

# Distributed Adaptive Fault-Tolerant Formation Control of Second-Order Multi-Agent Systems with Actuator Faults

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## ABSTRACT

This paper presents an adaptive fault-tolerant control (FTC) scheme for leader-follower formation of uncertain second-order mobile agents with actuator faults. A local FTC component is designed for each agent in the distributed system by using local measurements and suitable information exchanged between neighboring agents. Each local FTC component consists of a fault detection module and a reconfigurable controller module comprised of a baseline controller and an adaptive fault-tolerant controller activated after fault detection. Under certain assumptions, the closed-loop system stability and leader-follower formation properties of the distributed system are rigorously established under different modes of behavior of the FTC system. A simulation example is used to illustrate the effectiveness of the FTC method.

## 1. INTRODUCTION

The study of distributed multi-agent systems focuses on the development of control algorithms that enable a team of interconnected agents to accomplish desired team missions. One unique feature of these algorithms is their distributed nature, where each agent takes actions based on information obtained from its local neighbors. This distributed nature has numerous benefits, such as scalability and robustness. The research on distributed multi-agent systems has received increasing attention due to its broad application in numerous areas, such as spacecraft formation flying (Ren & Beard, 2004), smart grid (Pipattanasomporn, Feroze, & Rahman, 2009), and sensor networks (Cortes, Martinez, Karatas, & Bullo, 2004). One key concept in the study of distributed multi-agent systems is to have the team exchange information in order to achieve the desired goal.

Since such distributed multi-agent systems are required to operate reliably at all times, despite the possible occurrence of faulty behaviors in some agents, the development of fault diagnosis and accommodation schemes is a crucial step in achieving reliable and safe operations. In the last two decades, significant research activities have been conducted in the design and analysis of fault diagnosis and accommodation schemes (see, for instance, (Blanke, Kinnaert, Lunze, & Staroswiecki, 2006)). Most of these methods utilize a centralized architecture, where the diagnostic module is designed based on a global mathematical model of the overall system and is required to have real-time access to all sensor measurements. Because of limitations of computational resource and communication overhead, such centralized methods are not suitable for large-scale distributed interconnected systems. As a result, in recent years, there has been a significantly increasing research interest in the development of distributed fault diagnosis schemes for multi-agent systems (see, for instance, (Keliris, Polycarpou, & Parisini, 2013; Yan & Edwards, 2008; Reppa, Polycarpou, & Panayiotou, 2015; Ferrari, Parisini, & Polycarpou, 2012; Shames, Teixeira, Sandberg, & Johansson, 2011)).

This paper presents a method for detecting and accommodating actuator faults in a class of distributed nonlinear uncertain second-order multi-agent systems. A fault-tolerant control component is designed for each agent in the distributed system by utilizing local measurements and certain information exchanged between neighboring agents. Each local FTC component consists of two main modules: 1) an online fault detection scheme; and 2) the controller (fault accommodation) module consists of a baseline controller and an adaptive fault-tolerant controller employed after fault detection. Under certain assumptions, the closed-loop system's stability and leader-following formation properties are established for the baseline controller and adaptive fault-tolerant controller.

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A simulation example is used to illustrate the effectiveness of the FTC method.

In a previous paper, a centralized adaptive fault-tolerant control scheme is presented in (Zhang, Parisini, & Polycarpou, 2004) for a class of nonlinear uncertain systems, where the centralized fault-tolerant controller has access to all the measurements in the overall system. The distributed FTC problem considered in this paper is much more challenging than the centralized problem in (Zhang et al., 2004) because in the distributed communication topology of the leader-following multi-agent systems considered in this paper, the leader only communicates to a small number of agents in the overall system, and each agent exchanges measurement information only with its neighbors. In addition, a distributed fault-tolerant control scheme for first-order multi-agent systems is presented in (Khalili, Zhang, Polycarpou, Parisini, & Cao, 2015) and another previous paper (Khalili, Zhang, Cao, Polycarpou, & Parisini, 2015) considers the problem of distributed FTC design for second-order multi-agent systems addressing the case of process faults but not actuator fault. This paper extends the fault-tolerant control method to the case of leader-follower formation of distributed second-order multi-agent systems, considering actuator faults in the agent dynamics.

The rest of this paper is organized as follows. Section 2 provides the graph theory notations. Problem formulation for fault-tolerant leader-follower formation control of second-order multi-agent systems with actuator faults is described in Section 3. The closed-loop system stability and performance before fault occurrence is presented in Section 4. The distributed fault detection is described in Section 5. The design and analysis of the fault-tolerant control scheme after fault detection is rigorously investigated in Section 6. In Section 7, a simulation example is used to illustrate the effectiveness of the FTC method. Finally, Section 8 provides some concluding remarks.

