Remaining Useful Life Prognostics for Lithium-ion Battery Based on Gaussian Processing Regression Combined with the Empirical Model

Shan Yin, Jingyue Pang, Datong Liu, Yu Peng

Department of Automatic Test and Control, Harbin Institute of Technology, Harbin, 150080, China

yy.yinshan@163.com
pjy19880909@163.com
liudatong@hit.edu.cn
pengyu@hit.edu.cn

ABSTRACT

Data-driven techniques based on Bayesian framework like Gaussian Process Regression (GPR) can not only predict the lithium-ion battery Remaining Useful Life (RUL), but also provide the uncertainty representation. However, it is always difficult to choose the covariance function of GPR and the confidence bound is usually large if the training data are not enough. In order to solve this problem, a combining method is proposed, it is a prognostic framework based on GPR model combined with Empirical Model (EMGPR) to realize the lithium-ion battery RUL prediction. EMGPR has the advantages of predicting the tendency and uncertainty management for RUL estimation. The modeling process of EMGPR consists of two steps. The self-deterministic part, which reflects the real physical process of battery degradation, is approximated by the empirical model. And the disturbance part, which is caused by random noise such as measurement and environment noise, is expressed by the GPR model. In application, two key factors of EMGPR are focused. Firstly, the prediction result is not accurate enough if the training data are not very reliable. In this case, more reliable training data should be selected optimized. Secondly, the characteristic of the disturbance is involved to determine the kernel function of GPR model. With this EMGPR framework, the RUL result is estimated with uncertainty representation, as well, the covariance function of GPR is easy to choose. Experiments with NASA PCoE and CALCE battery data show the satisfactory result can be obtained with the EMGPR approach.

1. INTRODUCTION

Lithium-ion batteries have been widely used in the domains of portable designs, notebook computers, electric vehicles and airplanes and spacecrafts because of their high energy density, low self-discharge rates, wide operating temperature ranges, and high charge-discharge rates. Lithium-ion batteries which act as energy storage components are critical to the safety of electric devices or systems (Yang, Ye, Guo & Ma, 2012). However, the lithium-ion battery will degrade over time on account of aging, environmental impacts and dynamic loading, etc. Failure of battery may lead to loss of operation, decreased output, and it might even bring danger to the operators. Hence, it is meaningful to detect the underlying degradation and take measures to prevent the potential failures and ultimately prevent the disastrous failures. Prognostics and health management (PHM), is to predict how soon a system or component will loss efficacy or reach the failure threshold (Zhang & Lee, 2011) (Widodo, Shim, Caesarendra & Yang, 2011). Effective precaution measures could be taken in advance if we predict the failures successfully. For health state monitoring, battery parameters included voltage, current, temperature and capacity are measured to estimate the state of charge (SOC), the State of Health (SOH), the end of life (EoL) and the remaining useful life (RUL) (here we only focus on the remaining cycle life) of lithium-ion battery (Saha, Goebel & Christophersen, 2009).

In order to estimate lithium-ion battery RUL well and make an optimized design of battery-systems, both model-based and data-driven techniques are applied. Gao *et al.* presents a dynamic model which is suitable for virtual-prototyping of portable battery-powered systems. The model takes nonlinear equilibrium potentials, rate, temperature-variation, thermal effects and transient power demand into consideration (Gao & Liu, 2002). Rong *et al.* introduces an analytical model to predict the remaining capacity of a lithium-ion battery, which is in view of the cycle-aging and temperature effects (Rong, 2006). Erdinc *et al.* proposes a dynamic model which cares about the significant temperature and capacity fading (Erdinc, Vural &

Uzunoglu, 2009). The model-based technologies are direct and intelligible. However, the performances and characters of batteries are highly influenced by their complex operating conditions and internal inaccessible. Hence, it is difficult to establish an exact model to describe whole information of lithium-ion batteries.

With the advancement of sensor and data storage technologies, the data-driven prognostics are emphasized. The data-driven approach Auto-Regressive Integrated Moving Average model (ARIMA) is utilized for RUL estimation. Without taking any physical process into consideration, it is possible to deal with the non-stationary monitoring data. But the ARIMA model is unsuitable for long-term prediction. Then the Extended Kalman Filter (EKF) is proposed to handle the nonlinear and non-stationary modeling. The EKF cannot accommodate the non-modeled process (Saha, Goebel & Christophersen, 2009) (Do, Forgez, Benkaha & Friedrich, 2009) (He, Williard, Osterman & Pecht, 2011). Lots of data-driven methods cannot manage the uncertainty of prognostics.

