Distribution Free Prediction Interval for Uncertainty Quantification in Remaining Useful Life Prediction

Huimin Chen [1]

[1] *Department of Electrical Engineering, University of New Orleans, New Orleans, LA, 70148, U. S. A.*
hchen2@uno.edu

ABSTRACT

Remaining useful life (RUL) prediction is an important component for system health monitoring and prognosis. Ideally, one expects the prediction algorithm to provide the complete distribution of the RUL prediction over time taking various uncertainties into account. However, the dynamic model being used to characterize state estimation and future loading uncertainties is often simplified through various approximations, leading to non-credible predicted distribution. Nevertheless, certain algorithm may only provide a point estimate of the RUL, making it difficult to quantify the uncertainty of the prediction. In this paper, we focus on interval prediction with high probability that guarantees finite sample validity without the knowledge of statistical distribution of the noise. The key idea is to leverage the newly proposed conformal prediction framework with non-parametric conditional density estimation. Under certain regularity conditions, the proposed interval estimator converges to an oracle band at a minimax optimal rate. In addition, we apply a data driven method to automatically select the bandwidth in the kernel density estimator. We discuss practical approximations to speed up the computation. The proposed method can be used to predict the RUL interval with physics-based model in a distribution free manner. It can also be applied to assess the validity of other prognostic algorithms from experimental data. We demonstrate the effectiveness of the RUL prediction for Li-Ion batteries using both simulated and experimental data.

1. INTRODUCTION

Remaining useful life (RUL) prediction is an important component for prognosis and system health monitoring. Ideally, one expects the prediction algorithm to provide the complete distribution of the RUL prediction over time taking various uncertainties into account. However, the dynamic model being used to characterize state estimation and future loading uncertainties is often simplified with various approximations, resulting in non-credible predicted distribution. Nevertheless, certain algorithms often provide point estimates of the RUL, making it difficult to quantify the uncertainty of the prediction. Existing efforts for uncertainty quantification can be largely classified into two categories: point-based and density-based credibility tests. In point-based tests, one wants to find the probability that the RUL is longer than the estimate so that the critical component can be replaced before its failure. In many cases, one has to ensure that the probability is above a certain confidence level, say 0.95, in order to declare that the point estimate of the RUL is valid. Among all the valid RUL predictions, it is desirable to find the least conservative one, i.e., the largest RUL prediction for decision-making. However, a prediction point with a desirable confidence level does not fully characterize the uncertainty of the RUL estimate. The complete description requires the posterior probability density function of the RUL, which is hard to be fully specified when one has the uncertainty of future loading that will affect the system dynamics during the predicted time horizon. Density-based tests address whether the posterior distribution of the RUL provided by a prognostic algorithm is valid in certain desirable notions. For example, one can check wether the whole distribution is valid with a desirable significance level by Kolmogorov-Smirnov test (Justel, Pena, & Zamar, 1997). This may require a large number of samples for any statistically meaningful result. Alternatively, one may test the statistical significance at some value of the cumulative distribution function of the RUL estimate using Fisher's exact test (Fisher, 1954). However, the method only ensures validity of a point estimate in the RUL and throws away other useful information contained in the probability density function.

It is clear that a point estimate of the RUL seems to be inadequate to quantify the uncertainty. However, the whole posterior density of the RUL is hard to come by. Nevertheless, density-based prognostic methods have been used by people engaged in various density-based state estimation techniques, e.g., particle filters (Saha & Goebel, 2011). Note that the posterior density is valid only when the assumed process noise distribution is true across the entire RUL prediction horizon.

In practice, the prediction interval of the RUL with high probability is often useful for monitoring a mission critical component. The lower limit serves as the valid point estimate of the RUL while the upper limit shows the full potential of the component in account for the future loading uncertainty. For a given confidence level, one seeks the smallest interval to contain the RUL with a probability no less than the desired confidence level. This boils down to the problem of interval prediction with high probability that guarantees finite sample validity without the knowledge of statistical distribution of the noise. The key idea is to leverage the newly proposed conformal prediction framework (Shafer & Vovk, 2008) with non-parametric conditional density estimation (Rosenblatt, 1956). Under certain regularity conditions, the proposed interval estimator converges to an oracle band at a minimax optimal rate. In addition, we apply a data driven method to automatically select the bandwidth in the kernel density estimator and discuss practical approximations to speed up the computation. The method can be used to predict the RUL interval with physics-based model in a distribution free manner. It can also be applied to assess the validity of any point-based prognostic algorithm from the experimental data. We demonstrate the effectiveness of the RUL prediction for Li-Ion batteries using both simulated and experimental data.