## 2. GRAPH THEORY MAIN NOTATIONS

A directed graph  $\mathcal{G}$  is a pair  $(\mathcal{V}, \mathcal{E})$ , where  $\mathcal{V} = \{v_1, \dots, v_m\}$  is a set of nodes,  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$  is a set of edges, and  $m$  is the number of nodes. An edge is an ordered pair of distinct nodes  $(v_j, v_i)$ , meaning that the  $i$ th node can receive information from the  $j$ th node. For an edge  $(v_j, v_i)$ , node  $v_j$  is called the parent node, node  $v_i$  is the child node, and  $v_j$  is a neighbor of  $v_i$ . A sequence of distinct edges in the directed graph  $\mathcal{G}$  creates a directed path between two distinct nodes. An undirected graph can be considered as a special case of a directed graph where  $(v_i, v_j) \in \mathcal{E}$  implies  $(v_j, v_i) \in \mathcal{E}$  for any  $v_i, v_j \in \mathcal{V}$ .

The set of neighbors of node  $v_i$  is denoted by  $N_i = \{j : (v_j, v_i) \in \mathcal{E}\}$ . The weighted adjacency matrix  $\mathcal{A} = [o_{ij}] \in \mathbb{R}^{m \times m}$  associated with the directed graph  $\mathcal{G}$  is defined by

$o_{ii} = 0$ ,  $o_{ij} > 0$  if  $(v_j, v_i) \in \mathcal{E}$ , and  $o_{ij} = 0$  otherwise. An interaction graph  $\mathcal{G}$  is said to be fixed, if each node has a fixed neighbor set and  $o_{ij}$  is fixed. It is clear that for undirected graphs  $o_{ij} = o_{ji}$ . The Laplacian matrix  $L = [l_{ij}] \in \mathbb{R}^{m \times m}$  is defined as  $l_{ii} = \sum_{j \in N_i} o_{ij}$  and  $l_{ij} = -o_{ij}$ ,  $i \neq j$ . Both  $\mathcal{A}$  and  $L$  are symmetric for undirected graphs, and  $L$  is positive semidefinite. More detailed description of graph theory can be found in (Ren & Beard, 2008).

## 3. PROBLEM FORMULATION

### 3.1. Distributed Multi-Agent System Model

Consider a set of  $M$  interconnected agents where the second-order dynamics of the  $i$ th agent,  $i = 1, \dots, M$ , is described by:

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= \phi_i(x_i) + u_i + \eta_i(x_i, t) + \beta_i(t - T_i)f_i(u_i), \end{aligned} \quad (1)$$

where  $x_i = \begin{bmatrix} p_i \\ v_i \end{bmatrix} \in \mathbb{R}^2$  and  $u_i \in \mathbb{R}$  are the state vector and the input of the  $i$ th agent, respectively. Additionally,  $\phi_i : \mathbb{R}^2 \mapsto \mathbb{R}$ ,  $\eta_i : \mathbb{R}^2 \times \mathbb{R}^+ \mapsto \mathbb{R}$ ,  $f_i : \mathbb{R}^2 \times \mathbb{R} \mapsto \mathbb{R}$  are smooth vector fields.

The model given by

$$\dot{x}_i = \begin{bmatrix} 0 \\ \phi_i(x_i) \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i \quad (2)$$

represents the *known nominal* dynamics of the  $i$ th agent with  $\phi_i$  being the known nonlinearity. The healthy system is described by

$$\dot{x}_i = \begin{bmatrix} 0 \\ \phi_i(x_i) \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (u_i + \eta_i(x_i, t)). \quad (3)$$

The difference between the nominal model Eq. (2) and the actual (healthy) system dynamics Eq. (3) is due to  $\eta_i(x_i, t)$  characterizing the modeling uncertainty in the state dynamics of the  $i$ th agent.

The term  $\beta_i(t - T_i)f_i(u_i)$  in Eq. (1) denotes the changes in the dynamics of  $i$ th agent due to the occurrence of an actuator fault. Specifically,  $\beta_i(t - T_i)$  represents the time profile of an actuator fault which occurs at some unknown time  $T_i$ , and  $f_i(u_i) = \theta_i u_i$  is an actuator fault function representing partial loss of effectiveness of the actuators, where the fault parameter  $\theta_i \in (-1, 0]$ , characterizes the unknown magnitude of the actuator fault. In this paper,  $\beta_i(\cdot)$  is assumed to be a step function (i.e.,  $\beta_i(t - T_i) = 0$  if  $t < T_i$ , and  $\beta_i(t - T_i) = 1$  if  $t \geq T_i$ ) which denotes an abrupt fault.

The objective of this paper is to develop a robust distributed fault-tolerant leader-following formation control scheme for the class of distributed second-order multi-agent systems de-

scribed by Eq. (1). It is assumed there is only a single fault in each agent at any time. However, the system model Eq. (1) allows the occurrence of faults in multiple agents. The following assumptions are needed:

**Assumption 1.** The unstructured modeling uncertainty, represented by  $\eta_i(x_i, t)$  in Eq. (1), has a known upper bound, i.e.,  $\forall x_i \in \mathbb{R}^2$

$$|\eta_i(x_i, t)| \leq \bar{\eta}_i(x_i, t), \quad (4)$$

where the bounding function  $\bar{\eta}_i$  is known and uniformly bounded with respect to  $(x, t)$ .

