Psi\zeta^p = \sum_{i=1}^{M}\sum_{j\in N_i} k_{ij}(y_{ip}-y_{jp})\eta_{ip} . \tag{19}$$

By using the same reasoning logic and knowing that $\bar{\eta}_{(M+1)p} = \bar{d}_{(M+1)p} = \tilde{\phi}_{(M+1)p} = 0$, we can obtain the following from Eqs. (14), (15) and (16):

$$y^{pT}\Psi\bar{\zeta}^p = \sum_{i=1}^{M}\sum_{j\in N_i} k_{ij}\tilde{y}_{ij}\bar{\zeta}_{ij} , \tag{20}$$

$$y^{pT}\Psi\tilde{\phi}^p = \sum_{i=1}^{M}\sum_{j\in N_i} k_{ij}\tilde{y}_{ij}\tilde{\phi}_{ip} , \tag{21}$$

$$y^{pT}\Psi\dot{d}^p = \sum_{i=1}^{M}\sum_{j\in N_i} k_{ij}\tilde{y}_{ij}\dot{d}_{ip} . \tag{22}$$

Using the property that $k_{ij} = k_{ji}$ for $j \in N_i$, $j \neq M+1$ (based on Assumption 3), we know that $\sum_{i=1}^{M}\sum_{j\in N_i, j\neq M+1} k_{ij}(y_{ip} - y_{jp}) = 0$. Therefore, we have

$$2\dot{y}_p^r \sum_{i=1}^{M} k_{i(M+1)}(y_p^r - y_{ip}) = -2\dot{y}_p^r \sum_{i=1}^{M}\sum_{j\in N_i} k_{ij}\tilde{y}_{ij} . \tag{23}$$

By substituting Eqs. (19), (20), (21), (22) and (23) into Eq. (18), we have

$$\dot{V}_p = -y^{pT}\bar{\mathcal{L}}\, y^p + 2\sum_{i=1}^{M}\sum_{j\in N_i} k_{ij}\tilde{y}_{ij}(\eta_{ip} + \tilde{\phi}_{ip} + \dot{d}_{ip} - \dot{y}_p^r) - 2\sum_{i=1}^{M}\sum_{j\in N_i} k_{ij}\tilde{y}_{ij}\bar{\zeta}_{ip} . \tag{24}$$

Based on the Assumptions 1 and 2, we have

$$(\eta_{ip} + \tilde{\phi}_{ip} + \dot{d}_{ip} - \dot{y}_p^r)\sum_{j\in N_i} k_{ij}\tilde{y}_{ij} - \bar{\zeta}_{ip} \leq 0 . \tag{25}$$

Therefore, by applying the above inequality to Eq. (24), we obtain

$$\dot{V}_p \leq -y^{pT}\bar{\mathcal{L}}\, y^p .$$

Therefore, using Lemma 2, we know that $\dot{V}_p$ is negative definite with respect to $y_{ip} - y_{jp}$, because the only y^p that makes $-y^{pT}\bar{\mathcal{L}}\, y^p$ zero is $y^p = \mathbf{1}_{M+1}c$, where c is a constant. Therefore, consensus with respect to the agents' outputs is reached asymptotically, i.e., $y_{ip} - y_{jp} \to 0$ as $t \to \infty$. More specifically, $y_{ip} - y_p^r \to 0$ as $t \to \infty$ and therefore, the leader-follower consensus is reached asymptotically. In the presence of the output measurement uncertainty d_{ip}, by using Eq. (3), we have $x_{ip} - x_{jp} \to d_{jp} - d_{ip}$ as $t \to \infty$. □

5. DISTRIBUTED FAULT DETECTION

The distributed fault detection architecture is comprised of M local fault detection components designed for each of the M agents. The objective of each local fault detection component is to detect faults in the corresponding agent. Under normal conditions, each local fault detection estimator (FDE) monitors the corresponding local agent to detect the occurrence of any fault.

Based on the agent model described by Eq. (1), the FDE for each agent is chosen as:

$$\begin{aligned} \dot{\hat{x}}_i &= \phi_i(y_i) + u_i + H_i(y_i - \hat{y}_i) \\ \hat{y}_i &= \hat{x}_i , \end{aligned} \tag{26}$$

where $\hat{x}_i \in \Re^n$ and $\hat{y}_i \in \Re^n$ denote the estimated local state and output, $H_i = \mathrm{diag}\{h_{i1}, \cdots, h_{in}\}$ is a positive definite matrix, where $-h_{ip} < 0$ is the estimator pole, $p = 1, \cdots, n$, $i = 1, \cdots, M$. Without loss of generality, let the observer gain be $H_i = h_i I_n$ where I_n is a $n \times n$ identity matrix. It is worth noting that the distributed FDE Eq. (26) for the ith agent is constructed based on local input and output variables (i.e. u_i and y_i) and certain communicated information y_j from the FDE associated with the jth agent.

