A reliability-based prognostics framework for railway track management

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ABSTRACT

Railway track geometry deterioration due to traffic loading is a complex problem with important implications in cost and safety. Without appropriate maintenance, track deterioration can lead to severe speed restrictions or disruptions, and in extreme cases, to train derailment. This paper proposes a physics-based reliability-based prognostics framework as a paradigm shift to approach the problem of railway track management. As key contribution, a geo-mechanical elasto-plastic model for cyclic ballast settlement is adopted and embedded into a particle filtering algorithm for sequential state estimation and RUL prediction. The suitability of the proposed methodology is investigated and discussed through a case study using published data taken from a laboratory simulation of train loading and tamping on ballast carried out at the University of Nottingham (UK).

1. INTRODUCTION

Given the rampant demand of use of the railways in many developed countries, there is an increasing need to better understand the long-term degradation of railway track systems. For ballasted tracks, which represent the vast majority of the railway network world-wide, geometry degradation represents the main ageing factor requiring periodic interventions to restore the initial geometry of the track, like *tamping* or *ballast blowing* (Selig & Waters, 1994). These interventions not only represent a significant part of the railway operation expenses, but also imply temporal line closures and a reduction of the effective network capacity. As a result, track geometry maintenance typically needs to be planned with months in advance. It is in this context of anticipated maintenance where the benefits of prognostics can be fully exploited for improved and more cost-efficient maintenance decisions and optimum asset availability.

However, to confer the required predictability of the prognostics solution, a suitable model for long-term track asset degradation is needed, taking into consideration the current and future mechanical, loading and operational conditions. Track geometry degradation is a complex process driven by several mechanisms, like ballast settlement, fatigue of sleepers, and rail wear, among others. The settlement of the granular layers, which is a consequence of the frictional sliding and compaction between aggregate particles and the breakage of them caused by fatigue, is known to be the main controlling factor for track degradation (Selig & Waters, 1994; Suiker & Borst, 2003). Several models for track settlement have been proposed in the literature over the last decades. Section 2.1 provides a selective overview of the main models in the literature, which are categorized into stochastic, phenomenological, and physics-based formulations. A more systematic review can be found in (Dahlberg, 2001), and more recently in (Soleimanmeigouni, Ahmadi, & Kumar, 2016).

As evident from the literature, track degradation modelling and maintenance to date has a strong empirical retrospective character, mainly based on data-based (stochastic or phenomenological) models. The prediction accuracy of these models depends on the quality and quantity of the available historic data, and thus they are prone to misjudgments specially under uncertain (future) loading and operating scenarios. In contrast, physics-based models are grounded on the first principles about track degradation, they are transparent to geo-mechanical input variables, and require much less training data to provide accurate predictions.

In this paper, a physics-based filtering-based prognostics framework for track geometry degradation is proposed. Instead of making maintenance decisions based on a retrospective historical analysis of the track, a prospective approach that fuses historical data and a geo-mechanical model for track deterioration within a state-of-art prognostics algorithm is proposed in this research. In particular, the elasto-plastic model by Indraratna, Thakur, Vinod, and Salim (2012) for cyclic ballast densification is adopted and embedded into a particle filtering algorithm for sequential state estimation and future state prediction. Then, probability-based estimations about the remaining time to reach a predefined functional limit for track settlement are subsequently obtained using the reliability-based prognostics methodology by (Chiachío, Chiachío, Sankararaman, Saxena, & Goebel, 2015). This physics-based reliability-based prognostics methodology in application to the railway track asset is precisely the key contribution of this paper, in relation to a similar contribution from the literature (Mishra, Odelius, Thaduri, Nissen, & Rantatalo, 2017). As a case study, the proposed methodology is tested against experimental data taken from Aursudkij, McDowell, and Collop (2009) about permanent axial strain in a ballasted railway track carried out at the Nottingham Railway Test Facility (Brown, Brodrick, Thom, & McDowell, 2007). Results are presented in Section 4, and discussed in Section 5.

2. Railway track degradation modelling

Several families of models for track settlement have been proposed in the literature. This section provides an overview of the main models in the literature, classified into stochastic, phenomenological, and physics-based formulations. Among them, a physics-based model is selected for the proposed filtering-based prognostics approach presented in Section 3. The key formulation of this model is given in Section 2.2.

2.1. Overview of track settlement models

2.1.1. Stochastic models

Hamid and Gross (1981) proposed an empirical approach to investigate the statistical dependencies between several track geometric defects. In addition, they developed empirical degradation formulations based on autoregressive mathematical models to relate an artificially defined *track quality index* (TQI) with physical measurements, as follows:

$$Y = a_0 + bY' + \sum_{i=1}^{N_m} a_i X_i \tag{1}$$

where Y' is the previous TQI index, $X_i,\ i = 0, \ldots, N_m$ are the measurements, and $a_i,\ i = 0, \ldots, n$ and b are fitting parameters. Bing and Gross (1983) predicted how the *track quality index* defined in Eq. (1) changes as a function of causal parameters such as traffic, track type, maintenance, etc. Shafahi and Hakhamaneshi (2009) represented the variation of TQI as a Markov model, establishing a mapping between TQI and a finite number of states for the Markov chain. A similar approach has been recently proposed by Bai, Liu, Sun, Wang, and Xu (2015).

Hamid and Gross (1981) used time-series analysis techniques to statistically represent track-geometry variations including random waviness, periodic behaviour at joints, and isolated variations. In fact, they represented some track geometry defects as a stationary random process, and other as periodic processes. Iyengar and Jaiswal (1995) proposed a Gaussian random field to represent the vertical irregularity of the track based on data from Indian railways. Vale and Lurdes (2013) used time-series degradation data from the Portuguese high-speed line Lisboa-Porto and fitted them to a probabilistic model (Dagum distribution), taking the standard deviation of the longitudinal level as degradation parameter.