The rest of the paper is organized as follows. Section 2 formulates the RUL prediction problem. Section 3 presents the distribution free interval estimation of the component's end-of-life and discusses its asymptotic properties. Section 4 provides the experimental study on the state-of-charge estimation of Li-Ion battery using the predicted RUL intervals of a desired confidence level. Concluding summary is in Section 5.

2. The Formulation of RUL Prediction Problem

State space model has been commonly used to describe the dynamics of a component such as the aging process during repeated usage (Luo et al., 2003). In general, one can apply the principle of physics to model the dynamics of a component by

$$\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{p}(t), \mathbf{u}(t), \mathbf{v}(t)) \tag{1}$$

where $\mathbf{x}(t)$ is the state vector, $\mathbf{p}(t)$ is the parameter vector that may change over time but not governed by the differential equation as the state $\mathbf{x}(t)$, $\mathbf{u}(t)$ is the input vector, and $\mathbf{v}(t)$ is the process noise vector. The state and parameter can be observed through a generic continuous time model

$$\mathbf{y}(t) = \mathbf{h}(t, \mathbf{x}(t), \mathbf{p}(t), \mathbf{u}(t), \mathbf{w}(t)) \tag{2}$$

where $\mathbf{w}(t)$ is the measurement noise vector. In practice, measurements are made at discrete time time instants t_1, ..., t_k, ... and the measurement model becomes

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{p}_k, \mathbf{u}_k, \mathbf{w}_k) \tag{3}$$

where the subscript k indicates that the measurement $\mathbf{y}_k$ is obtained at time t_k. Denote by $\mathbf{Y}_k$ the measurement sequence $\mathbf{y}_1$, ..., $\mathbf{y}_k$. The state estimation problem is to obtain the posterior distribution $p(\mathbf{x}_k|\mathbf{Y}_k)$ at any time t_k using the prior knowledge $p(\mathbf{x}_0)$ and the measurements $\mathbf{Y}_k$. The desired performance of the component can be characterized by a set of constraints from the state, parameter and input space to some set that fully characterizes the normal condition (Sankararaman & Goebel, 2013). Specifically, we let $c_i(T) = 1$ if the constraint is satisfied, i.e., $\mathbf{g}_i(\mathbf{x}(t), \mathbf{p}(t), \mathbf{u}(t)) \in B_i$ for $t \in [0, T]$, where $\mathbf{g}_i$ is a known mapping and B_i is the set of acceptable values. Otherwise, $c_i(T) = 0$. The end-of-life (EOL) of a component is defined as the earliest time that one of the n-constraints is violated. Denote by t_{EOL} the end-of-life of the component given the state, parameter and input vectors, i.e.,

$$t_{\text{EOL}} = \inf_t \left\{ t \left| \prod_{i=1}^{n} c_i(t) = 0 \right. \right\} \tag{4}$$

The remaining useful life (RUL) of the component at time t_P is given by $t_{\text{EOL}} - t_P$. Owing to the uncertainty in the state dynamics, the state $\mathbf{x}(t)$ is a random process. Thus the constraints have to be assessed in a probabilistic sense. Let $\alpha \in [0, 1]$ be the significance level and we define the EOL as a function of α by

$$t_{\text{EOL}}(\alpha) = \inf_t \left\{ t \left| P\left(\prod_{i=1}^{n} c_i(t) = 0 \right) \leq \alpha \right. \right\} \tag{5}$$

where the constraint c_i is evaluated by $\mathbf{g}_i(\hat{\mathbf{x}}(t), \hat{\mathbf{p}}(t), \mathbf{u}(t))$ using the state and parameter estimates with the measurements up to time t_P. Note that one can propagate the state, parameter and input uncertainties into the future and assess the constraint satisfaction c_i probabilistically. For small α, $t_{\text{EOL}}(\alpha)$ may be conservative especially when the uncertainty of the state dynamics is large initially with limited number of measurements. The RUL prediction becomes a density estimation problem if one wants to fully characterize $t_{\text{EOL}}(\alpha)$ at any time t_P. It is computationally demanding and does not have a closed form expression for a problem with either nonlinear dynamics or nonlinear constraint.

