Experimental Approach for Estimating Mesh Stiffness in Faulty States of Rotating Gear

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ABSTRACT

Gear mesh stiffness (GMS) is a principal factor in understanding a dynamic behavior and estimating a health condition of the gear system. Lots of methodologies have been proposed to estimate GMS in normal and abnormal states. However, most of them are performed in an analytical way, therefore experimental studies are limited. Moreover, previous experimental studies have limitations that they were only performed either in a static state or for a normal gear. In this study, we develop a methodology to estimate GMS of a rotating gear in faulty states, root crack and spalling. In the procedures, we employ transmission error (TE) which is defined as the difference between rotation of input and output gear. The methodology proposes the concepts of relative stiffness to remove the effect of low frequency component from shaft motion and variability of individual teeth, and corrected stiffness to exactly estimate GMS of cracked gear. Meanwhile, the study proposes a differentiating algorithm of gear faults between root crack and spalling considering the failure mechanisms of each fault. The developed algorithm is validated measuring the TE from a test-bed of a spur gear. Consequently, the algorithm has differentiated the gear in root crack and surface failure, and estimated the GMS of the gear in faulty states.

1. INTRODUCTION

Gear systems are widely used in many engineering applications like wind turbines, industrial robots, helicopters, etc. In gear systems, gear mesh stiffness (GMS) is a key parameter to understand a dynamic behaviors as it is one of the main sources of excitation for the systems. Therefore, the GMS has been widely studied, especially when the gear is in faulty states. Chaari et al. (2008) investigated the effect of spalling and tooth breakage on the stiffness and vibratory motions by an analytic method. The effects of tooth root crack on the stiffness were also studied by Chaari et al. (2009). In the study, after the time-varying profiles of GMS are analytically evaluated, they are demonstrated using a finite element method. Chen and Shao (2013) studied the effect of tooth root crack under the tooth profile modification. Liang et al. (2014) calculate the mesh stiffness of a planetary gear with a crack using the potential energy method.

On the other hand, experimental studies for estimating GMS are limited. The GMS is estimated measuring transmission error (TE) over the range of path of contact in gear teeth (Munro, Palmer & Morrish. 2001). In the proposed methodology, however, gear teeth near the measured tooth are artificially modified to minimize the effect of the teeth on TE. Yesilyurt, Fengshou, and Andrew (2003) developed the modal testing apparatus to estimate reduction ratio of GMS in wear conditions. The severity of the faulty states was assessed by calculating the peaks of frequency response functions. However, due to the characteristics of modal testing, the method cannot be applied to rotating gears. The GMS was also evaluated for a spline coupling with teeth (Curà & Andrea. 2013). A hexapod specially designed for measuring angular deformation of the tooth pairs is developed. In the study, the effects of angular misalignment on the stiffness were also inspected. However, the device cannot be adopted for a spline coupling in operation.

The objective of this study is, therefore, to develop experimental procedures to evaluate GMS of faulty gears in rotating condition, which have not been performed in previous studies. To this aim, TE, which is defined as the difference between rotation of input and output gear, is adopted to estimate GMS in faulty states, root crack and spalling.

In the procedures, relative stiffness and corrective stiffness are proposed. TE, which we employed for estimation, is subject to shaft motion and variability arising from gear teeth. Therefore, the relative stiffness is proposed to remove the effect of shaft and variability of gear teeth. Then, relative reduction ratio of GMS at each tooth for each fault can be quantified from relative stiffness. Next, corrective stiffness is proposed to exactly estimate the GMS for a root crack fault. Due to peculiar fault mechanism of rotating gear in path of contact, calculated relative stiffness could be underestimated for root crack of gear tooth. Therefore, the underestimated values of GMS in root crack are compensated using corrective stiffness. The proposed two stiffness lead to experimentally estimate GMS of a fault gear in operating conditions.

The rest of this paper is organized as follows. Section 2 reviews the characteristics of TE which we adopted as an estimation signal. Then, the proposed methodologies are introduced in Section 3. After the methodologies are demonstrated using a case study in Section 4, Section 5 concludes this paper with recommendations for future work.

