## A Study on the parameter estimation for crack growth prediction under variable amplitude loading

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## ABSTRACT

Bayesian formulation is presented to address the parameters estimation under uncertainty in the crack growth prediction subjected to variable amplitude loading. Huang's model is employed to describe the retardation and acceleration of the crack growth during the loadings. Model parameters are estimated in probabilistic way and updated conditional on the measured data by Bayesian inference. Markov Chain Monte Carlo (MCMC) method is employed for efficient sampling of the parameter distributions. As the model under variable amplitude loading is more complex, the conventional MCMC often fails to converge to the equilibrium distribution due to the increased number of parameters and correlations. An improved MCMC is introduced to overcome this failure, in which marginal PDF is employed as a proposal density function. A centercracked panel under a mode I loading is considered for the feasibility study. Parameters are estimated based on the data from specimen tests. Prediction is carried out afterwards under variable amplitude loading for the same specimen, and validated by the ground truth data.

**Key Words** : Prognostics and Health Management (PHM), Markov Chain Monte Carlo (MCMC), Crack growth, Variable amplitude loading.

## 1. INTRODUCTION

Although the reliability-based design technology for lifecycle is in its mature stage, it has its limited value due to the inability to account for the unexpected incidences during the in-service condition. Besides, critical systems such as aircraft tend to be operated without retirement even after the design lives. In such cases, efficient maintenance techniques should be incorporated during the operation. Frequent preventive maintenance can, however, increase operating cost significantly, especially for aging aircraft. Recently, prognostics and health management (PHM) techniques are drawing considerable attention, which detect, monitor and predict the damage growth, from which only the faults indicating impending failure are repaired. As a result, condition-based maintenance (CBM) can be achieved, which significantly reduce the number of maintenance visits and repairs.

Prognosis of crack growth is one of the active research topics in the PHM study because the physical model underlying the feature is relatively well known. Numerous literatures have been devoted to this topic, mainly focused on the probabilistic methods to address the associated uncertainties. Orchard and Vachtsevanos (2007) introduced an on-line particle-filtering-based framework for failure prognosis, and applied to a crack growth problem of UH-60 planetary carrier plate. They assumed that the crack growth is described by a simple Paris model and the model parameters are known a priori, which is questionable in practical applications. Cross et al. (2007) developed a Bayesian technique for simultaneous estimation of the equivalent initial flaw size (EIFS) and crack growth rate distributions. AFGROW is used for the crack growth calculation for the fastener hole crack under constant amplitude load. Coppe et al. (2009, 2010) employed Bayesian formulation using the Paris model in which the model parameters are estimated and updated conditional on the measured crack data. A center-cracked panel under a mode I loading is considered for the study. An et al. (2011) conducted similar study by introducing Markov Chain Monte Carlo (MCMC) method for more efficient sampling of the parameters' distribution. They payed particular attention to the parameters correlation as well as the imprecise data due to the noise and bias, which may make

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the Bayesian estimation more difficult. It should be noted that all the previous studies have employed Paris model for the sake of simplicity which holds good under a constant amplitude loading.

In this paper, the study by An et al. (2011) is extended to the case of variable amplitude loading, which involves more parameters in the crack growth model. The feasibility of Bayesian approach is studied to cope with the increased parameters in which the correlations are encountered. We have experienced that the MCMC does not work well, i.e., fails to converge at the equilibrium distribution. An improved MCMC method is introduced to relieve this problem by employing marginal PDF as a proposal density function. Feasibility of the method is illustrated by a centercracked panel under a mode I loading with constant and variable amplitudes, respectively. In the case of variable amplitude loading, the unknown model parameters are estimated based on the crack data by lab specimens under multiple set of constant amplitude loadings. The prognosis under variable loading is then conducted for the same specimen using the obtained parameter samples, and the remaining useful life (RUL) is predicted accordingly.