3. Distribution Free Prediction Interval

Consider a generic RUL prediction problem where one observes $\mathbf{Y}_k$ and $\mathbf{U}_k$ up to time t_k and applies state estimation algorithm to obtain $\hat{\mathbf{x}}_k$ with the associated error covariance P_k. In order to estimate t_{EOL}, one has to propagate the dynamic model (1) from t_k with either known (deterministic) future input or random future input with the antic-

ipated uncertainty given by a probabilistic model. Clearly, exact characterization of (5) is computationally challenging and one may only have a point estimate of the component's EOL, namely, $\hat{t}_{k\mathrm{EOL}}$ at time t_k. In principle, the estimate of t_{EOL} can be based on the maximum a posteriori (MAP) criterion, which does not necessarily need the complete knowledge of the posterior distribution. However, a prediction interval $[t_{k\,\min}, t_{k\,\max}]$ of the t_{EOL} conditioned on $\mathbf{Y}_k$ and $\mathbf{U}_k$ with a guaranteed confidence level α is often desirable. Existing criterion to evaluate an RUL prediction algorithm using the prediction interval such as the α-λ metric (Saxena, Celaya, Saha, Saha, & Goebel, 2009) requires the complete distribution from the prediction algorithm, which is often unavailable from the point-based estimators. Nevertheless, the performance assessment can be misleading when the predictive distribution is not credible. To circumvent the above challenge, we take an alternative route to generate distribution free prediction interval using only the point estimate $\hat{t}_{k\mathrm{EOL}}$. The main idea is to generate conformal prediction interval so that

$$P(t_{\mathrm{EOL}} \in [t_{k\,\min}, t_{k\,\max}]) \geq \alpha$$

for any distribution of the state and future input. The usual way to obtain a non-parametric prediction interval leads to the form

$$\left[\hat{t}_{k\mathrm{EOL}} - Z_{\alpha/2}\sqrt{\hat{\sigma}_k^2 + s^2}, \hat{t}_{k\mathrm{EOL}} + Z_{\alpha/2}\sqrt{\hat{\sigma}_k^2 + s^2}\right]$$

where $\hat{t}_{k\mathrm{EOL}}$ is the point estimate from the prediction algorithm, $Z_{\alpha/2}$ is the normal quantile, $\hat{\sigma}_k^2$ is the estimated conditional variance, and s^2 is the estimated error of the prediction algorithm using bootstrapping (Efron & Tibshirani, 1993). However, such a prediction interval does not have distribution free finite sample validity.

3.1. Constructing Valid Prediction Interval

We apply the recently proposed conformal prediction method (Shafer & Vovk, 2008) to ensure the finite sample validity. Specifically, let $\hat{t}_{1\mathrm{EOL}}$, ..., $\hat{t}_{k\mathrm{EOL}}$ be a random sequence following an unknown distribution. We can estimate the density of the prediction sequence and denote the estimated density by $\hat{p}_k(t_{\mathrm{EOL}})$. The p-value from k samples is given by

$$\pi_k(t) = \frac{1}{k+1}\sum_{i=1}^{k+1} 1\left(\hat{p}_i(t) \leq \hat{p}_k(t)\right) \tag{6}$$

where $1(\cdot)$ is the indicator function. The prediction interval for the $(k+1)$-th sample with confidence level α can be obtained by

$$C_{k+1}(\alpha) = \{t : \pi_k(t) \geq 1 - \alpha\} \tag{7}$$

Intuitively, we want to test the hypothesis $H_0 : \hat{t}_{k+1\mathrm{EOL}} = t$ for arbitrary value t and use the height of the density estimate as a test statistic. Since the vector $(\hat{p}_1, ..., \hat{p}_k)$ is exchangeable, $\pi_k(t)$ is uniformly distributed in $[0, 1]$ and is a valid p-value for the significance test of H_0. The set $C_k(\alpha)$ contains all values t that are not rejected by the test, thus

$$P(\hat{t}_{k+1\mathrm{EOL}} \in C_{k+1}) \geq \alpha$$

for any distribution.

