Health-Informed Uncertainty Quantifications via Bayesian Filters with Markov Chain Monte Carlo Simulations for Fatigue Critical Rotorcraft Components

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ABSTRACT

This paper presents the applications of Bayesian-filters (BF) with Markov chain Monte Carlo (MCMC) simulations for probabilistic lifing assessment of aircraft fatigue critical components. Uncertainties in damage growth parameters are updated with new information obtained through structural health monitoring (SHM) systems and the remaining useful life (RUL) are predicted. State transition function representing virtual damage growth of a component and measurement function representing the SHM measurements of the component are defined. State transition function is described by a typical Paris equation for fatigue crack propagation. In the equation, the initial crack size and crack growth rate are updated by incoming SHM measurements. Measurement functions are assumed in this study which describe the relationship between the damage features derived from SHM signals and the damage sizes. Damage tolerance (DT) and risk-based remaining useful life of fatigue critical structural components are determined at various reliability levels. The variabilities of RUL are also quantified for various magnitudes of random measurement noise and various measurement frequencies. It is found that the variability of the RUL is proportional to that of the measurement noise. In addition, more frequent measurements will result in less variability in RUL.

1. INTRODUCTION

Army is pursuing next generation fatigue design and maintenance methodologies for rotorcraft fatigue critical components. The goal is to reduce the operating and manufacturing cost as well as vehicle weights for performance improvement while maintaining high reliability during operations. Currently, fatigue critical components are retired at their respective design life (which is also called life limited components) based on traditional safe-life approach. This approach results in a big waste that many retired components still have long and healthy remaining useful life after retirement. However, their respective remaining useful life (RUL) and associated reliability are unknown and non-quantifiable under current design and assessment methodologies. If the remaining useful life of those life limited components can be accurately and reliably determined, then the retirement of those parts could be postponed. As a result, enormous savings can be realized for the stake holders.

Army Research Laboratory is conducting researches to develop a probabilistic approach and framework to determine the critical information for real time risk assessment and risk management regarding aircraft life cycle management of new and existing life-limited components. Using health state information from health monitoring systems, combined with physics-based damage model, a comprehensive life assessment is formulated. This new approach allows regulators to make informed judgments regarding potential extending the current design life of critical rotorcraft components while operating under safe and reliable conditions. It also provides new tools to design and manage fatigue critical components of next generation rotorcraft for low cost, light weight and high reliability.

The assessment requires uncertainty modeling, physics-based modeling, health state quantification, and probabilistic methodologies development. Uncertain design parameters such as material properties, usages, geometries, damage propagation, manufacture-related uncertain parameters, etc., as well as uncertainties related to health state quantification need to be statistically determined. One of the major issues for this research is the technical challenge in applying probabilistic lifing methods to quantify the health state while the component is continuously degraded. In this paper, we use the probabilistic lifing approach based on Bayesian filters

(BF) and MCMC (Markov chain Monte Carlo simulations) for individual component risk tracking.

BF has become a broad topic involving many scientific areas. There exists a number of excellent tutorial papers on Bayesian filters and Monte Carlo filters (Arulampalam, Maskell, Gordon & Clapp, 2002; Baraldi, Compare, Sauco & Zio, 2012; Chen, 2003; Doucet, Godsill & Andrieu, 2000; Doucet, de Freitas, Murphy & Russell, 2000; Tanizaki, 2000). There are also researches for BF applications on fatigue crack/damage growth scenarios (An, Choi & Kim, 2013; He, Bechhoefer, Demsey & Ma, 2012; Orchard & Vachtsevanos, 2009). BF is a probabilistic approach to quantify the uncertain damage state of a component recursively over time using incoming measurements and a mathematical damage progression model. In this paper, we investigate and demonstrate the applications of Bayesian filtering-based lifing approach with Markov chain Monte Carlo (MCMC) for uncertainty updating and lifing prediction. MCMC methods (Khan, Balch & Dellaert, 2004; Renard, Garreta & Lang, 2006) including random walk Monte Carlo are a class of algorithms for sampling based on construction of Markov chain from the target probability distribution for which direct sampling is difficult. This motivates our current research.