For each local FDE, let $\tilde{x}_i \triangleq x_i - \hat{x}_i$ denote the state estimation error of the ith agent. Then, before fault occurrence (i.e., for $0 \leq t < T_i$), by using Eqs. (1) and (26), the estimation error dynamics are given by

$$\begin{aligned} \dot{\tilde{x}}_i &= -H_i\tilde{x}_i + \phi_i(x_i) - \phi_i(y_i) - H_i d_i + \eta_i(x_i,t) \\ \tilde{y}_i &= \tilde{x}_i + d_i \,. \end{aligned} \tag{27}$$

The presence of uncertainties $\eta_i(x_i,t)$ and d_i cause a nonzero estimation error. A bounding function on the state estimation error $\tilde{x}_{ip}$, before the occurrence of the fault can be derived. Specifically, based on Assumptions 1-2, for $0 \leq t < T_i$, each component of the state estimation error $\tilde{x}_{ip}$ satisfies

$$|\tilde{x}_{ip}| \leq \int_0^t e^{-h_i(t-\tau)}(\bar{\eta}_{ip} + (h_i + \sigma_{ip})\bar{d}_{ip})d\tau + \bar{x}_{ip}e^{-h_i t}\,,$$

where $\bar{x}_{ip}$ is a conservative bound on the initial state estimation error (i.e., $|\tilde{x}_{ip}(0)| \leq \bar{x}_{ip}$). Therefore, for each component of the output estimation error (i.e., $\epsilon_{ip} = y_{ip} - \hat{y}_{ip}$), by using Eq. (27) and applying the triangle equality, we have $|\epsilon_{ip}| \leq \nu_{ip}$, where

$$\begin{aligned} \nu_{ip}(t) \;\triangleq\; & \int_0^t e^{-h_i(t-\tau)}\big(\bar{\eta}_{ip}(y_i,\tau) + (h_i + \sigma_{ip})\bar{d}_{ip}\big)d\tau \\ & + \bar{x}_{ip}e^{-h_i t} + \bar{d}_{ip}\,. \end{aligned} \tag{28}$$

Note that the integral term in the above threshold can be easily implemented as the output of a linear filter with the input given by $\bar{\eta}_{ip}(y_i,t) + (h_i + \sigma_{ip})\bar{d}_{ip}$.

Thus, we have the following:

Fault Detection Decision Scheme: The decision on the occurrence of a fault (detection) in the ith agent is made when the modulus of at least one component of the output estimation error (i.e., $\epsilon_{ip}(t)$) generated by the local FDE exceeds its corresponding threshold $\nu_{ip}(t)$ given by Eq. (28).

The fault detection time T_d is defined as the first time instant such that $|\epsilon_{ip}| > \nu_{ip}$, for some $T_d \geq T_i$ and some $p \in \{1, \cdots, n\}$, that is,

$$T_d \triangleq \inf \bigcup_{p=1}^{n} \{t \geq 0 \,:\, |\epsilon_{ip}(t)| > \nu_{ip}(t)\}$$

6. FAULT-TOLERANT CONTROLLER MODULE

In this section, the design and analysis of the fault-tolerant control scheme is rigorously investigated for the closed-loop system after fault detection. After the fault is detected at time $t = T_d$, the nominal controller is reconfigured to ensure the system stability and tracking performance after fault detection. In the following, we describe the design of the fault-tolerant controller using adaptive tracking techniques.

For $t \geq T_d$, in the case of an actuator fault, the dynamics of the system takes on the following form: for $p = 1, \cdots, n$,

$$\begin{aligned} \dot{x}_{ip} &= \phi_{ip}(x_{ip}) + (1+\theta_{ip})u_{ip} + \eta_{ip}(x_i,t) \\ y_{ip} &= x_{ip} + d_{ip}\,. \end{aligned} \tag{29}$$

Without loss of generality, let the leader be agent number $M+1$ with a set of neighborhoods N_{M+1}. The control objective is to force the output y_i, $i = 1, \cdots, M$, to track the output of the leader with a known time-varying output y^r.

