An Energy-Based Prognostic Framework to Predict Fatigue Damage Evolution in Composites

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Abstract

In this work, a prognostics framework to predict the evolution of damage in fiber-reinforced composites materials under fatigue loads is proposed. The assessment of internal damage thresholds is a challenge for fatigue prognostics in composites due to inherent uncertainties, existence of multiple damage modes, and their complex interactions. Our framework, considers predicting the balance of release strain energies from competing damage modes to establish a reference threshold for prognostics. The approach is demonstrated on data collected from a run-to-failure tension-tension fatigue experiment measuring the evolution of fatigue damage in carbon-fiber-reinforced polymer (CFRP) cross-ply laminates. Results are presented for the prediction of expected degradation by micro-cracks for a given panel with the associated uncertainty estimates.

1. Introduction

Composites are high-performance materials with a wide range of engineering applications like aerospace, automotive, and construction because of their high specific stiffness and strength in relation to their weight. Most of these applications involve components subject to cyclic loadings that make them susceptible to fatigue degradation. Unlike metals, fatigue damage degradation in composites is a complex multi-scale process driven by several internal fracture mechanisms such as matrix-cracks, local and global delaminations, fiber breakage, etc. The progression of these damage modes and their interactions are significantly influenced by the material properties, stacking sequence, stress level and other loading conditions (Reifsnider & Talug, 1980; R. D. Jamison, Schulte, Reifsnider, & Stinchcomb, 1984; Beaumont, Dimant, & Shercliff, 2006). Among them, transverse matrix cracking[1] holds a central position as a precursor of other damage modes in adjacent plies, such us local delaminations[2] and fiber breakage (Lee, Allen, & Harris, 1989; Beaumont et al., 2006). It is generally accepted that the matrix crack density in off-axis plies tends asymptotically to an upper bounded value corresponding to a spacing of aspect ratio unity[3], termed as Characteristic Damage State (CDS) (Reifsnider & Talug, 1980). This state is usually concomitant with more severe damage scenarios that may lead to a subsequent catastrophic failure.

In addition to the CDS, damage progression may exceed other "subcritical damage states" before ultimate failure, corresponding for instance with the onset of local delaminations or global delaminations respectively. These damage states define tolerance limits that can be used as thresholds for prognostics. However, establishing a deterministic damage progression path to these subcritical states is not an easy task because of the uncertainties in the growth and interactions of internal fracture modes. It is exactly in this context, where the benefits of the proposed prognostic framework can be fully exploited to deal with the uncertainty of the damage accumulation process during fatigue conditions. This framework al-

[1]The terms matrix micro-cracks, transverse cracks or intralaminar cracks can be invariably used to refer to the cracks growing along fiber directions in off-axis plies.
[2]Local delaminations are small inter-laminar fractures growing from the tips of matrix cracks.
[3]Ratio between average crack spacing $(2l)$ and ply thickness, t.

lows us to sequentially update the information about the existing damage modes by fusing probabilistic information from sensors and models and, ultimately, redefining the path to the subsequent subcritical damage states.

Prognostics is concerned with determining the health of system components and making end of life (EOL) predictions based on meeting specific critical thresholds, given an evolutionary damage model. As with diagnostics, prognostics methods are typically categorized as either model-based or data-driven, depending on whether the damage evolution model is based on physical first principles, or, alternatively uses run-to-failure data to capture trends of degradation. In the recent decades, a large number of articles have been reported to cover data-driven approaches for damage evolution under the topics of stochastic modeling of fatigue damage growth for several applications (Wu & Ni, 2004). First attempts involve modeling the damage evolution as a Markov process (Lin & Yang, 1983). Rowatt and Spanos (1998); Ganesan (2000); Wei, Johnson, and Haj-Ali (2010) proposed Markov chain models to describe the progression of fatigue degradation in composites from laminate compliance measurements, as an extension of the pioneering work of Bogdanoff and Kozin (1985). More recently Chiachio, Chiachio, and Rus (2011) proposed an enhancement of (Rowatt & Spanos, 1998) by a model parameterization that allows accounting for the non-stationarity of the damage process.

On the other hand, model-based approaches provide EOL estimates that are more accurate and precise than data-driven approaches, if good models are available (M. Daigle & Goebel, 2010). Specifically, these models have the ability to adapt to different systems (specimen, materials, conditions, etc.) without much training, they are transparent to human understanding, and furthermore, they can incorporate monitoring data in a Structural Health Monitoring (SHM) context. Particularly in composites, where multiple fracture modes may co-exist, a model-based prognostics framework allows dynamically assessing the dominant damage mode and establishing the thresholds of each of the competing damage modes, by means of a comparison of the energy spent by each single mode (Nairn & Hu, 1992).