The paper is structured as follows. Section 2 discusses the probabilistic lifing methods using Bayesian filters with MCMC as the numerical solver. Section 3 discusses and demonstrates the procedure of generating random samples using MCMC from the unknown posterior joint probability distribution. The random samples generated are inputs to a probabilistic damage tolerance analysis framework to determine the remaining useful life for a specified reliability. Section 4 provides summary of the study.

2. PROBABILISTIC LIFING METHODS USING BAYESIAN FILTERS (BF) AND MARKOV CHAIN MONTE CARLO (MCMC) SIMULATIONS

Bayesian filters are methods to solve Bayesian inference numerically where probability distributions are intractable in closed form. BF with MCMC approximates the probability distributions of unknowns using a large number of samples. As the number of samples increases toward infinity, the approximated probability distributions will converge to actual. Bayesian filtering-based probabilistic lifing methodology requires the information of the evolution of damage signals and damage extent in time to estimate the relationship between features extracted from monitoring signals and damage extent for RUL prediction of a degrading system. Since it is a process to forecast future risks based on current system health states, uncertainty associated with the forecasting needs to be quantified and propagated. This approach describes the system state in terms of discrete probability distribution using a set of simulations representing state values (realizations). Each simulation has an assigned probability representing its probability of occurrence. The general process of BF is based on a state transition function f and a measurement function h shown in Eqs. 1 and 2.

$$a_j = f(a_{j-1}, \theta, \nu_j) \tag{1}$$

and

$$z_j = h(a_j, \omega_j) \tag{2}$$

where j is the time step index, a_j is the damage state, θ is a vector of model parameters, z_j is a SHM feature extracted from health monitoring signals, and ν_j and ω_j are process and measurement Gaussian noises with zero-mean respectively. The state function f is referred to as a damage model. The measurement function h describes the relationship between the collected SHM features and damage states. BF estimates and updates unknown parameters as a form of the probability density function (PDF) based on the Bayes' theorem

$$p(\Theta | z) \propto p(z | \Theta)\ p(\Theta) \tag{3}$$

where Θ is a vector of all unknown parameters to be updated, z is a vector of observed features, $p(z|\Theta)$ is the PDF value of z conditional on the given Θ, $p(\Theta)$ is the prior PDF of Θ. Eqs. 1 to 3 will be used recursively at each time step j to propagate and update various uncertainties by BF. The RUL for a specified reliability can be determined by a general purpose probabilistic methodology considering all uncertainties including those with posterior PDFs.

In our previous study (Shiao, Chen & Ghoshal, 2016), samples are randomly generated by traditional Monte Carlo simulations based on prior probability distributions. Posterior probability distributions are determined via simulations using Eqs. 1 to 3. In this paper, we employ Markov chain Monte Carlo (MCMC) methods to generate random samples from unknown posterior probability distributions. Markov chain Monte Carlo (MCMC) is a family of algorithms used to produce approximate random samples from a probability distribution too difficult to sample directly. The MCMC method used in this paper is Metropolis algorithm for obtaining a sequence of random samples from the posterior joint probability distribution. This algorithm has been widely used in Bayesian applications for its simplicity and efficiency.

The Metropolis algorithm is performed based on the following iterative procedure:

a). At iteration $i=0$, select a vector X_0 (arbitrarily or based on prior knowledge) consisting of realizations of random variables in the study as the initial sample.

b). For each following iteration $i>0$,

1. Generate a candidate vector X' randomly from the joint probability distribution function $g(X'|X_{i-1})$ where

function *g* is referred to as the proposal density function or jumping distribution. In the Metropolis algorithm, *g* is symmetric. A usual choice is to let *g* be a Gaussian distribution with mean = X_{i-1}.