2. REVIEW OF TRANSMISSION ERROR

In this study, we use TE to estimate GMS of rotating gears in faults. Although TE is a physically meaningful signal in relation with GMS, it could show some biased behaviors in measuring the signal. To fully utilize physical properties of the signal for GMS estimation, characteristics of the signals are explained in this section.

2.1. General Behaviors of TE

TE is usually measured by encoder, which calculates the rotational displacement. As mentioned above, TE is defined as the difference between rotation of input and output gear, and can be formulated as below:

$$TE = \theta_h - R \times \theta_l \quad (1)$$

where θ_h and θ_l are the rotational displacements of high speed and low speed shaft; R is gear ratio. When TE is measured from an encoder, it is usually composed of shaft motion and tooth motion like Figure 1 which shows simulated TE signals. Shaft motion happens due to pitch line run-out error while showing large fluctuations in the motions (Inalpolat, Handschuh & Kahraman. 2015), and tooth motions happens due to GMS while showing small fluctuations in the motions. To closely observe the effect of gear meshes on TE, the TE

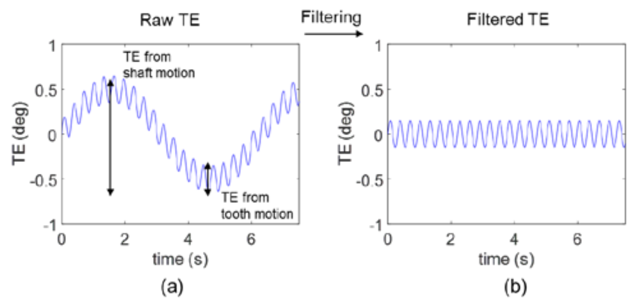


Figure 1. Simulated TE signals (a) before filtering and (b) after filtering.

from shaft motion should be removed by filtering techniques like in Figure 1 (b). In the tooth motion of an actual gear, however, the behaviors of TE are not consistent unlike Figure 1 (b). Lots of unpredictable factors like machining errors, tip relief errors, and indexing errors affects the behaviors of TE in an actual condition (Inalpolat et al. 2015, Kahraman & Blankenship. 1999, Li. 2007). Therefore, the TE behavior of each tooth from the individual gear is not consistent.

2.2. TE Behaviors in Tooth Root Crack and Spalling

Faults in gear teeth degrade the GMS. Then, due to the degraded GMS, TE shows distinct behavior when faults arise in the gear teeth. However, each fault has its own fault mechanism, and degraded GMS also shows distinct behaviors at each fault. GMS is composed of three components; (i) stiffness of the tooth, (ii) stiffness of the gear body, (iii) contact stiffness between gear teeth. Among the components, the tooth root crack mainly degrades the stiffness of the tooth (Wu, Zuo & Parey. 2008). Therefore, the effect of the fault lasts all over the contact regions. On the other hand, the localized faults like spalling would alter the contact stiffness (Tan, Irving & Mba. 2007). Then, the spalling would affect the only single portion of the whole contact regions.

Figure 2 and 3 show the change of GMS and TE from gear path of contacts with tooth root crack and spalling.

In these figures, TE arising from shaft motion is not included which is shown in Figure 1 (a). Since root crack affect overall region of contact path as mentioned above, GMS and TE are modified at whole region of contact path, which is shown in Figure 2. For the case of spalling, the fault modified GMS and TE only at limited region of contact path, at single contact region, which is shown in Figure 3. Although these fault mechanisms can be different according to a contact ratio of gear (Pandya & Parey. 2013), the behaviors are common in usual gears (Endo, Randall & Gosselin 2009).