In addition to track degradation, several authors have attempted to represent the interaction between track degradation and maintenance strategy using stochastic degradation models. Meier-Hirmer, Riboulet, Sourget, and Roussignol (2009) proposed a Gamma process to represent the track deterioration within an optimization framework including an intervention efficiency model and a cost model for various maintenance actions. The model was applied to track maintenance data from the French National Railways. Quiroga and Schnieder (2012) proposed a stochastic degradation-restoration model, representing the degradation as a track quality measure that evolves following an exponential function. They used the Monte Carlo method to investigate cost-effective maintenance strategies. Prescott and Andrews (2015) proposed a Markov model to investigate the evolution of track degradation for a given asset management strategy. The model represents not only the deterioration of the asset, but also its dependence on the maintenance history. Authors, however, acknowledged practical limitations of the Markov modelling approach in real life scenarios which prevents its further development for track asset management modelling (Andrews, 2012; Andrews, Prescott, & De Rozières, 2014). Notwithstanding, track degradation models embedded within the Petri net paradigm (Andrews, 2012; Prescott & Andrews, 2013; Andrews et al., 2014) have emerged as a suitable and promising modeling approach, since it allows to efficiently incorporate complex deterioration processes and their dependence on the maintenance history together with several intervention options for inspection, repair, and renewal.

2.1.2. Phenomenological models

Sato (1995) suggested that the settlement y of the track can be calculated as a function of the number of loading cycles n as

$$y(n) = \gamma\left(1 - \exp(-\alpha n)\right) + \beta n \tag{2}$$

where γ, α, and β are fitting parameters. The first term in Eq. (2) is intended to describe the settlement of the track immediately after tamping, while the second captures the long-term behaviour, which is hypothesized to evolve linearly with the number of cycles. Alva-Hurtado and Selig (1981) proposed a logarithmic model for track settlement based on the permanent strain ϵ^p after n loading cycles, as follows:

$$\epsilon^p(n) = (1 + C \log n)\, \epsilon^p(1) \tag{3}$$

where $\epsilon^p(1)$ is the permanent vertical strain after the first load cycle, and C is a fitting constant. This model originally assumes that the loading cycles are identical. Hettler (1984) suggested also a logarithmic model for track settlement given by:

$$y(n) = y_1(1 + c \ln n) \tag{4}$$

where c is a fitting parameter taking values within the interval $[0.25, 0.55]$, and y_1 is the initial settlement after the first cycle, which is calculated as $y_1 = sA_P^{1.6}$, with A_P the amplitude of the load and s a scaling factor. A similar logarithmic function was presented by Indraratna, Salim, Christie, et al. (2002) to model the plastic settlement of ballast stabilised with geosynthetics, as:

$$y(n) = a + b \log n \tag{5}$$

where a and b are empirical parameters.

Based on laboratory and fields experiments, Shenton (1984) discussed the validity of the logarithmic models explained above for large values of load cycles N and proposed a settlement equation of the form:

$$y(n) = k_1 n^{0.2} + k_2 n \tag{6}$$

where k_1 and k_2 are fitting parameters which implicitly depend on geometrical and mechanical inputs such as the axle loads, track and subgrade stiffness, rail section, among others. Similarly, Chrismer and Selig (1993) proposed a power function for ballast strain of the form:

$$\epsilon^p(n) = \epsilon^p(1) n^b \tag{7}$$

where $\epsilon^p(1)$ is the plastic strain after the first cycle and b is a fitting parameter. They concluded that the power equation represents track settlement better than logarithmic models.

A common limitation of the formulations given by Eqs. (2) to (7) is that they do not explicitly account for the effect of the magnitude of loads in the track degradation, only accounting for the number n of loading cycles. In this sense, Ford (1995) investigated the effects of varying loads in the track settlement and suggested how the model in Eq. (3) can be extended to account for different amplitude loading cycles. In the same context, the ORE (Office for Research and Experiments of the International Union of Railways) proposed a phenomenological model to predict the track settlement given by (Hecke, 1998):

$$y = y_0 + hT^\alpha P^\beta v^\gamma \tag{8}$$

where h is a constant, T is the traffic volume, P is the dynamic axle load, v is the speed, and α, β, γ are fitting parameters. As evident from Eq. (8), ORE's model explicitly accounts for loading conditions, however there are no track parameters (e.g., geo-mechanical parameters) involved in this model, which bounds its applicability to sections with the same geo-mechanical configuration than those covered by the historic data.

2.1.3. Physics-based models

Going more in depth about the causes of track degradation, ballast densification caused by particle rearrangement produced by cyclic loading immediately after tamping, and inelastic behaviour of the ballast (due to micro-slip between particles) at long-term loading, are key elements in track settlement. Over the last years several authors are attempting to capture these mechanisms using physics-based models in order to achieve more rational and accurate long-term predictions. For example, Shi (2009) presented a Finite Element Model (FEM) to predict the vertical settlement of the ballast beneath a sleeper under monotonic loading. This model was calibrated and validated using experimental measurements using the Nottingham Railway Test Facility (Brown et al., 2007). A limitation of this model is that it does not account for cyclic loadings, only for monotonic loads. Lim and McDowell (2005) explored the suitability of the Discrete Element Model (DEM) to investigate the mechanisms of degradation underlying the settlement of the ballast for both monotonic and cycling loads. The method was shown to improve the limitations of the physical tests to accurately monitor the ballast response at the particle level and the mechanisms underlying the degradation of the ballast. However, for a real engineering application, the number of equations to be solved becomes considerably large for each model evaluation. This problem is extremely exacerbated in the context of a particle-filter prognostics framework like the one proposed herein, where thousands of model evaluations are usually required at every load cycle.

