# Health Monitoring and Prognosis of Hybrid Systems

Said Zabi[1], Pauline Ribot[2], and Elodie Chanthery[3]

[1,2,3] *CNRS, LAAS, 7 avenue du colonel Roche, F-31400 Toulouse, France*

[1,2] *Univ de Toulouse, UPS, LAAS, F-31400 Toulouse, France*
*said.zabi@laas.fr*
*pauline.ribot@laas.fr*
[3] *Univ de Toulouse, INSA, LAAS, F-31400 Toulouse, France*
*elodie.chanthery@laas.fr*

## Abstract

Maintenance and repair of complex systems are an increasing part of the total cost of final product. Efficient diagnosis and prognosis techniques have to be adopted to detect, isolate and anticipate faults. Moreover the recent industrial systems are naturally hybrid: their dynamic behavior is both continuous and discrete. This paper presents an architecture of health monitoring and prognosis for hybrid systems. By using model and experience-based approach we propose an implementation of an integrated diagnosis/prognosis process based on Weibull probabilistic model. This article focuses particularly on the prognosis algorithm description. The process has been implemented and tested on Matlab. Simulation results on a water tank system show how prognosis and diagnosis interact into the architecture.

## 1. Introduction

Due to new technologies, the development of industrial systems is increasingly complex. It becomes difficult to manage and anticipate the behavior of these systems, especially when they are exposed to failures. In time of economic crisis, it is particularly essential to improve the system availability by reducing maintenance and repair costs. Efficient diagnosis and prognosis techniques have to be adopted to detect, isolate and anticipate faults leading to failures. Health monitoring of industrial systems aims at determining the health state of systems at any time in order to optimize their functioning and act in case of malfunctions. Diagnosis helps to determine the current health state of a system. Malfunctions or failures may be anticipated by a prognostic reasoning on the system. The PHM community defines the prognosis as the calculation of the remaining useful life (RUL). This temporal prediction gives the date at which the system is not operational anymore and must to be repaired. When the system is in such a state, it is said to be in a *failure mode*. Before this state, it can be either in a *nominal mode*, or after the occurrence of one or several faults in a *faulty mode* or degraded mode. Prognosis requires the knowledge of the current health state of the system through a diagnosis, hence the need of a health monitoring method integrating diagnosis and prognosis.

Recent industrial systems exhibit an increasing complexity of behaviors that are both continuous and discrete. It has become difficult to ignore the fact that most systems are hybrid. Therefore this paper proposes to use the techniques of model-based diagnosis and prognosis in the framework of hybrid systems. We propose to enrich the commonly used modeling framework for hybrid systems with available knowledge about aging or degradation of the system. Systems are continuously degrading according to operational conditions. According to information available on the system, it is possible to establish physical aging laws or time-dependent fault probabilities based on the feedback. This temporal and/or stochastic information should be taken into account in the model of the hybrid system.

The main ideas on the integrated architecture for diagnosis and prognosis of hybrid systems we proposed have been published in Chanthery and Ribot (2013). This paper focuses particularly on the prognosis algorithm description. It begins with a brief overview of the related work in diagnosis and prognosis on hybrid systems. Section 3 gives an overview of our architecture for interleaving diagnosis and prognosis processes. The diagnosis process is briefly described. Then section 4 presents the hybrid system modeling for prognosis. Section 5 is the core of the article. It details the prognosis algorithm. The algorithm has been implemented. Simulation results are given on Section 6, illustrated by a water tank system. Finally, Section 7 concludes the paper and proposes some future work.

## 2. RELATED WORK

There has been considerable work on diagnosis of hybrid systems on one hand, and on prognosis on the other hand. However, to the best of our knowledge, very few studies succeed in coupling diagnosis and prognosis and the authors could not find related work dealing with prognosis on hybrid systems.

A formal generic modeling framework for a complex system is presented in Ribot, Pencolé, and Combacau (2009) that encapsulates the knowledge used by diagnosis and prognosis. In this work, the authors establish a coupling of diagnosis and prognosis based on a characterization of complex system modes but no algorithm and implementation have been proposed. Another approach has been proposed in Roychoudhury and Daigle (2011). The authors propose a common framework for diagnosis and prognosis thanks to a state representation that describes the nominal behavior of the system and fault progression. However, there is absolutely no hybrid or discrete aspect in this work. The model used is a state model that specifies the system behavior in nominal modes and in faulty modes. A parameter vector and an associated evolution equation are used to represent fault progression over time. The method consists in building an observer from the nominal behavior to perform fault detection. The identification is made from a set of observers that are built for each fault. Prognosis consists in predicting the remaining useful life (RUL) for each fault using an estimator based on a fault progression model.

Most of the works on discrete event systems consider prognosis as a prediction of an event trajectory or fault event occurrences (Genc & Lafortune, 2009). The term "predictability" of a fault event introduced by Cao (1989) is based on the system observability property. It is clearly related to the diagnosability notion in discrete event systems: "it is certain that a critical event will take place". Genc and Lafortune (2009) demonstrates that any predictable event is diagnosable. An extension for the prediction of event patterns is proposed in Jéron, Marchand, Genc, and Lafortune (2008). In these studies, the system model is a classical automaton in which only ordered, undated and without delay event sequences are considered.

To perform prognosis, it is required to take the temporal aspect into account to compute the RUL of the system. Only Khoumsi (2009) uses a timed automata (TA) in order to prognose a fault event on the system. Clock ticks are added to transitions of the TA to determine the dated trajectories leading to fault events. No notion of uncertainty (neither by mean of probabilities nor intervals) is taken into account in these timed automata. However, uncertainty is intrinsically linked to prognosis.

In Zemouri and Faure (2006), the evolution of the system operating state is modeled by a stochastic timed automaton (STA). A stochastic distribution $f(t)$ is associated to each transition of the automaton. The distribution $f(t)$ gives the probability of occurrence for $x_{j+1}$ at time $t_{j+1}$ after the occurrence of $x_j$ at time $t_j$. In this study, events occurring in the system are represented in the states of the stochastic timed automaton that does not take the hybrid dynamics of the system into account.

Castaneda, Aubry, and Brinzei (2010) proposes a stochastic hybrid automaton to evaluate the system dynamic reliability. The stochastic hybrid automaton represents the possible behavioral modes of the system. The stochastic part helps to take faults and uncertainties about system knowledge into account. The system switches from one mode to another with events that may be deterministic or stochastic. Stochastic events occur when a threshold on their probability law has been reached. In this study, stochastic transitions have a constant rate. The model is simulated to obtain availability and reliability defined as the probabilistic evaluation of the hybrid system failure.

In order to complete previous works, the first issue investigated in this paper is the representation of the behavior of the hybrid system and the uncertainty of its degradation into a single model. The second issue concerns the implementation and the test of a health monitoring method coupling diagnosis and prognosis.

