Damage Identification in Frame Structures, Using Damage Index, Based on H_2-Norm

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ABSTRACT

A simulation method to detect and locate damage in frame structures by defining a damage index is proposed. Structural members are Timoshenko beam type. The method defines a damage index which is the reduction percentage of H_2 norm of the structure at certain locations in both healthy and damaged states. Structure modeling is done by finite element method.

1. INTRODUCTION

Defining a damage index (D.I.) has been on focus in many publications. Extensive literature reviews on vibration-based damage detection methods is published by Doebling, Farrar, Prime and Shevitz (1996) and Carden and Fanning (2004). Looking to these various vibration based techniques, particularly those using modal parameters, the D.I. method seems more promising. The basic idea behind defining damage indices is that changes in physical properties of a structure will eventually alter some of the system intrinsic properties such as some of natural frequencies, mode shapes or mode shape curvatures (Choi & Stubbs, 2004). A Damage Index is defined based on the changes of the *j*th mode curvature at location *i* (Stubbs, Kim, & Farrar, 1995). Choi and Stubbs (2004) used the strain energy of pre and post damaged structure to define D.I.. Also combination of D.I. and neural network method is used to identify damage in structures (Dackermann, Li, & Samali, 2010). In mode shape curvature based D.I.; changes in the damage index and relating these changes with the potential locations are assessed by statistical methods. Normal distribution of damage indices in different locations is extracted and D.I. values which are two or more standard deviation away from the mean D.I. value are reported to be most probable location of damage (Stubbs, et al., 1995). An extension to mode shape curvature method is that one can take into account all frequencies in the measurement range and not just the modal frequencies. In other words one may use Frequency Response Function (FRF) instead of mode shape data. It is claimed that this method can detect, localize and assess damage extent. The theory is fostered with some experimental results (Sampaio, Maia, & Silva, 1999).

Nevertheless development of suitable and reliable damage metrics and identification algorithms is still an issue to be investigated. D.I. as a scalar quantity is a damage metric that gives a criterion to judge the extent of damage of a structure (Giurgiutiu, 2008). Although these methods are well applicable in some cases but are not usually applicable to the cases that the sizes of cracks are small relative to the structure, or the crack is somewhere in a wide area of the structure. The main reason is that small cracks do not change the lower modal properties appreciably and thus they are not easily detectable using experimental data. It should be noted that this limitation is not due to lack of sensitivity of the method, but it is due to the practical limitations of exciting higher modes. Excitation of higher modes requires significant amount of energy which may not be viable to large structural systems (Ginsberg, 2001).

2. PROBLEM STATEMENT

A 2D frame type structure as shown in Figure 1 is studied. A D.I. based on H_2 norm, as discussed in next section, is formulated to compare the healthy and damaged state of the structure and localize the damage. The structure is modeled using 16 two-node Timoshenko beam element in which each node has 3 degrees of freedom (DOF). Timoshenko beam theory has proved to give more accurate results when the length of the beam element is relatively short (Reddy, 2004). Damage is modeled by reducing the stiffness in the element confined between nodes 11 and 4 by 80%. The

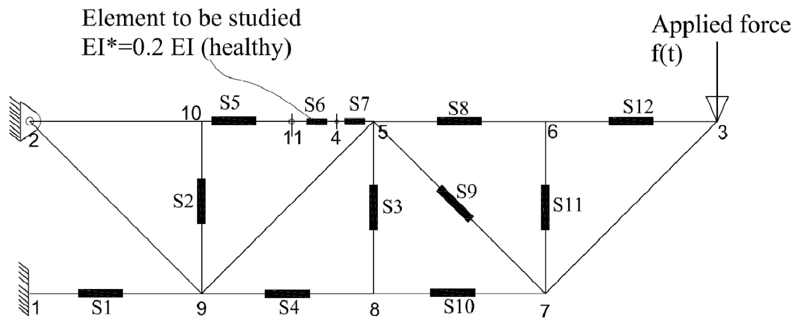

Figure 1. Frame structure configuration and strain gauge sensors placement

material properties of the members are considered as:

$$E = 200\ GPa\ ,\ \ G = 80\ GPa,\ \ \rho = 7800\,\frac{kg}{m^3} \qquad (1)$$

The cross section of members is $3cm\ \times 3cm$ and the length of each horizontal or vertical member is 1 m (Figure 1).

