On-Line Parameter and RUL Updating for Degradation Processes with Three-Source Variability

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ABSTRACT

Degradation model based remaining useful life (RUL) prediction is often used in prognostics and health management (PHM). In practice, three-source of variability, i.e., the temporal variability, unit-to-unit variability, and measurement variability, is often encountered in degradation modeling, leading to complex degradation models and great challenge for the parameter estimation and RUL prediction. Commonly, off-line methods are used, which, however, cannot fulfill the real-time requirement of decision-making in the PHM. In this extended abstract, a generic degradation model is introduced, which can characterize the three-source variability and provide a flexible for on-line parameter and RUL updating. An integrated simulation-based filtering method is introduced by feeding the output of a Markov chain Monte Carlo simulation into an extended particle filter, which can fuse the historical trajectories and condition monitoring observations and update the parameter and RUL simultaneously. Critical aspects of the generic model and the filtering method are presented.

1. INTRODUCTION

The capability of real-time updating of model parameters and performance state based on newly observed condition monitoring observations is of critical importance for an effective degradation based on-line RUL predictions (Si *et al.*, 2011, Ye & Xie, 2014). In traditional degradation based RUL prediction, the degradation model and parameter estimation method are both tailored specifically to achieve the real-time updating capability (Jouin *et al.*, 2016). However, these tailored model and estimation methods can hardly deal with general situations encountered in real practice, where both historical degradation trajectories and real-time condition monitoring needed to be counted in and three-source variability exists as well (Si *et al.*, 2014). In this paper, a general stochastic process based degradation model is introduced in the form of state-space model, where temporal variability, unit-to-unit variability and measurement variability can be simultaneously characterized. The Gaussian process, gamma process, and inverse Gaussian (IG) process models with random effect and measurement errors are included as special cases. The Markov chain Monte Carlo (MCMC) and particle filter are coupled to construct a method for parameter estimation and state updating. The MCMC is adopted to fusing the historical degradation trajectories and condition monitoring observations. The posterior samples of model parameters generated from the MCMC is further used as input particles of particle filtering for real-time updating of model parameters and product state simultaneously when newly observations are available.

2. THE GENERIC DEGRADATION MODEL

To characterize the three-source variability, the stochastic process, the random effect model, and the measurement error model are combined in this paper to formulize a generic degradation models. Within the generic model, the stochastic process, such as the Wiener process, gamma process, inverse Gaussian process etc., are used to characterize the degradation pattern and temporal variability. The random effect model is used to describe the unit-to-unit variability. The measurement error model is incorporated for the measurement variability. In addition, the generic model is delivered through a combination of a state model and a measurement model given as follows.

The state model:

$$\begin{aligned} &x_{t+\Delta t} = x_t + \Delta x_{t+\Delta t}, \\ &\Delta x_{t+\Delta t} \sim \mathrm{Z}\left(\Delta x_{t+\Delta t}, t, t+\Delta t \mid \boldsymbol{\theta}^F, \boldsymbol{\theta}^R\right), \\ &\boldsymbol{\theta}^R \sim G_{\boldsymbol{\theta}}\left(\boldsymbol{\theta}^R \mid \boldsymbol{\theta}^H\right), \end{aligned} \quad (1)$$

where x_t is the degradation state at time t, $\Delta x_{t+\Delta t}$ is the degradation increment within the time interval $[t, t+\Delta t]$, $\mathrm{Z}\left(\Delta x_{t+\Delta t}, t, t+\Delta t \mid \boldsymbol{\theta}^F, \boldsymbol{\theta}^R\right)$ is a probability distribution with covariates t and parameters $\boldsymbol{\theta}$, $\boldsymbol{\theta}^F$ includes the parameters without random effects, $\boldsymbol{\theta}^R$ includes the parameters with random effect, $G_{\boldsymbol{\theta}}\left(\boldsymbol{\theta}^R \mid \boldsymbol{\theta}^H\right)$ is a probability distribution of random effect parameter $\boldsymbol{\theta}^R$, and $\boldsymbol{\theta}^H$ includes the hyper-parameters of the random distribution of $\boldsymbol{\theta}^R$.