## 3. ARCHITECTURE FOR INTERLEAVING DIAGNOSIS AND PROGNOSIS PROCESSES

In this study, a system is supposed to start operating in nominal behavior. A set of critical faults has been anticipated for the system and each one of them may occur at any time from each nominal state. Faults are supposed to be permanent: it means that once a fault has occurred, the system evolves in what is called a *faulty mode*. This degradation can evolve into a worst degraded mode. Finally, when the system is not operational anymore, it is said to be in a *failure mode*. Obviously, it is possible that the system directly evolves from a nominal mode towards a failure mode. It is a particular case of what is described in the article. Without maintenance or repair action, the evolution of a system is then supposed to be unidirectional. This evolution of the system from a nominal mode to a failure mode is illustrated in Figure 1.

The combination of faults leading to a failure can be established from a fault tree analysis (Rausand & Hoyland, 2004). We define a prognosis as a prediction of the sequence of fault dates. With the analysis of the fault tree and what we call a prognosis, it is simple to obtain the system RUL that corresponds to the remaining time until the system failure. This fault analysis allows to link our prognosis definition to the one commonly used in the PHM community.

This section begins by describing the architecture interleav-

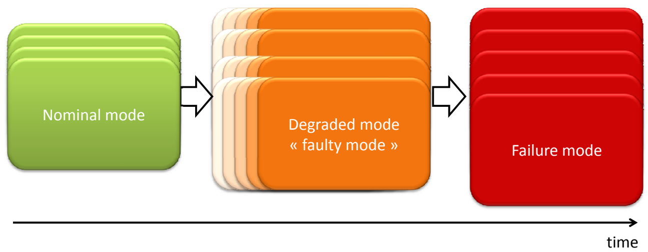



Figure 1. Unidirectional system evolution without maintenance or repair action.

ing diagnosis and prognosis. It will then go on the description of the diagnosis process.

### 3.1. Overview of the architecture

The originality of our work is to interleave diagnosis and prognosis processes that are too often considered separately. This section presents an architecture coupling diagnosis and prognosis into a single new process called *InterDP*. This architecture, illustrated in Figure 2, has already been described in Chanthery and Ribot (2013).

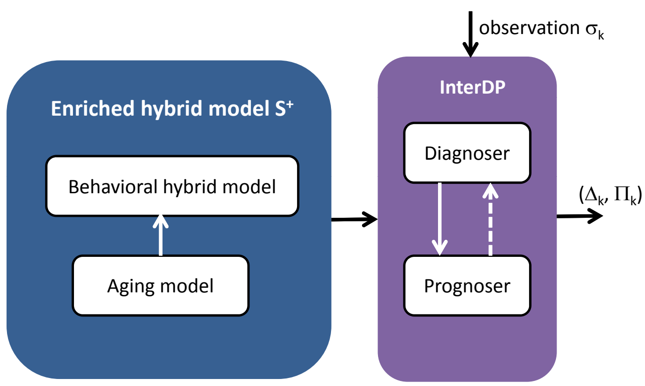



Figure 2. *InterDP* process interactions

Inputs of the *InterDP* process are an enriched hybrid model $S^+$ defined for our methodology and observations that will be formally defined in the next section. To put it in simple terms, the enriched hybrid model contains all the knowledge about the system behavior and degradation that is required to perform diagnosis and prognosis. Observations are a set of observable events. These events are built from information issued for example from the sensors that are embedded in the system.

The output of InterDP at each clock tick $t_k$ is a couple $(\Delta_k, \Pi_k)$ of diagnosis and prognosis results. Diagnosis $\Delta_k$ is performed by a hybrid diagnoser (Bayoudh, Travé-Massuyès, & Olive, 2008). It is built off-line from the hybrid automaton part of the enriched hybrid model. On-line, it takes as input the observations and provides a vector containing all possible modes (nominal, faulty or failure modes) for the system that are consistent with observations:

$$\Delta_k = \begin{bmatrix} \Delta_k^1 \\ \Delta_k^2 \\ \vdots \\ \Delta_k^D \end{bmatrix}$$

where $D$ is the number of diagnosis hypotheses. $\Delta_k$ can be seen as a belief state of the hybrid system. The diagnoser may take also as input information from the prognoser (dashed arrow in the figure), in order to disambiguate some diagnosis hypotheses. This link is not the core of this article and will be studied in future works.

Prognosis $\Pi_k$ is performed by a prognoser. On-line, at each clock tick $t_k$, the prognoser takes as input the enriched hybrid model and the result of the diagnosis process and provides a vector:

$$\Pi_k = \begin{bmatrix} \Pi_k^1 \\ \Pi_k^2 \\ \vdots \\ \Pi_k^D \end{bmatrix}$$

where $\Pi_k^i$ is associated to a diagnosis $\Delta_k^i$ and represents the most likely sequence of dated faulty modes leading to the system failure:

$$\Pi_k^i = (\{f_1, d_1\}, \ldots, \{f_j, d_j\}, \ldots, \{f_N, d_N\}).$$

where $d_i$ is the date occurrence of fault $f_j$ and $N$ represents the number of degraded modes before the failure mode.

One hypothesis in our work is that the system is assumed to be diagnosed after each new observation, that is to say when a change in measurements is detected. As diagnosis consists in monitoring the diagnoser, the diagnosis computation duration can be considered as instantaneous. It is also supposed that between two different observations, both diagnosis and prognosis can be performed. Let $t_k$ be an occurrence of an observation, $t_{k+1}$ be the occurrence of the next observation and $CT_p$ be the computation time for prognosis.

**Hypothesis 1.** *The computation time of the prognosis process is smaller than the interval between two different observations.*

$$CT_p \leq (t_{k+1} - t_k) \tag{1}$$

### 3.2. Diagnosis process

The diagnosis process in *InterDP* has been described in detail in Chanthery and Ribot (2013) and is not the focus point of this paper. We just recall here the main steps of the process. The hybrid diagnoser is built from the hybrid behavioral automaton of $S^+$ that is formally defined is the next section. On-line it takes as input the set of observations on the system. The diagnosis method for hybrid systems that is adopted for our approach is the one developed in Bayoudh (2009).

It interlinks a standard diagnosis method for continuous systems, namely the parity space method, and a standard diagnosis method for DES, namely the diagnoser method (Sampath, Sengputa, Lafortune, Sinnamohideen, & Teneketsis, 1995). The diagnosis part of the methodology may be decomposed into three parts:

- diagnose the continuous part of the system,
- abstract the continuous part in terms of discrete events and enrich the discrete part of the system with discrete events that come from the abstraction of the continuous part,
- then apply the diagnoser method on the resulting discrete event system in order to build a diagnoser able to follow on-line the behavior of the system according to the observable events.