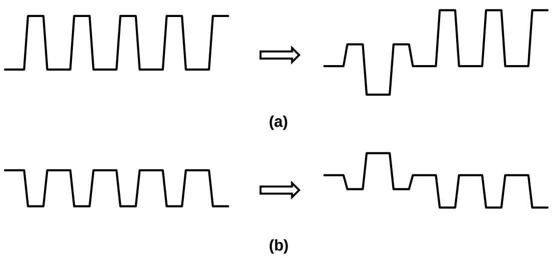


Figure 2. The change of (a) GMS and (b) TE with a root crack in a gear

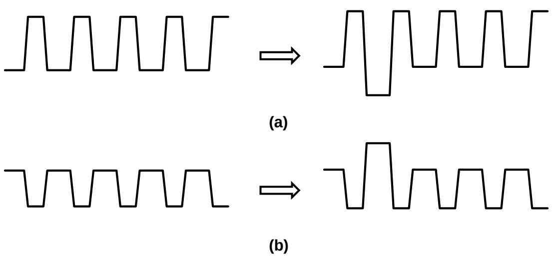


Figure 3. The change of (a) GMS and (b) TE with a spalling in a gear

3. RELATIVE STIFFNESS AND CORRECTIVE STIFFNESS

Due to the characteristics of TE mentioned in Section 2, direct estimation of GMS using TE is impossible. Therefore, this section proposes the ideas of relative stiffness and corrective stiffness which enable correct estimation of GMS in faults.

3.1. Relative Stiffness

As mentioned in Section 2.1, TE is composed of two components, shaft motion and tooth motion. To estimate a health condition of each tooth using TE, first of all, shaft motion should be removed from the measured TE. Moreover, TE of each tooth is affected by many factors like machining errors, tip relief errors, and indexing errors. This cause different magnitude of TE at each normal tooth according to its machining state from manufacturing errors even in a same gear. Therefore, the health condition of each tooth is estimated by its relative magnitude in waveforms of current TE to that of previously measured TE. The overall procedures of estimating relative stiffness are described in Figure 4.

First of all, raw TE signals are calculated using Equation (1). The detailed procedures for calculating TE is explained in Remond and Mahfoudh (2005). The raw signals pass through filters to remove shaft motions. One wave motion of the filtered signals represents an effect of TE from one tooth in gear path of contact indicated in Figure 1 (in Section 2.2). Then, the effect of TE from one tooth is quantified using peak-to-peak (P2P) values of waveforms. Higher magnitudes of P2P imply degraded conditions of tooth. However, the health states of teeth cannot be evaluate from magnitudes of

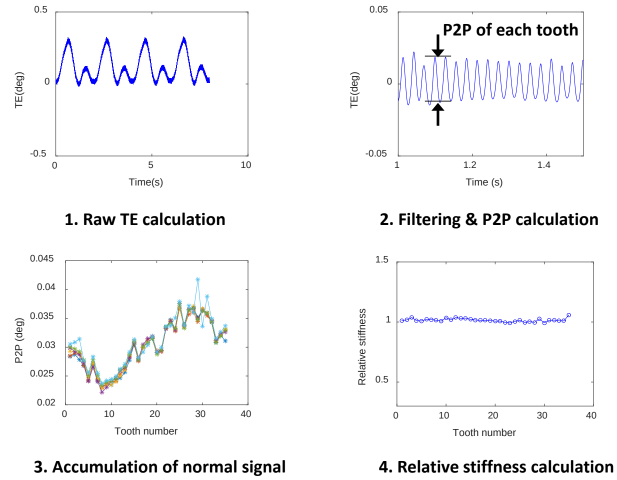


Figure 4. The procedures for calculating the relative stiffness

P2P in different teeth, but from those of P2P in same tooth due to different machining states. Hence, the reference signals for estimating health condition are calculated by a mean of accumulated magnitudes of P2P from each tooth. Then, the relative stiffness is calculated from the ratio between currently measured signals and reference signals like below:

$$GMS^{i}_{rel} = \frac{\frac{1}{n-1}\sum_{k=1}^{n-1} A(TE^{i}_{k})}{A(TE^{i}_{n})} \tag{2}$$

where GMS^{i}_{rel} is relative stiffness of the i^{th} tooth and $A(TE^{i}_{n})$ is P2P values of the i^{th} tooth at the n^{th} test set. Using this equation, health state of each gear can be quantified with normalized value, where one means perfectly normal state and zero means perfectly faulty state. In Figure 4, the P2P value at each tooth from reference signals is different, which does not imply different health conditions. As the reference signals are accumulated in normal states, different P2P values at each tooth represent individual normal states. However, when the faults happens, the P2P value gets higher than reference signals, which induces the reduction of relative stiffness.

3.2. Corrective Stiffness for tooth root crack

Using the relative stiffness, the health condition of gear teeth could be normalized with values from zero to one. However, due to the fault mechanisms explained in Section 2.2, the relative stiffness of the root crack could show biased behaviors. The reason of biased behaviors for root crack is explained in Figure 5.