In this work, a model-based prognostic framework is formulated to predict in real time the accumulation of damage in composites under fatigue conditions. A tension-tension fatigue experiment in a cross-ply CFRP laminate is used for case study, measuring the evolution of matrix-cracks density. Damage thresholds for remaining useful life (RUL) prediction are sequentially updated by means of a model parameterization of the energy release rate due to microcracking. To this end, a particle filter algorithm (Arulampalam, Maskell, Gordon, & Clapp, 2002; Cappé, Godsill, & Moulines, 2007), is used for the joint state-parameter sequential estimation.

The rest of the paper is organized as follows. Section 2 describes the model of damage propagation and introduces the basis for establishing damage thresholds based on energy release. The transformation of the physics-based evolutionary model into a probabilistic state space model is covered in Section 3. Section 4 formally defines the prognostics problem and describes the algorithmic architecture. Section 5 presents the demonstration of the approach on real data of fatigue considering a cross-ply CFRP laminate. Finally, some concluding remarks are presented in Section 6.

2. Damage propagation models

Behind the versatility of the Markov chain models and other stochastic models for fatigue damage evolution, their lack of physical meaning has been the main source of criticism. Instead, the Paris' law[4] (Paris, Gomez, & Anderson, 1961), which relates the crack growth rate to the range in applied stress intensity factor, has been widely used for being better associated with the physics of the damage growth process. However in contrast to metals, there is no observation of a single crack growth in composites under fatigue loadings but a densification of multiple interlaminar and intralaminar microcracks. The crack tip stress intensity factor is therefore meaningless, instead, the range in microcracking energy release rate (ΔG) can be incorporated within the "traditional" power law formulation, leading to the modified Paris' Law (Nairn & Hu, 1992):

$$\frac{\mathrm{d}\rho}{\mathrm{d}n} = A(\Delta G)^{\alpha} \tag{1}$$

In the equation above, ρ is the crack density which increases with the fatigue cycles n, and A and α are fitting parameters. The energy release rate ΔG represents the energy released due to the formation of a new crack between two existing cracks for a specific stress amplitude: $\Delta G = G_{|\sigma_{max}} - G_{|\sigma_{min}}$. This energy is intimately connected with the effective laminate stiffness, i.e. the stiffness due to the current damage state, which requires a number of theoretical assumptions to be made for composite materials and needs the hard to get information about the geometrical crack pattern per ply, damage extension, etc (Talreja & Singh, 2012). See the next section for further information. It follows that analytical expressions for ΔG are quiet complex, hence a closed-form solution for the modified Paris' Law is difficult to obtain. To overcome this drawback, the resulting differential equation is solved by approximating the derivative by finite differences as:

$$\frac{\Delta\rho}{\Delta n} = \frac{\rho_n - \rho_{n-1}}{1} = A\left(\Delta G(\rho_{n-1})\right)^{\alpha} \tag{2}$$

hence

$$\rho_n = \rho_{n-1} + A\left(\Delta G(\rho_{n-1})\right)^{\alpha} \tag{3}$$

[4] $\frac{\mathrm{d}l}{\mathrm{d}n} = A(\Delta K)^{\alpha}$, where l: crack length, A and α: fitting parameters, and ΔK: stress intensity factor.

2.1. Energy release rate

Several families of models can be found in the composites literature addressing the relation between the (macro-scale) effective stiffness E, and the (micro-scale) matrix-crack density ρ, which essentially forms the basis for the underlying physics behind the energy release rate, ΔG. Therefore, there is a closed connection between micro-scale damage propagation models (i.e., the modified Paris' Law) and macro-scale stiffness models. These models are generally classified as 1) computational methods, 2) semi-analytical methods and 3) analytical methods. The recent work of Talreja and Singh (2012) provides a thorough review on this topic. The computational models are mostly based on the finite element method (FEM), the finite difference method (FDM), and the boundary element method (BEM). In all these models, a homogenization approach is needed to incorporate the microstructure of damage within the formulation of the numerical element, which requires several assumptions to be adopted. The main drawback of this kind of models from the prognostic point of view, is that they are very demanding computationally. Surrogate models may alleviate that problem by adopting data-driven techniques but are beyond the scope of this work. The aim of semi-analytical methods relies on improving the numerical predictions of the former by means of adding experimentally-fitted parameters, or by using numerical expansion of some of the model coefficients (Aboudi, Lee, & Herakovich, 1988; Lee & Hong, 1993). However, these approaches are mostly extensions of the previously developed ideas (Talreja & Singh, 2012), and also, the computational cost is still high. The details of these methods are not relevant to the discussions in this paper and the readers are referred to the cited articles and in particular to the critical review provided in Talreja and Singh (2012) and Talreja (2008).