**Assumption 2.** The communication topology among followers is undirected, and the leader has directed paths to all followers.

Assumption 1 characterizes the class of modeling uncertainty under consideration. The bound on the modeling uncertainty is needed in order to distinguish between the effects of faults and modeling uncertainty during the fault diagnosis process (Emami-Naeini, Akhter, & Rock, 1988). Assumption 2 is needed to ensure that the information exchange among agents is sufficient for the team to achieve the desired team goal. Note that the leader is only required to be a neighbor of a subset of followers but it has at least one path to each follower through the intercommunication topology.

### 3.2. Fault-Tolerant Control Structure

In this paper, we investigate the FTC problem of leader-following formation. The objective is to develop distributed robust FTC algorithms such that each agent's output converges to a given predefined formation with a time-varying bounded leader even in the presence of modeling uncertainty and faults. Without loss of generality, let the leader be identified as agent number 0 (i.e.,  $x_0(t) = [p_0(t) \ v_0(t)]^T$  where  $p_0 = \dot{v}_0$ ). Therefore, the distributed FTC control algorithm is designed to ensure  $p_i(t) - p_0(t) \rightarrow \bar{p}_i$  and  $v_i(t) \rightarrow v_0(t)$ , where  $\bar{p}_i$  is the constant desired relative position between the leader and agent  $i$ , for  $i = 1, \dots, M$ . It is worth noting that each agent (including the leader) only communicates with its neighbors. An example of the distributed FTC architecture considered is shown in Figure 1.

First of all, we define two important time-instants:  $T_i$  is the (unknown) fault occurrence time;  $T_d > T_i$  is the time-instant when a fault is detected. The structure of the fault-tolerant controller for the  $i$ th agent takes on the following general form (Zhang et al., 2004):

$$\begin{aligned} \dot{\omega}_i &= \begin{cases} g_0(\omega_i, x_i, x_{J_i}, t), & \text{for } t < T_d \\ g_D(\omega_i, x_i, x_{J_i}, t), & \text{for } t \geq T_d \end{cases} \\ u_i &= \begin{cases} h_0(\omega_i, x_i, x_{J_i}, t), & \text{for } t < T_d \\ h_D(\omega_i, x_i, x_{J_i}, t), & \text{for } t \geq T_d \end{cases} \end{aligned} \quad (5)$$

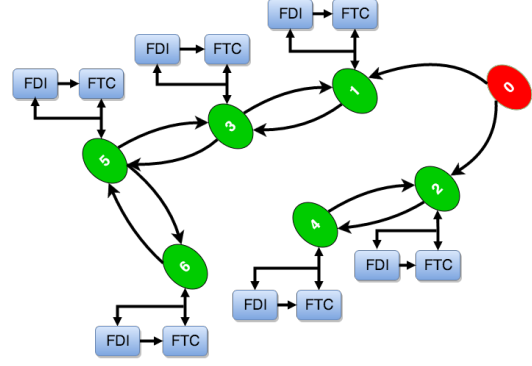


Figure 1. An example of distributed FTC architecture

where  $\omega_i$  is the state vector of the distributed controller,  $x_{J_i}$  contains the state variables of neighboring agents that directly communicate with agent  $i$ , i.e.,  $J_i = \{j : j \in N_i\}$ ;  $g_0, g_D, h_0$  and  $h_D$  are nonlinear functions to be designed according to the following qualitative objectives:

1. In a fault free mode of operation, a baseline controller guarantees that the output of  $i$ th agent  $x_i(t)$  should track the predefined formation with a time-varying leader  $x_0(t)$ , even in the possible presence of plant modeling uncertainty.
2. If an actuator fault is detected, the baseline controller is reconfigured to compensate for the effect of the fault. This new controller should guarantee the boundedness of system signals and leader-following formation, even in the presence of fault.

**Remark 1:** The distributed aspect of the control algorithm is reflected by the communication topology. Specifically, the leader only communicates to a small subset of followers, and each follower only communicates to its directly connected neighbors. However, the distributed FTC algorithm guarantees the velocity of each follower converges to that of the leader, and the position of each follower converges to a specified distance from the leader's position.

### 4. DISTRIBUTED BASELINE CONTROLLER DESIGN

In this section, that has been presented in (Khalili, Zhang, Cao, et al., 2015), we describe the distributed baseline controller for each follower and present the closed-loop stability and performance result of the overall system before fault occurrence. Based on the system model Eq. (1), the baseline controller for the  $i$ th agent is designed as follows:

$$\begin{aligned} u_i &= - \sum_{j \in N_i} k_{ij} \left( \ell(p_i - \bar{p}_i - p_j + \bar{p}_j) + \gamma(v_i - v_j) \right) \\ &\quad - \phi_i(x_i) - (\bar{\eta}_i + \kappa) \operatorname{sgn}(\Xi_i), \end{aligned} \quad (6)$$

where  $\bar{p}_i$  and  $\bar{p}_j$  are the constant desired relative position between the leader and agents  $i$  and  $j$ , respectively,  $\kappa$  is a posi-