The structure is fixed in all DOF (X, Y, θ) at node 1 and only in translational DOF (X, Y) at node 2. Hence the structure has 28 free DOFs. There are 12 strain gauge sensors placed in different locations of the structure. There are relatively more strain gauges near the damaged link to have more accuracy in finding damage. The input force is applied on node number 3 as shown in the Figure 1. Mass and stiffness matrices of the structure are found after assembling the global stiffness and consistent mass matrix of all elements using finite element technique. The system damping is assumed to be proportional to the system stiffness and mass matrices based on Rayleigh damping as:

$$D = \alpha M + \beta K \qquad (2)$$

The parameters α and β are considered here to be 0 and 0.001, respectively.

3. PROBLEM FORMULATION

The governing equations of a linear structure in the finite element form can be described as (Gawronski, 2004)

$$M\ddot{q} + D\dot{q} + Kq = B_o u \qquad (3)$$

For the 2D frame structure discussed in previous section, M, D and K are 28×28 mass, damping and stiffness matrices, respectively. B_o is input vector and q is nodal displacement vector and both are 28×1 vectors. u is the input force magnitude.

The desired output is the strains in specified members. This output is a linear combination of system nodal displacements. For example, for the element with strain gauge S4:

$$S4\colon \varepsilon_x = \frac{q_{x8} - q_{x9}}{L_{8-9}} \qquad (4)$$

q_{x8}, q_{x9}: Displacement of node 8 and 9 in x- direction

L_{8-9}: Length of member 8-9

ε_x: Strain in member 8-9

Thus the output vector y has 12 strain components which can be related to the nodal displacement vector q as

$$y = C_q q \qquad (5)$$

where C_q is a 12×28 matrix.

3.1 Modal model

Modal model in structures is a standard modeling procedure in which modal displacement vector (q_m) is related to the original nodal displacement vector q as

$$q = \Phi q_m \qquad (6)$$

in which (Φ) is the system modal matrix whose columns are eigenvectors (normal modes) of the system.

Now by substituting Eq. (6) into Eq. (3) an then multiplying the resulting equation from left side by transpose of (Φ), one may write:

$$M_m \ddot{q}_m + D_m \dot{q}_m + K_m q_m = \Phi^{\mathrm{T}} B_o u \qquad (7)$$

in which

$$M_m = \Phi^{\mathrm{T}} \mathrm{M} \Phi$$

$$D_m = \Phi^{\mathrm{T}} \mathrm{D} \Phi \qquad (8)$$

$$K_m = \Phi^{\mathrm{T}} \mathrm{K} \Phi$$

are modal mass, modal damping and modal stiffness matrices which are diagonal due to orthogonality of eigenvectors. (Rao, 2007)

Also the output vector described in Eq. (5) can be written as:

$$y = C_{mq} q_m \tag{9}$$

in which C_{mq} is the modal system output matrix written as:

$$C_{mq} = C_q \Phi \tag{10}$$

Multiplying Eq. (7) by the inverse if the modal mass, M_m^{-1}, from the left side yields:

$$\ddot{q}_m + M_m^{-1} D_m \dot{q}_m + M_m^{-1} K_m q_m = M_m^{-1} \Phi^{\mathrm{T}} B_o u \tag{11}$$

or

$$\ddot{q}_m + 2\mathrm{Z}\Omega \dot{q}_m + \Omega^2 q_m = B_m u \tag{12}$$

in which $\Omega = M_m^{-1/2} K_m^{1/2}$ is the diagonal matrix of eigenvalues (natural frequencies):

$$\Omega = \begin{bmatrix} \omega_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \omega_n \end{bmatrix} \tag{13}$$

and Z is the diagonal modal damping matrix defined as:

$$Z = \frac{D_m}{2\sqrt{M_m K_m}} \tag{14}$$

Modal system input matrix B_m is also defined by

$$B_m = M_m^{-1} \Phi^{\mathrm{T}} B_o \tag{15}$$

3.2 H_2 norm

Based on modal representation of the linear system, and derived system modal matrices, the H_2 norm of the system is defined. Norms are employed to quantify the intensity of system response to standard excitations, such as unit impulse, or white noise of unit standard deviation. H_2 norm is used to compare two different situations. It should be noted that H_2 norm of a mode with multiple inputs (or outputs) can be broken down into the rms sum of norms of that mode with a single input (or output) (Gawronski, 2004).