Computing $C_{k+1}(\alpha)$ is expensive since one has to find the p-value $\pi_k(t)$ for every t. An approximation is made assuming that $\hat{p}_k(t)$ can be well estimated by a kernel density estimator as the number of samples increases. With the exchangeability assumption, we can order the predictions $\hat{t}_{1\mathrm{EOL}}$, ..., $\hat{t}_{k\mathrm{EOL}}$ increasingly such that $\hat{p}_k(\hat{t}_{1\mathrm{EOL}}) \leq ... \leq \hat{p}_k(\hat{t}_{k\mathrm{EOL}})$. Let $j = \lfloor k\alpha \rfloor$ and define

$$C_{k+1}^{+}(\alpha) = \left\{t : \hat{p}_k(t) \geq \hat{p}_k(t_{k\mathrm{EOL}}) - \frac{K(0)}{kh}\right\} \tag{8}$$

where $K(\cdot)$ is the kernel function used to estimate $\hat{p}_k$ and h is the bandwidth of the kernel density estimator. It can be shown that $C_{k+1}^{+}(\alpha)$ also has finite sample validity and it has the same efficiency as $C_{k+1}(\alpha)$ asymptotically if the bandwidth h is chosen appropriately (Lei, Robins, & Wasserman, 2011).

3.2. Asymptotic Properties

Let $\hat{p}_k(t)$ be the kernel density estimate conditioned on $\hat{t}_{1\mathrm{EOL}}$, ..., $\hat{t}_{k\mathrm{EOL}}$. We can recursively update the estimate by

$$\hat{p}_{k+1}(t) = \frac{k}{k+1}\hat{p}_k(t) + \frac{1}{(k+1)h}K\left(\frac{\hat{t}_{k+1\mathrm{EOL}} - t}{h}\right)$$

with some smooth kernel $K(\cdot)$. The p-value at an arbitrary time is estimated by

$$\pi_i = \frac{1}{k+1}\sum_{j=1}^{k+1} 1\left(\hat{p}_j \leq \hat{p}_k\right), 1 \leq i \leq n+1$$

where we dropped the time argument for simplicity. We have the distribution free prediction interval satisfying

$$P(t_{\mathrm{EOL}} \in C_{k+1}^{+}) \geq \alpha$$

for any chosen $\alpha \in (0, 1)$. However, such a prediction interval is not unique and we would hope to shrink the interval without losing the validity conditioned on the input and observation.

Consider a partition $\mathcal{A} = \{A_j, j \geq 1\}$ of length s_k. Let $n_j = \sum_{i=1}^{k} 1(\hat{t}_{i\mathrm{EOL}} \in A_j)$ be the histogram counts. A local marginal kernel density estimate is

$$\hat{p}(t|A_j) = \frac{1}{n_j h_k}\sum_{i=1}^{k} 1(\hat{t}_{i\mathrm{EOL}} \in A_j)K\left(\frac{\hat{t}_{i\mathrm{EOL}} - t}{h_k}\right)$$

where h_k is the kernel bandwidth. The local conditional den-

sity rank can be defined as

$$\pi_{k,j} = \frac{1}{n_j+1}\sum_{i=1}^{k+1} 1(\hat{t}_{i\mathrm{EOL}} \in A_j)\cdot 1(\hat{p}(\hat{t}_{i\mathrm{EOL}}|A_j) \le \hat{p}(\hat{t}_{k+1\mathrm{EOL}}|A_j))$$

and the prediction interval with confidence α is

$$C_{k,j}(\alpha) = \{t : \pi_{k,j}(t) \le 1-\alpha\}.$$

We can see that the density estimate changes very little when t varies inside A_j. When A_j has a small diameter and the kernel function is smooth enough, the local sample approximates independent observations drawn inside the partition A_j. Thus we can optimize the kernel bandwidth h_k adaptively to achieve the smallest valid prediction interval asymptotically. The tuning of the partition size s_k and kernel bandwidth h_k depends on the smoothness of the conditional density. It can be trained by using a subset of the samples to construct a local conformal prediction interval and then increasing the bandwidth until C_{k+1} can not be further reduced.