Hence, the uncertainty caused by the measurement errors, the environmental noise and the model noise is addressed in prognostics. Especially, for long-term state prediction, the uncertainty must be represented and managed effectively (He, Williard, Osterman & Pecht, 2011). Bayesian framework based data-driven approaches such as Particle Filter (PF) (Dalal, Ma & He, 2011) (Xing, Tsui & Pecht, 2012), Relevance Vector Machine (RVM) and Gaussian Process Regression (GPR) can provide the uncertainty reprepsentation of the RUL value (Goebel, Saha & Saxena, 2009) (Saha, Goebel, Poll & Christophersen, 2009) (Goebel, Saha, Saxena, Celaya & Christophersen, 2009) (Chen & Pecht, 2012).

Among these algorithms, the GPR model based on the Bayesian framework is flexible to be adopted in the non-linear regression of stochastic time series. And it can predict the RUL of lithium-ion batteries, offerring the confidence interval of predicted value and manage the uncertainty (Rasmussen & Williams, 2006) (Saha, Goebel & Christophersen, 2009). The GPR algorithm provides variance around its means predictions, and combines prior knowledge with observed data (Li & Zhang, 2010) (Cristianini & Taylor, 2000) (Scholkopf & Smola, 2002).

Although the GPR provides a theoretically framework for prognostics, there are some limitations in practical usage. First, choosing proper kernel function (covariance function) is critical. However, it is sometimes difficult to make an optimal selection due to lack of knowledge about the actual process. In addition, the GPR model can predict the mean function and variance function with hyper-parameters, if the training data is not available enough, the prediction confidence bound will become so large and the result is not reliable (Goebel, Saha & Saxena, 2009).

In this work, a novel empirical mode combined with GPR is proposed to predict the RUL of lithium-ion battery (it is as EMGPR). Firstly, by the combination, we can conquer the inconvenience of choosing the prediction covariance function. Secondly, the optimization of the selection of the training data is focused to reduce the prediction uncertainty.

In the EMGPR framework, training data are divided into two parts. One is self-deterministic and could be estimated by empirical model. This part reflects the real physical deterioration of lithium-ion battery. The other is the disturbance components which reflects the random noise including measure noise, model noise and environment noise could be expressed by GPR. The final prediction result of RUL is the sum of the two parts. Experiments have been done with data set of NASA and the University of Maryland to illustrate the effectiveness of EMGPR prognostics framework for lithium-ion battery.

This paper is organized as follows. The GPR method is depicted in Section 2. In Section 3, the lithium-ion prognostic method of EMGPR is introduced. Experiments of lithium-ion battery RUL estimation with EMGPR are discussed in details in Section 4. The conclusion and future work are described in Section 5 and 6.

2. Gaussian Processing Regression Model

The GPR model affects input variables to output crack growth by probabilistically inferring the nonlinear relationship between input and output (Mohanty, Das, Chattopadhyay & Peralta, 2009). It has been widely applied in machine learning (Rasmussen & Williams, 2006) (Snelson, 2007), data mining, image processing, pattern recognition and prognostics of both metallic and electronics systems. Particularly, GPR model is utilized for the prognostics of lithium-ion battery (Liu, Pang, Zhou, & Peng, 2010).

The basic idea of GPR modeling is to define the Gaussian Processing (GP) to describe a function distribution. The GP is a collection of random and finite stochastic variables which follows to Gaussian distribution. GP is fully described by its mean function $m(x)$ and the covariance function $k(x,x')$.

$$f(x) \sim GP(m(x), k(x,x')) \tag{1}$$

$$m(x) = E[f(x)] \tag{2}$$

$$k(x,x') = E[(f(x)-m(x)),(f(x')-m(x'))] \tag{3}$$

where the symbol E means the expectation.