2. Calculate the acceptance ratio $\bar{\alpha} = q_x(X')/q_x(X_{i-1})$. The ratio will be used to decide whether to accept or reject the candidate. $q_x(X')$ is a function proportional to the posterior probability density of model parameters θ.
3. If $\bar{\alpha} \geq 1$, then the candidate is more likely than X_{i-1}; automatically accept the candidate by setting $X_i = X'$.
4. If $\bar{\alpha} < 1$, accept the candidate with probability $\bar{\alpha}$; if the candidate is rejected, set $X_i = X_{i-1}$, instead.

In this paper, we use a Gaussian jump distribution with variance matrix $\Sigma' = C^2\Sigma$ where C is a constant and Σ is the covariance matrix of the samples. Studies have shown that robustness of the Metropolis algorithm is heavily affected by the choice of the constant *C*. If *C* is too large which results in a large jump, most candidate vectors will be far away from the high density area of the posterior distribution. As a result, higher rejection rate will be observed. If *C* is small, almost all the candidates will be close to the precedent, leading to high acceptance rate. It has been found that either *C* is too large or *C* is too small, the samples generated with those *C* values will be highly correlated which is an undesirable outcome for simulations. In the paper, *C* is initially set to be $2.4/\sqrt{D}$ where D is the dimension of the matrix. *C* is then chosen adaptively by keeping the acceptance rate between 0.23 and 0.44. The covariance matrix is assumed to consist of diagonal terms only at the starting point. The variance is then updated at every 1000 iterations.

One of the issues related to Metropolis algorithm is the burn-in period. Although the Markov chain eventually converges to the desired distribution, the initial samples may follow a very different distribution, especially if the starting point is in a region of low density. As a result, a burn-in period is typically necessary, where an initial number of samples are thrown away. In our study, we will investigate the burn-in period and auto- and cross- correlation functions of the random variables a_0 and $log(c)$ by MCMC where a_0 is the initial crack size and *c* is the crack growth rate in the state transition function.

In the following section, we will discuss and demonstrate the procedure of generating random samples using MCMC from the unknown posterior joint probability distribution. The random samples generated are then input to a probabilistic damage tolerance analysis framework to determine the remaining useful life for a specified reliability.

3. CASE STUDY

In order to use Bayesian filter to solve Bayesian inference, a state transition function and a measurement function need to be defined first. In this study, a simple crack growth model is used to define the state transition function as shown in Eq. 4.

$$a_t = a_{t-1} + c\,(\Delta K_{t-1})^m \Delta t,\ \ \Delta K_{t-1} = \Delta\sigma\sqrt{\pi\, a_{t-1}} \qquad (4)$$

where *c* is the random crack growth rate and *m* is a constant. a_0 is the random crack size at time 0. $\Delta\sigma$ is the stress range and ΔK is the range of stress intensity factor. The process noise *v* in state transition function in Eq. 1 is not used since it can be handled through the uncertainty in model parameters.

Next, a general process to define a measurement function is briefly described below. Typically one would establish a relationship between the inspection feature (usually termed $\hat{a}$) and the 'true' measured crack length, a . This relationship is established using linear regression as follows (Department of Defense, 2009; Ihn, Pado, Leonard, Desimio & Olson, 2011; Kabban, Greenwell, Desimio & Derriso, 2015). Let $x = q_1(a)$ and $y = q_2(\hat{a})$ where q_1 and q_2 are either linear or nonlinear functions selected such that *x* and *y* are linearly related. The relationship between the a *and* $\hat{a}$ is then estimated as:

$$y = \alpha + \beta * x + \omega \qquad (5)$$

where α and β are constants and ω is the measurement noise. In our study, without losing generality we let $x = a$ and $y = \hat{a}$ for simplicity. The measurement function in Bayesian filter thus becomes

$$\hat{a}_t = \alpha + \beta * a_t + \omega_t \qquad (6)$$

The initial crack size a_0, crack growth rate c in Eq. 4 and ω_t are random variables for the study. ω_t are normally distributed random variables with 0 mean and standard deviation σ_ω . The unknown parameters Θ in Eq. 3 include the damage state a, model parameters $\theta = (a_0, c)$ and measurement noise ω .