tive bound on  $|\dot{v}_0|$  (i.e.,  $\kappa \geq |\dot{v}_0|$ ),  $\text{sgn}(\cdot)$  is the sign function defined to take zero value at zero,  $\Xi_i = \sum_{j \in N_i} k_{ij} (\epsilon(p_i - \bar{p}_i - p_j + \bar{p}_j) + \rho(v_i - v_j))$ ,  $N_i$  is the set of neighboring agents that directly communicate with the  $i$ th agent. Note that if the leader directly communicates with agent  $i$ , then the leader is represented as agent number 0 with  $\bar{p}_0 = 0$ ,  $k_{ij}$  are positive constants for  $j \in N_i$ , and  $\ell$ ,  $\gamma$ ,  $\rho$ , and  $\epsilon$  are positive constants to be determined in Lemma 1. Note that  $k_{il} = 0$ , for  $l \notin N_i, l = 0, \dots, M$ .

The following Lemma is needed for the design and analysis of the distributed formation control algorithm:

**Lemma 1.** (Khalili, Zhang, Cao, et al., 2015) Consider a positive definite square matrix  $\Psi \in \mathbb{R}^{M \times M}$ . Define

$$A = \begin{bmatrix} 0_{M \times M} & I_M \\ -\ell\Psi & -\gamma\Psi \end{bmatrix}, P = \begin{bmatrix} \rho\Psi & \epsilon\Psi \\ \epsilon\Psi & \rho\Psi \end{bmatrix}, \quad (7)$$

where  $I_M$  is the identity matrix of order  $M$ , and  $\rho$ ,  $\epsilon$ ,  $\gamma$ , and  $\ell$  are positive constants satisfying  $\rho > \epsilon$ . The matrix  $Q = PA + A^T P$  is negative definite if the following conditions are satisfied:

$$\gamma\epsilon = \ell\rho, \quad \frac{\epsilon}{\ell\epsilon + \rho\gamma} < \mu_{\min}, \quad \frac{\rho^2}{4\ell(\rho^2 - \epsilon^2)} < \mu_{\min}, \quad (8)$$

where  $\mu_{\min}$  is the smallest eigenvalue of  $\Psi$ .

The following result characterizes the closed-loop stability and leader-following formation performance properties of the overall multi-agent system prior to any fault occurrence. Detailed stability analysis has been already considered in (Khalili, Zhang, Cao, et al., 2015). For the sake of completeness of presentation in this paper we include the following result.

**Theorem 1.** (Khalili, Zhang, Cao, et al., 2015) Suppose that Assumptions 1 and 2 hold. In the absence of faults in the multi-agent system Eq. (1), the baseline controller Eq. (6) guarantees that the leader-follower formation control is achieved asymptotically with a time-varying reference state, i.e.  $p_i(t) - p_0(t) \rightarrow \bar{p}_i$  and  $v_i(t) - v_0(t) \rightarrow 0$  as  $t \rightarrow \infty$ .

**Remark 2:** The robustness to modeling uncertainty is achieved by the distributed controller Eq. (6) using the bounding control method (Farrell & Polycarpou, 2006). Note that, in general, the control law Eq. (6) is discontinuous at  $\sum_{j \in N_i} k_{ij} (\epsilon(p_i - \bar{p}_i - p_j + \bar{p}_j) + \rho(v_i - v_j)) = 0$ . This may lead to the switching control law, thus creating chattering problems. Using a smooth approximation to the sign function (Farrell & Polycarpou, 2006) (for instance, the hyperbolic tangent function (Zhang et al., 2004)) can remedy the chattering issue.

## 5. DECENTRALIZED FAULT DETECTION

The decentralized fault detection architecture is comprised of  $M$  local fault detection components designed for each of the  $M$  agents. The objective of each local fault detection component is to detect faults in the corresponding agent. Under normal conditions, each local fault detection estimator (FDE) monitors the corresponding local agent to detect the occurrence of any fault.

Based on the agent model described by Eq. (1), the FDE for each agent is chosen as:

$$\dot{\hat{x}}_i = \Lambda_i^0(x_i - \hat{x}_i) + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (\phi_i(x_i) + u_i), \quad (9)$$

where  $\hat{x}_i \in \mathbb{R}^2$  denotes the estimated local state,  $\Lambda_i^0 = \begin{bmatrix} \lambda_{p_i}^0 & 0 \\ 0 & \lambda_{v_i}^0 \end{bmatrix} \in \mathbb{R}^{2 \times 2}$  is a positive definite estimator gain matrix.