Next, we argue that the convergence rate of the optimized local conformal prediction interval is asymptotically minimax optimal compared with the oracle prediction interval. Assume that the density of t_{EOL} satisfies $0 < c_0 \le p_{t_{\mathrm{EOL}}}(t) \le c_1 \le +\infty$ for any input and observation sequences. In addition, we assume that $p_{t_{\mathrm{EOL}}}(t)$ is β times differentiable and uniformly bounded by L. In particular, the conditional density is Lipschitz in t, i.e.,

$$||p_{t_{\mathrm{EOL}}}(\cdot|t_1) - p_{t_{\mathrm{EOL}}}(\cdot|t_2)||_\infty \le L|t_1 - t_2|$$

to ensure that the kernel function $K(\cdot)$ is a smooth approximation of $p_{t_{\mathrm{EOL}}}(t)$ of order β. Let $C^*(\alpha)$ be the oracle prediction interval given by

$$C^*(\alpha) = \{t : p_{t_{\mathrm{EOL}}}(t|\mathbf{Y}_\infty, \mathbf{U}_\infty) \ge t(\alpha)\}$$

where $t(\alpha)$ is the appropriately chosen threshold to achieve $P(t_{\mathrm{EOL}} \in C^*(\alpha)) = \alpha$. If we choose $s_k \sim r_k$ and $h_k \sim r_k^{1/\beta}$, then the prediction interval $C_k^+(\alpha))$ satisfies

$$P\left(\sup_t \mu\left(C_k^+(\alpha) \bigcap \neg C^*(\alpha)\right) \ge c_\lambda r_k\right) \sim O(k^{-\lambda})$$

for any $\alpha \in (0,1)$, $\lambda > 0$ and some constant c_λ being independent of k. The measure μ shows the difference between the kernel density estimate and the oracle prediction intervals while the critical rate r_k is

$$r_k = \left(\frac{\log k}{k}\right)^{\frac{\beta}{2\beta+1}}.$$

As the sample size increases, the prediction interval by kernel density estimate converges to the oracle prediction interval and the rate can not be improved in the minimax sense, i.e.,

$$\inf_{C_k^+(\alpha)} \sup_{p_{t_{\mathrm{EOL}}}} E_{p_{t_{\mathrm{EOL}}}}\left[\mu\left(C_k^+(\alpha) \bigcap \neg C^*(\alpha)\right)\right] \ge c r_k$$

for some constant $c > 0$. The proof invokes generalized Fano's lemma (Tsybakov, 2009) where the supremum is over all distribution $p_{t_{\mathrm{EOL}}}$ such that $p_{t_{\mathrm{EOL}}}(\cdot|x)$ is Lipschitz in x in the sup-norm sense and $p_{t_{\mathrm{EOL}}}(t)$ is smooth enough, i.e., β times differentiable. The kernel bandwidth $h = cr_k$ with a small enough constant c will converge to the oracle prediction interval and guarantee the finite sample validity simultaneously.

4. Battery State-of-Charge Estimation

Lithium-ion battery is the core of new plug-in hybrid-electrical vehicles as well as considered in many 2nd generation hybrid electric vehicles. The lithium-ion battery performance plays an important role for the energy management of these vehicles as high-rate transient power source cycling around a relatively fixed state of charge (SOC). The estimation of state-of-charge and state-of-health of the battery cell has drawn significant attention in battery health management (Charkhgard & Farrokhi, 2010; Chiasson & Vairamohan, 2005; Kim & Cho, 2011; Klein et al., 2013; Saha, Goebel, & Christophersen, 2009).

4.1. Battery Dynamic and Measurement Models

We adopt the enhanced self-correcting cell model (Plett, 2004a) which contains both state and unknown parameters. The model is simplified from the detailed electro-chemical model (Klein et al., 2013) and it includes open-circuit voltage, internal resistance, voltage time constant, and hysteresis. Assume the sampling interval is T_s. At time k, the current i_k is nearly a constant. The state-of-charge (SoC) c_k is governed by

$$c_{k+1} = c_k - \left(\frac{\eta T_s}{C}\right) i_k$$

where η is the Coulombic efficiency factor at current level i_k and C is the cell capacity in Ampere-seconds. The time constants of the cell voltage response are captured by several internal states. Let $\mathbf{z}_k$ be the internal state vector at time k. A linear model was suggested in (Plett, 2004a) given by

$$\mathbf{z}_{k+1} = A_{\mathbf{z}}\mathbf{z}_k + B_{\mathbf{z}} i_k$$

where $A_{\mathbf{z}}$ is a diagonal matrix with real valued entries and $B_{\mathbf{z}}$ is chosen to have all 1s. The hysteresis level is modeled by

$$h_{k+1} = e^{-\left|\frac{\eta i_k \gamma T_s}{C}\right|} h_k + \left[1 - e^{-\left|\frac{\eta i_k \gamma T_s}{C}\right|}\right] \mathrm{sgn}(i_k)$$