The analytical models are expressed through closed-form relations grounded in the physics of the stress-strain relationships in the presence of damage. Some assumptions about damage distribution are used such as crack pattern, interaction between plies and the presence of local delamination among others. Depending on the level of assumptions, the body of literature classifies them into shear-lag models (1D), variational models (2D), and crack opening displacement models (3D). Among them the variational models are presumably the most referred ones in the literature, because of their efficiency in accounting for the complexity of damage in relation to the assumptions adopted, compared to the rest of cited analytical models. Additionally, this class of models allows considering the interaction between different damage modes, such as matrix cracks, local and global delaminations. Therefore, they are adopted in this work to study the energy release rate term in the Paris' Law, ΔG.

Originally proposed by Hashin (1985), these models establish the relationship between stresses and strains considering a two-dimensional damage pattern of matrix cracks, local delamination, and global delamination (Nairn & Hu, 1992). The released energy is further obtained by integrating the stresses and strains fields over the laminate volume. Equations 4a and 4b provide analytical expressions for the energy release rate for (a) the formation of a new microcrack between two cracks separated an average dimensionless distance of $\bar{l}$, and (b) the growth of local delamination with a dimensionless length δ between two existing microcracks separated by a distance $\bar{l}$.

$$\Delta G_{TC} = C_3 t_{90} \left(\frac{E_x^{(90)}}{E_0} \Delta\sigma_0 \right)^2 \left(2\chi\left(\bar{l}/2\right) - \chi\left(\bar{l}\right) \right) \tag{4a}$$

$$\Delta G_{LD} = C_3 t_{90} \left(\frac{E_x^{(90)}}{E_0} \Delta\sigma_0 \right)^2 \left(\frac{\chi'(0) - \chi'(\bar{l}-\delta)}{2} \right) \tag{4b}$$

For the case of global delamination, the model for the energy release rate is adopted from Nairn and Hu (1992) and (Hosoi, Takamura, Sato, & Kawada, 2011):

$$\Delta G_{GD} = h \left(\frac{\Delta\sigma_0}{E_{eff}} \right)^2 \left(E_x^{(\text{center})} - E_x^{(\text{edge})} \right) \tag{5}$$

See a detailed description of all the terms in these equations in the Nomenclature section. The functions χ, χ' and C_3 are defined in the Appendix. Notice that ΔG_{LD} depends on the magnitude $(\bar{l} - \delta)$, which expresses the separation between the tips of two growing delaminations starting from the tips of the matrix micro-cracks. Without lack of generality, thermal stresses are not considered in Eq. 4 and 5, given that the data used in this paper were collected in a temperature controlled environment.

2.2. Competing damage modes

Due to the dependency upon the laminate stacking sequence, ply properties and experimental conditions for the damage progression, an energy-based framework is used to establish thresholds of damage without much previous experimental evidences. Matrix microcracks are the most plausible damage mode in the first stages of fatigue damage for angle-ply, quasi-isotropic and cross-ply laminates (R. Jamison, 1985). Hence, we focus on predicting the saturation of matrix microcracks and the onset of the subsequent damage mode, which is a priori unknown.

Based on a balance of energies between different plausible damage modes by Eq. 4 and 5, one can address the question of whether the next increment in damage will be through another transverse crack or a different damage mode (i.e., local delamination, global delamination, etc) induced by the existing microcracks (Nairn & Hu, 1992). Figure 1 illustrates this concept through a case study for a cross-ply laminate. See the

Table 1. Ply properties used in the calculations.