For each local FDE, let  $\varepsilon_i \triangleq x_i - \hat{x}_i = [\varepsilon_{p_i} \ \varepsilon_{v_i}]^T$  denote the state estimation error of the  $i$ th agent. Then, before fault occurrence (i.e., for  $0 \leq t < T_i$ ), by using Eq. (1) and (9), the estimation error dynamics are given by

$$\dot{\varepsilon}_i = -\Lambda_i^0 \varepsilon_i + \begin{bmatrix} 0 \\ \eta_i(x_i, t) \end{bmatrix}. \quad (10)$$

Therefore, using Eq. (10), we have  $|\varepsilon_{v_i}| \leq \nu_i$ , where

$$\nu_i(t) \triangleq \int_0^t e^{-\lambda_{v_i}^0(t-\tau)} \bar{\eta}_i(x_i, \tau) d\tau + \bar{x}_i e^{-\lambda_{v_i}^0 t}, \quad (11)$$

and  $\bar{x}_i$  is a possibly conservative bound on the initial state estimation error (i.e.,  $|\varepsilon_{v_i}(0)| \leq \bar{x}_i$ ). Note that the integral term in the above thresholds can be easily implemented as the output of a first-order linear filter  $H(s) = 1/(s + \lambda_{v_i}^0)$  with the input given by  $\bar{\eta}_i(x_i, t)$ .

Thus, we have the following decision scheme:

**Fault Detection Decision Scheme:** The decision on the occurrence of a fault (detection) in the  $i$ th agent is made when the absolute value of the estimation error (i.e.,  $\varepsilon_{v_i}$ ) generated by the local FDE exceeds its corresponding threshold (i.e.,  $|\varepsilon_{v_i}(t)| > \nu_i(t)$ ) where  $\nu_i(t)$  is given by Eq. (11).

The fault detection time  $T_d$  is defined as the first time instant such that  $|\varepsilon_{v_i}(T_d)| > \nu_i(T_d)$ , for some  $T_d \geq T_i$ , that is,

$$T_d \triangleq \inf \bigcup_{i=1}^M \{t \geq 0 : |\varepsilon_{v_i}(t)| > \nu_i(t)\}. \quad (12)$$

## 6. FAULT-TOLERANT CONTROLLER MODULE

In this section, the design and analysis of the fault-tolerant control scheme is rigorously investigated for the closed-loop system after the detection of actuator fault. After the fault

is detected at time  $t = T_d$ , the nominal controller is reconfigured to ensure the system stability and leader-follower formation after the detection of actuator fault. In the following, we describe the design of the fault-tolerant controller using adaptive techniques.

After the occurrence of an actuator fault, i.e., for  $t \geq T_d$ , the dynamics of the system takes on the following form:

$$\dot{x}_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (\phi_i(x_i) + (1 + \theta_i)u_i + \eta_i(x_i, t)). \quad (13)$$

Without loss of generality, let the leader be agent number 0. The control objective is to force the states  $x_i, i = 1, \dots, M$ , to track the known time-varying state of the leader  $x_0$ .

After the detection of the actuator fault, i.e.,  $t \geq T_d$ , the following adaptive fault-tolerant controller is adopted:

$$u_i = \frac{1}{1 + \hat{\theta}_i} \bar{u}_i, \quad (14)$$

$$\begin{aligned} \bar{u}_i &\triangleq -\phi_i(x_i) - \sum_{j \in N_i} k_{ij} (\ell \tilde{p}_{ij} + \gamma \tilde{v}_{ij}) \\ &\quad - (\bar{\eta}_i + \kappa_p) \text{sgn} \left( \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) \right), \quad (15) \end{aligned}$$

$$\dot{\hat{\theta}}_i = \mathcal{P}_{\hat{\theta}_i} \left\{ \Gamma_i \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) u_i \right\}, \quad (16)$$

where  $\tilde{p}_{ij} \triangleq (p_i - \bar{p}_i) - (p_j - \bar{p}_j)$  and  $\tilde{v}_{ij} \triangleq v_i - v_j$ ,  $\hat{\theta}_i$  is an estimation of the unknown actuator fault magnitude parameter  $\theta_i$  with the projection operator  $\mathcal{P}$  restricting  $\hat{\theta}_i$  to the corresponding set  $[\bar{\theta}_i, 0]$  for  $\bar{\theta}_i \in (-1, 0)$ , and  $\Gamma_i$  is a positive learning rate constant.

**Remark 3:** Compared with the baseline controller Eq. (6), the adaptive term  $\frac{1}{1 + \hat{\theta}_i}$  and the adaptive law Eq. (16) are utilized to compensate for the effect of actuator fault occurred to the agent  $i$ .

Then, we have the following theorem:

**Theorem 2.** *Suppose that Assumptions 1-2 hold. Assume that an actuator fault occurs at time  $T_i$  and that it is detected at time  $T_d$ . Then, the fault-tolerant controller Eq. (14) and fault parameter adaptive law Eq. (16) guarantee that the leader-follower formation is achieved asymptotically with a time-varying reference state, i.e.,  $p_i(t) - p_0(t) \rightarrow \bar{p}_i$  and  $v_i(t) \rightarrow v_0(t)$  as  $t \rightarrow \infty$ .*

*Proof.* Using some algebraic manipulations, we can rewrite Eq. (14) as  $u_i = \bar{u}_i - \hat{\theta}_i u_i$ . Note that  $(1 + \theta_i)u_i = u_i + \theta_i u_i = \bar{u}_i - \hat{\theta}_i u_i + \theta_i u_i$ . Therefore, by substituting  $(1 + \theta_i)u_i$  in Eq.