To start the BF process using MCMC, model parameters $\theta_i = (a_{0,i}, c_i)$ are selected arbitrarily or based on prior knowledge for *i=0*. The likelihood of measurement (observation) $\hat{a}_k$ at time *k* of i^{th} iteration with model parameters $\theta_i = (a_{0,i}, c_i)$ is shown in Eq. 7.

$$\begin{aligned} &p(\hat{a}_k \mid a_{k,i}(a_{0,i}, c_i, k)) = \\ &\frac{1}{\sqrt{2\pi\sigma_\omega}} * \exp\left[-\frac{1}{2}\left(\frac{\hat{a}_k - (\alpha + \beta * a_{k,i}(a_{0,i}, c_i, k))}{\sigma_\omega} \right)^2 \right] \\ &where\ \ k = 1, \ldots\ldots, t \end{aligned} \qquad (7)$$

The posterior probability of each realization pair $(a_{0,i}, c_i)$ at time k with a new measurement $\hat{a}_k$ is updated recursively in time using Eqs. 4 and 7 as shown in Eq. 8.

$$p(a_{0,i}, c_i \mid \hat{a}_{1:k}) \propto$$
$$p(\hat{a}_k \mid a_{k,i}(a_{0,i}, c_i, k))\ p(a_{0,i}, c_i \mid \hat{a}_{1:k-1}) \quad 8)$$
$$where\ k = 1,......,t$$

where $\hat{a}_{1:k} = [\hat{a}_1, \hat{a}_2, ...\hat{a}_k]$. By successive substitution of Eq. (8), the updated joint probability density function of $(a_{0,i}, c_i)$ at k=t is shown in Eq. (9).

$$p(a_{0,i}, c_i \mid \hat{a}_{1:t}) \propto \left\{ \prod_{k=1}^{t} p(\hat{a}_k \mid a_{k,i}(a_{0,i}, c_i, k)) \right\} p(a_{0,i}, c_i) \quad (9)$$

The function q_x in Metropolis algorithm is thus approximated by

$$q_x(a_{0,i}, c_i) = \left\{ \prod_{k=1}^{t} p(\hat{a}_k \mid a_{k,i}(a_{0,i}, c_i, k)) \right\} p(a_{0,i}, c_i) \quad (10)$$

Once the joint probability density function of initial crack size a_0 and crack growth rate c is updated using Eq. (10) with BF/MCMC procedure, crack size a_t at any time t after the last SHM measurement at time t' can be determined using Eq. (4). Our next step is to determine the reliability of the component as an event where the stress intensity factor K_t at time t is greater than the fracture toughness K_{Ic} where

$$K_t = \sigma_{\max} \sqrt{\pi a_t} \quad (11)$$

Both K_{Ic} and maximum stress $\sigma_{\max}$ are normally distributed random variables with means and standard deviations defined in Table 1.

Table 1. Mean and Standard Deviation of Fracture Toughness and Maximum Stress

	Mean	Standard Deviation
K_{Ic}	80	12
$\sigma_{\max}$	75	1.5

R_t is determined by Eq. (12).

$$R_t = 1 - POF(K_t > K_{Ic}) \quad (12)$$

POF in Eq. (12) represents the probability of failure (POF) where $K_t > K_{Ic}$. The remaining useful life T for a specified reliability $\overline{R}$ after the last SHM measurement at time t' is thus determined by finding the T in Eq. (13)

$$\overline{R} = 1 - POF(K_{t'+T} > K_{Ic}) \quad (13)$$

In the following, case studies will be performed for a simulated ground truth crack growth representing a virtual component. Various measurement sets representing respective SHM measurements of a component were simulated using two σ_ω (standard deviation of the measurement noise) and two Δt_m (measurement intervals) as shown in Table 2.