### 2. CRACK GROWTH MODEL

When the load is applied in a constant amplitude, Paris model best describes the crack growth:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\left(\Delta K\right)^m, \ \Delta K = \Delta \sigma \cdot \alpha \sqrt{\pi a} \tag{1}$$

where a is the half crack size, N is the number of cycles (flights),  $\Delta K$  the range of stress intensity factor (SIF) and  $\alpha$  the geometric correction factor. In the case of the variable amplitude loading, however, the crack growth behavior is significantly different from that under constant loading, presenting the crack growth retardation and acceleration caused by the overload. Numerous models have been developed to adequately describe this behavior. A model based on the crack closure approach, which considers plastic deformation and crack face interaction in the wake of the crack, was proposed by Eiber (1971). Willenborg (1971) and Wheeler (1972) proposed other models based on the calculations of the yield zone size ahead of the crack tip. In this paper, crack growth model by Huang et al. (2007) is used, which is based on a modified Wheeler model to account for the overload and underload effect. Huang's model consists of two parts, one being the scaling factor  $M_R$  which accounts for the crack growth under constant amplitude loading and the other the correction factor  $M_p$ which accounts for the loading sequence interaction such as retardation and acceleration under variable amplitude. The expression is given as follows.

$$\frac{da}{dN} = C[(\Delta K_{eq0})^m - (\Delta K_{th0})^m]$$
<sup>(2)</sup>

$$\Delta K_{eq0} = M_R M_P \Delta K \tag{3}$$

$$M_{R} = \begin{cases} (1-R)^{-\beta_{1}} & (-5 \le R < 0) \\ (1-R)^{-\beta} & (0 \le R < 0.5) \\ (1.05 - 1.4R + 0.6R^{2})^{-\beta} & (0.5 \le R < 1) \end{cases}$$
(4)

Here, *R* is the stress ratio,  $\Delta K_{eq0}$  and  $\Delta K_{th0}$  are equivalent and threshold SIF range respectively, *C*,*m* are the Paris model parameters, and  $\beta$ ,  $\beta_1$  are the shaping parameters for  $M_R$ . The parameters *C*, *m*,  $\Delta K_{th0}$ ,  $\beta$  and  $\beta_1$  are the fitting parameters under a constant amplitude loading, which determines the relationship between the crack growth rates da/dN and SIF range  $\Delta K$ . The correction factor  $M_P$  is given by

$$M_{p} = \begin{cases} \left(\frac{r_{y}}{a_{OL} + r_{OL} - a - r_{\Delta}}\right)^{n} & \left(a + r_{y} < a_{OL} + r_{OL} - r_{\Delta}\right) \\ 1 & \left(a + r_{y} \ge a_{OL} + r_{OL} - r_{\Delta}\right) \end{cases}$$
(5)

where

$$r_{y} = \alpha \left(\frac{K_{\max}}{\sigma_{y}}\right)^{2}$$
(6)

$$r_{OL} = \alpha \left(\frac{K_{\max}^{OL}}{\sigma_y}\right)^2 \tag{7}$$

where  $r_v$  is the plastic zone size ahead of the crack tip,  $r_{\Delta}$  is the increment in the plastic zone size due to the underload following an overload, n is a shaping parameter determined by fitting to the test data under variable amplitude loading, and parameters with the subscript OL denote those under the overload. In Eq.(6) and Eq.(7),  $\alpha$  is the plastic zone size factor which is dependent upon the constraints around the crack tip and the maximum applied stress, yield strength of the material, and specimen thickness (Voorwald et al. 1991). The size of the each plastic zone is calculated in terms of the applied maximum SIF and yield strength  $\sigma_{y}$ . The crack growth under variable amplitude loading is accounted for by incorporating the correction factor  $M_{p}$ after decomposing the variable loading into the successive series of different constant amplitude loadings. Consequently, only the parameters C, m,  $\Delta K_{th0}$ ,  $\beta$  and  $\beta_1$  are the unknown parameters to be estimated in this study.

# 3. MARKOV CHAIN MONTE CARLO FOR PARAMETER ESTIMATION

In this study, Bayes rule is used to account for the uncertainties in the parameters estimation (Bayes, 1763):

$$p(\mathbf{\theta} | \mathbf{y}) \propto L(\mathbf{y} | \mathbf{\theta}) p(\mathbf{\theta})$$
 (8)

where  $L(\mathbf{y} | \boldsymbol{\theta})$  is the likelihood of observed data y conditional on the given parameters  $\theta$ ,  $p(\theta)$  is the prior distribution of  $\boldsymbol{\theta}$ , and  $p(\boldsymbol{\theta}|\mathbf{y})$  is the posterior distribution of  $\boldsymbol{\theta}$  conditional on  $\mathbf{y}$ . The equation states that our degree of belief on the parameter  $\theta$  is expressed as posterior PDF in light of the given data y. In general, the posterior distribution is given by complex expression in terms of the parameters, of which the sample drawing is cumbersome, and prohibiting the use of standard techniques of probability functions. MCMC has been recognized as an effective sampling method, which is based on a Markov chain model of random walk with the stationary distribution being the target distribution. Metropolis-Hastings is the most typical variants of the MCMC algorithm: 1. Initialise  $x^{(0)}$ 