Now let us consider a flexible structure with one actuator (or one input) and n modes (n=system DOF), the modal input matrix B is then:

$$B_m = \begin{bmatrix} B_{m1} \\ B_{m2} \\ . \\ . \\ . \\ B_{mn} \end{bmatrix} \tag{16}$$

For the 2D frame structure discussed before, B_m has 28 rows and one column and B_{mi} corresponds to the actuator effect on ith mode.

Similar to the actuator properties, for r sensors installed on a n DOF structure, the output matrix is as follows:

$$C_m|_{r\times n} = [C_{m1}, C_{m2}, \dots, C_{mn}] \tag{17}$$

For mode number j

$$C_{mj} = \begin{bmatrix} C_{m1j} \\ C_{m2j} \\ . \\ . \\ . \\ C_{mrj} \end{bmatrix} \tag{18}$$

The H_2 norm of the ith mode of a structure with a set of r sensors is the rms sum of the H_2 norms of the mode with each single sensor from this set. Norm of a structure with one actuator and multiple sensors is defined as (Gawronski, 2004)

$$\|G_{mi}\|_2 \cong \frac{\|B_{mi}\|_2 \|C_{mi}\|_2}{2\sqrt{\zeta_i \omega_i}} \tag{19}$$

The jth sensor H_2 norm of the structure corresponding to each sensor could be derived similar to modal H_2 norm as (Gawronski, 2004):

$$\|G_{sj}\|_2 \cong \frac{\|B_{mj}\|_2 \|C_{msj}\|_2}{2\sqrt{\zeta_j \omega_j}} \tag{20}$$

4. DAMAGE INDEX (D.I.)

To localize damaged elements of a structure, a damage index attributed to the sensor (sensor damage index) is defined (Gawronski, 2004). By denoting the norm of the jth sensor of the healthy structure by $\|G_{sj}^h\|_2$, and the norm of the jth sensor of the damaged structure by $\|G_{sj}^d\|_2$. The jth sensor index of the structural damage is defined as a weighted difference between the jth sensor norms of a healthy and damaged structure as:

$$DI_j^s = \left| \frac{\|G_{sj}^h\|_2^2 - \|G_{sj}^d\|_2^2}{\|G_{sj}^h\|_2^2} \right| \tag{21}$$

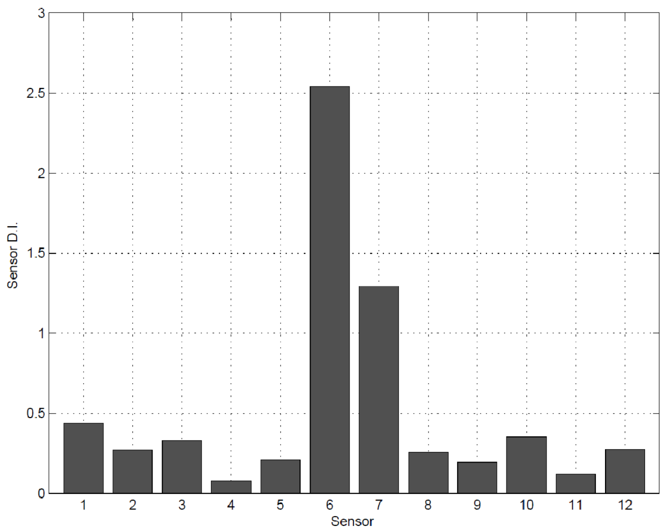

Figure 2. Sensor damage indices

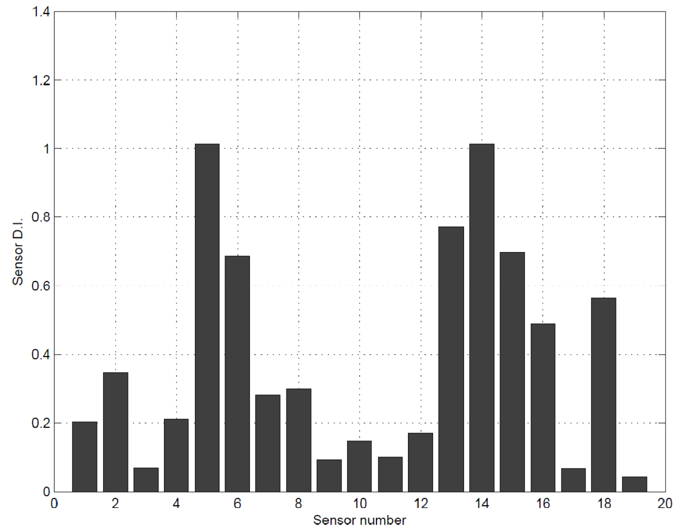

Figure 4. Sensor damage index for structure with two damage spots.