Table 2. Measurement Interval and Measurement Noise for Case Studies

Case	1	2	3	4
Measurement Intervals Δt_m	20	20	10	10
Measurement Noise σ_ω	0.0020	0.0010	0.0020	0.0010

Also $\alpha = 0.04$ and $\beta = 0.4$ in measurement function shown in Eq. 6 are assumed throughout the study. At t'=2500 flight cycles, the remaining useful life (RUL) of the virtual component for each case is assessed by a probabilistic damage tolerance analysis (Chen & Shiao, 2015; Shiao, Chen, Wu & Ghoshal, 2016). The ground truth crack growth for the study is generated based on a_0 =1.08E-02 and c=4.31E-10 with m=3.4 as shown in Figure 1.

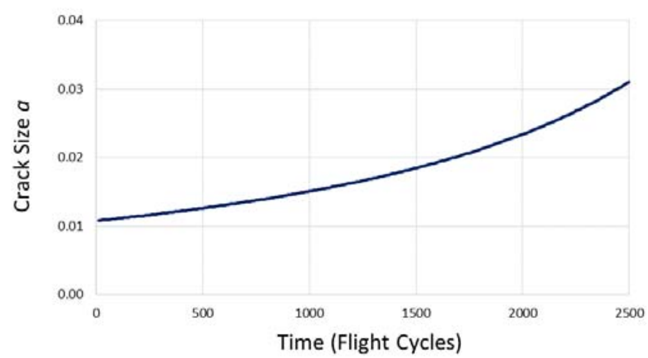

Figure 1. Ground Truth Crack Growth Curve for Case Study

Typical measurement sets simulated by Eq. 6 for four selected cases using the given ground truth are shown in Figure 2. Each measurement set randomly generated for a given case represents a possible virtual measurements of the component for a given σ_ω and Δt_m. In our study, 1000 virtual measurement sets for each case are randomly generated in order to study the effect of σ_ω and Δt_m on the variability of predicted RUL.

MCMC-based Bayesian filter is now applied. The first sample set at i = 0 is arbitrarily selected. In our study, a_0 and c are 1.40E-02 and 7.58E-10 respectively. The samples generated by MCMC for the measurement set (Case 1 in Figure 2) are shown in Figure 3. As shown, the burn-in period is about 2000 samples. 10000 samples from sample 5001 to sample 15000 are then used for further reliability analysis. The auto- and cross correlation function of a_0 and *log(c)* are plotted in Figure 4. In the figure, S_1 represents a_0 and S_2 represents *log(c)*. As noticed in the figure, at lag is about 10, auto and cross- correlations are reduced to 0.