In this context, analytical or semi-analytical methods for track settlement like the those proposed by Suiker and Borst (2003) and Indraratna et al. (2012) represents an advanced step for the suitability of the physics-based approaches in the context of prognostics. Suiker and Borst (2003) developed an elasto-plastic model to simulate the cyclic accumulation of the permanent strains in the granular substructure (ballast and sub-ballast) using the classical plasticity theory as point of departure. The plastic flow rule was explicitly decomposed into a *frictional sliding* component and a *volumetric compaction* component, leading to a governing equation of the form:

$$\frac{d\epsilon_{ij}^p}{dn} = \frac{d\epsilon_s^p}{dn} m_{ij}^f + \frac{d\epsilon_v^p}{dn} m_{ij}^c \tag{9}$$

where ϵ_{ij}^p is the plastic strain tensor, ϵ_v^p and ϵ_s^p are the volumetric and deviatoric plastic strains, and m_{ij}^f and m_{ij}^c are the plastic flow directions for frictional sliding and volumetric compaction, respectively. In addition, the authors propose a method to numerically integrate the differential equation in Eq. (9).

In Indraratna et al. (2012), authors proposed an elasto-plastic model based on Pender's postulates about plastic deformation of overconsolidated soils (Pender, 1978), and on critical state soil mechanics (Roscoe, Schofield, & Wroth, 1958). This model is rigorously grounded in the geo-mechanical nature of the railway track, and it is the one selected in this research. Further details are provided in the next section.

2.2. Adopted track settlement model

As stated in the last section, an elasto-plastic geo-mechanical model is adopted herein to represent the evolution of the permanent deformation of ballast with cyclic loading (Indraratna et al., 2012). To avoid literature repetition, the reader is referred to Indraratna et al. (2012) for the particular details of the model, however, the key formulation is reproduced here for the sake of clarity and better readability. In essence, the adopted model predicts the cyclic accumulation of permanent deformations in the granular substructure as a function of the applied stress invariants p and q, along with some input parameters (refer to Appendix), as:

$$\frac{d\epsilon_v^p}{d\epsilon_s^p} = \frac{9(M - \eta p/p_{cs})}{9 + 3M - 2\eta M p/p_{cs}} \tag{10a}$$

$$\frac{d\epsilon_s^p}{d\eta} = \frac{2\phi\kappa\,(1 - p_{0,i}/p_{cs,i})\,(p/p_{cs})}{M^2(1+e_0)\,(2p_0/p - 1)} \frac{d\epsilon_s^p}{d\epsilon_v^p}\eta \tag{10b}$$

where ϵ_v^p and ϵ_s^p are the plastic volumetric and deviatoric deformation, respectively, and $\eta = q/p$ is the applied stress ratio[1]. The rest of input parameters are defined in the Nomenclature Section and in the Appendix. It should be noted that the terms representing the contribution of ballast breakage in the plastic deformation of ballast proposed in Indraratna et al. (2012) have been neglected here. The reason is twofold: (1) there is empirical evidence supporting that, for the expected range of confining pressures and applied stresses in a real track, the influence of ballast breakage on track settlement becomes negligible (Suiker, Selig, & Frenkel, 2005), and (2), if any, this contribution would be subsumed within the modelling and measurement errors in the proposed state-space model in Section 3.1.

For each loading cycle, the differential constitutive equations in Eq. (10) can be numerically integrated by finite differences, as following:

$$\epsilon_s^p(n) = \epsilon_s^p(n-1) + \sum_{j=1}^{N_s} \left.\frac{d\epsilon_s^p}{d\eta}\right|_j \Delta\eta_j \tag{11a}$$

$$\epsilon_v^p(n) = \epsilon_v^p(n-1) + \sum_{j=1}^{N_s} \left.\frac{d\epsilon_v^p}{d\epsilon_s^p}\right|_j \left.\frac{d\epsilon_s^p}{d\eta}\right|_j \Delta\eta_j \tag{11b}$$

where $\epsilon_s^p(n)$ and $\epsilon_v^p(n)$ are the remanent deviatoric and volumetric strains after cycle n, respectively, and N_s the number of discretizing steps for the loading ramp within the cycle (the unloading ramp is regarded as elastic). The vertical plastic deformation, which is the management variable of interest, is straightforwardly obtained as a function of the component plastic strains in Eq. (11), as:

$$\epsilon_1^p(n) = \epsilon_s^p(n) + \frac{1}{3}\epsilon_v^p(n) \tag{12}$$

3. Track degradation prognostics

Prognostics deals with the estimation of the remaining time of a system to reach a functional limit based on (uncertain) knowledge about the current damage state and an estimation of the future degradation process of the system. This time is typically referred to as the *remaining useful life* (RUL) by the Prognostics and Health Management (PHM) community. A complete prognostics solution requires a model for system degradation (Section 2.2), a quantifiable criterion for what constitutes an unacceptable degradation or *failure*, and an algorithm to propagate the model forward in time until the degradation reaches a predefined degradation threshold, while quantifying the underlying uncertainty. In the following sections, details are provided to define the constitutive elements for prognostics in the context of the railway track geometry degradation problem investigated herein.