5. RESULTS

The H_2 norm damage index defined in Section 4 has been evaluated for the 2D frame structure as described before in section 2. Using the modal finite element formulation elaborated in Section 3, Figure 2 indicates the sensor D.I. in all 12 sensors.

As it can seen from Figure 2, the sensor number 6 (S6) has the highest D.I. value indicating that the most probable place to have damage is member between nodes 4 and 11 (member 11-4) which is indeed the location of the defined damage.

The developed algorithm can be easily applied to identify multiple damage locations in the case that structure has more than one damaged spot. Naturally, more sensors should be added to reasonably accurate results and increase the algorithm sensitivity.

In this example it is assumed that the structural member between nodes 5 and 7 (member 5-7) is divided into 4 elements and members 5-6 and 5-8 are also divided into 3 elements and new strain gauges are installed on these new elements as shown in Figure 3. Damage is introduced to element 13-14 (S14) as well as previous member 11-4 (S5).

It is assumed that both members have 80% reduction in stiffness EI. Figure 4 indicates the sensor D.I. for this new damage configuration. It could be seen that the algorithm has accurately identified the exact damage locations because the damage index in 5th and 14th locations are the two highest.

6. CONCLUSION

A methodology to detect and locate damage in frame structures by defining a damage index is formulated. Structural members are modeled as Timoshenko beams type. The method defines a damage index which is the reduction percentage of H_2 norm of the structure at certain locations where strain gauges are installed and compares both healthy and damaged states. However to have accurate results one should install enough number of sensors. There is room to extend this work by installing different types of sensors such as accelerometers or to find the minimum number of required sensors to have accurate results as possible.

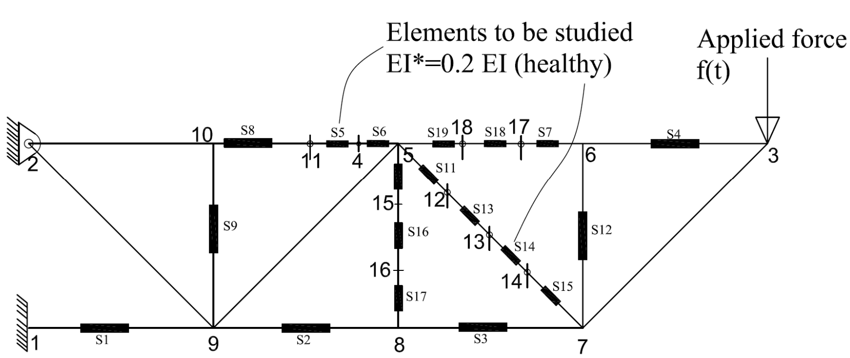

Figure 3. Frame structure with two damage spots

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REFERENCES

Carden, E. P., & Fanning, P. (2004). Vibration based condition monitoring: a review. *Structural Health Monitoring, 3*(4), 355.

Choi, S., & Stubbs, N. (2004). Damage identification in structures using time-domain response. *Journal of Sound and Vibration, 275*(Copyright 2004, IEE), 577-590.

Dackermann, U., Li, J., & Samali, B. (2010). Technical Papers: Dynamic-Based Damage Identification Using Neural Network Ensembles and Damage Index Method. *Advances in Structural Engineering, 13*(6), 1001-1016.

Doebling, S. W., Farrar, C. R., Prime, M. B., & Shevitz, D. W. (1996). *Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review*: Los Alamos National Lab., NM (United States).

Gawronski, W. K. (2004). *Advanced structural dynamics and active control of structures*: Springer Verlag.

Ginsberg, J. H. (2001). *Mechanical and structural vibrations: theory and applications*: John Wiley & Sons.

Giurgiutiu, V. (2008). *Structural health monitoring with piezoelectric wafer active sensors*: Academic Pr.

Rao, S. S. (2007). *Vibration of continuous systems*: Wiley Online Library.

Reddy, J. N. (2004). *Mechanics of laminated composite plates and shells: theory and analysis*: CRC.

Sampaio, R., Maia, N., & Silva, J. (1999). Damage detection using the frequency-response-function curvature method. *Journal of Sound and Vibration, 226*(5), 1029-1042.

Stubbs, N., Kim, J. T., & Farrar, C. R. (1995). *Field verification of a nondestructive damage localization and severity estimation algorithm*.