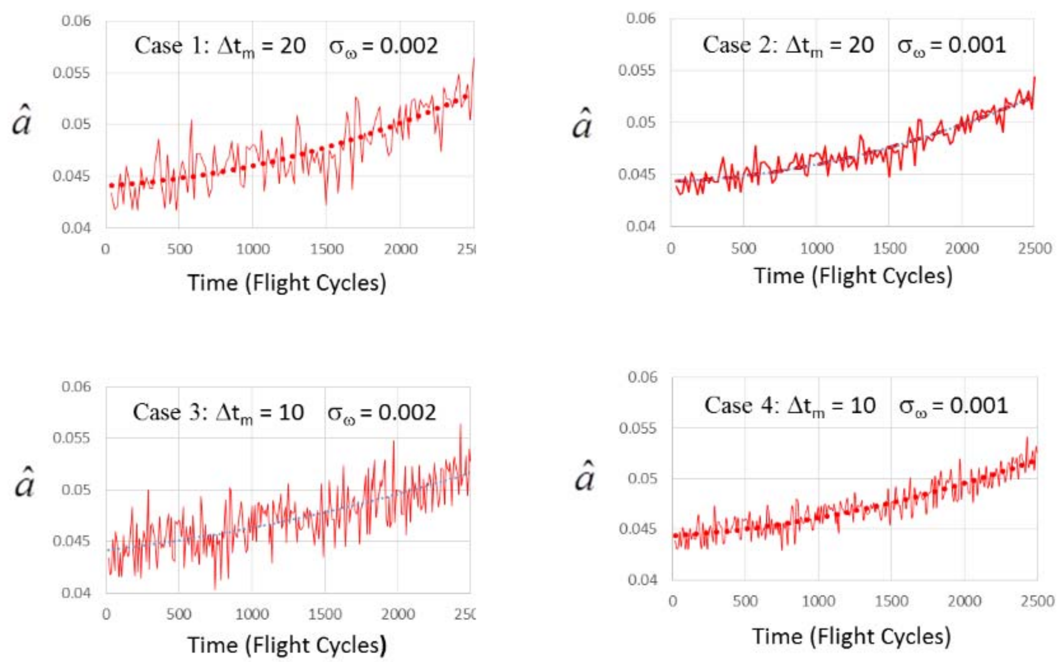

Figure 2. Typical Measurement Sets for Respective Cases

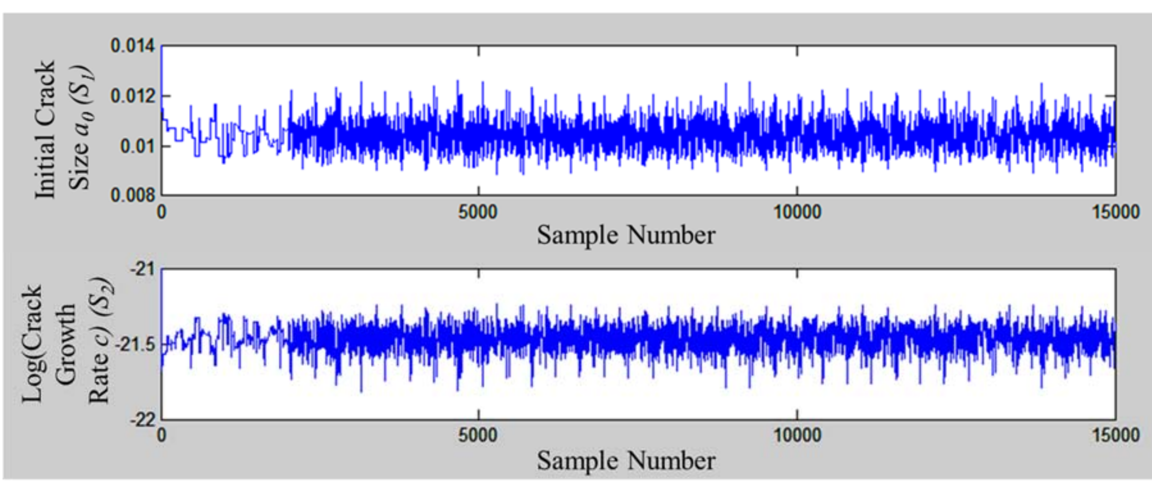

Figure 3. Samples of Crack Size a_0 and Log of Crack Growth Rate c for Measurement Set of Case 1 in Figure 2