## 4. Hybrid system modeling for prognosis

### 4.1. Hybrid formalism

The modeling framework that is adopted for hybrid systems is based on a hybrid automaton (Henzinger, 1996). The hybrid automaton is defined as a quintuple $S = (\zeta, Q, \Sigma, T, C, (q_0, \zeta_0))$ where:

- $\zeta$ is the set of continuous variables that comprises input variables $u(t) \in R^{n_u}$, state variables $x(t) \in R^{n_x}$, and output variables $y(t) \in R^{n_y}$. The set of directly measured variables is denoted by $\zeta_{OBS}$.
- $Q$ is the set of discrete system states. Each state $q_i \in Q$ represents a behavioral mode of the system. It includes nominal and anticipated faulty modes, including failure modes. The anticipated faulty modes are faulty modes that are known to be possible on the system. The unknown mode can be added to model all the non-anticipated faulty situations.
- $\Sigma$ is the set of events that correspond to discrete control inputs, autonomous mode changes and fault occurrences. $\Sigma = \Sigma_{uo} \cup \Sigma_o$, where $\Sigma_o \subseteq \Sigma$ is the set of observable events and $\Sigma_{uo} \subseteq \Sigma$ is the set of unobservable events.
- $T \subseteq Q \times \Sigma \to Q$ is the partial transition function. The transition from mode $q_i$ to mode $q_j$ with associated event $\sigma_{ij}$ is noted $t(q_i, \sigma_{ij}, q_j)$ and we have $T(q_i, \sigma_{ij}) = q_j$. $T$ also denotes the set of transitions.
- $C = \bigcup_i C_i$ is the set of system constraints linking continuous variables. $C_i$ denotes the set of constraints associated to the mode $q_i$. $C$ represents the set of differential and algebraic equations modeling the continuous behavior of the system. The continuous behavior in each mode is assumed to be linear.
- $(\zeta_0, q_0) \in \zeta \times Q$, is the initial condition.

The occurrence of a fault is modeled by a discrete event $f_i \in \Sigma_F$. $\Sigma_F$ is the set of fault events associated to the anticipated faults of $F$. Without loss of generality it is assumed that $\Sigma_F \subseteq \Sigma_{uo}$. The discrete part of the hybrid automaton is given by $M = (Q, \Sigma, T, q_0)$, which is called the *underlying discrete event system (DES)* and the continuous behavior of the hybrid system is modeled by the so-called *underlying multi-mode system* $\Xi = (\zeta, Q, C, \zeta_0)$. An example of a hybrid system is given in Figure 3.

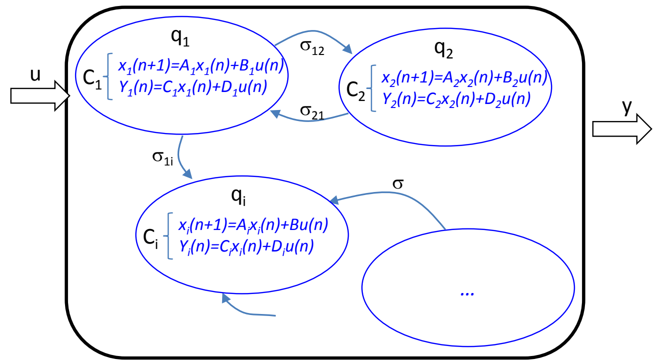



Figure 3. Example of an hybrid system

This hybrid automaton describes the set of knowledge useful to achieve model-based diagnosis. In order to perform prognosis, it is necessary to enrich the hybrid model by adding the available knowledge about the aging or the degradation of the system. A way to take the uncertainty on the degradation function into account is to introduce probability measures for each state that represents a mode of the system.

### 4.2. Aging modeling

The modeling framework that is adopted for the system degradation is based on the Weibull probabilistic model (Ribot & Bensana, 2011). A particular way for representing the remaining useful life of systems is to establish a fault probability from reliability analyses at different stress levels (operating conditions) (Hall & Strutt, 2003; Vachtsevanos, Lewis, Roemer, Hess, & Wu, 2006). Stress is defined as the set of internal and external conditions/factors that may have an impact on the system behavior. The parametrized Weibull model is often used in reliability for life data analyses due to its flexibility (Ferreiro & Arnaiz, 2008):

$$W(t, \beta, \eta, \gamma) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{(\beta-1)} e^{-(\frac{t-\gamma}{\eta})^{\beta}} \quad (2)$$

where $t \geq 0$, $\beta \geq 0$, $\eta \geq 0$ and $\gamma \in [-\infty; \infty]$. The scale characteristic $\eta$ defines the characteristic life of the system and corresponds to the mean life expectancy for a studied population sample. The shape characteristic $\beta$ modifies the probability density function (pdf) nature and allows to model the different life phases of a system defined by the idealized bathtub curve of reliability. The location characteristic $\gamma$ shifts the curve from the origin. It defines the system minimal life. The case $\gamma > 0$ means that the fault probability is zero until a date $\gamma$. In most cases, we assume $\gamma = 0$. The characteristic $\eta$ is stress-dependent while $\beta$ is assumed to remain constant across different stress levels.

Weibull characteristics $\beta_j^{q_i}, \eta_j^{q_i}, \gamma_j^{q_i}$ model the aging evolution of a system that leads to a fault $f_j$ in a behavioral mode $q_i$ and totally define the fault probability distribution $f_j^{q_i}$:

$$f_j^{q_i}(t) = \int_0^t W(t, \beta_j^{q_i}, \eta_j^{q_i}, \gamma_j^{q_i})dt. \quad (3)$$

The fault probability density function $W(t, \beta_j^{q_i}, \eta_j^{q_i}, \gamma_j^{q_i})$ has to give at any time the probability that the fault $f_j$ occurs in the system from a mode $q_i$. Weibull characteristics $\beta_j^{q_i}$ and $\eta_j^{q_i}$ are fixed by the mode $q_i$ of the system. The location characteristic $\gamma_j^{q_i}$ can be used to memorize the degradation evolution of the system in the past modes from the operation start of the system (Ribot & Bensana, 2011). At first, the system is in a mode $q_0$. If the system has never been used, $q_0$ obviously represents the nominal mode and we suppose that $\forall f_j, \;\; \gamma_j^{q_0} = 0$ as previously explained. This characteristic $\gamma_j^{q_i}$ will be modified to take degradation in each behavioral mode into account during the system operation.

The occurrence date $d_{f_j}$ of a fault event $f_j$ for the system in mode $q_i$ can be determined from a decision criterion $P_{max_{f_j}}$ that corresponds to a probability threshold beyond which the risk becomes unacceptable:

$$d_{f_j} \text{ such that } \int_0^{d_{f_j}} W(t, \beta_j^{q_i}, \eta_j^{q_i}, \gamma_j^{q_i})dt = P_{max_{f_j}}. \quad (4)$$

### 4.3. Enriched hybrid model

In each mode $q_i$, the system is subject to different aging laws. The set of aging laws is supposed to be accurately known.

**Hypothesis 2.** *An aging law of a system is supposed to be continuous over time.*

The consequence of this hypothesis is that the initial condition for an aging law at time $t^+_{k+1}$ is the value at $t^-_{k+1}$, when the system has not yet commuted between two modes.

To take into account the different aging laws, the hybrid system is then described as an enriched hybrid automaton $S^+ = (\zeta, Q, \Sigma, T, C, \mathcal{F}, (q_0, \zeta_0))$, where $\mathcal{F} = \{F^{q_i}, i \in \{1, \ldots, card(Q)\}\}$ is the set of aging laws associated to behavioral modes or the system. $F^{q_i}$ is a vector of aging laws for each anticipated fault in the mode $q_i$. For example, in a system where $N_F$ faults are considered:

$$F^{q_i}(t) = \begin{bmatrix} f_1^{q_i}(t) \\ f_2^{q_i}(t) \\ \ldots \\ f_{N_F}^{q_i}(t) \end{bmatrix} \quad (5)$$

where $f_j^{q_i}$ represents the probability distribution of the fault $f_j$ at any time in mode $q_i$.