After the detection of the actuator fault, i.e., $t \geq T_d$, the following adaptive fault-tolerant controller is adopted:

$$u_{ip} = \frac{1}{1+\hat{\theta}_{ip}}\bar{u}_{ip} \tag{30}$$

$$\bar{u}_{ip} = -\phi_{ip}(y_{ip}) - \sum_{j\in N_i}\big(k_{ij}\tilde{y}_{ij}\big) + \bar{\zeta}_{ip} \tag{31}$$

$$\dot{\hat{\theta}}_{ip} = \mathcal{P}_{\bar{\theta}_{ip}}\Big\{\Gamma_{ip}\sum_{j\in N_i}k_{ij}\tilde{y}_{ij}u_{ip}\Big\} \tag{32}$$

where $\hat{\theta}_{ip}$ is an estimation of the unknown actuator fault magnitude θ_{ip} with the projection operator $\mathcal{P}$ restricting $\hat{\theta}_{ip}$ to the corresponding set (i.e., $\hat{\theta}_{ip} \in [\bar{\theta}_{ip}, 0]$, with $\bar{\theta}_{ip} \in (-1, 0)$) is used to ensure that $\hat{\theta}_{ip}$ remains within a certain region to guarantee that the denominator of the control law does not approach zero (Ioannou & Sun, 1996), and Γ_{ip} is a symmetric positive definite learning rate matrix.

The following theorem characterizes the stability and leader-following performance properties of the adaptive fault-tolerant controller for $t \geq T_d$:

Theorem 2. *Assume that a fault occurs at time T_i and that it is detected at time T_d. Then, the fault-tolerant controller Eq. (30) and fault parameter adaptive law Eq. (32) guarantee that*

1. *The leader-follower consensus is achieved asymptotically with a time-varying reference state, i.e. $y_i - y^r \to 0$ as $t \to \infty$;*
2. *All states are bounded, and $x_i - x_j \to d_j - d_i$ as $t \to \infty$.*

Proof. Using some algebraic manipulations, we can rewrite Eq. (30) as $u_{ip} = \bar{u}_{ip} - \hat{\theta}_{ip}u_{ip}$. Therefore, substituting u_{ip} in Eq. (29) and using Eq. (31), the closed-loop system dynamics are given by

$$\begin{aligned} \dot{y}_{ip} \;=\; & -\sum_{j\in N_i}(k_{ij}\tilde{y}_{ij}) + \eta_{ip}(x_i,t) + \dot{d}_{ip} - \bar{\zeta}_{ip} \\ & + \phi_{ip}(x_{ip}) - \phi_{ip}(y_{ip}) + \tilde{\theta}_{ip}u_{ip}\,. \end{aligned}$$

We can represent the collective output dynamics as

$$\dot{y}^p = -\mathcal{L}y^p + \tilde{\phi}^p + \zeta^p - \bar{\zeta}^p + \dot{d}^p + \xi^p \tag{33}$$

where $y^p \in \Re^{M+1}$, $p = 1, \cdots, n$, is comprised of the pth component of the M agents and the leader as the $(M+1)$th agent, i.e., $y^p = [y_{1p}, y_{2p}, \cdots, y_{Mp}, y^r_p]^T$, and the terms $\zeta^p \in \Re^{M+1}$, $\bar{\zeta}^p \in \Re^{M+1}$, $\tilde{\phi}^p \in \Re^{M+1}$, $\dot{d}^p \in \Re^{M+1}$ are defined in Eqs. (13), (14), (15) and (16), and the term ξ^p is defined as

$$\xi^p \;\triangleq\; \big[\tilde{\theta}_{1p}u_{1p}, \;\; \cdots, \;\; \tilde{\theta}_{Mp}u_{Mp}, \;\; 0\big]^T, \tag{34}$$

where $\tilde{\theta}_{ip} = \theta_{ip} - \hat{\theta}_{ip}$ is the actuator fault magnitude estimation error.

We consider the following Lyapunov function candidate:

$$V_p = y^{pT} \Psi y^p + \tilde{\theta}^{pT} (\Gamma^p)^{-1} \tilde{\theta}^p , \quad (35)$$

where Ψ is defined in Lemma 2, $\tilde{\theta}^p = [\tilde{\theta}_{1p}, \cdots, \tilde{\theta}_{Mp}]^T$ is the collective actuator fault magnitude parameter estimation errors, and $\Gamma^p = \text{diag}\{\Gamma_{1p}, \cdots, \Gamma_{Mp}\}$ is a positive definite adaptive learning rate matrix. Then, using Eqs. (19), (20), (21) and (22), and the same reasoning logic for Eq. (34), the time derivative of the Lyapunov function Eq. (35) along the solution of Eq. (33) is given by