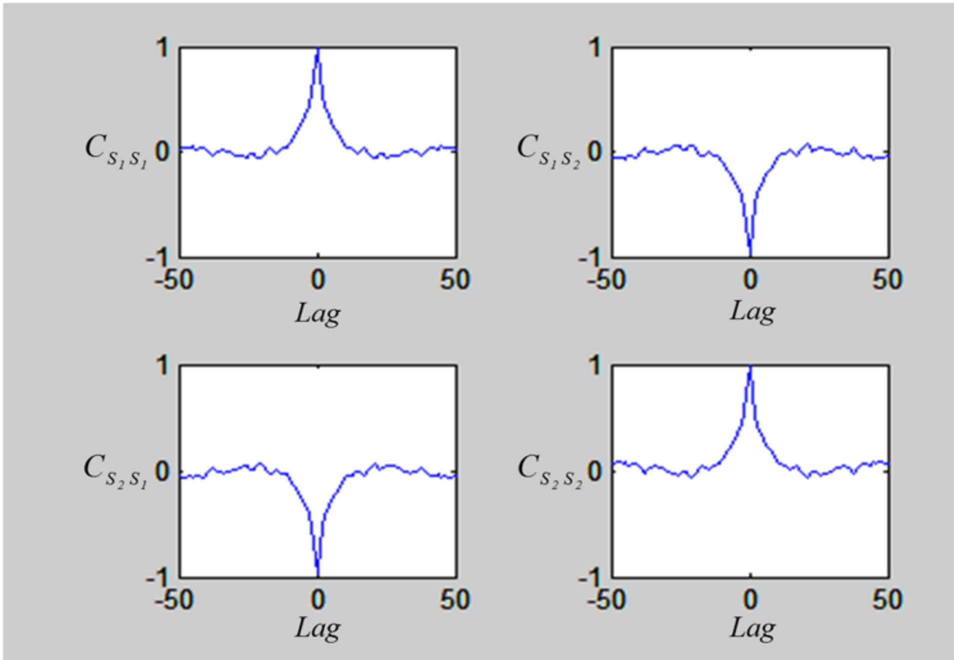

Figure 4. Auto- and Cross- Correlation of a_0 and log(c)

The posterior histogram for Case 1 with measurement set shown in Figure 2 are plotted in Figure 5. As can be seen in the figure, a_0 and *log(c)* are highly correlated. The correlation of a_0 and *log(c)* from the analysis is about -0.95 for all the cases being studied.

For each measurement set, the probability of failure of remaining useful life T is determined using the samples generated from MCMC. The failure occurs when the fracture toughness is greater than the stress intensity factor. The probability of failure (POF) for Case 1 with the measurement set shown in Figure 2 is plotted in Figure 6. In the figure, the RUL with 0.999 and 0.99 reliability are shown to be 760 and 1010 respectively.

The same procedure for each case is repeated 1000 times. The mean of the RUL at 0.999 and 0.99 reliability levels for all 4 cases are shown in Table 2. The table indicates that better RUL prediction can be obtained for the case with smaller measurement noise and more frequent SHM measurements. Coefficient of variation (CV) of RUL are shown in Figure 7. The figure shows that the coefficient of variation of RUL is linearly proportional to the standard deviation (STDV) of the measurement noise.

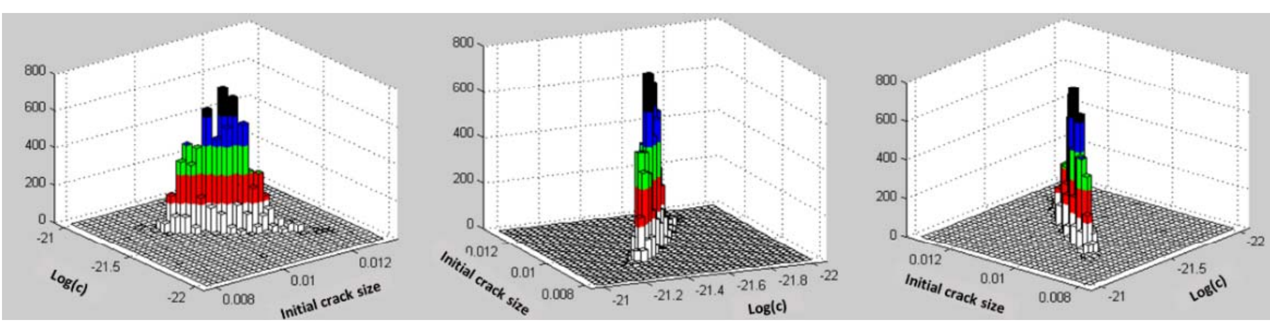

Figure 5. 2D Histogram of Samples Generated by Metropolis Algorithm

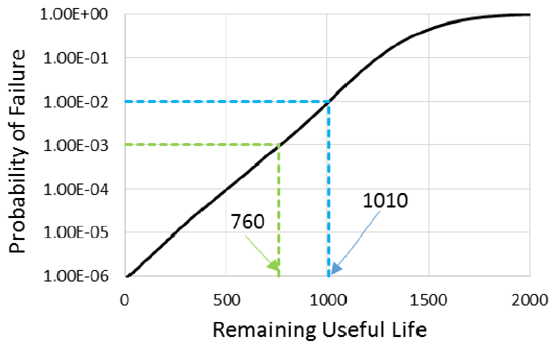

Figure 6. The Probability of Failure of Remaining Useful Life for Measurement Set in Figure 2 Case 1