Long. Modulus E_x [GPa]	Trans. Modulus E_y [GPa]	In-plane Poisson ν_{xy}	Out-of-plane Poisson ν_{yz}	Shear modulus G_{xy} [GPa]	Out-of-plane-Shear modulus G_{yz} [GPa]	Thickness t [mm]
127.55	8.41	0.309	0.49	6.2	2.82	0.152

material parameters used for this exercise in Table 1. Observe that initially, the energy release for transverse cracks is larger than the rest of damage modes. Therefore, matrix microcracks are expected to accumulate at a faster rate at earlier cycles. Results also show that until the final stage of the process, the local delamination mode of damage releases more strain energy than the global delamination mode. Therefore, local delaminations are expected much earlier than global delaminations along the fatigue damage process. These conclusions agree with the experimental evidence obtained for cross-ply laminates (R. Jamison, 1985). Observe also that the point where TC and LD curves intersect defines a critical value for the matrix-crack density. At this point local delaminations are more likely to appear than another matrix crack. This point can be computed from Equation 4b using $\delta = 0$. Another important reference-point is the intersection of the LD and GD curves, where local delaminations are supposed to cease starting a global delamination process. Notice that global delamination is unlikely to appear since the energy release rate associated to its growth is too small. Therefore, the required damage for global delamination onset is so severe, that the specimen usually fail before that point.

Based on this reasoning, prognostic thresholds can be establish by predicting the position of these reference points as the fatigue process continues. It is important to remark that the energy term ΔG depends on model parameters which are sequentially updated as new data arrives. Hence, the intersection points defining the thresholds dynamically shift their position until a convergence stage. All these aspects are covered in the next sections.

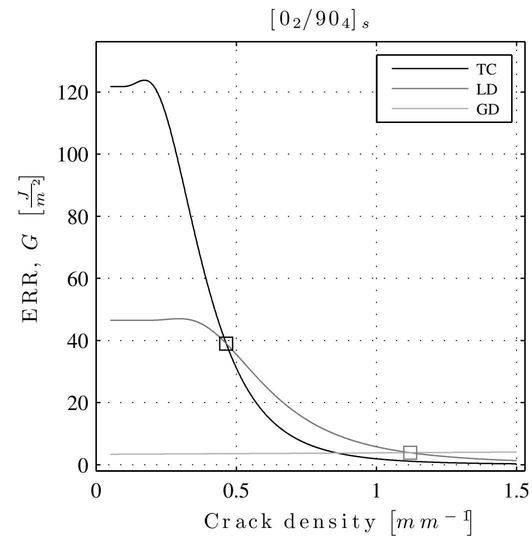

Figure 1. Energy release rate term as a function of the matrix crack density. TC, LD and GD are referring to transverse cracks, local delamination and global delamination, respectively. The two markers correspond to the points where a change in the dominant fracture mode is expected.

3. STOCHASTIC EMBEDDING

Let assume a physical system that may be idealized by an I/O model as:

$$\mathbf{x}_n = g(\mathbf{x}_{n-1}, \mathbf{u}_n, \boldsymbol{\theta}_n) + \mathbf{v}_n \tag{6a}$$

$$\mathbf{y}_n = h(\mathbf{x}_n) + \mathbf{w}_n \tag{6b}$$

where g and h are the state equation and observation equation, respectively; $\mathbf{x}_n \in \mathbb{R}^{n_x}$ is the state vector, $\mathbf{u}_n \in \mathbb{R}^{n_u}$ is the input vector and $\boldsymbol{\theta} \in \mathbb{R}^{n_\theta}$ is the model parameter vector. $\mathbf{v}_n$ and $\mathbf{w}_n$ are the process noise vector and measurement noise vector, which can be defined[5] as zero mean Gaussians, $\mathcal{N}(0, \sigma_{v_n})$ and $\mathcal{N}(0, \sigma_{w_n})$ respectively.

The model g is dependent upon a set $\boldsymbol{\theta}$ of model parameters that can be updated to sequentially improve the model predictability as more experimental data are available. In this context, standard Sequential Monte Carlo (SMC) methods (Doucet, De Freitas, & Gordon, 2001) fail and it is necessary to rely on more sophisticated algorithms. Although this problem is still open in the specific literature (Liu & West, 2001; Storvik, 2002; Kantas, Doucet, Singh, & Maciejowski, 2009; Patwardhan, Narasimhan, Jagadeesan, Gopaluni, & Shah, 2012), here we choose the "artificial dynamics" approach (Liu & West, 2001) due to its pragmatism and simplicity, by which model parameters performs a random walk by introducing a small (and decreasing with n) artificial noise term, $\boldsymbol{\xi}_n$, as:

$$\boldsymbol{\theta}_n = \boldsymbol{\theta}_{n-1} + \boldsymbol{\xi}_n \tag{7}$$

To sequentially reduce the variance of this artificial error sequence $\boldsymbol{\xi}_n$, there are many alternative methods (Kantas et al., 2009), however the recent method proposed by (M. Daigle & Goebel, 2010; M. J. Daigle & Goebel, 2013) is chosen by its efficiency. In our problem, the damage state variable is defined as: $\mathbf{x}_n = \rho_n$, where ρ_n is the matrix crack density at