It can be noticed that as opposed to Ribot and Bensana (2011), the hybrid automaton we propose represents behavioral modes and not operational modes based on function availability.

For example, for a system with two nominal modes $q_{01}$, $q_{02}$, two possible actions $a_1$, $a_2$ that are observable events, and two faults $f_1$ and $f_2$, a possible model is given in Figure 4. This system is in a failure mode when $f_1$ and $f_2$ have occurred. If only one fault occurred, then the system is in a faulty mode.

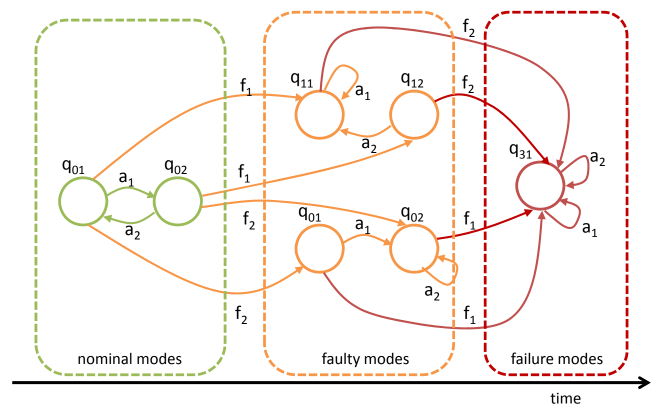



Figure 4. Example for a system with 2 nominal modes

## 5. Prognosis

The focus of this article is on the prognosis process. It consists, at each clock tick $t_k$, in computing the most likely fault sequence $\Pi_k$ until the system failure. Algorithm 1 describes how the prognosis process is structured and introduces the three main functions of the process.

Prognosis takes as input information on aging laws in $S^+$ associated to the set of anticipated faults $\mathcal{F}$ and the set of behavioral models $Q$. It takes also as input the current result of the diagnosis $\Delta_k$ to update on-line the system aging laws according to the operation time in each behavioral mode. After each observable event, the appropriate aging laws are selected (line 11) according to the mode that is estimated by the diagnosis and the fault probability value reached in previous modes. Then the prognosis process predicts the most probable fault sequence (lines 6 to 14) supposing that the system

**Algorithm 1**: Prognosis of a hybrid system

**Inputs:** enriched model $S^+$, on-line diagnosis $\Delta_k$
**Outputs:** $RUL$, fault sequences $\Pi_k$

1: k = 0
2: $q- \leftarrow q_0$ {Mode initialization}
3: **for** each anticipated fault $f_j \in \Sigma_f$ **do**
4: $(f_j^{q-}, d_{f_j}) \leftarrow$ InitializeAgingParameters($q-$)
5: **end for**
6: $(\Pi_0, RUL) \leftarrow$ PredictFaultSequence($S^+$, $q-$, $\{d_{f_j}\}$)
7: **repeat**
8: k++
9: **if** $\Delta_k \neq q-$ **then**
10: $q+ \leftarrow \Delta_k$
11: $\{(f_j^{q+}, d_{f_j})\} \leftarrow$ UpdateAgingParameters($S^+, q-, q+$)
12: $q- \leftarrow q+$
13: $(\Pi_k, RUL) \leftarrow$ PredictFaultSequence($S^+$, $q-$, $\{d_{f_j}\}$)
14: **end if**
15: **until** $RUL = 0$

remains in the current mode. It is possible to ”disconnect” prognosis from the diagnosis in the architecture and to execute the prognoser module independently. Obviously in this case the system is supposed to remain in the current mode.

Thus three main functions may be distinguished: the aging parameters initialization (line 3), the aging parameters update (line 11) and the fault sequence prediction (lines 6 to 14). The following subsections describe precisely how the prognosis process is built and interleaved with diagnosis.

### 5.1. Aging function parameters initialization

The system is degrading in different ways leading to fault occurrences that may provoke a failure. This degradation depends on the mode of the hybrid system. For each mode, the degradation embodies the impact of stress factors. We recall that the aging dynamic of the system exposed to a fault $f_j$ in mode $q_i$ is modeled by a set of Weibull parameters $\beta_j^{q_i}$,$\eta_j^{q_i}$ and $\gamma_j^{q_i}$ in the enriched model $S^+$.

For the simplicity of the presentation, we first assume that there is no problem of diagnosability in the system that is studied. It means that at each clock tick, the system mode is totally known, i-e. non ambiguous and is given by the diagnosis process. This is a high hypothesis, and the case of ambiguity in the system state has to be studied in the future. Nevertheless, an easy solution to transform an ambiguous case into a non ambiguous one is to consider that the system is in its most probable state.

When the prognosis process is started, the system is in initial state $(q_0, \zeta_0)$. The aging law $f_j^{q_0}(t)$ associated to each anticipated fault $f_j$ is initialized. Parameters $\beta_j^{q_0}$ and $\eta_j^{q_0}$ are assumed to be fixed and derived from reliability analyses. If the studied system has never been used before, $q_0$ obviously represents the nominal mode and we suppose that $\forall f_j,\ \gamma_j^{q_0} = 0$ as previously explained. If $q_0$ is not nominal, the enriched model has to give information about the initial values of $\gamma_j^{q_0}$.

The occurrence date $d_{f_j}$ of each anticipated fault $f_j$ for the system in mode $q_0$ is then determined from a decision criterion $P_{max_{f_j}}$ :

$$\int_0^{d_{f_j}} W(t, \beta_j^{q_0}, \eta_j^{q_0}, \gamma_j^{q_0}) dt = P_{max_{f_j}}. \tag{6}$$

All along its operation, the aging probability of the system exposed to a fault $f_j$ is denoted by $P_{f_j}$. It is evaluated on-line with diagnosis and predicted to determine the RUL. Knowing the aging dynamic of the system in the initial mode and the different aging threshold, the prognosis process predicts the most probable fault sequence until the failure mode. This prediction process is done each time the diagnosis process updates the current mode of the system. The next two sections describe how the aging parameters need to be updated after a new diagnosis result and how fault sequences are then predicted.

### 5.2. Aging parameters update

Algorithm 2 describes how the aging functions are updated after each new diagnosis result.