(13), the closed-loop system dynamics are given by

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= - \sum_{j \in N_i} k_{ij} (\ell \tilde{p}_{ij} + \gamma \tilde{v}_{ij}) + \eta_i - (\bar{\eta}_i + \kappa) \text{sgn}(\Xi_i) \\ &\quad + \tilde{\theta}_i u_i, \end{aligned}$$

where  $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$  is the actuator fault parameter estimation error corresponding to the  $i$ th agent. We can represent the collective output dynamics as

$$\dot{\tilde{x}} = A \tilde{x} + \begin{bmatrix} \mathbf{0}_M \\ \zeta - \bar{\zeta} - \mathbf{1}_M \dot{v}_0 + \varpi \end{bmatrix}, \quad (17)$$

where  $A$  is defined in Lemma 1 with the stable matrix  $\Psi = \mathcal{L} + \text{diag}\{k_{10}, k_{20}, \dots, k_{M0}\}$  (Cao & Ren, 2012),  $\mathcal{L}$  is the communication graph Laplacian matrix,  $\mathbf{1}_M$  is a  $M \times 1$  column vector of ones,  $\tilde{x} = [\tilde{p}^T \ \tilde{v}^T]^T \in \mathfrak{R}^{2M}$  in which  $\tilde{p}$  is the column stack vector of  $\tilde{p}_i \triangleq p_i - \bar{p}_i - p_0$  and  $\tilde{v}$  is the column stack vector of  $\tilde{v}_i \triangleq v_i - v_0$ , the terms  $\zeta \in \mathfrak{R}^M$ ,  $\bar{\zeta} \in \mathfrak{R}^M$  and  $\varpi \in \mathfrak{R}^M$  are defined as

$$\zeta \triangleq [\eta_1 \ \dots \ \eta_M]^T, \quad (18)$$

$$\bar{\zeta} \triangleq [\bar{\zeta}_1 \ \dots \ \bar{\zeta}_M]^T, \quad (19)$$

$$\varpi \triangleq [\tilde{\theta}_1 u_1 \ \dots \ \tilde{\theta}_M u_M]^T, \quad (20)$$

and  $\bar{\zeta}_i = (\bar{\eta}_i + \kappa_i) \text{sgn}(\Xi_i), i = 1, \dots, M$ .

We consider the following Lyapunov function candidate:

$$V = \tilde{x}^T P \tilde{x} + \tilde{\theta}^T (\Gamma)^{-1} \tilde{\theta}, \quad (21)$$

where  $P$  is defined in Lemma 1,  $\tilde{\theta} = [\tilde{\theta}_1 \ \dots \ \tilde{\theta}_M]^T$  is the collective parameter estimation errors, and  $\Gamma = \text{diag}\{\Gamma_1, \dots, \Gamma_M\}$  is a positive definite learning rate matrix. Then, the time derivative of the Lyapunov function Eq. (21) along the solution of Eq. (17) is given by

$$\dot{V} = \tilde{x}^T Q \tilde{x} + 2\tilde{x}^T P \begin{bmatrix} \mathbf{0}_M \\ \zeta - \bar{\zeta} - \mathbf{1}_M \dot{v}_0 + \varpi \end{bmatrix}, \quad (22)$$

where  $Q$  is defined in Lemma 1. Based on Eq. (7), (18), (19), and (20), and by using  $\tilde{p}_i = \tilde{p}_{i0}$  and  $\tilde{v}_i = \tilde{v}_{i0}$  we have

$$\begin{aligned} \tilde{x}^T P \begin{bmatrix} \mathbf{0}_M \\ \zeta \end{bmatrix} &= \epsilon \tilde{p}^T \Psi \zeta + \rho \tilde{v}^T \Psi \zeta \\ &= \sum_{i=1}^M \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) \eta_i, \quad (23) \\ \tilde{x}^T P \begin{bmatrix} \mathbf{0}_M \\ \mathbf{1}_M \dot{v}_0 \end{bmatrix} &= \epsilon \tilde{p}^T \Psi \mathbf{1}_M \dot{v}_0 + \rho \tilde{v}^T \Psi \mathbf{1}_M \dot{v}_0 \\ &= \sum_{i=1}^M \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) \dot{v}_0, \quad (24) \end{aligned}$$

$$\begin{aligned}\tilde{x}^T P \begin{bmatrix} \mathbf{0}^M \\ \tilde{\zeta} \end{bmatrix} &= \epsilon \tilde{p}^T \Psi \tilde{\zeta} + \rho \tilde{v}^T \Psi \tilde{\zeta} \\ &= \sum_{i=1}^M \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) (\bar{\eta}_i + \kappa_i) \text{sgn}(\Xi_i),\end{aligned}\quad (25)$$

$$\begin{aligned}\tilde{x}^T P \begin{bmatrix} \mathbf{0}^M \\ \varpi \end{bmatrix} &= \epsilon \tilde{p}^T \Psi \varpi + \rho \tilde{v}^T \Psi \varpi \\ &= \sum_{i=1}^M \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) \tilde{\theta}_i u_i.\end{aligned}\quad (26)$$