[5]A rational way to define a probability model for the error term could be to select it such that it produces the most uncertainty (largest Shannon entropy). The maximum-entropy PDF for an unbounded variable given its mean and variance is a Gaussian distribution.

cycle n, i.e, $\rho_n = 1/2l_n$, being l_n the half-separation between cracks at cycle n, expressed in millimeters. Hence Eq. 6 can be rewritten as:

$$\rho_n = g(\rho_{n-1}, \mathbf{u}_n, \boldsymbol{\theta}_n) + v_n \tag{8a}$$

$$\hat{\rho}_n = \rho_n + w_n \tag{8b}$$

where $g(\rho_{n-1}, \mathbf{u}_n, \boldsymbol{\theta}_n)$ is the modified Paris' law. Given that for a particular cycle n the error term v_n is a Gaussian PDF, the probability model for the state transition equation (Eq. 8a) will be also a Gaussian:

$$p(\rho_n|\rho_{n-1}, \mathbf{u}_n, \boldsymbol{\theta}_n) = \mathcal{N}(g(\rho_{n-1}, \mathbf{u}_n, \boldsymbol{\theta}_n), \sigma_{v_n}) \tag{9a}$$

Similarly, the expression for measurement equation (8b) and the artificial evolution of parameters are obtained as follows:

$$p(\hat{\rho}_n|\rho_n) = \mathcal{N}(\rho_n, \sigma_{w_n}) \tag{10a}$$

$$p(\boldsymbol{\theta}_n|\boldsymbol{\theta}_{n-1}) = \mathcal{N}(\boldsymbol{\theta}_{n-1}, \boldsymbol{\xi}_n) \tag{10b}$$

Finally, to select the set of most sensitive model parameters $\boldsymbol{\theta}$ among the set of parameters that defines the modified Paris' law, a global sensitivity analysis was done following the methodology proposed by (Saltelli et al., 2008). By means of this study, the ply properties $\{E_x, E_y, t\}$ together with the Paris' law fitting parameters $\{A, \alpha\}$ emerged as the key parameters in terms of model output uncertainty. Then the set of updatable parameters was defined by adding the standard deviation of the model error and measurement error to the last choice, i.e., $\boldsymbol{\theta} = \{A, \alpha, E_x, E_y, t, \sigma_v, \sigma_w\}$. The rest mechanical and geometrical parameters act as static non-updatable input parameters, i.e., $\mathbf{u} = \{\sigma_0, B, G_{xy}, G_{yz}, \nu_{xy}, \nu_{yz}\}$.

4. Damage prognostics

For predicting remaining useful life (RUL) of a composite structure, we are interested in predicting the time when the damage grows beyond a predefined acceptable threshold (Saxena, Celaya, Saha, Saha, & Goebel, 2010). The time at which it occurs is known as the expected end of life (EOL). Using the most current knowledge of the system state at cycle n estimated by particle filters (Eq. 11), the goal now is to estimate the EOL$_n$, as probability: $p(EOL_n|\mathbf{y}_n)$. The particle filter computes the joint state-parameter distribution $p(\rho_n, \boldsymbol{\theta}_n|\mathbf{y}_n, \mathbf{u}_n)$, which can be approximated by a set of N discrete weighted *particles*, $\{(\rho_n^i, \boldsymbol{\theta}_n^i), \omega_n^i\}_{i=1}^N$, as

$$p(\rho_n, \boldsymbol{\theta}_n|\mathbf{y}_n) \approx \sum_{i=1}^{N} \omega_n^i \delta(\rho_n - \rho_n^i)\delta(\boldsymbol{\theta}_n - \boldsymbol{\theta}_n^i) \tag{11}$$

In our problem, we consider a sequence of measurements, $\mathbf{y}_n = \hat{\boldsymbol{\rho}}_{0:n}$, where $\hat{\boldsymbol{\rho}}_{0:n} = \{\hat{\rho}_0, \hat{\rho}_1, \ldots, \hat{\rho}_n\}$. Hence, Eq. 11 can be rewritten as:

$$p(\rho_n, \boldsymbol{\theta}_n|\hat{\boldsymbol{\rho}}_{0:n}) \approx \sum_{i=1}^{N} \omega_n^i \delta(\rho_n - \rho_n^i)\delta(\boldsymbol{\theta}_n - \boldsymbol{\theta}_n^i) \tag{12}$$

For simpler notation the conditioning on the model input $\mathbf{u}_n$ is dropped from Eq. 11. Applying Bayes' Theorem, the importance weights ω_n^i can be updated as:

$$\omega_n^i \propto p(\hat{\rho}_n|\rho_n)\omega_{n-1}^i \tag{13}$$

Here we assume that the system model is Markovian of order one and that the observations are conditionally independents of the state. To this end, we use the sampling importance resampling (SIR) particle filter, using systematic resampling (M. Daigle & Goebel, 2010).