**Algorithm 2**: UpdateAgingParameters

**Inputs:** Enriched model $S^+$, previous behavioral mode $q-$, new behavioral mode $q+$ provided by diagnosis
**Outputs:** New aging laws $f_j^{q+}$ for each anticipated fault $f_j$, new dates of fault occurrences $d_{f_j}$

**for** each anticipated fault $f_j \in \Sigma_f$ **do**
$P_{f_j} \leftarrow$ ComputeAgingProbability($f_j^{q-}$) {with Equation (7)}
$\gamma_j^{q+} \leftarrow$ ComputeLocationParameter($P_{f_j}$,$q+$) {with Equations (8) (9)}
**end for**

On receipt of a new mode estimation $q+$ at time $t^+$, the Weibull aging functions associated to faults in the mode $q+$ are updated according to the time spent by the system in previous mode $q-$. The aging probability associated to a fault $f_j$ that the system has reached in past mode(s) at $t^+$ is computed with

$$P_{f_j} = \int_0^{t^+} W(t, \beta_j^{q-}, \eta_j^{q-}, \gamma_j^{q-}) \mathrm{d}t. \tag{7}$$

To memorize this aging probability $P_{f_j}$, a new value for characteristic $\gamma_j^{q+}$ of aging model associated to the fault $f_j$ in the new mode $q+$ is computed, knowing the values of $\beta_j^{q+}$ and

$\eta_j^{q+}$ thanks to the enriched model:

$$\gamma_j^{q+} = (t^+ - \delta) \text{ such that}$$
$$\int_0^{\delta} W(t, \beta_j^{q+}, \eta_j^{q+}, 0)\mathrm{d}t = P_{f_j}. \quad (8)$$

With the above equation, we introduce a mathematical manipulation to memorize the aging probability $P_{f_j}$ reached in past modes into new aging models for mode $q+$ from $t^+$. By this calculation, the continuity of any aging function is guaranteed in all mode change points and

$$\int_0^{t^+} W(t, \beta_j^{q+}, \eta_j^{q+}, \gamma_j^{q+})\mathrm{d}t = P_{f_j} \quad (9)$$

Figure 5 illustrates how Weibull pdf associated to a fault $f_1$ evolves in order to describe the fault aging in two behavioral modes $q_{01}$ and $q_{02}$. The two curves on the top illustrate the Weibull pdf associated to fault $f_1$ in mode $q_{01}$ (on the left) and in mode $q_{02}$ (on the right). Let suppose that the system is in mode $q_{01}$ at $t = 0$, then $P_{f_1}$ corresponds to the area under the curve. When the system switches into mode $q_{02}$, the continuity condition on the aging law (Hypothesis 2) implies that the area under the first curve is equal to the area under the second one. So the second curve is translated on the left.

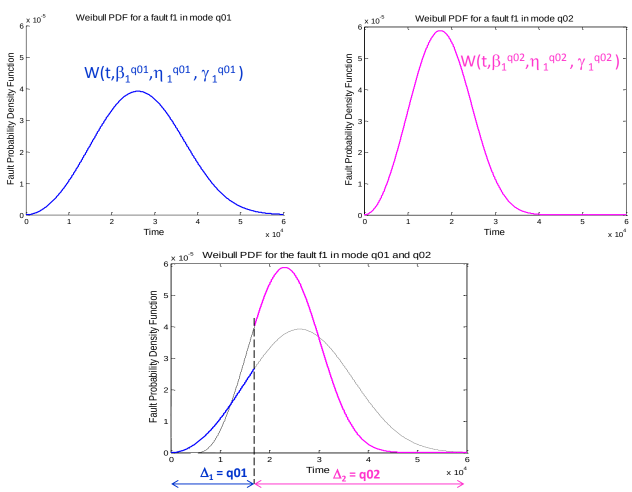



Figure 5. Weibull pdf associated to a fault $f_1$ for a system in two behavioral modes

## 5.3. Fault sequence prediction

To determine the sequence of fault events that leads to the system failure, we need to define a discrete fault event automaton (line 4) to extract the system faulty evolutions. This automaton describes all the possible fault transitions between operating modes until the system failure. The Discrete Fault Event System (DFES) is obtained by the projection of the underlying DES on the fault events (see Section 4.1). It corresponds to an abstraction of the hybrid system which contains fault events only. The DFES is a finite state machine $M_f = (Q_f, \Sigma_f, T_f, q_{f0})$ formally defined as follows:

- $Q_f$ is the set of discrete states of the system,
- $\Sigma_f$ is set of the fault events,
- $T_f \subseteq (Q_f \times \Sigma_f \to Q_f)$ is the partial transition function,
- $q_{f0} = q_0$ is the initial state corresponding to the nominal mode of the system.

The DFES of the example described in Figure 4 is illustrated by Figure 6.

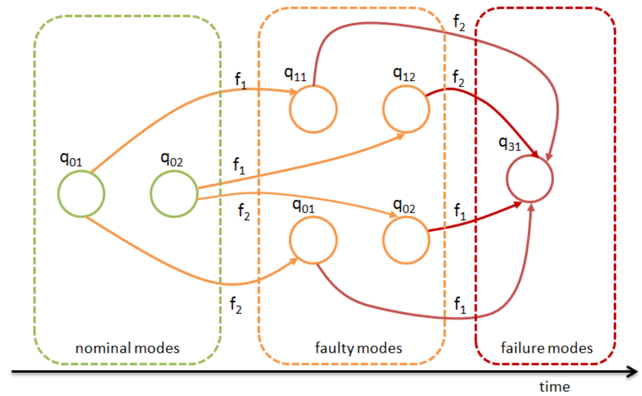



Figure 6. The DFES

Once a mode change is detected by diagnosis, the idea is to run through the DFES and predict the fault sequences until the system failure. A state $q_c$ in a DFES is a deadlock if

$$\forall f \in \Sigma_f,\ T_f(q_c, f) = \emptyset \quad (10)$$

The stop criterion for the prediction function is a dead-lock (for example mode $q_{31}$ in Figure 6).

For each fault $f_j$ that has not occurred yet, the algorithm computes the date of occurrence of $f_j$. The minimum value in this set of dates is denoted $d_{min_1}$: it corresponds to the date of occurrence of the next fault, denoted $f_{min_1}$ (line 6). The set of faults whose occurrence has not been predicted yet is denoted by $\Sigma_{ff}$, then $\Sigma_{ff}$ is updated and $\Sigma_{ff} = \Sigma_f \backslash f_{min_1}$ (line 7). It then contains faults whose occurrence date is superior to $d_{min_1}$. At $d_{min_1}$, the system is predicted to switch into fault mode $q_{f_{min_1}}$. New aging models in fault mode $q_{f_{min_1}}$ (described by the Weibull pdf $W(t, \beta_j^{q_{f_{min_1}}}, \eta_j^{q_{f_{min_1}}}, \gamma_j^{q_{f_{min_1}}})$) have to be updated for each fault $f_j$ in $\Sigma_{ff}$ (line 12). The mode change predicted at $d_{min_1}$ may result in a modification of fault dates $\{d_{f_j}\}$ that have been previously computed.