Table 3. Mean of Remaining Useful Life at 0.999 and 0.99 Reliability

Case	1	2	3	4	*GT
Measurement Intervals Δt_m	20	20	10	10	
Measurement Noise σ_ω	0.002	0.001	0.002	0.001	
$\overline{R}$ =0.999	992	1028	1047	1044	1040
$\overline{R}$ =0.99	1260	1312	1335	1331	1340

*Ground Truth

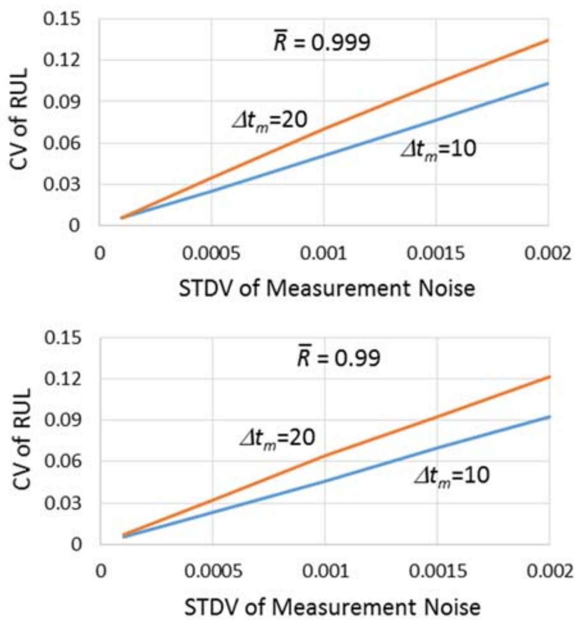

Figure 7. CV (Coefficient of Variation) of RUL vs STDV of Measurement Noise

4. Summary

Bayesian-filters (BF) and Markov chain Monte Carlo (MCMC) based probabilistic lifing method was explored for its robustness in probabilistic life prediction. Uncertainties in damage growth parameters are updated and reduced with new information obtained through structural health monitoring (SHM) systems. State transition function is described by a typical Paris equation for fatigue crack propagation for demonstration purpose. In the equation, the initial crack size and the crack growth rate are random parameters to be updated. Measurement functions are assumed in this study which describe the relationship between the features derived from SHM signals and the damage sizes. Damage tolerance and risk-based remaining useful life (RUL) of fatigue critical structural components are determined at various reliability levels. In our study, burn-in period from MCMC procedure was around 2000 samples. 10,000 samples after the burn-in period was selected for updating and prediction. The variabilities of predicted RUL are also studied for various magnitudes of measurement noise and measurement frequencies. It is found the variability of the predicted RUL is proportional to the variability of the measurement noise. The results with more frequent measurement are closer to the ground truth.

Nomenclature

a_0:	crack size at time 0
c:	crack growth rate in Paris equation
C:	constant is for jump distribution in Metropolis algorithm
BF:	Bayesian Filter
CDF:	cumulative distribution function
CV:	coefficient of variation
DT:	damage tolerance
m:	Paris Law constant
MCS:	Monte Carlo simulation
MCMC:	Markov chain Monte Carlo
p:	probability
PDF:	probability density function
POF:	probability of failure
$\overline{R}$:	a specified reliability
R_t:	reliability at time t
RUL:	remaining useful life
SHM:	structural health monitoring
STDV:	standard deviation
S_1:	initial crack size a_0
S_2:	crack growth rate c
t':	time of last SHM measurement
T:	remaining useful life
z:	SHM measurements
σ_ω :	standard deviation of measurement noise ω
Δt_m:	SHM measurement interval

ΔK :	the range of stress intensity factor
$\hat{a}$:	damage feature extracted from SHM signals
α	constant in measurement function
β:	constant in measurement function
$\overline{\alpha}$:	acceptance ratio in Metropolis algorithm
θ :	model parameters
Θ :	uncertain parameters
v :	process noise
ω :	measurement noise

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