By substituting Eq. (23), (24), (25) and (26) into Eq. (22), we have

$$\begin{aligned}\dot{V} &= \tilde{x}^T Q \tilde{x} + 2 \sum_{i=1}^M \left[ \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) (\eta_i - \dot{v}_0) \right. \\ &\quad - \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) (\bar{\eta}_i + \kappa) \text{sgn}(\Xi_i) \\ &\quad \left. + \tilde{\theta}_i^T \left( \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) u_i - (\Gamma_i)^{-1} \dot{\hat{\theta}}_i \right) \right],\end{aligned}\quad (27)$$

where  $Q$  is defined in Lemma 1. Based on Assumption 1, we have

$$\begin{aligned}(\eta_i - \dot{v}_0) \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) \\ - (\bar{\eta}_i + \kappa) \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) \text{sgn}(\Xi_i) \leq 0.\end{aligned}$$

By applying the above inequality to Eq. (27), and choosing the adaptive law as Eq. (16) we obtain

$$\dot{V} \leq \tilde{x}^T Q \tilde{x} \leq 0. \quad (28)$$

It is worth noting that since the parameter projection modification can only make the Lyapunov function derivative more negative, the stability properties derived for the standard algorithm still hold (Farrell & Polycarpou, 2006). Thus, we conclude that  $\tilde{x}_i = [\tilde{p}_i \tilde{v}_i]^T$  and  $\tilde{\theta}_i$  are uniformly bounded. By integrating both sides of Eq. (28), it can be easily shown that  $\tilde{x}_i \in L_2$ . Additionally,  $x_i$  is bounded because  $\tilde{x}_i$  and the leader's state  $x_0$  are bounded. Therefore, based on Eq. (15), (13), and the smoothness of the function  $\phi_i$ , we have  $\bar{u}_i \in L_\infty$  and  $\dot{x}_i \in L_\infty$ . Since  $\tilde{x}_i \in L_\infty \cap L_2$  and  $\dot{\tilde{x}}_i \in L_\infty$ , based on Barbalat's Lemma (Ioannou & Sun, 1996), we can conclude that the leader-following formation is reached asymptotically, i.e.,  $\tilde{p}_i \rightarrow 0$  and  $\tilde{v}_i \rightarrow 0$  as  $t \rightarrow \infty$ .  $\square$

## 7. SIMULATION RESULTS

In this section, a simulation example of 5 agents is considered to illustrate the effectiveness of the distributed fault-tolerant formation control method. The dynamics of the  $i$ th vehicle, motivated by the automated highway system considered in (Spooner & Passino, 1996), is given by

$$\begin{aligned}\dot{x}_i &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{m} (-A_\rho v_i^2 - d_f v_i + u_i + \eta_i \\ &\quad + \beta_i (t - T_i) f_i(u_i)), \quad i = 1, \dots, 5,\end{aligned}\quad (29)$$

where  $x_i = [p_i \ v_i]^T$  is the state of the  $i$ th agent consisting of the position  $p_i$  and velocity  $v_i$ ,  $u_i$  is the input of  $i$ th agent representing the applied force in longitudinal direction,  $m$  is the mass of the vehicle,  $A_\rho$  is aerodynamic drag coefficient and  $d_f$  is a constant friction coefficient. The model Eq. (29) can be easily put into the general form Eq. (1) by letting the nominal term in the dynamics of each agent  $\phi_i(x_i) = \frac{1}{m} (-A_\rho v_i^2 - d_f v_i)$ .

Figure 2 shows the communication graph of the agents. As we can see, the leader only communicates to a small subset of followers, and each follower only communicates to its directly connected neighbors. The objective is to design the controller  $u_i$  to have each agent follow a virtual leader  $x_0$  and also keep a predefined formation around the leader even in the presence of modeling uncertainty  $\eta_i$  and possible occurrence of faults  $f_i(u_i)$ . The unknown modeling uncertainty in the local dynamics of the agents are assumed to be a sinusoidal signal  $\eta_i = 0.5 \sin(t)$  bounded by  $\bar{\eta}_i = 0.6$ . The virtual leader  $x_0$  is given by  $\dot{x}_0 = \begin{bmatrix} v_0 \\ 0.5 \sin(t) \end{bmatrix}$  with zero initial condition. The constant desired relative positions between the leader and agents are  $\bar{p}_1 = -4$ ,  $\bar{p}_2 = -2$ ,  $\bar{p}_3 = 0$ ,  $\bar{p}_4 = 2$ ,  $\bar{p}_5 = 4$ . The other model parameters used in the simulation example are  $m = 1 \text{ kg}$ ,  $A_\rho = 0.5 \text{ N s}^2/\text{m}^2$  and  $d_f = 0.6 \text{ N s}/\text{m}$ .