As for an aging parameter update resulting from a change in diagnosis, characteristic $\gamma_j^{q_{f_{min_1}}}$ of aging models in future mode $q_{f_{min_1}}$ has to be computed from the fault probability $P^1_{f_j}$ the system could have reached at predicted commutation time $d_{min_1}$. Let $q_c$ denotes the current system mode, for each fault $f_j$ in $\Sigma_{ff}$ :

$$P_{f_j} = \int_0^{d_{min_1}} W(t, \beta_j^{q_c}, \eta_j^{q_c}, \gamma_j^{q_c})\mathrm{d}t, \quad (11)$$

and $\gamma_j^{q_{f_{min_1}}} = (d_{min_1} - \delta)$ such that

$$\int_0^{\delta} W(t, \beta_j^{q_{f_{min_1}}}, \eta_j^{q_{f_{min_1}}}, 0)\mathrm{d}t = P_{f_j}. \quad (12)$$

Characteristic $\gamma_j^{q_{f_{min_1}}}$ allows to memorize the system aging in all past modes from $q_0$ and guarantee the continuity of aging laws. The date $d_{f_j}$ of fault occurrences in $\Sigma_{ff}$ are modified as follows:

$$\int_0^{d_{f_j}} W(t, \beta_j^{q_{f_{min_1}}}, \eta_j^{q_{f_{min_1}}}, \gamma_j^{q_{f_{min_1}}})dt = P_{max_j}. \quad (13)$$

The next possible fault $f_{min_2}$ after $f_{min_1}$ is determined from the minimal predicted fault date $d_{min_2}$ for faults in $\Sigma_{ff}$. Then $\Sigma_{ff} = \Sigma_{ff} \backslash f_{min_2}$. Fault propagation is studied as explained above to compute $\gamma_j^{q_{f_{min_2}}}$ for faults that have not reached their probability threshold at $d_{min_2}$ using new aging models for mode $q_{f_{min_2}}$ and the process reiterates.

The prognosis process computes the most likely future sequence $\Pi_k$ of dated fault events according to a diagnosis $\Delta_k$:

$$\Pi_k = (\{f_{min_1}, d_{min_1}\}, \{f_{min_2}, d_{min_2}\}, \ldots, \{f_{min_N}, d_{min_N}\}). \quad (14)$$

Algorithm 3 sums up the procedure of fault sequence prediction.

**Algorithm 3**: PredictFaultSequence

**Inputs:** Enriched model $S^+$, Current mode $q_c$, Dates of fault occurrence $\{d_{f_j}\}$
**Outputs:** Fault sequence $\Pi_k$, RUL

$\Pi_k = \emptyset$
$\Sigma_{ff} \leftarrow \Sigma_f$ {$\Sigma_{ff}$ is the set of faults to be predicted}
$i \leftarrow 1$
ConstructDFES($S^+$)
**while** $\exists f \in \Sigma_{ff} \mid T_f(q_c, f) \neq \emptyset$ {$q_c$ is not a dead-lock} **do**
  $(f_{min}(i), d_{min}(i)) \leftarrow$ PredictNextFault($\Sigma_{ff}$, $\{d_{f_j}\}$)
  $\Sigma_{ff} \leftarrow \Sigma_{ff} \backslash f_{min}$
  $\Pi_k = \Pi_k \cup \{(f_{min}(i), d_{min}(i)\}$
  RUL $\leftarrow d_{min}(i)$
  $q_f \leftarrow T_f(q_c, f_{min}(i))$ {System is predicted to switch in mode $q_f$ at $d_{min}$}
  **for** each anticipated fault $f_j \in \Sigma_{ff}$ **do**
    $(f_j^{q_f}, d_{f_j}) \leftarrow$ UpdateAgingParameters($q_c$,$q_f$) {with Equations (11) (12) (13)}
  **end for**
  $i \leftarrow i + 1$
  $q_c \leftarrow q_f$
**end while**

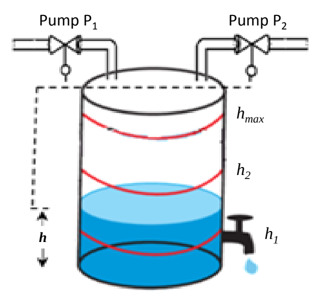


Figure 7. Water tank system

## 6. Experimental results

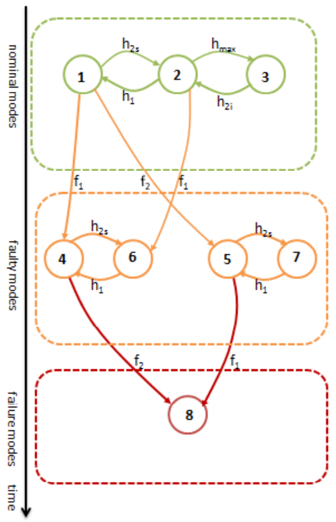
HYDIAG is a software program on MATLAB developed by the DISCO team. It performs diagnosis of hybrid systems Bayoudh et al. (2008). The idea was to enrich it with Weibull aging models to performs prognosis, to implement the prognosis algorithm and interleave diagnosis and prognosis processes into a single one module named *InterDP*. This has been implemented and tested on a water tank system.

### 6.1. Modeling of a Water tank system

*InterDP* has been applied to the case study of a water tank system (Figure 7). This system is composed of one tank with two hydraulic pumps ($P_1$, $P_2$) providing water with different rates. Water flows through a valve at the bottom of the tank depending on the system control. Three sensors located at different tank levels ($h_1$, $h_2$, $h_{max}$) detect the water level and allows to set the control of the pumps (on/off). If the water level $h$ is between $h_1$ and $h_2$, both pumps $P_1$ and $P_2$ are turned on. If $h_2 < h < h_{max}$, only $P_1$ is on and when $h \leq h_{max}$, the pumps are turned off. It is assumed that the pumps may fail only if they are on.

The discrete model of water tank and the controls of pumps are given in Figure 8. Eight discrete behavioral modes has been identified $Q = \{q_1, q_2, \ldots, q_8\}$. Discrete events in $\Sigma = \{h_1, h_{2s}, h_{2i}, h_{max}, f_1, f_2\}$ allow the system to switch into different modes. Observable events are $\Sigma_o = \{h_1, h_{2s}, h_{2i}, h_{max}\}$. They result from sensor measurements of water level in the tank. Two faults are anticipated $\Sigma_f = \{f_1, f_2\}$ and are not observable: $f_1$ corresponds to the failure of $P_1$ and $f_2$ is the failure of $P_2$. The parameter values of aging models $\mathcal{F} = \{F^{q_i}\}$ associated to every behavioral mode $q_i$ are reported in Table 1. Let us recall that the Weibull characteristics $\gamma$ is assumed to be equal to zero at the system start.