where γ is the hysteresis rate constant. The voltage is

$$v_k = \mathrm{OCV}(c_k) + G\mathbf{z}_k - Ri_k + Mh_k$$

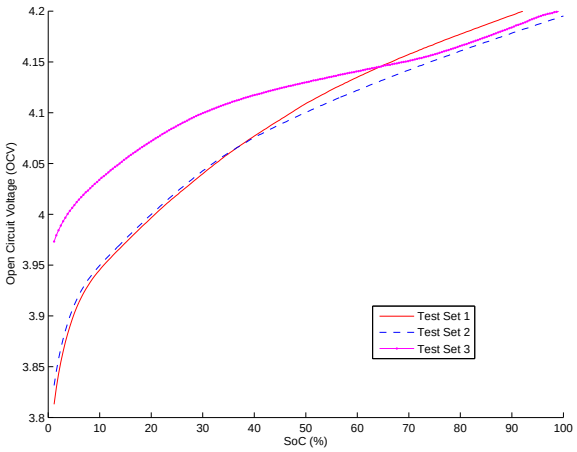

Figure 1. Empirical function of open circuit voltage vs. state-of-charge of Li-Ion battery sets

where $\mathrm{OCV}(\cdot)$ is an empirical function found by battery cell testing; G is a vector of unknown parameters related to the battery aging status; R is the internal resistance; and M is the maximum hysteresis level. Thus the overall state vector is $\mathbf{x}_k = [c_k \ \mathbf{z}_k \ h_k]^T$. The dynamic model contains unknown parameters η, γ, and G. The measurements are voltage v_k and current i_k. The primary goal is to accurately estimate the state-of-charge c_k during the battery usage with variable load.

4.2. Experimental Results

The Li-Ion battery cells used for experimental study were divided into three sets. Set 1 of two batteries was used to tune the cell model parameters (e.g., $\mathrm{OCV}(\cdot)$ function). Set 2 and Set 3 of four batteries were used in evaluating the joint state and parameter estimation to see how well the filters perform under different dynamics. The sampling interval T_s=1s. The voltage measurement accuracy is $\pm$5mV and the current measurement accuracy is $\pm$100mA.

The open-circuit voltage as a function of the state-of-charge (SoC) for three sets of the Li-Ion battery cells is plotted in Fig. 1. First, the cell was fully charged to 4.2 V with a constant current. Then, the cell was discharged at at a constant rate until 3.0V. The cell voltage as a function of the SoC under discharge and under charge were averaged to compute the OCV. This has the effect of eliminating, to the greatest extent, the presence of hysteresis and Ohmic resistance in the OCV function. In Set 1, the batteries were put in a chamber with controlled temperature of 25°C. In Set 2, the batteries were put on an open table with an electronic fan turned on. In Set 3, the batteries were put on an open table with the fan turned off. We can see that Set 2 has a closer empirical OCV function to Set 1 than Set 3 to Set 1.

The real test comprised a sequence of 40 charge and discharge cycles for each battery in three sets. The battery was connected to a potentiometer load, separated by 2A discharge pulses and 10-min rests, and spread over the 20%-90% SoC range. Set 1 was used to estimate the OCV function and calibrate the 4th order model of $A_{\mathbf{z}}$. Sets 2 and 3 were used to evaluate the SoC estimation accuracy with the model state and parameters initialized from the same conditions as in Set 1. We compare the adaptive cubature Kalman filter (CKF) (Chen, 2012) with the dual extended Kalman filter (EKF) (Plett, 2004b) in terms of the root mean square (RMS) error and maximum error over the whole duration with approximately 5000s for each cycle. The adaptive CKF and dual EKF have the same initial condition. In adaptive CKF, the forgetting factor sequence was chosen by

$$\lambda_k = 1 - 0.05 \cdot 0.95^k.$$

Table 1. Comparison of SoC Estimation Accuracy

Method	Test Set	RMS error (%)	Maximum error (%)
dual EKF	1	0.32	1.33
adaptive CKF	1	0.29	1.34
dual EKF	2	2.14	9.8
adaptive CKF	2	1.33	3.2
dual EKF	3	7.42	13.3
adaptive CKF	3	2.53	5.2

The SoC estimation results are listed in Tab. 1. The dual EKF has similar SoC estimation accuracy to the adaptive CKF on Set 1 batteries since the model parameters have been well calibrated. The slight increase of the error by the dual EKF is mainly due to linearization of the dynamic model. It is interesting to note that the adaptive CKF performs much better in Set 2 and Set 3 where the OCV functions and unknown parameters are different from those in Set 1. In Set 3, η has more than 20% of variation from the nominal value in Set 1. The dual EKF yields more than twice of the SoC estimation error made by the adaptive CKF.