2. For 
$$i = 0$$
 to  $nm - 1$   
- Sample  $u \sim U_{[0,1]}$   
- Sample  $x^* \sim q\left(x^* \mid x^{(i)}\right)$   
- if  $u < A\left(x^{(i)}, x^*\right) = \min\left\{1, \frac{p(x^*)q\left(x^{(i)} \mid x^*\right)}{p\left(x^{(i)}\right)q\left(x^* \mid x^{(i)}\right)}\right\}$ 
(9)  
 $x^{(i+1)} = x^*$ 
else

 $x^{(i+1)} = x^{(i)}$ 

In this equation,  $x^{(0)}$  is the initial value of an unknown parameter to estimate, nm is the number of iterations or samples, U is a uniform distribution, p(x) is the posterior distribution (target PDF), and  $q(x^* | x^{(i)})$  is an arbitrary chosen proposal distribution which is used when a new sample  $x^*$  is to be drawn conditional on the current point  $x^{(i)}$ . Uniform or Gaussian distribution at the current point are the most common choices for the proposal distribution. Success and failure of the algorithm relies heavily on a proper design of the proposal distribution. In order to illustrate this, a target distribution of x is considered (Andrieu et al, 2003):

$$p(x) \propto 0.3 \exp(-0.2x^2) + 0.7 \exp(-0.2(x-10)^2)$$
 (10)

As the candidates of proposal distribution, normal distributions with three different standard deviation,  $\sigma = 1$ ,  $\sigma$  =10 and  $\sigma$  =100, are attempted. The shapes of each distribution are compared in Figure 1(a). The MCMC sampling results using each three proposal distributions with the number of samples nm = 5000 are shown in Figure 1(b)~(d), respectively. Only the proposal distribution with  $\sigma = 10$  gives acceptable result. In the general case with increased parameters and correlations, however, this would be much more difficult.

An improved MCMC method is introduced in this study, which is to employ a marginal PDF as a proposal distribution:

1. Initialise 
$$x^{(0)} = mean(q(x))$$
  
2. For  $i = 0$  to  $nm - 1$   
- Sample  $u \sim U_{[0,c]}$   
- Sample  $x^* \sim q(x)$   
- if  $u < A(x^{(i)}, x^*) = min\left\{1, \frac{p(x^*)}{p(x^{(i)})}\right\}$ 
(11)  
 $x^{(i+1)} = x^*, i = i + 1$   
else

where q(x) is the marginal PDF of x defined by

$$q(x_{i}) = \int \cdots \int \int \cdots \int p(x_{1}, \dots, x_{i-1}, x_{i}, x_{i+1}, \dots, x_{np}) dx_{1} \cdots dx_{i-1} dx_{i+1} \cdots dx_{np}$$
(12)

i = i

Conventional way to construct the marginal PDF requires intensive computation which requires large number of joint PDF evaluation. In this paper, a simpler approach, which employs Latin Hypercube Sampling (LHS), is used to facilitate efficiency because the marginal PDF needs not be precise in view of the proposal density function.

In the algorithm (11), unlike the conventional MCMC, if the new sample  $x^*$  is not accepted, the i+1 'th sample is not assigned and the sampling is repeated until i+1 th sample satisfies the MH criteria, which results in a little longer



MCMC sampling results of the target PDF Figure 1. given by Eq. (10)

computing time. The uniform distribution,  $U_{[0,1]}$  in the conventional MCMC is replaced here by  $U_{[0,c]}$  where c is a constant less than 1. By the authors' experience, it was found that as c gets smaller, the overall time was decreased dramatically, while the obtained samples distribution did not change much.

#### **CRACK GROWTH UNDER CONSTANT** 4. AMPLITUDE LOADING

In order to verify the new MCMC method, the data generated with fixed parameter values are used. Crack growth of a center-cracked panel of Al 7075-T6 under a mode I loading as shown in Figure 2 is considered. Assuming the effect of finite plate size is ignored, Paris model predicts the crack growth in terms of the fatigue cycles in the closed form expression as:

$$a(N) = \left[ NC \left( 1 - \frac{m}{2} \right) \left( \Delta \sigma \sqrt{\pi} \right)^m + a_i^{1 - \frac{m}{2}} \right]^{\frac{2}{2-m}}$$
(13)