$$\dot{V}_p = -y^{pT} \bar{\mathcal{L}} y^p + 2 \sum_{i=1}^{M} \tilde{\theta}_{ip} \bigg(\sum_{j \in N_i} k_{ij} \tilde{y}_{ij} u_{ip} - (\Gamma_{ip})^{-1} \dot{\hat{\theta}}_{ip} \bigg)$$
$$+ 2 \sum_{i=1}^{M} \sum_{j \in N_i} k_{ij} \tilde{y}_{ij} \bigg((\tilde{\phi}_{ip} + \eta_{ip} + \dot{d}_{ip} - \dot{y}^r_p) - \bar{\zeta}_{ip} \bigg) ,$$

where $\bar{\mathcal{L}}$ is defined in (10). Therefore, by choosing the adaptive law as Eq. (32), and after some algebraic manipulations, we have

$$\dot{V}_p \leq -y^{pT} \bar{\mathcal{L}} y^p = -2 \sum_{i=1}^{M} \bigg(\sum_{j \in N_i} k_{ij} (y_{ip} - y_{jp}) \bigg)^2 .$$

It is worth noting that since the parameter projection modification can only make the Lyapunov function derivative more negative, the stability properties derived for the standard algorithm still hold (Farrell & Polycarpou, 2006). Because $\bar{\mathcal{L}}$ is positive semidefinite, referring to the proof of Theorem 1, we know that $\dot{V}_p \leq 0$ with respect to $y_{ip} - y_{jp}$ and $\tilde{\theta}_{ip}$. Integrating both sides of $\dot{V}_p$, we know that $(y_{ip} - y_{jp}) \in L_2$. Since $(y_{ip} - y_{jp}) \in L_\infty \cap L_2$ and $\dot{y}_{ip} - \dot{y}_{jp} \in L_\infty$, we can conclude that consensus is reached asymptotically, i.e., $y_{ip} - y_{jp} \to 0$ as $t \to \infty$. More specifically, $y_{ip} - y^r_p \to 0$ as $t \to \infty$ and therefore, the leader-follower consensus is reached asymptotically. In the presence of the output measurement uncertainty d_{ip}, by using Eq. (3), we have $x_{ip} - x_{jp} \to d_{jp} - d_{ip}$ as $t \to \infty$. □

Remark 3: Note that the convergence of tracking errors does not require the convergence of fault parameter estimation error, which requires the condition of persistency of excitation (Farrell & Polycarpou, 2006). In this paper, we do not assume persistency of excitation.

7. SIMULATION RESULTS

In this section, a simulation example of a networked multi-agent system consisting of 5 agents is considered to illustrate the effectiveness of the distributed fault-tolerant control method. The dynamics of each agent is given by

$$\begin{aligned} \dot{x}_i &= u_i + \eta_i + \beta_i(t - T_i) f_i(u_i) \\ y_i &= x_i + d_i \end{aligned} , \quad (36)$$

where, for $i = 1, \cdots, 5$, the state vector $x_i = [x_{i1}, x_{i2}]^T$ represents the ith agent's position in a two-dimension coordinate, y_i and $u_i = [\bar{\nu}_i cos(\bar{\psi}_i), \bar{\nu}_i sin(\bar{\psi}_i)]^T$ are the output and input vectors, and $\bar{\psi}_i$ and $\bar{\nu}_i$ in the input vector u_i are the orientation and the linear velocity of each agent representing a ground vehicle, respectively.

The ground vehicle model given in (36) is a standard unicycle-like model that can be controlled with the orientation $\bar{\psi}_i$ and vehicle linear velocity $\bar{\nu}_i$. Using the developed algorithms, the desired orientation and linear velocity of the ground vehicle robot can be obtained uniquely. Then, a low level controller can be designed to track the desired orientation and linear velocity for driving the ground vehicles to desired positions.

The unknown modeling uncertainty in the local dynamics of the agents are assumed to be sinusoidal signals $\eta_i = [0.5sin(t),\ 0.5sin(t)]^T$ which is assumed to be bounded by $\bar{\eta}_i = [0.6,\ 0.6]^T$. There is also an unknown uncertainty in the sensor measurement $d_i = [-0.5\,cos(t),\ -0.5\,cos(t)]^T$ which is assumed to be bounded by $\bar{d}_{i1} = \bar{d}_{i2} = 0.6$. The objective is for each agent to follow the leader described by $y^r = [y^r_1,\ y^r_2]^T = [4 + sin(t),\ 4 + cos(t)]^T$.