Next, we apply the SoC estimation to estimate the end-of-life (EOL) and remaining useful life (RUL) of the battery during a discharge cycle based on the proposed conformal interval estimation using a Gaussian kernel. For convenience, we chose $\alpha = 0.99$ and calculated the true t_{EOF} when the cell voltage dropped below 3.0V. The adaptive CKF was used to jointly estimate the state and parameter and the resulting prediction interval of the EOL is shown in Fig. 2. We can see that more than 99% of the predicted intervals cover the true EOL, indicating the validity of the prediction. Note that the predicted EOL interval does not always decrease over time owing to the future loading uncertainty as well as the evolvement of the dynamic state. Note also that the lower and upper limit of the prediction EOL may not be symmetric around the best point estimate, indicating possibly an asymmetric posterior distribution. Nevertheless, the predicted interval reduces to less than 20s fairly quickly. Fig. 3 shows the RUL prediction interval from 100s to 700s with $\alpha = 0.99$. We can see that the prediction interval covers the true RUL nearly all the time with the upper limit being close to the true RUL. Thus the

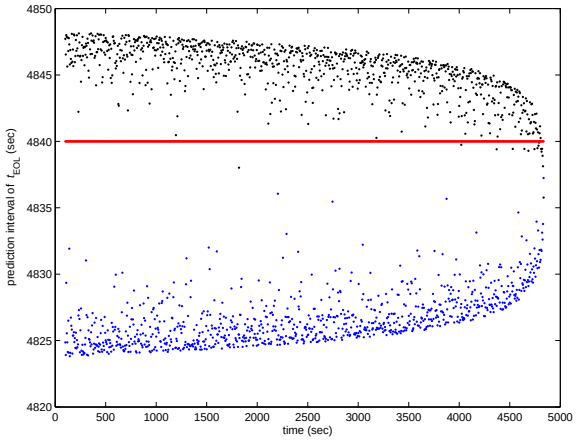

Figure 2. The end-of-life (EOL) prediction interval with $\alpha = 0.99$ using CKF for one discharge cycle of Set 2.

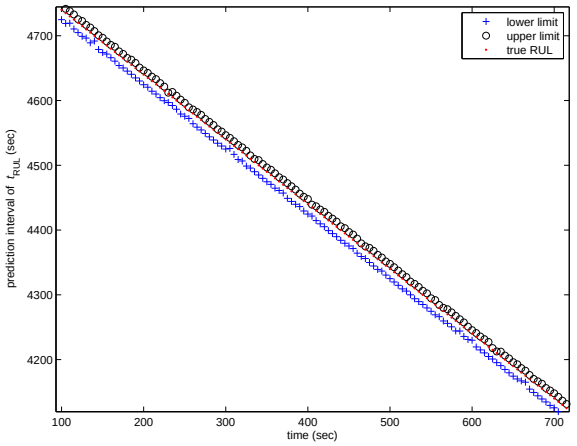

Figure 3. The remaining useful life (RUL) prediction interval with $\alpha = 0.99$ using CKF for one discharge cycle of Set 2.

algorithm seems to be practical for monitoring the battery's SoC and fully utilizing its capacity.

5. Conclusion

We presented a generic framework for distribution free interval estimation to quantify the uncertainty of the end-of-life or the remaining useful life (RUL) prediction of a system component. The method combines the conformal prediction and non-parametric density estimation to ensure the finite sample validity with arbitrarily chosen confidence level. Under certain regularity conditions, the proposed interval estimator converges to an oracle band at a minimax optimal rate. In addition, we used a data driven method to automatically select the bandwidth in the kernel density estimation and worked out a practical approximation to speed up the computation. The proposed method was used to predict the RUL interval for Li-Ion batteries with the joint state and parameter estimation using nonlinear filtering methods. The results reveal that the 99% confidence interval shrinks quickly when the dynamic model captures the discharge cycle fairly accurately. In addition, the lower and upper limit of the RUL prediction interval is often non-symmetric of the true RUL indicating the asymmetric nature of the posterior distribution.