As described in section 2.2, TE at root crack increased at overall regions of contact path. Therefore, when calculating

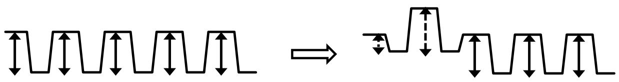


Figure 5. The change of P2P values due to crack: Solid and dotted arrows means P2P values with and without a crack

P2P values of each waveform, the P2P value before crack shows a smaller value than reference signals due to increased P2P of a prior tooth like Figure 5. Moreover, the P2P value at crack is relatively underestimated as much as reduced magnitude of a P2P value before a root crack as indicated in Figure 5 with dotted arrows. Therefore, corrective stiffness is calculated to compensate the lost P2P values as below:

$$GMS^{i}_{cor.} = \frac{\frac{1}{n-1}\sum_{k=1}^{n-1} A(TE^{i}_{k})}{A(TE^{i}_{n}) + \left(\frac{1}{n-1}\sum_{k=1}^{n-1} A(TE^{i-1}_{k}) - A(TE^{i-1}_{n})\right)} \quad (3)$$

Where $GMS^{i}_{cor.}$ is corrective stiffness of the i^{th} tooth. The difference between Equation (2) and (3) is the compensation term in the denominator in Equation (3). The term means the elevated values of P2P in the (i-1)th tooth which cause the underestimation of P2P in the i^{th} tooth. This corrective term could compensate the underestimated values of P2P for the tooth root crack while not affecting the values of P2P for normal and other fault cases.

This section explains the concept of relative stiffness and corrective stiffness. The relative stiffness could quantify the health state of individual gear tooth while considering the variability of the individual tooth and shaft motion. And the corrective stiffness could improve the accuracy of reduction ratio of stiffness for root crack as it considers the effect of adjacent tooth to the measured tooth. The following section demonstrates the described procedures of calculating GMS using a test-bed of a spur gear.

4. CASE STUDY

The concepts of relative and corrective stiffness are verified using a case study in this section. After introducing the test set-up and specimens used in this study, the TE measured from the test-bed are transformed into forms of relative stiffness and corrective stiffness. Then, we discussed the results comparing with results from other studies.

4.1. Test Set-up and specimens

As mentioned above, the GMS is estimated using TE. Therefore, we constructed the test-bed that can measure the rotational displacements using encoders while applying the inverse torque using a brake system. An overview of the test-bed is shown in Figure 6. The parameters of the gears used in this study are specified in Table 1.

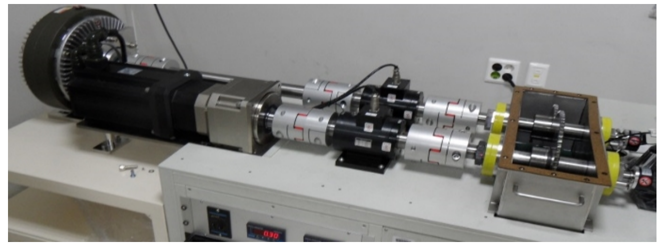
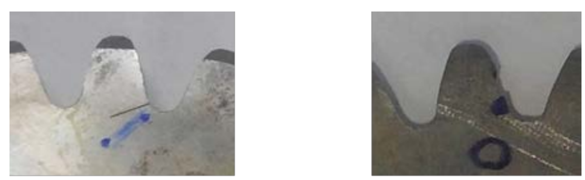


Figure 6. Overview of the test-bed

Table 1. Parameters of the gears

Gear data	Wheel	Pinion
Number of teeth	70	35
Pressure angle (deg)	20	20
Module (mm)	4	4
Face width (mm)	10	10
Pitch circle diameter (mm)	280	140

And the faults are seeded in a pinion gear like Figure 7. The types of seeded faults are tooth root crack and spalling. The length of the crack is 5 mm and the width of the spalling is 2.6mm. The faults are seeded by a wire-cutting method not to deteriorate the gear teeth shapes. And the crack and spalling were seeded in the 34th and 12th tooth, respectively.