For the regression, we model as,

$$y = f(x) + \varepsilon \tag{4}$$

Here x is the input vector, f is the function output and y is the observed values with noise. The noise ε is usually assumed to follow the Normal distribution.

$$\varepsilon \sim N(0,\sigma_n^2) \tag{5}$$

The prior distribution of y is,

$$y \sim N(m(x), K(X,X)+\sigma_n^2 I_n) \tag{6}$$

The prior joint distribution of y and the prediction value f_* is described as follows:

$$\begin{bmatrix} y \\ f_* \end{bmatrix} \sim N\left(m(x), \begin{bmatrix} K(X,X)+\sigma_n^2 I_n & K(X,x_*) \\ K(x_*,X) & k(x_*,x_*) \end{bmatrix} \right) \tag{7}$$

The parameter $K(X,X)=K_n=(k_{ij})$ is a symmetric positive definite covariance matrix. The element in the matrix k_{ij} means the correlation of x_i and x_j. The equation $K(X,x_*)=K(x_*,X)^T$ is the covariance matric of test data x_* and training data X. $k(x_*,x_*)$ is the covariance of x_* itself. The symbol I_n is a unity matrix.

We can compute the posterior distribution of prediction value f_*:

$$f_* \mid X,y,x_* \sim N(\overline{f}_*, \mathrm{cov}(f_*)) \tag{8}$$

$$\overline{f}_* = m + K(x_*,X)[K(X,X)+\sigma_n^2 I_n]^{-1}(y-m) \tag{9}$$

$$\mathrm{cov}(f_*) = k(x_*,x_*) - K(x_*,X)\times[K(X,X)+\sigma_n^2 I_n]^{-1} K(X,x_*) \tag{10}$$

Hence, the mean of the prediction output is,

$$\hat{\mu}_* = \overline{f}_* \tag{11}$$

And the variance of the prediction output is,

$$\hat{\sigma}_*^2 = \mathrm{cov}(f_*) \tag{12}$$

Different mean functions and covariance functions contain some unknown parameters, they are hyper-parameters. Based on marginal likelihood Bayesian theory, we can identify the optimal hyper-parameters with a numerical optimization routine such as conjugate gradients (Rasmussen & Williams, 2006) (Li & Zhang, 2010).

By analyzing Eq. (9) to Eq. (12), the main challenge is to determine the covariance function for the prognostics with GPR.

3. PROGNOSTICS FRAMEWORK OF EMGPR

The remaining useful capacity of lithium-ion battery is predicted in this paper to calculate the RUL. A fused framework of EMGPR is proposed to predict the RUL of lithium-ion battery. Here two important steps are involved. Firstly, the characteristic of battery is analyzed to set the kernel function of GPR. Secondly, experiments are implemented to optimize the preferable training data.

Theoretically, any time series can be represented as consisting of two parts, a self-deterministic part and a disturbance component (Saha, Goebel & Christophersen, 2009). The self-deterministic part depends on the real physical process, while the disturbance component mainly influenced by the random noise containing measurement noise, process noise, surrounding environment noise, etc. In the EMGPR framework, the self-deterministic part is described by the empirical model such as double-exponential model and Gaussian model, which influent the output by curve fitting algorithms. The disturbance component is expressed by GPR Model. The final prediction result is the fusion of the two parts. The prognostics flowchart is shown in Figure 1.

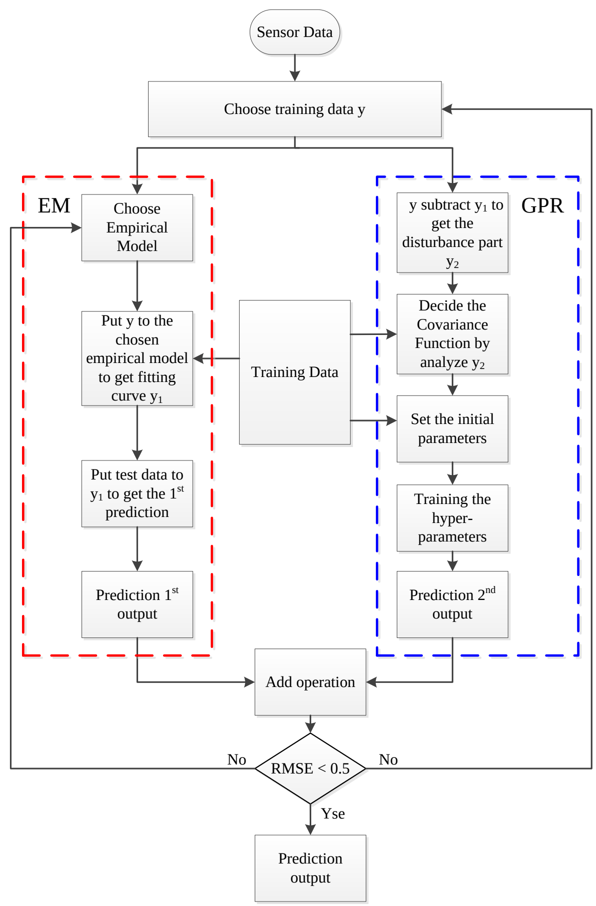

Figure 1. The fusion EMGPR framework based on combined GPR and Empirical Model

The detail steps of the EMGPR algorithm are as follows:

Step 1. Choose the training data. This step is executed repeatedly until a satisfied training accuracy is obtained. In this work, we assume that the training ends while the Root Mean Square Error (RMSE) value is less than 0.5.

Step 2. Choose a proper empirical model to describe the real physical deterioration of lithium-ion battery. Similarly, this step is carried out repeatedly to identify the suitable empirical model. The criterion of choosing the training data and empirical model is an experienced setting with experiments.

Step 3. Put the training data into the identified empirical model to get a fitting curve y_1 (In this step, the parameters of empirical model are determined).

Step 4. Predict with the model in Step 3 to obtain the 1st prediction output.

Step 5. Subtract y_1 from y to get the disturbance part, denoted as the variable y_2.

Step 6. Analyze the characteristics of disturbance part and choose the covariance function of GPR model.

Step 7. Initialize parameters of mean function and covariance function of GPR model.

Step 8. Train the hyper-parameters of covariance function.

Step 9. Compute the prediction results of disturbance with GPR model (it is as the 2nd prediction output).

Step 10. Fuse the 1st prediction output and the 2nd prediction output together to obtain the final estimated value.

4. EXPERIMENTS AND DISCUSSION

4.1. Raw Data of Lithium-ion Batteries

The data set used in this work to perform the lithium-ion battery prognostics are obtained from the data repository of NASA Ames Prognostics Center of Excellence (PCoE) and the Center for Advanced Life Cycle Engineering (CALCE) of the University of Maryland.

The battery data from NASA were run through 3 different operation profiles (charge, discharge and impedance) at room temperature. Charging was carried out in a constant current mode at 1.5A until the battery voltage reached 4.2V and then continued charging in a constant voltage mode until the charge current dropped to 20mA. Discharging was performed at a constant current level of 2A until the battery voltage falling to 2.7V, 2.5V, 2.2V and 2.5V for batteries B0005, B0006, B0007 and B0018 respectively. Impedance measurement was carried out through an electrochemical impedance spectroscopy frequency sweep from 0.1Hz to 5kHz. Repeated charge and discharge cycles result in accelerated aging of the batteries while impedance measurements provide insight into the internal battery parameters that change with aging processes. The experiments were stopped when the batteries reached end-of-life criteria, which was a 30% fade in rated capacity (from 2Ahr to 1.4Ahr). This data set offers us the discharge capacity of each cycle.

Figure 2 shows the capacity degradation of battery from NASA, assuming that the capacity threshold is 1.41Ah. The horizontal axis represents the number of charge and discharge cycles. The vertical axis represents the capacity (Ah).

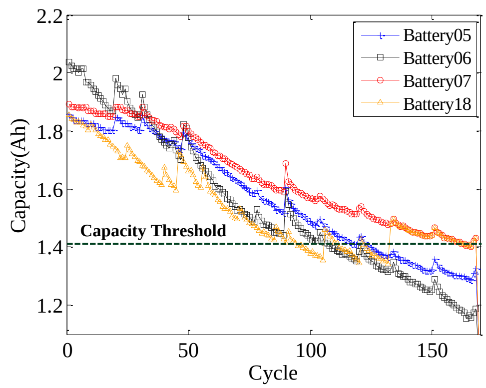

Figure 2. Capacity Degradation of Battery from NASA PCoE

Another data set is obtained from CALCE of the Maryland University, which is tested on the BT2000 lithium-ion battery experimental system. The experiment data contain two groups. The rated capacity is 1.35Ah and 1.1Ah separately. The experiments were done at 20℃ to 25℃, and the time, charging current/voltage, discharging current/voltage and charging/discharging capacity values are offered. Charging was carried out in a constant current mode at 0.675A until the battery voltage reached 4.2V and discharge was carried out at a constant current level of 1.35A until the battery voltage felt to 2.7V. The discharging rate of battery CS2_8, CS2_21, CS2_33 and CS2_34 are 0.5C. The experiments were stopped when the batteries reached end-of-life criteria, which was a 20% fade in rated capacity (from 1.1Ahr to 0.88Ahr).