4.1. Prognostics threshold

The damage space itself may be defined by means of a set of thresholds $\mathbf{C} = \{\mathcal{C}_1, \ldots, \mathcal{C}_c\}$ on more than one critical parameters. In such cases, these thresholds can be combined into a *threshold function* $T_{EOL} = T_{EOL}(\rho, \boldsymbol{\theta})$, that maps a given point in the joint state-parameter space to the Boolean domain $\{0, 1\}$ (M. Daigle & Goebel, 2011). For instance, when a given particle i starting from cycle n performs a random walk and hits any of the thresholds in $\mathbf{C}$, then $T_{EOL}^i = 1$, otherwise $T_{EOL}^i = 0$. The time $n' \geqslant n$ at which that happens defines the EOL$_n$ for that particle. Mathematically:

$$EOL_n^i = \inf\{n' \in \mathbb{N} : n' \geqslant n \wedge T_{EOL}^i(x_{n'}^i, \boldsymbol{\theta}_{n'}^i) = 1\} \tag{14}$$

Using the updated weights at the starting time n, a probabilistic estimation of the EOL is given as:

$$p(EOL_n|\hat{\boldsymbol{\rho}}_{0:n}) \approx \sum_{i=1}^{N} \omega_n^i \delta(EOL_n - EOL_n^i) \tag{15}$$

Once EOL$_n$ is estimated, the remaining useful life can be simply obtained as $RUL_n = EOL_n - n$. An algorithmic description of the prognostic procedure is provided as Algorithm 1.

Algorithm 1 RUL prediction

Requires: $\{(\rho_n^i, \boldsymbol{\theta}_n^i), \omega_n^i\}_{i=1}^N$, $\mathbf{C} = \{\mathcal{C}_1, \ldots, \mathcal{C}_c\}$
Output: $\{EOL_n^i, \omega_n^i\}_{i=1}^N$
for $i = 1 \to N$ **do**
  Calculate: $T_{EOL}^i(\rho_n^i, \boldsymbol{\theta}_n^i)$
  while $T_{EOL}^i = 0$ **do**
    Simulate: $\boldsymbol{\theta}_{n+1}^i \sim p(\cdot|\boldsymbol{\theta}_n^i)$; $\rho_{n+1}^i \sim p(.|\rho_n^i, \boldsymbol{\theta}_{n+1}^i)$
    $n \leftarrow n + 1$
    $(\rho_n^i, \boldsymbol{\theta}_n^i) \leftarrow (\rho_{n+1}^i, \boldsymbol{\theta}_{n+1}^i)$
  end while
  $EOL_n^i \leftarrow n$
  $RUL_n^i = EOL_n^i - n$
end for

5. RESULTS

The proposed framework is applied to fatigue cycling data for cross-ply graphite-epoxy laminates. Torayca T700G unidirectional carbon-prepreg material was used for 15.24 [cm] x 25.4 [cm] coupons with dogbone geometry. See the laminate ply properties in Table 1.

The tests, as reported in (Saxena et al., 2011), were conducted under load-controlled tension-tension fatigue loading with a frequency of $f = 5$ [Hz], a maximum stress of 80% of their ultimate stress, and a stress ratio $R = 0.14$. Lamb wave signals were periodically recorded using a PZT sensor network to estimate internal micro-crack density. A mapping between PZT raw data and micro-crack density was developed following the methodology proposed in Larrosa and Chang (2012).

Results for sequential damage state estimation and prognostics are presented in Figure 2. Based on the reasoning provided in Section 2.1, we initially expect for this laminate that the matrix cracks should saturate around the value of 0.427 $[mm]^{-1}$ (see Figure 1). Observe in Figure 2a that the threshold of matrix cracks density converges from its initial value to 0.419 $[mm]^{-1}$, as model parameters $\boldsymbol{\theta}$ are sequentially updated with new data.