3.1. Stochastic embedding of physics-based model

Let assume that our physical system can be represented by a state-space I/O model, as follows (Arulampalam, Maskell, Gordon, & Clapp, 2002):

$$x_n = g(x_{n-1}, \boldsymbol{\theta}_n) + v_n \tag{13a}$$

$$d_n = h(x_n) + w_n \tag{13b}$$

where $g(x_{n-1}, \boldsymbol{\theta}_n) : \mathbb{R}^{n_x} \times \mathbb{R}^{n_\theta} \to \mathbb{R}^{n_x}$ is the *state transition equation* given by Eq. (11), $h(x_n) : \mathbb{R}^{n_x} \to \mathbb{R}$ is the *observation equation* given by Eq. (12), and $\boldsymbol{\theta} \in \mathbb{R}^{n_\theta}$ is a model parameter vector. It follows that the state (degradation) variable at time or cycle n is given by $x_n = (\epsilon_s^p(n), \epsilon_v^p(n)) \in \mathbb{R}^2$, whereas the measurement variable is denoted by $d_n = \epsilon_1^p(n) \in \mathbb{R}$. In Eq. (13), $v_n \in \mathbb{R}^2$ and $w_n \in \mathbb{R}$ represent the modelling error and measurement noise, respectively, which, following the Principle of Maximum Information Entropy (Jaynes, 1957a, 1957b) are conservatively mod-

[1]For the ease of notation, p is adopted instead of the most commonly used p' to denote the mean stress invariant in drained conditions.

eled[2] as zero mean Gaussians, i.e.:, $v \sim \mathcal{N}(0, \sigma_v)$, $w \sim \mathcal{N}(0, \sigma_w)$. Thus, the dynamical model defined in Eq. (13) in state-space form can be probabilistically rewritten as:

$$p(x_n|x_{n-1}, \boldsymbol{\theta}) = \mathcal{N}(g(x_{n-1}, \boldsymbol{\theta}_n), \sigma_v) \tag{14a}$$

$$p(d_n|x_n) = \mathcal{N}(h(x_n), \sigma_w) \tag{14b}$$

The probabilities in Eq. (14) constitute the key elements in the proposed filtering based prognostics approach for track settlement, as shown below.

3.2. Filtering-based state estimation

Two main steps are required for prognostics, first, an estimation of the current degradation state of the system given a sequence of measurements (e.g., track settlement), and second, a propagation forward in time of the updated state estimate (in absence of new data) until a degradation threshold is reached. A methodology for these two constitutive steps for prognostics using *particle filters* is summarized in Sections 3.2.1 and 3.2.2.

3.2.1. Sequential state estimation

Sequential state estimation aims at recursively obtaining an updated estimate of the actual state of the system as long as new measurements (e.g., track settlement) are collected. Thus, given a sequence of measurements up to time or cycle n, $d_{1:n} = \{d_1, \ldots, d_n\}$, where d_i denotes a track settlement measurement at load cycle i, the goal is to estimate the *updated* probability density function (PDF) of the state of the system at current time n. This is given by Bayes' Theorem as follows:[3]

$$\begin{aligned} p(x_{0:n}|d_{1:n}) &= \frac{p(d_n|x_n)p(x_{0:n}|d_{1:n-1})}{\int_{\mathcal{X}} p(d_n|x_n)p(x_{0:n}|d_{1:n-1})dx_{0:n}} \\ &\propto \underbrace{p(d_n|x_n)}_{\text{Eq. (14b)}}\underbrace{p(x_n|x_{n-1})}_{\text{Eq. (14a)}}p(x_{0:n-1}|d_{1:n-1}) \end{aligned} \tag{15}$$

where $p(x_{0:n-1}|d_{1:n-1})$ is the last system update at cycle $n-1$. In Eq. (15), it is assumed that the system model is Markovian of order one, and that the observations are conditionally independents of the state (Chiachío, Chiachío, Sankararaman, et al., 2015). Note that Eq. (15) is analytically intractable except some especial cases using linear models and Gaussian uncertainties. A followed solution for the general case of both nonlinear and non-Gaussian state-space models is by the adoption of *particle methods* like *particle filters* (PF) (Gordon, Salmond, & Smith, 1993; Arulampalam et al., 2002) to obtain an approximation for the required posterior PDF by means of a set of K samples or *particles* with associated weights $\{\omega_n^{(i)}\}_{i=1}^K$, as follows:

$$p(x_{0:n}|d_{1:n}) \approx \sum_{i=1}^{K} \omega_n^{(i)} \delta(x_{0:n} - x_{0:n}^{(i)}) \tag{16}$$

where δ is the Dirac delta. Given that the posterior density is seldom known exactly, it is not possible to obtain samples from it directly. For this reason, a sequential importance sampling (SIS) approach is adopted whereby samples are obtained from an *importance density* $q(x_{0:n}|d_{1:n})$ (Arulampalam et al., 2002). To compensate for the difference between the importance density and the true posterior density, the unnormalized weights are computed as follows:

$$\hat{\omega}_n^{(i)} = \frac{p(x_{0:n}|d_{1:n})}{q(x_{0:n}|d_{1:n})} \tag{17}$$

whereby $\omega_n^{(i)} = \frac{\hat{\omega}_n^{(i)}}{\sum_{i=1}^K \hat{\omega}_n^{(i)}}$, $i = 1, \ldots, K$. Then, by substituting Eq. (15) into Eq. (17), and by assuming $q(x_n|x_{n-1}) = p(x_n|x_{n-1})$ (Gordon et al., 1993; Tanizaki & Mariano, 1998), the unnormalized importance weight for the i-th particle at cycle n rewrites as:

$$\hat{\omega}_n^{(i)} \propto \omega_{n-1}^{(i)} p(d_n|x_n^{(i)}) \tag{18}$$

The sampling importance resampling (SIR) algorithm with systematic resampling (Arulampalam et al., 2002) is adopted to obtain samples from Eq. (18). A pseudocode implementation for the SIR algorithm is provided as Algorithm 1.