Figure 3 shows the capacity degradation of battery from CALCE of. Here the capacity threshold we set is 0.88Ah.

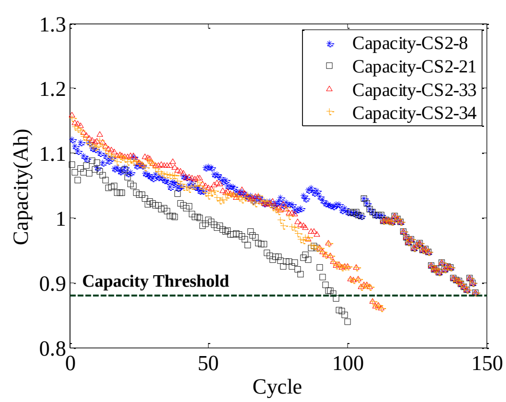

Figure 3. Capacity degradation of battery from CALCE

4.2. RUL prediction with different size of training data

The prediction result with large confidence bound using the non-sufficient available data is analyzed here. Experiments are implemented NASA battery B0007 to determine the better size of the training data. The training data (cycle) are from cycle 2 to 140, from cycle 50 to 140 and from cycle 100 to 140, respectively. The predicted results are shown in Figure 4. The red line of circle is the real test data, and the blue line with plus sign is the prediction result with training data from cycle 20 to cycle 140, the grey line with triangle is the prediction result with training data from cycle 50 to cycle 140 and the green line with square is the prediction result with training data from cycle 100 to cycle 140. The grey shade represents the prediction confidence bound.

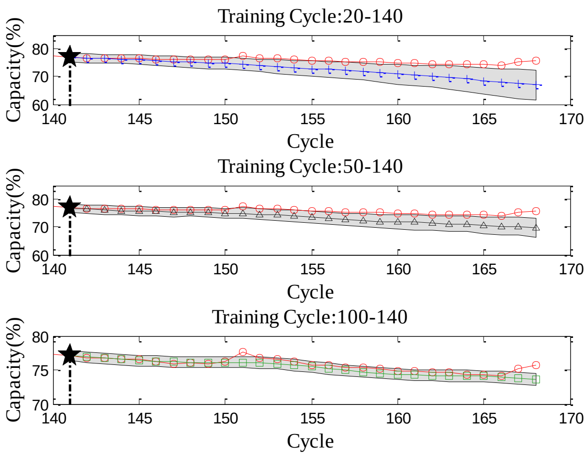

Figure 4. Prediction with different size of training data

Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) are utilized to evaluate the accuracy of the estimation, which are defined as Eq. (13) and Eq. (14).

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[x(i)-\bar{x}(i)]^2} \tag{13}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|x(i)-\bar{x}(i)\right| \tag{14}$$

Here $x(i)$ and $\bar{x}(i)$ represent the actual test data and the predicted result.

The quantified comparison is shown in Table 1.

Table 1 RMSE and MAE with different test data

Training Cycle	RMSE Value	MAE Value
20-140	0.3873	0.0581
50-140	0.2719	0.0430
100-140	0.0676	0.0090

From Table 1, we can find that better prediction result can be obtained and the prediction confidence bound is smaller with local training data set, although the local data set is with less data points.

4.3. RUL prediction with different covariance functions

In this section, experiments are executed to illustrate the influence of covariance function in the lithium-ion battery prognostics based on GPR. Predictions are implemented with two common covariance functions, conSEiso function and covMaterniso function. Based on the experiments above, training data are chosen from 100 to 140 cycles. Prediction result is shown in Figure 5.