where a is the half crack size at cycle N, C and m are the two damage growth parameters to be estimated,  $a_i$  is the initial crack size which is assumed to be known, and  $\Delta\sigma$  is the stress range due to the fatigue loading. Synthetic curve is generated for the case  $a_i = 10$  mm and  $\Delta \sigma$  =78.6MPa. Assuming that the true parameters,  $m_{true}$ and  $C_{true}$  are given by 3.8 and 1.5E-10 respectively, crack sizes are calculated according to Eq. (13) for a given N. Then, measurement errors with a deterministic bias b =-2mm and random noise  $N(0, \sigma = 1.33)$  are added intentionally to the synthetic curve for the generated data. 10 sets of generated data are made at the interval of 100 cycles. In this case, the unknown parameters consist of the two model parameters m, C and the two measurement errors  $b, \sigma$ . The joint posterior distribution of these parameters is given by

$$p(m,C,b,\sigma) \propto \left(\frac{1}{\sqrt{2\pi\sigma}}\right)^{10} f \cdot p(m) \cdot p(C)$$
 (14)



Figure 2. Specimen geometry (t=4.1, b=152.5, a=6 (mm))



Prediction of the crack growth Figure 3.

where f and p(m), p(C) are the likelihood and prior PDFs of the two parameters respectively, given by

$$f = \exp\left[-\frac{1}{2\sigma^2} \sum_{k=1}^{10} \left(a_{meas_k} - a\left(N_k\right) - b\right)^2\right]$$
  

$$p(m) = U_{[3.3,4.3]}, \quad p(C) = U_{[\log(5 \times 10^{-11}),\log(5 \times 10^{-10})]}$$
(15)

The synthetic curve and the generated data are plotted as black curve and solid dots with 10 numbers in Figure 3 respectively. The unknown parameters are to be estimated conditional on this data based on the MCMC process with the number of samples being 5000. Using the conventional MCMC, proper sampling could not be achieved in spite of lot of trials. One instance of such result is given in Figure 3(a). In Figure 3(a), the incorrect prediction using the failed samples is also given, in which the three dashed curves denote the median and 90% confidence bounds obtained from the distribution respectively. The green horizontal line denotes the critical crack size. On the other hand, the result of the improved MCMC is shown in Figure 3(b), which is instantly obtained at one attempt. The obtained PDF shapes look quite plausible and the correlation between m and Calso identified clearly. The posterior predictive is distribution of the crack growth obtained by the sampling results of the unknown parameters is shown inFigure 4. The improved MCMC predicts the crack growth quite well, following the synthetic curve by correcting the bias while the conventional MCMC could not. Therefore, the improved



Figure 4. Posterior PDFs of four parameters in the crack growth problem



Figure 5. Fatigue crack growth data under constant amplitude loading for Al 7075-T6 (Huang et al, 2007)

MCMC is verified by predicting the synthetic curve with correct parameter estimation.

## 5. CRACK GROWTH UNDER VARIABLE AMPLITUDE LOADING

In the prognosis of crack growth under variable amplitude loading, the unknown model parameters C, m,  $\Delta K_{ih0}$ ,  $\beta$  and  $\beta_1$  are to be estimated conditional on the measured crack data under study. In this study, the unknown model parameters are regarded as the intrinsic property of the material such as the Elastic modulus. Therefore, the unknown model parameters under constant amplitude loading are assumed as identical to those under variable amplitude loading. In view of this, data by Huang et al. (2007) are used for the prognosis, in which the cracks are grown for the lab specimens of Figure 2 under multiple sets of constant amplitude mode I loadings.

Assuming the error between the data and true crack growth model follows Gaussian distribution with  $N(0,\sigma)$ , the joint posterior distribution of the parameters is given by Eq. (8)

which  $\boldsymbol{\theta}$  denote  $C, m, \beta, \beta_1, \Delta K_{ih0}$  and  $\sigma$ , and  $\mathbf{y}$  are the measured crack data. *L* is the likelihood given by

$$L(y | C, m, \beta, \beta_1, \Delta K_{th0}, \sigma) = \left(\frac{1}{\sqrt{2\pi\sigma}}\right)^k \exp\left[-\frac{1}{2\sigma^2} \sum_{i=1}^k (y_{estimation}^i - y_{test}^i)^2\right]$$
(16)

MCMC simulation is implemented to obtain the samples that satisfy the distribution. In this case, the conventional MCMC does not work at all due to the large number of parameters, and fails to obtain the target distributions. Even the improved MCMC gives inadequate distributions as given in Figure 6(a). The reason may be attributed to the Eq. (2)~Eq.(4), in which the parameters  $\beta$ ,  $\beta_1$  exist only when  $R \neq 0$  whereas the data set include the case R = 0. Ignoring this characteristics and taking all three data set equally into account in Eq.(16) leads to the improper marginal PDF. In order to resolve this issue, following four steps are taken during the MCMC simulation.