Figures 2b shows the filtered-estimation of matrix micro-cracks together with the sequence of collected data. Every time new data arrive, a damage magnitude is estimated and the updated model is further used to propagate the damage into the future to compute the RUL, calculated as: $RUL_n = EOL_n - n$, using the methodology described in Section 4. These predictions are plotted against time in Figure 2c. Observe in Figure 2c that the RUL prediction is appreciably inaccurate within the first stage of fatigue before the threshold reaches its convergence value. In view of Figure 2a, this stage corresponds to the interval of cycles required for data to train model parameters. From this period, the prediction precision clearly improves with time. We use the two shaded cones of accuracy at 10% and 20% of true RUL, denoted as RUL* to help evaluating the prediction accuracy and precision. Notice also in Figure 2a that accuracy seems to depart from true RUL at the final stage, which indicates that the model and its variance structure do not fully capture the damage dynamics towards the end. Such behavior have been previously reported in (Saxena et al., 2010) and may be related with the asymptotic behavior of the micro-crack evolution, which requires more efficient algorithms for prognostics in such cases.

6. CONCLUSIONS

A model-based prognostics framework to predict the fatigue damage evolution in composites is proposed. We consider a stochastically embedded modified Paris' law, as a phenomenological model of damage evolution due to the benefits of the physical meaning of ΔG for estimating the RUL. We demonstrate the validity of this framework on data collected from a tension-tension fatigue experiment using CFRP cross-ply laminate. The following general concluding remarks are extracted:

- By means of balance of energies between different damage modes computed by ΔG, the more plausible damage mode can be elucidate together with the threshold for this mode.

- Due to the model parameterization, the threshold of damage dynamically changes as new data arrives. Hence this approach requires an initial period of cycles before the predictions reach an acceptable accuracy.

- More research effort is need to achieve more efficient prognostic algorithms to improve the accuracy at the final stage of the process, where most of the damage modes in composites typically accumulate in an asymptotic manner.

APPENDIX

The function $\chi(\bar{l})$ in the equations 4a and 4b accounts the change in the effective x-direction modulus caused by the formation of a new micro-crack midway between two existing micro-cracks. Expressions for $\chi(\bar{l})$ and its first derivate are given by:

$$\begin{aligned} \chi(\bar{l}) &= 2\alpha_1\alpha_2(\alpha_1^2+\alpha_2^2)\frac{\cosh(2\alpha_1\bar{l})-\cos(2\alpha_2\bar{l})}{\alpha_2\sinh(2\alpha_1\bar{l})-\sin(2\alpha_2\bar{l})} \\ \chi'(\bar{l}) &= 4\alpha_1\alpha_2(\alpha_1^2+\alpha_2^2)^2\frac{\sinh(2\alpha_1\bar{l})\sin(2\alpha_2\bar{l})}{\alpha_2\sinh(2\alpha_1\bar{l})+\alpha_1\sin(2\alpha_2\bar{l})} \end{aligned} \tag{16a}$$

$$\begin{aligned} \chi(\bar{l}) &= \alpha_1\alpha_2(\alpha_1^2-\alpha_2^2)\frac{\tanh(\alpha_2\bar{l})\tanh(\alpha_1\bar{l})}{\alpha_2\tanh(\alpha_2\bar{l})-\alpha_1\tanh(\alpha_1\bar{l})} \\ \chi'(\bar{l}) &= \alpha_1^2\alpha_2^2(\alpha_1^2-\alpha_2^2)\frac{\frac{\tanh(\alpha_2\bar{l})}{\cosh^2(\alpha_1\bar{l})}-\frac{\tanh(\alpha_1\bar{l})}{\cosh^2(\alpha_2\bar{l})}}{\big(\alpha_2\tanh(\alpha_2\bar{l})-\alpha_1\tanh(\alpha_1\bar{l})\big)^2} \end{aligned} \tag{17a}$$

where $\alpha_1 = \sqrt{\frac{-p}{2}+\sqrt{\frac{p^2}{4}-q}}$ and $\alpha_2 = \sqrt{\frac{-p}{2}-\sqrt{\frac{p^2}{4}-q}}$. The Eq. 16a applies for the case of $\frac{4q}{p^2} > 1$. Otherwise, Eq. 17a should be consider. p and q are relations of the ply properties and the stacking sequence defined by $p = {}^{(C_2-C_4)}/_{C_3}$, $q = {}^{C_1}/_{C_3}$. The terms $C_i,\ i:\{1,\ldots,4\}$,