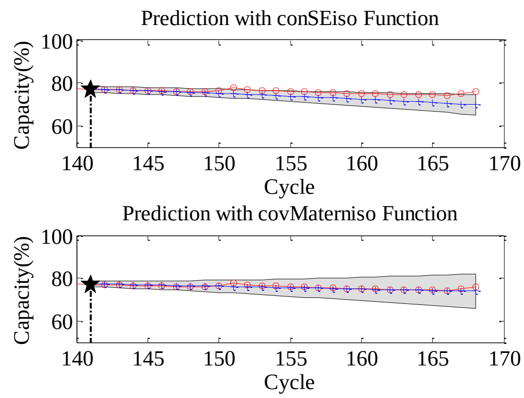

Figure 5. Predicted results with different covariance functions

In detail, the conSEiso function and the covMaterniso function are defined as Eqs. (15) and (16). In Eqs. (15) and (16), θ_1、θ_2、θ_3、ω、l、d are hyper-parameters. The valuables t and t' are the real value and prediction value.

$$k(t,t') = \theta_1^2 \exp(-\frac{(t-t')^2}{2\theta_2^2}) \tag{15}$$

$$k(t,t') = \theta_3 \exp(-\frac{1}{2}\sum_{l=1}^{d}\omega(t-t')^2) \tag{16}$$

The RMSE and MAE are shown in Table 2.

Table 2. Predicted RMSE and MAE with different covariance functions

Covariance function	RMSE	MAE
conSEiso	0.2520	0.0416
conMaterniso	0.0628	0.0083

We can find that the covariance function plays an important role in the prognostics of lithium-ion based on GPR model. Thus, it is necessary to take more efforts to the choice of covariance function.

4.4. Choice of covariance function

We assume that the battery data is composed by two parts: one reflects the inherent degradation regular (self-deterministic part) which can be described and analyzed as double-exponential or Gaussian empirical model. The other part is the disturbance component, which connects with environment factor, operating load etc., which are stochastic and can be estimated with GPR model.

Usually, the lifetime of electronics component degenerates with double-exponential or Gaussian curve. Hence, we firstly use a double-exponential model, described as Eq. (17), or Gaussian model, described as Eq. (18), to approximate the self-deterministic part.

$$y_1 = a_1 e^{b_1 x} + c_1 e^{d_1 x} \tag{17}$$

$$y_1 = a_2 e^{-(\frac{x-b_2}{c_2})^2} \tag{18}$$

Here, parameters a_1, b_1, c_1, d_1, a_2, b_2 and c_2 can be identified by fitting with the training data. The criterion of the choice of the empirical model is whether the prediction RMSE value is less than 0.5.

Then, we can construct the disturbance component (y_2) which can be predicted with GPR,

$$y_2 = y - y_1 \tag{19}$$

where y is the raw data value.

The disturbance parts are indicated in Figure 6.

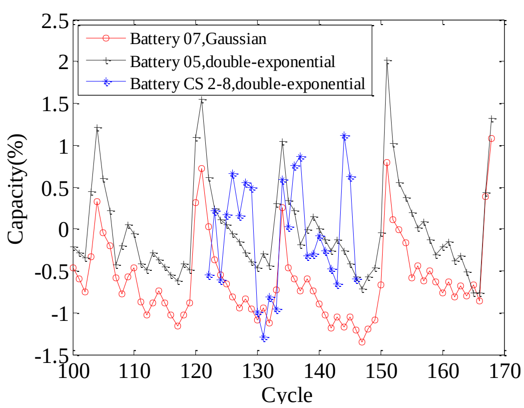

Figure 6. Disturbance parts

For battery B0007, we choose training data from cycle 100 to 140, the prediction output cycle is from 141 to 168 and the empirical model is Gaussian model. For battery B0005, the training data is from cycle 100 to 140, the prediction output cycle is from 141 to 168 and the empirical model is double-exponential model. For battery CS2_8, the training data is from cycle 122 to cycle 130, the prediction output cycle is from 131 to 146 and the empirical model is double-exponential model.

We can conclude that the training number is some extent periodical from Figure 6. Therefore, we use the covariance function as Eq. (20),

$$k_f = \sigma_f^2 \exp(-\frac{2}{l_2^2}\sin^2(\frac{\omega}{2\pi}(x-x'))) \tag{20}$$

With EMGPR framework, the regular of deterministic is represented, and the difficulty of choosing the covariance function of GPR model is overcome.

4.5. RUL Prediction with EMGPR

At last, experiments to predict the RUL of lithium-ion battery are realized. We predict the RUL of NASA batteries B0005 and B0007 and CACLE battery CS2_8 with the proposed EMGPR method.

Figure 7 shows the predicted result with NASA battery B0007. Here, the training data is from cycle 100 to cycle 140, and the estimated output is from cycle 141 to cycle 168. The covariance we use is the periodic type, as shown in Eq. (19), and the empirical model is the Gaussian model.