The underlying continuous behavior of every discrete mode




| pump / mode | Pump1 | Pump 2 |
|---|---|---|
| 1 | ON | ON |
| 2 | ON | OFF |
| 3 | OFF | OFF |
| 4 | Fail | ON |
| 5 | ON | Fail |
| 6 | Fail | OFF |
| 7 | OFF | Fail |
| 8 | Fail | Fail |

Figure 8. Water tank DES model

Table 1. Weibull parameters of aging models

| **Aging laws** | | $\beta$ | $\eta$ |
|---|---|---|---|
| $F^{q_1}$ | $f_1^{q_1}$ | 1.5 | 3000 |
| | $f_2^{q_1}$ | 1.5 | 4000 |
| $F^{q_2}$ | $f_1^{q_2}$ | 2 | 3000 |
| | $f_2^{q_2}$ | 1 | 7000 |
| $F^{q_3}$ | $f_1^{q_3}$ | 1 | 8000 |
| | $f_2^{q_3}$ | 1 | 7000 |
| $F^{q_4}$ | $f_1^{q_4}$ | NaN | NaN |
| | $f_2^{q_4}$ | 2 | 4000 |
| $F^{q_5}$ | $f_1^{q_5}$ | 2 | 3000 |
| | $f_2^{q_5}$ | NaN | NaN |
| $F^{q_6}$ | $f_1^{q_6}$ | NaN | NaN |
| | $f_2^{q_6}$ | 1 | 7000 |
| $F^{q_7}$ | $f_1^{q_7}$ | 1 | 8000 |
| | $f_2^{q_7}$ | NaN | NaN |
| $F^{q_8}$ | $f_1^{q_8}$ | NaN | NaN |
| | $f_2^{q_8}$ | NaN | NaN |

$q_i$ for $i \in \{1..8\}$ is represented by the same state space:

$$\begin{cases} X(k+1) &= AX(k) + BU(k) \\ Y(k) &= CX(k) + DU(k) \end{cases} \tag{15}$$

where the state variable $X$ is the water level in the tank, continuous inputs $U$ are the flows delivered by the pumps $P_1$, $P_2$ and the flow going through the valve, $A = (1)$, $B = \begin{pmatrix} eTe/S \\ eTe/S \\ eTe/S \end{pmatrix}$ with $Te$ the sample time, $S$ the tank base area and $e_i = 1$ (resp. 0) if the pump is turned on (resp. turned off), $C = (1)$ and $D = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

The continuous behavior is abstracted to build new observable discrete events $Rox\_y$ using the parity space approach. The enriched discrete event model of the hybrid system is used to build the diagnoser that will allow to track the system mode after each new observation.

The process *InterDP* was tested on this water tank hybrid model. Both diagnosis and prognosis are performed.

### 6.2. Simulation results

#### 6.2.1. Simulation parameters

The time horizon is fixed at $T_{sim} = 4000$h, the sampling period is $T_s = 36$s and the filter sensitivity for the diagnosis is set as $T_{filter} = 3$min. The residual threshold is $10^{-12}$ as in Bayoudh (2009).

The scenarios involve a variant use of water (max flow rate = 1200L/h) depending on user needs during 4000h. Pumps are automatically controlled to satisfy the specifications indicated above. Flow rate of $P_1$ and $P_2$ are respectively 750L/h and 500L/h.

The diagnoser issued from the diagnosis process is given in Figure 9. Its computation is done off-line. Each state of the diagnoser indicates the belief state in the model enriched by the abstraction of the continuous part of the system, with a tag that gives the set of faults that have occurred on the system. This set is empty in case of nominal mode. This diagnoser shows that the tested system is diagnosable.

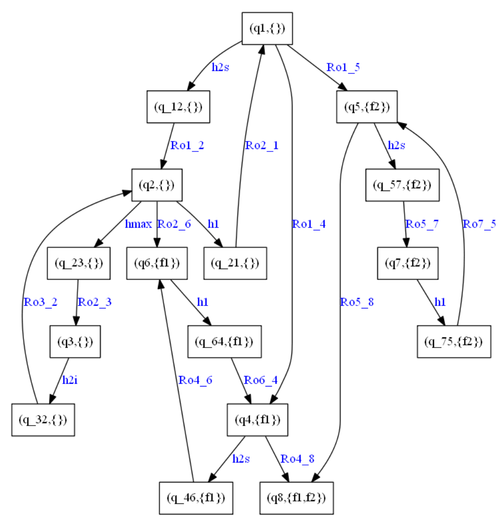



Figure 9. Diagnoser state tracker

Two fault scenarios have been simulated. In the Scenario 1, fault $f_1$ on the pump $P_1$ was injected after 3500h, fault $f_2$ is not injected. In the Scenario 2, fault $f_2$ is injected after 2000h, fault $f_1$ is not injected.

#### 6.2.2. Scenario 1

Figure 10 shows the diagnoser belief state for Scenario 1 just before and after the fault $f_1$ occurrence. Results are consistent with the scenario: before 3500h, the belief states of the diagnoser are always tagged with a nominal diagnosis. After 3500h, all the states are tagged with $f_1$.

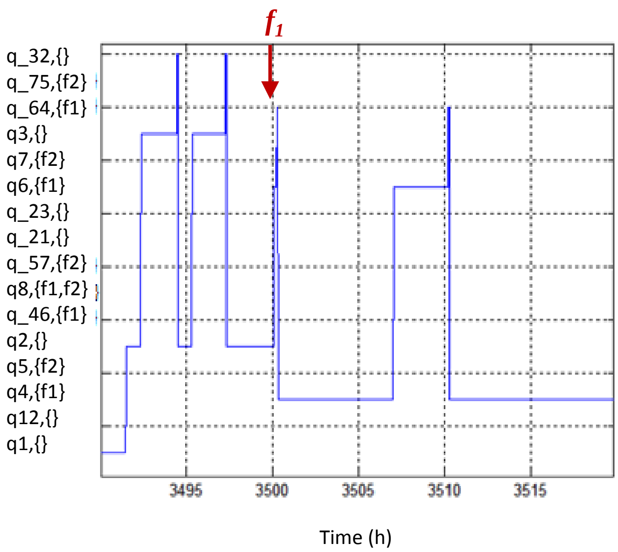



Figure 10. Diagnoser belief state for scenario 1

Figure 11 illustrates the predicted date of fault occurrence ($d_{f_1}$ and $d_{f_2}$). At the beginning of the process, the prognosis result is: $\Pi_0 = (\{f_1, 4120\}, \{f_2, 5105\})$. It can be noted that the predicted dates $d_{f_1}$ and $d_{f_2}$ of $f_1$ and $f_2$ globally increase. Indeed, the system oscillates between stressful modes and less stressful modes. To make it simple, we can consider that in some modes, the system does not degrade, so the predicted dates of $f_1$ and $f_2$ are postponed.

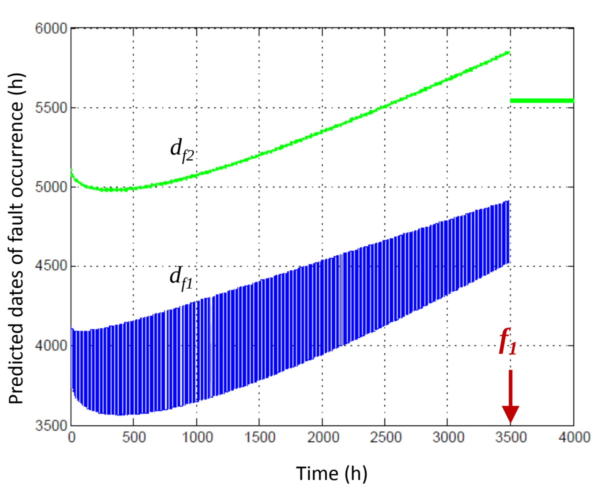



Figure 11. Prognosis results for scenario 1: $d_{f_1}, d_{f_2}$

Before 3500h, the predicted date of $f_1$ is lower than the one of $f_2$. After 3500h, the predicted date of $f_2$ is updated, knowing that the system is in a degraded mode. The prognosis result is $\Pi_{3501} = (\{f_2, 5541\})$.