Algorithm 1 SIR algorithm

At $n = 0$
Sample $\left\{x_0^{(i)}\right\}_{i=1}^K$ from prior PDF $p(x_0|\boldsymbol{\theta})$
Assign the initial weights: $\{\omega_0^{(i)} = 1/K\}_{i=1}^K$
At $n \geqslant 1$
for $i = 1 \to K$ **do**
    Sample from Eq. (14a): $x_n^{(i)} \sim p(\cdot|x_{n-1}^{(i)})$
    Update weights: $\hat{\omega}_n^{(i)} \propto \omega_{n-1}^{(i)} p(d_n|x_n^{(i)})$ (Eq. (18))
end for
Normalize: $\left\{\omega_n^{(i)}\right\}_{i=1}^K \leftarrow \left\{\hat{\omega}_n^{(i)}\right\}_{i=1}^K / \sum_{k=1}^K \hat{\omega}_n^{(k)}$
Resample: $\left\{x_n^{(i)}\right\}_{i=1}^K \leftarrow \left\{x_n^{(i)}, \omega_n^{(i)}\right\}_{i=1}^K$

3.2.2. Future prediction

Having estimated the updated state of the system at the time of prediction n, the next step for prognostics is to estimate the probability distribution of the future states of the system ℓ-steps forward in time in absence of new measurements. Using the up-to-date information of the system in Eq. (15) along the physics-based system dynamics, encapsulated in Eq. (14a), an estimation of the ℓ-step ahead state of the system can be

[2] The maximum-entropy PDF for the error terms is the one that produces the most prediction uncertainty (largest Shannon entropy).

[3] The conditioning on $\boldsymbol{\theta}$ has been dropped for simpler notation.

obtained by Total Probability Theorem, as:

$$p(x_{n+\ell}|d_{1:n}) = \int_{\mathcal{X}} \left[\prod_{t=n+1}^{n+\ell} p(x_t|x_{t-1})\right] p(x_n|d_{1:n}) dx_{n:n+\ell-1} \tag{19}$$

Replacing $p(x_n|d_{1:n})$ in Eq. (19) by its PF approximation, a particle estimation of the predictive PDF $p(x_{n+\ell}|d_{1:n}, \boldsymbol{\theta})$ can be obtained by conditional sampling (Chiachío, Chiachío, Sankararaman, et al., 2015), as:

$$p(x_{n+\ell}|d_{1:n}) \approx \sum_{i=1}^{N} \omega_n^{(i)} \delta(x_{n+\ell} - x_{n+\ell}^{(i)}) \tag{20}$$

where $x_{n+\ell}^{(i)} \in x_{n+1:n+\ell}^{(i)}$, $i = 1, \ldots, K$, being $x_{n+1:n+\ell}^{(i)}$=$\{x_{n+1}^{(i)}, x_{n+2}^{(i)}, \ldots, x_{n+\ell}^{(i)}\}$ a conditional sample sequence from each of the K multidimensional integrals in Eq. (19).

3.3. Reliability based prognostics

The reliability-based prognostics methodology proposed by Chiachío, Chiachío, Sankararaman, et al. (2015) is adopted herein to predict the remaining useful life based on sequentially updated predictions of the time-dependent reliability of the system. Although the key formulation is summarized here, the interested reader is referred to (Chiachío, Chiachío, Sankararaman, et al., 2015) for the particular details and the foundation of such methodology.

As a first step, let us define the *useful domain* as the non empty subset $\mathcal{U} \subset \mathcal{X}$ of "authorized" degradation states of the system, and the failure domain $\bar{\mathcal{U}} = \mathcal{X} \setminus \mathcal{U}$, the subset of states where the system turns to behave unacceptably, or simply, where system failure occurs. Then, the time-dependent reliability can be defined as the probability of the system to belong to the useful domain $\mathcal{U}$ at general time $n+\ell$. Using the most up-to-date information about the system states at cycle n, this reliability is shown to be given by:

$$R_{n+\ell|n} \approx \sum_{i=1}^{K} \omega_n^{(i)} \mathbb{I}_{(\mathcal{U})}(x_{n+\ell}^{(i)}) \tag{21}$$

where $\omega_n^{(i)}$ is the updated weights of the state particles at current time or cycle n, $x_{n+\ell}^{(i)}$ is a sample particle of the predicted state at cycle $n+\ell$ (following Eq. (20)), and $\mathbb{I}_{(\mathcal{U})}$ is a function defined such that:

$$\mathbb{I}_{(\mathcal{U})}(x_n) = \begin{cases} 1, & \text{if } x_n \in \mathcal{U} \\ 0, & \text{if } x_n \in \bar{\mathcal{U}} \end{cases} \tag{22}$$

Then, the interest is in predicting the minimum time ℓ when the system enters within the failure domain, i.e.:

$$RUL_n = \inf\{\ell \in \mathbb{N} : x_{n+\ell} \in \bar{\mathcal{U}}\} \tag{23}$$

Table 1. Parameters adopted in calculations

Γ	M	λ_{cs}	κ	e_0	α	β
2.99	1.9	0.194	0.007	0.81	4.06	−0.412

Using the axioms of Probability Logic (Jaynes, 2003), Chiachío, Chiachío, Sankararaman, et al. (2015) demonstrated that the events $\left[\mathbf{z}_{n+\ell} \in \bar{\mathcal{U}}\right]$ and $[RUL_n \leqslant \ell]$ occur with the same probability. Thus, the probability of RUL is shown to be given by:

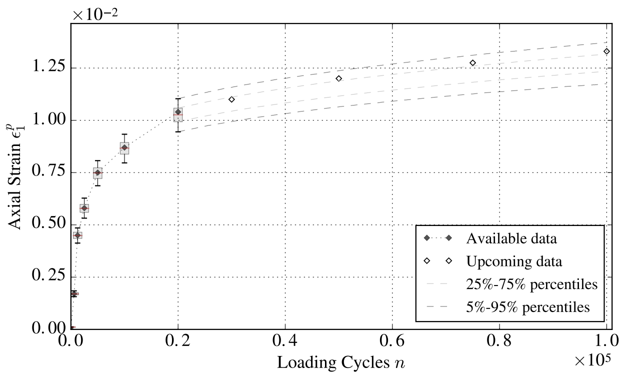
$$P\big(RUL_n \leqslant \ell|d_{1:n}\big) \approx 1 - \sum_{i=1}^{K} \omega_n^{(i)} \mathbb{I}_{(\mathcal{U})}(x_{n+\ell}^{(i)}) \tag{24}$$

Observe that it is possible to compute the entire probability distribution of RUL_n by evaluating Eq. (24) for different values of $\ell \geqslant 1$, until the value $R_{n+\ell|n} = 0$ is reached. The calculation of the time-dependent reliability can be updated each time n new data are collected.

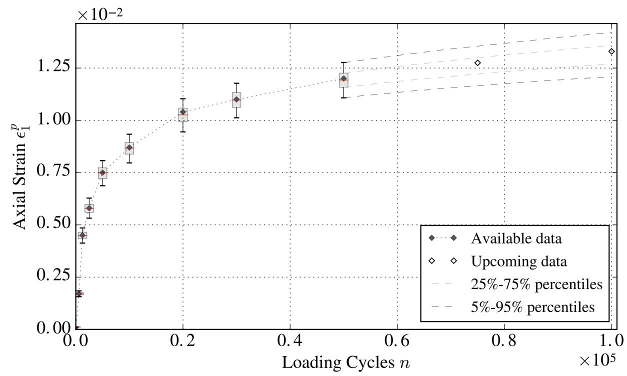
4. Case study

The reliability based prognostics methodology explained before is exemplified here using data about permanent axial strain in a ballasted railway track taken from the the literature (Aursudkij et al., 2009). The test, as reported in Aursudkij et al. (2009), was conducted on the Railway Test Facility of the University of Nottingham (Brown et al., 2007), and simulates an axle load of approximately 20 tonnes. According to (Aursudkij et al., 2009), the maximum applied vertical stress in each cycle is $\sigma_{1,max} = 210$ [KPa], and the confining pressure $\sigma_3 = 30$ [KPa], leading to a maximum stress ratio $(q/p)_{max} = 2$ according to Eq. (27). After some initial fitting tests, the material parameters describing the governing model in Eq. (11) were set to the values specified in Table 1.

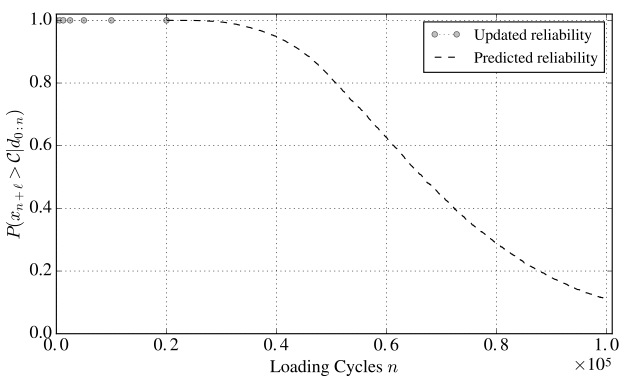
The results for sequential state estimation and reliability prediction are presented in Fig. 1. Every time new data is available, the state variable $x_n = (\epsilon_s^p(n), \epsilon_v^p(n))$ is updated using a SIR algorithm (refer to Algorithm 1) with $K = 10^4$ particles. This information is further used to propagate the model forward in time to compute the RUL using the methodology described in Section 3.3. Observe in figure 1 that the prediction gradually improves as more data are available. For this example, the useful domain is conveniently defined as $\mathcal{U} = \{(\epsilon_1^{(p)} \in [0, 0.012]\} \subset \mathbb{R}^2$. The predictions of RUL are plotted against time in Fig. 2.



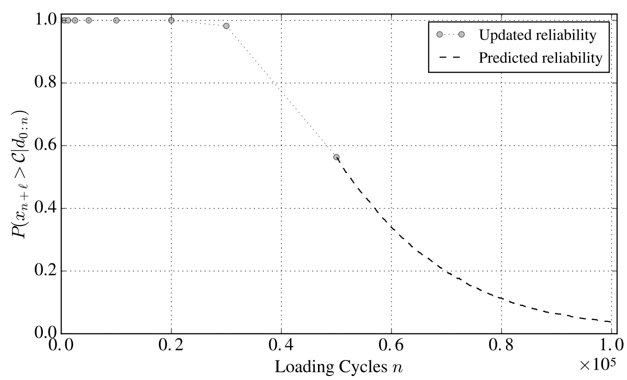
(a) Sequential updating and prediction at $n = 20000$