Figure 8 shows the experiment result on CALCE battery CS2_8. The training data is from cycle 122 to cycle 130. The predicted output is from cycle 134 to cycle 146. The covariance function is the same with that in Eq. (20). And the empirical model is double-exponential model.

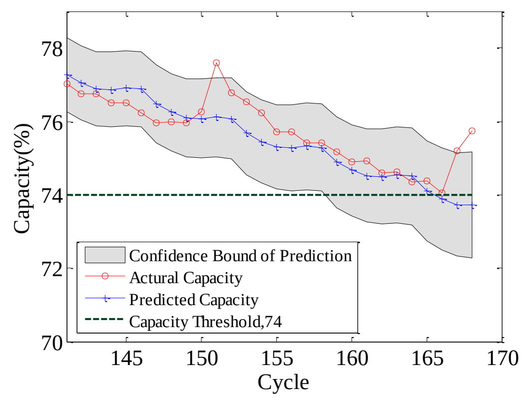

Figure 7. Predicted result of NASA battery B0007 based on EMGPR method

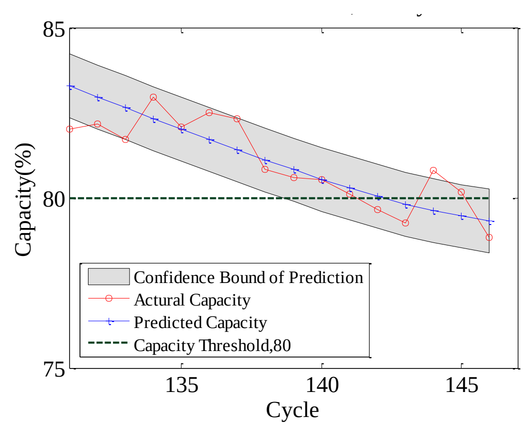

Figure 8. Predicted result of CALCE battery CS2_8 based on EMGPR method

The RMSE and MAE of predicted results for battery B0005, battery B0007 and battery CS2_8 are shown in Table 3.

Table 3. Comparison of RMSE and MAE with different prognostic methods

Battery index	RMSE	MAE
B0005	0.0805	0.0112
B0007	0.0663	0.0089
CS2_8	0.0314	0.0067

From the results above, it can be concluded that the prognostic framework of EMGPR can predict the RUL of lithium-ion battery satisfied. With this method, the estimated result is offered with uncertainty. The uncertainty expression parameter confidence bound is small. Moreover, the covariance function is easy to choose.

5. Conclusions

In this paper, a fusion prognostic framework of the combination of the GPR model and the empirical model (EMGPR) is proposed. The main contribution of this paper can be concluded as follows. (1) The GPR characteristics is studied and experiments have implemented to illustrate that the confidence bound is smaller if the training data keep closer to the test data. Thus, an important step of prediction with EMGPR approach is to choose the proper training data. (2) In the framework of the EMGPR approach, the training data are divided into two parts. One is self-deterministic which can be approximated by the empirical model (indicates the degradation trend). The other part is the disturbance components which turns out to be periodic and can be predicted with the GPR model. The periodicity of the disturbance components has a positive influence on the decision of the covariance function of GPR. As a result, the challenging selection of the covariance function can be solved. (3) The empirical models, proved to be effective in RUL prognostics of lithium-ion battery, are double-exponential model and Gaussian model. In actual application, the flow of choosing the proper empirical model should be considered. (4) Experimental results with both data from NASA PCoE and CALCE show that the EMGPR prognostic framework can predict the RUL of lithium-ion battery with high performance as well as indicated its uncertainty.

6. Future Work

In future, we will explore more effective empirical models. The more specific theory will be studied to choose the empirical model directly. Moreover, the idea of EMGPR can be extended. Other techniques such as filter, smooth theory can be combined with GPR method or EMGPR for lithium-ion battery prognostics. In addition, uncertainty representation such as probability density function (PDF) may be utilized.

Acknowledgement

This work is supported partly by National Natural Science Foundation of China under Grant No. 61301205, Twelfth Government Advanced Research Fund under Grant No. 51317040302, Research Fund for the Doctoral Program of Higher Education of China under Grant No. 20112302120027, Fundamental Research Funds for the Central Universities under Grant No. HIT.NSRIF.2014017 and China Scholarship Council.

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