(a) (b)

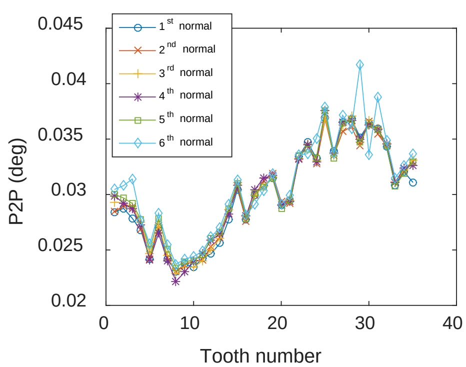
Figure 7. (a) Root crack of 5mm and (b) Spalling of 2.6mm seeded in a specimen gear

A 2.9kW servo motor drives the wheel gear with 30rpm and the brake implies 450Nm of the inverse torque. And the rotational displacements of the input and output shaft are achieved from encoders of 8192 pulse per revolution (PPR). After calculating TE from the rotational displacements, the TE data obtained are manipulated to calculate relative and corrective stiffness.

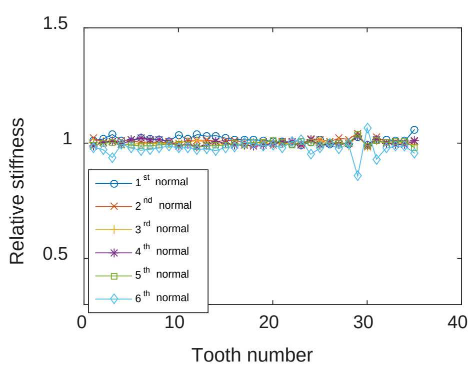
In this step, we performed multiple normal tests by re-assemblies processes. The re-assembly process of test-bed is inevitable during seeding the faults in the normal gear. Therefore, we could isolate the effect of fault-seeding on TE from re-assembly of test-bed by accumulating normal data by re-assemblies. In this study, we performed five times of re-assemblies, which lead to six data sets of normal data including the first data set. After accumulating the normal test data, root crack and spalling are seeded into the normal gears. And single set of a faulty data set is compared with the accumulated normal data sets.

4.2. Relative Stiffness and Corrective Stiffness Calculated from Measured TE

TE data are calculated using Equation (1) after obtaining the rotational displacement from the encoders. Then, the re-assemblies tests are performed for six times with a normal gear. Then, P2P of TE of each waveform from one tooth are calculated as shown in Figure 4. As mentioned in Section 3.1, the P2P values of TE are not consistent in each tooth due to the variability of teeth. Despite the uncertainty from re-assemblies, P2P values show consistent values in the same teeth except for the 6th test data. And the data are transformed into the relative stiffness using the Equation (2). The six accumulated normal data are used for calculating relative stiffness and calculated values of relative stiffness are shown in Figure 8. Most of the values shows values of one which means normal states. The abnormal changes of the 6th normal case might come from vibration of the test-bed.



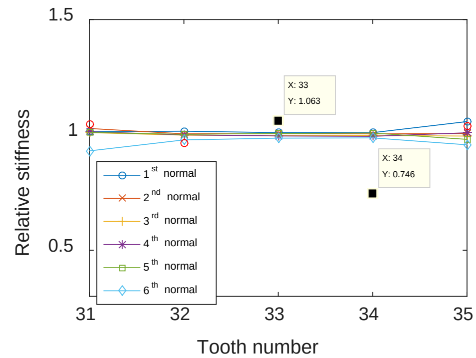
(a)



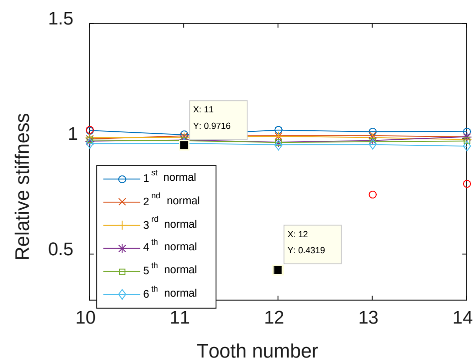
(b)

Figure 8. (a) P2P values of TE and (b) relative stiffness at each tooth of the spur gear for the six normal re-assembly tests