The Laplacian matrix of the intercommunication graph of agents plus leader, shown in figure 2, is chosen as

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 3 & 0 & 0 & -1 & -1 \\ 0 & 0 & 2 & -1 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ -1 & -1 & -1 & -1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} .$$

The fault considered here is an actuator fault function $f_i = \theta_i u_i$, where the magnitude of this fault is considered as $\theta_i \in [-0.8\ \ 0]$.

The observer gain for fault detection estimator is chosen as $h_i = 2$. After fault detection, the controller is reconfigured to accommodate the actuator fault occurred. We set the adaptive gain $\Gamma_i = 5$ with a zero initial condition (see Eq. (32)).

Figure 3 shows the fault detection results when actuator faults with a magnitude of -0.4 and -0.35 occur to agents 1 and 2 at $T_1 = 5$ and $T_2 = 8$ second, respectively. As can be seen from Figure 3, the residual corresponding to the output generated by the local FDE designed for agents 1 and 2 exceeds its threshold immediately after fault occurrence. Therefore, the actuator faults in agent 1 and 2 are timely detected. Note that the residual signals are time-varying because the uncertainties η_i and d_i in (27) are time-varying.

Regarding the performance of the adaptive fault-tolerant controllers, as can be seen from Figure 4, the tracking errors converge to zero. Thus, the leader-following consensus is achieved using the proposed adaptive FTC. On the other hand, the agents cannot follow the leader without the FTC controller (see Figure 5), since the tracking errors do not converge to zero. Therefore, the benefits of the FTC method can be clearly seen.

8. CONCLUSION

In this paper, we investigate the problem of a distributed FDI and FTC for a class of multi-agent uncertain systems. Under certain assumptions, adaptive thresholds are derived for distributed fault detection. Also, adaptive FTC controllers are developed to achieve the leader-following consensus in the

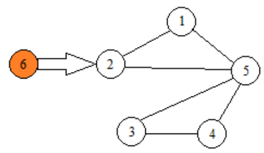

Figure 2. Communication graph

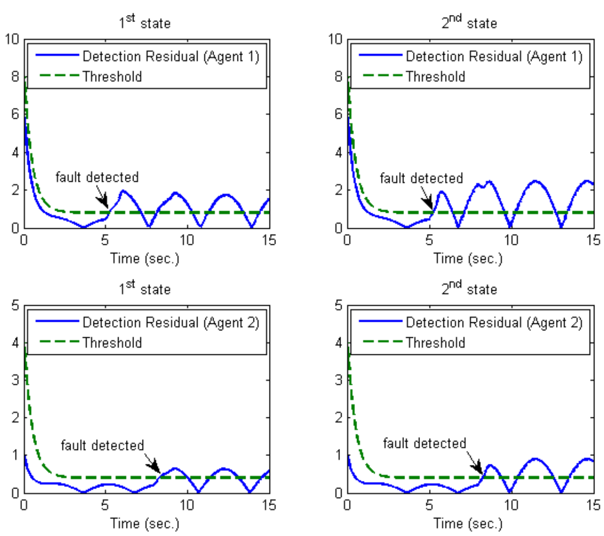

Figure 3. The case of actuator faults in agents 1 and 2: fault detection residuals (solid and blue line) and the corresponding threshold (dashed and green line) generated by the FDE of agents 1 and 2

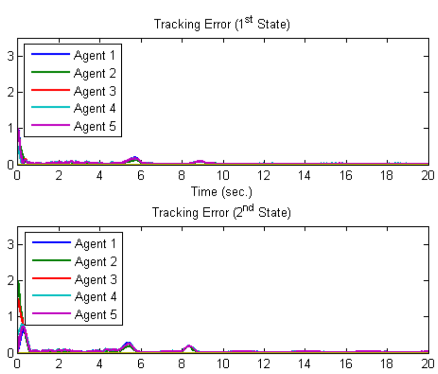

Figure 4. The tracking errors in the case of actuator faults: with adaptive fault-tolerant controllers

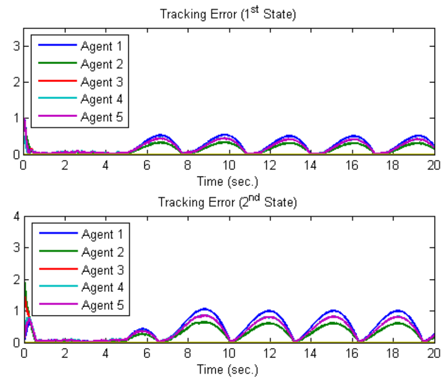

Figure 5. The tracking errors in the case of actuator faults: without adaptive fault-tolerant controllers

presence of actuator faults. The extension to systems with more general structure is an interesting topic for future research.

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