1. The marginal PDFs of  $C, m, \Delta K_{ih0}$  are constructed from R=0 data set. In this process,  $\beta, \beta_1$  is not necessary since R=0 makes  $M_R$  independent on  $\beta, \beta_1$ .

2. The ranges of  $C, m, \Delta K_{ih0}$  are given from the percentiles of the marginal PDF of  $C, m, \Delta K_{ih0}$ .

3. The marginal PDFs of  $\beta$  and  $\beta_1$  are constructed from the remaining two sets R=-1 and R=0.5 under the ranges of  $C, m, \Delta K_{th0}$  of the process 2.

4. All the marginal PDFs thus obtained are then used in the main process of improved MCMC as given by (11).

As a result, Figure 6(b) is obtained, in which the distributions of the parameters  $\theta$  exhibit plausible shape, and represent our degree of confidence due to the uncertainties caused by the insufficient data and measurement errors.

Once the distributions are obtained by the MCMC, the prognosis under variable amplitude loading is conducted using the obtained parameter samples. This is just to implement the crack growth simulation by integration of Eq.(2) to obtain the future crack size distribution using each of the parameter samples. The remaining useful life (RUL) can be predicted from this result. The same specimen is used in this study since the actual data of crack growth are available by Huang et al. (2007) under the variable loading condition as a ground truth data. The loading condition for prognosis process is given in Figure 7, in which a single cycle consists of the *p* numbers of repeated load between 3.48 - 68.13 *MPa* and the *q* numbers of overload with 3.48 - 103.02 *MPa*. This loading condition is repeatedly applied to the specimen generating total load cycles. Two



(a) Sample data from direct application of improved MCMC



- (b) Sample data after taking the four step process in the improved MCMC
- Figure 6. Histogram of samples for the parameters generated by the improved MCMC method

cases of p = 50, q = 1 and p = 50, q = 6 are considered. The results of the predictive simulation are shown in Figure 8, in which each blue curve represents a single result using realized parameters while the red curve represents the ground truth data made by the test of identical loading condition. Figure 9 also represents the confidence bounds obtained from the predictive distribution. The width of the curve in this figure may be attributed to the uncertainty of insufficient data and measurement errors. The RUL distribution shown in Figure 10 is obtained by calculating the cycles at which the crack of each sample grows to a critical crack size. 10% percentile as well as the true RUL



Figure 7. Variable amplitude loading

values are indicated by the marks respectively. Recall that in this study, the parameters were first estimated using the three specimens under constant amplitude loadings, followed by prognosis for the fourth specimen under variable loadings using the estimated parameters. The test data of the last specimen was used just for validation of the prognosis.

## 6. CONCLUSION

In this paper, Bayesian formulation is presented to identify the uncertain parameters in the crack growth problem under variable amplitude loading. Huang's model is employed to describe the retardation and acceleration of the crack growth during the loadings. As the conventional MCMC does not work well in the case of increased parameters and correlations as in this problem, improved MCMC method is introduced by employing marginal PDF as a proposal density function. Feasibility of the method is illustrated by a center-cracked panel under a mode I loading with constant and variable amplitudes, respectively. In the case of variable amplitude loading, parameters are first estimated based on the data from specimen tests under a multiple constant amplitude loadings, and prognosis is followed based on the parameters with another specimen under variable loading. The result is validated by the actual test data. The drawback of this approach is that the model parameters are identified by the lab experiments, and are used for the prognosis of a real part (although, in this case, the same specimen is chosen), of which the material and operating conditions may be somewhat different. Therefore, the estimated RUL has wide range to represent the general life of the entire specimen.

More desirably, the measured data from the real part undergoing variable amplitude loading may be utilized for the parameters estimation as well as the prognosis. Additional work toward this direction will be made in the final draft.



Figure 8. Crack growth simulation under variable amplitude loading using each sample of parameters (red curve :ground truth data)



Figure 9. Confidence bounds of crack growth simulation under variable amplitude loading



Figure 10. RUL distribution and its 10% percentile value

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