(b) Sequential updating and prediction at $n = 20000$



(c) Reliability updating and prediction at $n = 20000$



(d) Reliability updating and prediction at $n = 20000$

Figure 1. Sequential state estimation and prediction at different loading cycles

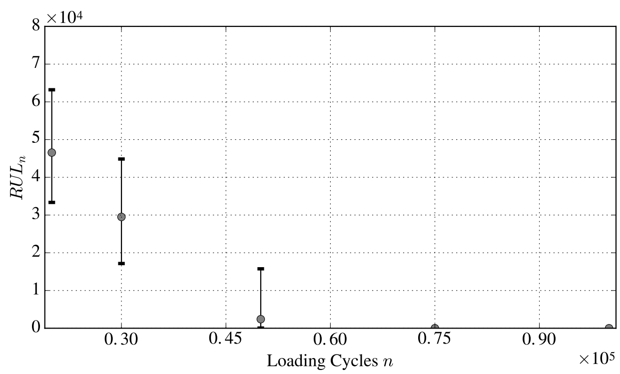


Figure 2. Particle-filter estimation of RUL at different cycles

5. Discussion and conclussions

The degradation of the railway track geometry results into a key asset management problem with important implications in safety and cost. This degradation is driven by a complex geo-mechanical phenomenon, and has been approached by different modelling perspectives over the last decades. Three families of models seem to come into view in the literature: stochastic models, (2) phenomenological models, and (3) physics-based models, of which a representation is summarized in Sections 2.1.1 to 2.1.3, respectively. A common feature of both stochastic and phenomenological models is that they are almost purely based on historic data. It implies that the prediction accuracy of these models would ultimately depend on the quality and quantity of the available data set. In addition, these models are usually ”blind” to most key inputs variables (load amplitude, track geo-mechanical properties, etc.), thus they can hardly take into consideration medium-to-long future scenarios (as those foreseen by the European Commission in (EC, 2011)), simply because there is no available data about such conditions, or it is very limited. In contrast, physics-based models, like the one adopted in this paper, are transparent to geo-mechanical input variables, and require much less training data to provide accurate predictions. Although a typical criticism of these models is that they are normally based on deterministic input-output relationships, however, they can be *stochastically embedded* (Beck, 2010; Chiachío, Chiachío, Saxena, et al., 2015) using the methodology provided in Section 3.1, and thus converted into full-probabilistic models.

A physics-based filtering-based prognostics framework for track settlement has resulted from this work. As apparent from the results in Fig. 1, the proposed methodology is able to anticipate the future evolution of the permanent axial strain with quantified uncertainty after an initial training stage with limited data. From these predictions, a sequential estimation of the time-dependent reliability is derived, whereby a probability-based estimation of the RUL is obtained following the methodology by (Chiachío, Chiachío, Sankararaman, et al., 2015). This information can be further embedded within a broader maintenance context for risk reduction in go/no-go decision, cost reduction through the scheduling of maintenance as-needed, and improved asset availability, among other asset management activities. A methodology for the integration of prognostics within an asset management framework using Petri Nets is presented by the authors as a separated piece of work in the Proceedings of the Annual Conference of the Prognostics and Health Management Society, 2017.

It should be noted that an apparent limitation of the proposed approach is that it is based on a modelling framework that predicts the plastic strains of the track, instead of other management variables like the Track Geometry Index (Mundrey, 2009), more commonly used by the railway industry. However, these management variables can be represented as functions of the plastic strains of the track over a predefined segment length. Then, these functions can be readily included within the state-space model in Eq. (13), from which the proposed filtering-based prognostics methodology is derived.

Notwithstanding, an important theoretical and computational challenge must be highlighted, which is related to the slow-asymptotic behaviour of the temporal evolution of the permanent axial strain (as observed in Figs. 1a and 1b). In terms of RUL estimation, this means that the predicted particles $x_{n+\ell}^{(i)}$ tend to approach the threshold asymptotically, making the event $\left[x_{n+\ell}^{(i)} \in \bar{\mathcal{U}}\right]$ a rare event, except when $\ell >> n$. This issue takes place precisely when the system is approaching the boundary of the useful domain, which requires faster and more accurate RUL predictions. A first solution is to considerably increase the amount of particles (K in Algorithm 1), but it is at the cost of heavy computation. A more suitable solution is the adoption of a dedicated prognostics algorithm for rare events, like the one recently developed by the authors in (Chiachío, Chiachío, Shankararaman, & Andrews, 2017). This, together with the adoption of management variables and data aligned with the current practice of the railway industry considering several maintenance cycles, constitute desirable further steps of this research.

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NOMENCLATURE

$d\epsilon_v^p$	plastic volumetric strain increment
$d\epsilon_s^p$	plastic distortional strain increment
e	voids ratio
e_0	Initial voids ratio
p	mean stress invariant
p_0	initial mean stress
q	deviatoric stress invariant
q_{max}	in-cycle maximum deviatoric stress
q_{min}	in-cycle minimum deviatoric stress
η	stress-ratio $\eta = {}^{q}/_{p}$
M	critical stress-ratio
Γ	critical state model parameter
λ_{cs}	critical state model parameter
κ	swelling/recompression constant
σ_1	applied vertical stress
σ_3	confining stress
α	empirical factor Eq. (28)
β	empirical factor Eq. (28)

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BIOGRAPHIES

Juan Chiachío is a Research Fellow in Infrastructure Asset Management in the Resilience Engineering Research Group at the University of Nottingham (UK). He received his PhD in Structural Engineering in 2014 (*Summa Cum Laude, International Mention*) by the University of Granada, Spain. In addition, he holds a MSc in Structural Engineering and a MSc in Civil Engineering, both by the University of Granada. His research is focused on translating reliability and prognostics methods into the life-cycle analysis of structural and infrastructural systems subjected in-service degradation. This research has led to several publications in highly ranked journals, a best-paper award and nominations in major conferences, and it has been awarded by the Spanish National Council of Education through one of the FPU annual fellowships, by the Andalusian Society of Promotion of Talent, by the Prognostics and Health Management Society with a Best Paper Award, and by the European Council of Civil Engineers. In addition, his work has attracted the interest of world-class institutions for collaborative research, like the Prognostics Center of Excellence of NASA, the California Institute of Technology, and the Hamburg University of Technology (Germany). His current research at the University of Nottingham deals with the development of a Bayesian prognostics framework for infrastructure asset management, under EPSRC project titled ”Whole-life cost assessment of novel material railway drainage systems (EP/M023028/1)”.