Figure 12 shows the RUL of the system. Because of the oscillation of the system between stressful modes and less stressful modes, the RUL value oscillates between two values that globally decrease. The maximum bound corresponds to the less stressful mode whereas the minimum bound corresponds to the more stressful one. After the occurrence of the fault, the RUL is updated.

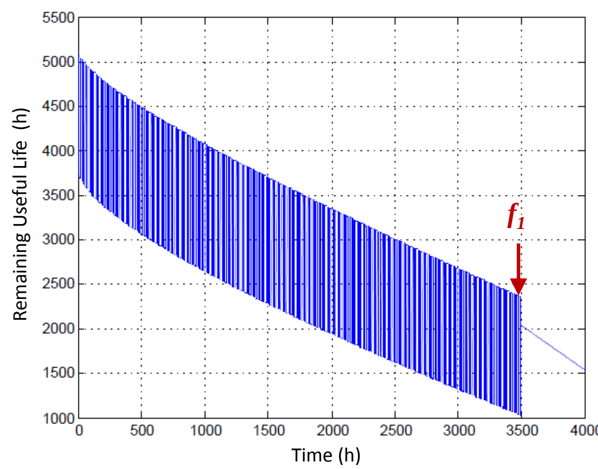



Figure 12. Prognosis results for scenario 1: RUL

#### 6.2.3. Scenario 2

Figure 13 shows the diagnoser belief state for Scenario 2 just before and after the fault $f_2$ occurrence. Results are coherent with the Scenario: before 2000h, the belief states of the diagnoser are always tagged with a nominal diagnosis. After 2000h, all the states are tagged with $f_2$.

Figure 14 illustrates the predicted date of fault occurrence ($d_{f_1}$ and $d_{f_2}$). At the beginning of the process, the prognosis result is: $\Pi_0 = (\{f_1, 4120\}, \{f_2, 5105\})$.

Before 2000h, the predicted date of $f_1$ is lower than the one of $f_2$. The prognosis result is After 2000h, the predicted date of $f_1$ is updated, knowing that the system is in a degraded mode. The prognosis result is $\Pi_{2010} = (\{f_1, 3947\})$.

Figure 15 shows the RUL of the system.

## 7. Conclusion and Future Work

This paper proposed an architecture of health monitoring and prognosis for hybrid systems. Diagnosis and prognosis are interleaved into a single process called *InterDP*. An original enriched hybrid model is proposed. It is used both for describing the hybrid behavior dynamic and for modeling the

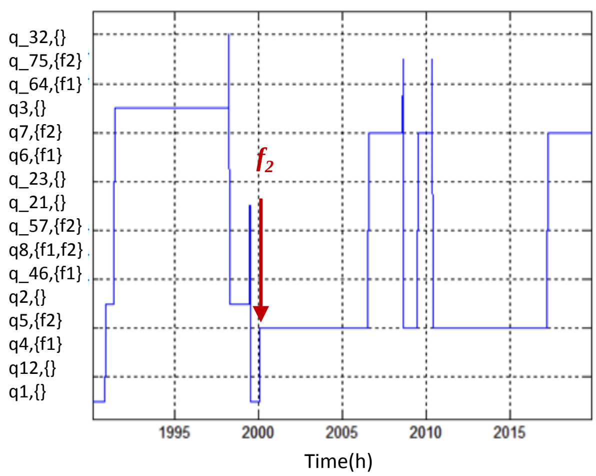



Figure 13. Diagnoser belief state for scenario 2

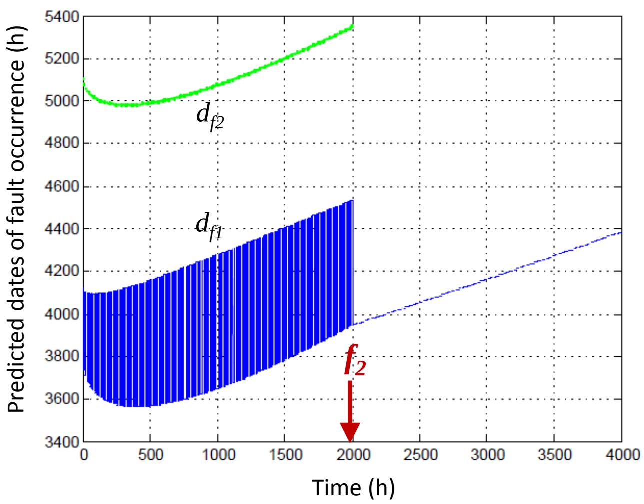



Figure 14. Prognosis results for scenario 2: $d_{f_1}$, $d_{f_2}$

degradation of the system. This paper is focused on the prognosis process whose algorithm is given in detail. An implementation has been done in Matlab and *InterDP* has been tested on a water tank system. Simulation results show how the prognosis is updated thanks to the diagnosis.

In future work, we will study the case of a non diagnosable system, where the diagnoser provides a set of belief states. This will detail the arrow from the prognoser to the diagnoser. It will lead to a complexity problem both for diagnosis and prognosis processes. If a fault sequence has to be predicted for each belief state, this could combinatorially explode. Another complexity problem concerns the number of anticipated faults in each behavioral state. For all these reasons, it seems rather complicated to fulfill the hypothesis of running both diagnosis and prognosis between two observations. A solution that will be developed is to do what we call "any-time diagnosis". It consists in elaborating a diagnosis in constrained

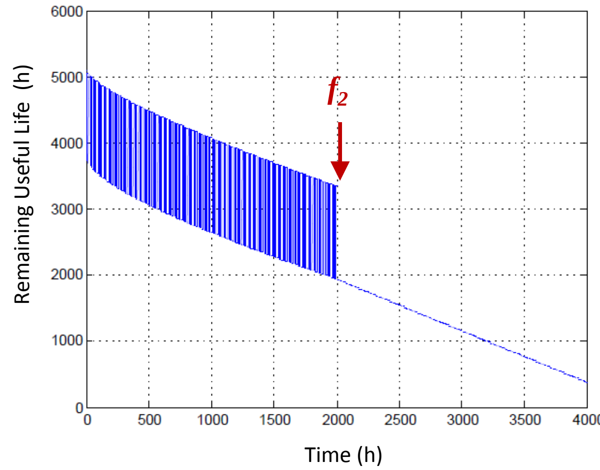



Figure 15. Prognosis results for scenario 2: RUL

time. The diagnoser gives its best result at the time it has to give a result. In the same time, the prognoser could also be improved by reducing the model size of the hybrid system, for example if only the most critical faults in each behavioral mode are taken into account.

Another perspective is a study on the model itself. A more detailed analysis of how to get a sufficient amount of information for the enriched model is required. Taking into account uncertainties on the available knowledge is a solution that will be investigated. Finally, future work will apply this approach to real-word examples. We plan to compare our approach with some others from the literature.

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