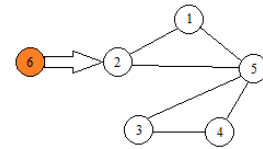


Figure 2. Communication graph

The Laplacian matrix of the intercommunication graph of agents, shown in figure 2, is chosen as

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & 0 & 0 & -1 \\ 0 & 0 & 2 & -1 & -1 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix}.$$

The virtual leader only communicates with the second agent (i.e.,  $k_{20} = 1$ ). The matrix  $\Psi = \mathcal{L} + \text{diag}\{0, 1, 0, 0, 0\}$  has the minimum eigenvalue of  $\mu_{min} = 0.13$ . We choose  $\ell = 3$ ,  $\gamma = 30$ ,  $\epsilon = 0.1$ , and  $\rho = 1$  so that the conditions given in Lemma 1 are satisfied.

The fault considered here is an actuator fault function  $f_i = \theta_i u_i$ , where the magnitude of this fault is considered as  $\theta_i \in [-0.8 \ 0]$ . The estimator gain for the fault detection estimator is chosen as  $\lambda_{p_i}^0 = \lambda_{v_i}^0 = 2$ . After fault detection, the controller is reconfigured to accommodate the actuator fault occurred. We set the adaptive gain  $\Gamma_i = 0.2$  with a zero initial condition (see Eq. (16)).

Figure 3 shows the fault detection results when actuator faults with a magnitude of  $-0.5$  and  $-0.4$  occur to agents 1 and 3 at  $T_1 = 40$  and  $T_3 = 60$  second, respectively. As can be seen from Figure 3, the residual corresponding to the output generated by the local FDE designed for agents 1 and 3 exceeds its threshold immediately after fault occurrence. Therefore, the actuator faults in agent 1 and 3 are timely detected. Note that the residual signals are time-varying because the uncertainty  $\eta_i$  in Eq. (10) is time-varying.

Regarding the performance of the adaptive fault-tolerant controllers, as can be seen from Figure 4, the tracking errors converge to zero. Thus, the leader-following formation in the presence of actuator faults is achieved using the proposed adaptive FTC. On the other hand, the agents cannot follow the leader and become unstable without the FTC controller (see Figure 5), since the tracking errors do not converge to zero. Therefore, the benefits of the FTC method can be clearly seen.

## 8. CONCLUSIONS

In this paper, we investigate the problem of a distributed FDI and FTC for a class of uncertain second-order multi-agent systems. By using on-line diagnostic information, adaptive FTC controllers are developed to achieve the leader-following formation with a time-varying leader in the presence of actuator faults. The closed-loop stability and leader-following formation properties at different stages of the fault diagnosis process are rigorously established under different modes of the FTC system. The extensions to systems with more general structures is an interesting topic for future research.

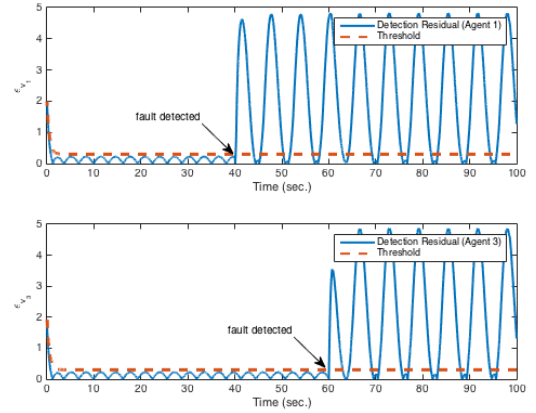


Figure 3. Fault detection residuals (solid and blue line) and the corresponding threshold (dashed and red line) generated by the local FDE of agents 1 and 3

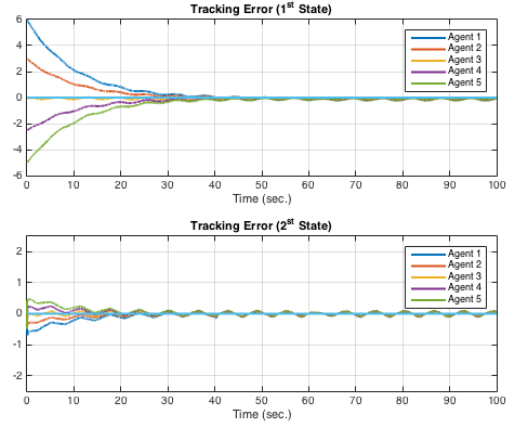


Figure 4. The tracking errors in the case of an actuator fault in agents 1 and 3: with adaptive fault-tolerant controllers

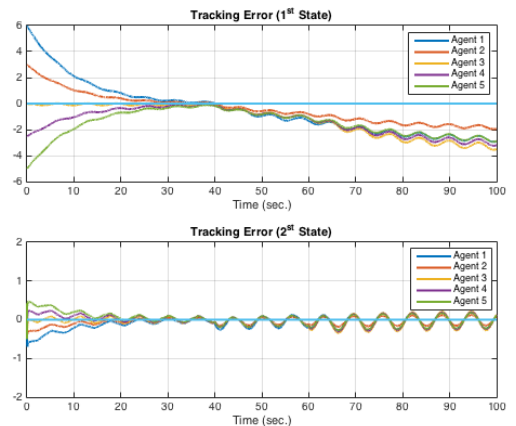


Figure 5. The tracking errors in the case of an actuator fault in agents 1 and 3: without adaptive fault-tolerant controllers

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