Manuel Chiachío is Postdoctoral Research Fellow at the Resilience Engineering Research Group, University of Nottingham, U.K. He holds a PhD in Structural Mechanics (*Summa Cum Laud, International Mention*) awarded by the University of Granada, (Spain), and a MSc in Civil Engineering (2007), and also a MSc in Structural Engineering (2011), by the same University. His research focuses on uncertainty quantification methods and algorithms, risk and reliability analysis, and artificial intelligence methods in application to a variety of engineering problems, which range from structural and mechanical engineering to bioengineering applications. During the course of his PhD work, Manuel worked as guest scientist at world-class universities and institutions, like Hamburg University of Technology (Germany), California Institute of Technology (Caltech), and NASA Ames Research Center (USA). This research has led to several publications in highly ranked journals. It has been awarded by the National Council of Education of Spain through one of the prestigious FPU fellowships, by the Andalusian Society of promotion of the Talent, by the European Council of Civil Engineers (ECCM) with the Silver Medal prize in the 1st European Contest of Structural Design (2008), and also by the Prognostics and Health Management Society with a Best Paper Award in 2014. Prior to joining the University of Granada in 2011, Manuel worked as structural engineer for four years in top engineering companies in Spain.

Darren Prescott Darren Prescott is Assistant Professor in Risk and Reliability Engineering in the Resilience Engineering Research Group at the University of Nottingham. The focus of his work is the development of models for the assessment of reliability, maintenance and asset management and the impact of each of these on system or network performance. His current research interests include: the development of asset management models for the railway network, with particular focus on track and track drainage; aircraft fleet maintenance modelling; the application of reliability modelling techniques to support decision making in autonomous systems; and the asset management of ageing infrastructure in the nuclear and offshore oil and gas industries. He is the current chair of the ESRA (European Safety and Reliability Association) Technical Committee on Aeronautics and Aerospace.

John Andrews is Head of the Resilience Engineering Research Group at the University of Nottingham where he holds a Royal Academy of Engineering Research Chair in Infrastructure Asset Management. Prior to this he worked for 20 years at Loughborough University where his final post was Professor of Systems Risk and Reliability The prime focus of his research has been on methods for evaluating the system resilience, unavailability, unreliability and risk. Much of this work has concentrated on the Fault Tree analysis technique and the use of Binary Decision Diagrams (BDDs) as an efficient and accurate solution method. Recently attention has turned more the degradation modelling and the effects of maintenance, inspection and renewal on asset performance. In this context the modelling he has carried out has extended the Petri net and Bayesian Network capabilities In 2005, John founded the Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability of which was the Editor-in-chief for 10 years. He is also a member of the Editorial Boards for 6 other international journals in this field .

APPENDIX

For granular materials like ballast and suballast under three-dimensional stresses, the following relationships are used to obtain the stress invariants p and q:

$$p = \frac{1}{3}\sigma_{kk} \tag{25a}$$

$$q = \sqrt{\frac{3}{2} s_{ij} s_{ij}} \tag{25b}$$

where σ_{ij} is the stress tensor, and s_{ij} the stress deviator tensor, defined as

$$s_{ij} = \sigma_{ij} - p\delta_{ij} \tag{26}$$

with δ_{ij} the Kronecker delta function. Under the assumption of axisymmetric stress state ($\sigma_2 = \sigma_3$), the stress invariants simplify to

$$p = \frac{1}{3}(\sigma_1 + 2\sigma_3) \tag{27a}$$

$$q = \|\sigma_1 - \sigma_3\| \tag{27b}$$

In regards to the governing equations, the function ϕ in Eq. (10) is a semi-empirical factor that accounts for complex phenomena observed in the yielding behaviour of granular materials under cyclic loading conditions, such as the Bauschinger effect, the effect of the stress ratio and loading history, among others (Mroz, Norris, & Zienkiewicz, 1978; Salim, 2004). In this research, the expression proposed by Indraratna et al. (2012) is adopted to account for such effects, which is given by:

$$\phi = \alpha \left(1 - \frac{\eta}{M}\frac{p}{p_{cs}}\right)\left(\frac{\langle p - p_e\rangle^2 + \langle q - q_e\rangle^2}{(\Delta p)^2 + (\Delta q)^2}\right)^{1/2} N^{\beta} \tag{28}$$

with $\Delta p, \Delta q$ being the in-cycle total stress increments, $\langle\cdot\rangle$ the Macauley brackets, and α an empirical fitting parameter. The elastic mean stress p_e is given by the expression

$$p_e = p_{min} + \left(1 - \frac{1}{\ln(N + 10)}\right)\Delta p \tag{29}$$

In Eqs. (28) and (10), p_{cs} is the value of p at the critical state, which can be obtained as (Roscoe et al., 1958)

$$p_{cs} = \exp\left(\frac{\Gamma - e}{\lambda_{cs}}\right) \tag{30}$$

where Γ, and λ_{cs} are material parameters, which, together with M and κ (see Nomenclature section), conform the parameters of the model (Indraratna et al., 2012). The rest of elements are defined in the Nomenclature section.

The reader is referred to the Nomenclature section for information of laminate constants involved in the last equations.