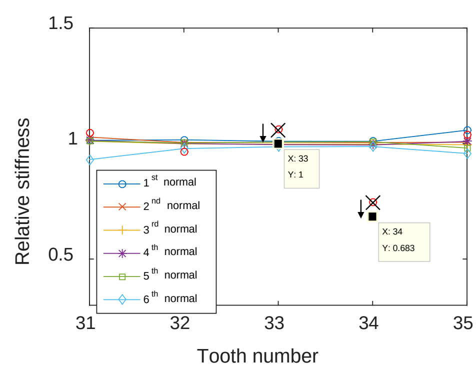
Then, relative stiffness with root crack and spalling are calculated using TE values measured from the test-bed. The calculated values of relative stiffness are shown in Figure 9. At the 34th tooth with the crack seeded, the relative stiffness is 0.746. And, at the 12th tooth where the spalling was seeded, the relative stiffness is 0.4319. Also, as noted in Section 3.2, the relative stiffness of the tooth before crack is increased due



(a)



(b)



(c)

Figure 9. The relative stiffness at (a) root crack (12th tooth), (b) spalling (34th tooth) and (c) corrective stiffness for the six normal re-assembly tests and one faulty test (marked as red circles)

to the decreased P2P value of TE, which causes the underestimation of the relative stiffness of the root crack. Therefore, the underestimated value is compensated to the relative stiffness of the cracked toot using the equation (3). Then, the corrective stiffness value is 0.683.

4.3. Discussion

From the relative stiffness, we could estimate the reduction ratio of GMS in the root crack and spalling. The reduction ratios are about 0.25 and 0.57 for root crack and spalling which can be obtained from relative stiffness in Figure 8. After calculating the corrective stiffness for root crack, the reduction ratio is 0.32. We could observe that spalling induced the more reduction in GMS than root crack. The results are also consistent with other studies that investigate the GMS and TE for root crack and spalling (Chaari et al. 2008, Chaari et al. 2009, Endo et al. 2009). As mentioned in Section 2.1, spalling deteriorated contact stiffness in GMS. Therefore, we could conclude that the contact stiffness which was degraded by spalling takes up the largest portion in GMS at the given operating conditions. And the results is consistent with the previous study by Kiekbusch, Sappok, Sauer & Howard (2011). The study showed that the proportions of contact stiffness get larger as magnitudes of torque become larger, which is similar with our operating conditions. And the calculated results implies the discriminating algorithms between root crack and spalling. A root crack is known to be a more serious fault of gear than surface damages like spalling and pitting (Fan & Zuo. 2006) as propagation of the crack could result in loss of the tooth. Therefore, the concept of corrective stiffness could make it possible to exactly estimate the reduction ratio of GMS by compensating the underestimated values of GMS. Moreover, the existence of the higher relative stiffness than one (or lower P2P values of TE than reference signals) before fault signals could be a feature that discriminates the faults between crack and spalling when the relative stiffness shows decreased values due to faults

5. CONCLUSION

This paper presents the experimental approach to estimate reduction ratio of GMS in faulty states. In the procedures, we employed the TE to develop relative stiffness and corrective stiffness. The relative stiffness could remove the biased behavior of TE from shaft motion and variability of gear teeth. And the corrective stiffness could compensate the underestimated value of reduction ratio in GMS from the fault mechanism of a root crack. The 2.6mm of a spalling reduces the 57 % of GMS, and the 5mm of a crack reduces the GMS 32 % after calculating corrective stiffness.

The main advantages of the proposed methodology are as follows. First, it provides experimental procedures to estimate reduction ratio of GMS in gear faults. Second, it differentiates the different types of gear faults; root crack and spalling. The described experimental procedures could be used to gears in a rotating state and health condition of a measured gear could be estimated using the calculated reduction ratio of GMS. Also, the differentiation method could provide maintenance strategies based on a fault type along with reduced ratio of measured signal.

This study was demonstrated using the test-bed measuring the TE signals of a spur gear in normal and faulty states. However, the operating conditions (e.g., rotating speed of a spur gear, inverse torque) and the sizes of faults are limited. In the future studies, the proposed methodology would be applied to various operating conditions and various sizes of faults. In the study, the methodology will be demonstrated in various operating conditions. Also, relationship between reduced ratio of GMS and faults sizes will be studied.

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