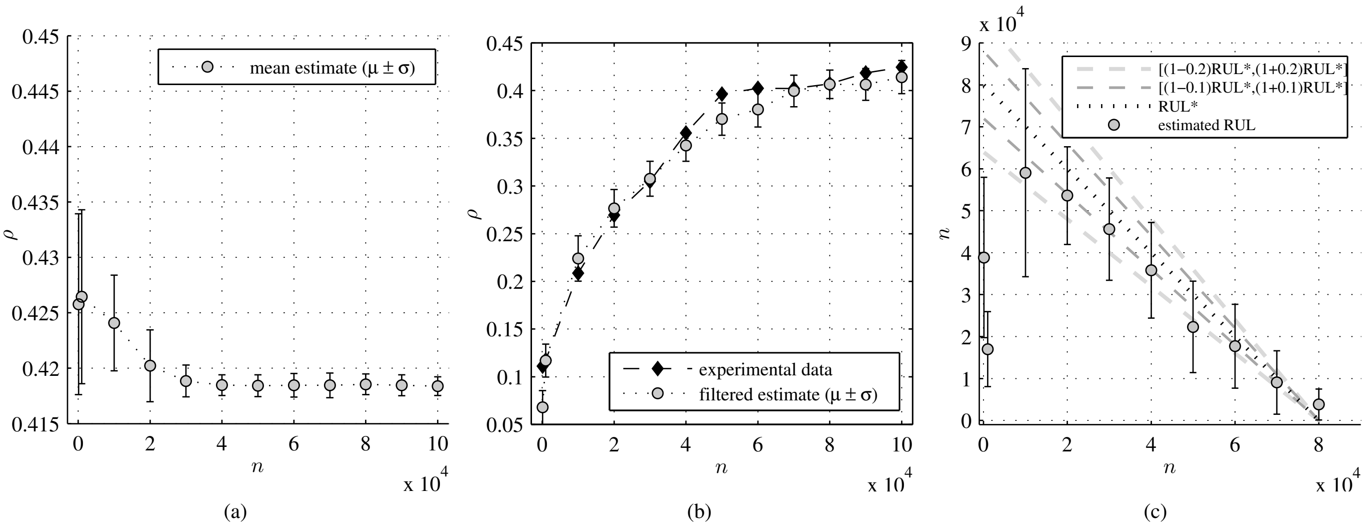

Figure 2. (a): Trace of the threshold value for crack density saturation over cycles. (b): Sequential estate estimation of micro-crack density by the particle filter. At each cycle n, the estimation is calculated using the data available up to that cycle. (c): Remaining useful life (RUL) prediction.

are known functions of the laminate properties defined as:

$$C_1 = \frac{1}{E_x} + \frac{1}{\lambda E_y} \tag{18a}$$

$$C_2 = \left(\lambda + \frac{2}{3}\right)\frac{\nu_{yz}}{E_y} - \frac{\lambda \nu_{xy}}{3E_x} \tag{18b}$$

$$C_3 = (1+\lambda)\left(3\lambda^2 + 12\lambda + 8\right)\frac{1}{60E_y} \tag{18c}$$

$$C_4 = \frac{1}{3}\left(\frac{1}{G_{yz}} + \frac{\lambda}{G_{xy}}\right) \tag{18d}$$

The reader is referred to the Nomenclature section for information of laminate constants involved in the last equations.

ACKNOWLEDGMENT

The two first authors would like to thank the Ministry of Education of Spain for the FPU grants AP2009-4641, AP2009-2390, to the European Union for the “Programa Operativo FEDER de Andalucia 2007-2013” for project GGI3000IDIB and to the Prognostics Center of Excellence at NASA Ames, which kindly hosted them during the course of this work. Authors would also like to thank Structures and Composites lab at Stanford University for experimental data and NASA ARMD/AvSafe project SSAT, which provided partial support for this work.

NOMENCLATURE

h	Laminate half-thickness
t_S	$[S]$ sublaminate thickness
t_{90}	$[90_n]$ sublaminate half-thickness
λ	Ply thickness ratio $\lambda = {}^{t_0}/_{t_{90}}$
$\bar{l}$	average dimensionless half spacing of cracks, $\bar{l} = \frac{l}{t_{90}}$
B	Laminate half-width
δ	Average local delamination length, $\delta = \frac{d}{t_{90}}$
E_x^{90}	Undamaged x-direction $[90_n]$ sublaminate modulus
E_0	Undamaged laminate Young's modulus
E_{eff}	Damaged x-direction laminate Young's modulus
$E_x^{(\text{center})}$	Damaged Young modulus of sublaminate from the center side.
$E_x^{(\text{edge})}$	Damaged Young modulus of sublaminate from the edge side.
σ_0	Applied stress

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