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References

Charkhgard, M., & Farrokhi, M. (2010). State-of-charge estimation for lithium-ion batteries using neural networks and ekf. *Industrial Electronics, IEEE Transactions on*, *57*(12), 4178–4187.

Chen, H. (2012). Adaptive cubature kalman filter for nonlinear state and parameter estimation. In *Information fusion (fusion), 15th international conference on* (pp. 1413–1420).

Chiasson, J., & Vairamohan, B. (2005). Estimating the state of charge of a battery. *Control Systems Technology, IEEE Transactions on*, *13*(3), 465–470.

Efron, B., & Tibshirani, R. (1993). *An introduction to the bootstrap* (Vol. 57). CRC press.

Fisher, R. (1954). *Statistical methods for research workers*. Oliver and Boyd.

Justel, A., Pena, D., & Zamar, R. (1997). A multivariate kolmogorov-smirnov test of goodness of fit. *Statistics & Probability Letters*, *35*(3), 251–259.

Kim, J., & Cho, B. H. (2011). State-of-charge estimation and state-of-health prediction of a li-ion degraded battery based on an ekf combined with a per-unit system. *Vehicular Technology, IEEE Transactions on*, *60*(9), 4249–4260.

Klein, R., Chaturvedi, N. A., Christensen, J., Ahmed, J., R., F., & Kojic, A. (2013). Electrochemical model based observer design for a lithium-ion battery. *Control Systems Technology, IEEE Transactions on*, *21*(2), 289–301.

Lei, J., Robins, J., & Wasserman, L. (2011). Efficient nonparametric conformal prediction regions. *Manuscript. http://arxiv.org/abs/1111.1418.*.

Luo, J., Namburu, M., Pattipati, K., Liu, Q., Kawamoto, M., & Chigusa, S. (2003). Model-based prognostic techniques. In *Ieee systems readiness technology conference* (pp. 330–340).

Plett, G. L. (2004a). Extended kalman filtering for battery management systems of lipb-based hev battery packs part 2 – modeling and identification. *Journal of Power Sources*, *134*(2), 262–276.

Plett, G. L. (2004b). Extended kalman filtering for battery

management systems of lipb-based hev battery packs part 3 – state and parameter estimation. *Journal of Power Sources*, *134*(2), 277–292.

Rosenblatt, M. (1956). Remarks on some nonparametric estimates of a density function. *The Annals of Mathematical Statistics*, *27*(3), 832–837.

Saha, B., & Goebel, K. (2011). Model adaptation for prognostics in a particle filtering framework. *International Journal of Prognostics and Health Management*, *2*(006).

Saha, B., Goebel, K., & Christophersen, J. (2009). Comparison of prognostic algorithms for estimating remaining useful life of batteries. *Transactions of the Institute of Measurement & Control*, *31*(3-4), 293–308.

Sankararaman, S., & Goebel, K. (2013). Uncertainty quantification in remaining useful life of aerospace components using state space models and inverse form. In *Aiaa/asme/asce/ahs/asc structures, structural dynamics and materials conference.*

Saxena, A., Celaya, J., Saha, B., Saha, S., & Goebel, K. (2009). On applying the prognostic performance metrics. In *Annual conference of the prognostics and health management society.*

Shafer, G., & Vovk, V. (2008). A tutorial on conformal prediction. *Journal of Machine Learning Research*, *9*, 371–421.

Tsybakov, A. (2009). *Introduction to nonparametric estimation*. Springer.

BIOGRAPHIES

Huimin Chen received the B.E. and M.E. degrees from Department of Automation, Tsinghua University, Beijing, China, in 1996 and 1998, respectively, and the Ph.D. degree from the Department of Electrical and Computer Engineering, University of Connecticut, Storrs, in 2002, all in electrical engineering. He was a post doctorate research associate at Physics and Astronomy Department, University of California, Los Angeles, and a visiting researcher with the Department of Electrical and Computer Engineering, Carnegie Mellon University from July 2002 where his research focus was on weak signal detection for single electron spin microscopy. He joined the Department of Electrical Engineering, University of New Orleans in Jan. 2003 and is currently a Don E. Wilson Chevron Associate Professor. His research interests are in general areas of signal processing, estimation theory, and information theory with applications to target detection and target tracking.