The measurement model:

$$\begin{aligned} &y_{t+\Delta t} = x_{t+\Delta t} + \varepsilon_{t+\Delta t}, \\ &\varepsilon_{t+\Delta t} \sim G_{\varepsilon}\left(\varepsilon_{t+\Delta t} \mid \boldsymbol{\theta}^E\right), \end{aligned} \quad (2)$$

where $y_{t+\Delta t}$ is the measurement of the degradation at time $t+\Delta t$, and $\varepsilon_{t+\Delta t}$ is the measurement error at time $t+\Delta t$, and $G_{\varepsilon}\left(\varepsilon_{t+\Delta t} \mid \boldsymbol{\theta}^E\right)$ is the probability distribution of $\varepsilon_{t+\Delta t}$ with parameters $\boldsymbol{\theta}^E$.

Within the model presented above, the temporal variability is modelled through the probability distribution of degradation increment, where $\mathrm{Z}\left(\Delta x_{t+\Delta t}, t, t+\Delta t \mid \boldsymbol{\theta}^F, \boldsymbol{\theta}^R\right)$ can be normal distribution $\mathrm{N}\left(\mu(t, t+\Delta t), \sigma^2 \Delta t\right)$ with mean function $\mu(t, t+\Delta t) = \mu(t+\Delta t) - \mu(t)$, gamma distribution $\mathrm{G}\left(\eta(t, t+\Delta t), \gamma\right)$ with $\eta(t, t+\Delta t) = \eta(t, t+\Delta t) - \eta(t)$ and IG distribution $\mathrm{IG}\left(\Lambda(t, t+\Delta t), \lambda \Lambda^2(t, t+\Delta t)\right)$ with mean function $\Lambda(t, t+\Delta t) = \Lambda(t+\Delta t) - \Lambda(t)$. The unit-to-unit variability is modelled through the random effect of model parameters $\boldsymbol{\theta}^R$. The measurement variability is modelled through the probability distribution $G_{\varepsilon}\left(\varepsilon_{t+\Delta t} \mid \boldsymbol{\theta}^E\right)$.

By specifying different probability distributions for $\mathrm{Z}\left(\Delta x_{t+\Delta t}; t, t+\Delta t \mid \boldsymbol{\theta}^F, \boldsymbol{\theta}^R\right)$, the generic model presented in Eq. (1) and (2) can include the Wiener process model, gamma process model and IG process model as special cases.

3. THE INTEGRATED SIMULATION-BASED FILTERING METHOD

Both historical trajectories and condition-monitoring observations may be available for RUL prediction. The Bayesian method is used to integrate the historical trajectories and condition-monitoring observations. An auxiliary particle filter is developed to update the model parameters and degradation state in-time. In addition, the MCMC method is used to generate posterior samples from the posterior distribution of model parameters obtained by the Bayesian method. These posterior samples are further fed into the auxiliary particle filter as initial particles for parameters and state updating.

The Bayesian method for fusing historical trajectories and condition-monitoring observations is given as follows.

$$\begin{aligned} p\left(\boldsymbol{\theta} \mid \mathbf{D}^H, \mathbf{D}^C\right) \propto\, & L\left(\mathbf{D}^C, \boldsymbol{\theta}^R \mid \boldsymbol{\theta}^F, \boldsymbol{\theta}^H\right) L\left(\mathbf{D}^T, \boldsymbol{\theta}^R \mid \boldsymbol{\theta}^F, \boldsymbol{\theta}^H\right) \\ & \times p\left(\boldsymbol{\theta}^F\right) p\left(\boldsymbol{\theta}^H\right) \end{aligned} \quad (3)$$

where $\boldsymbol{\theta}$ includes all the model parameters, $\mathbf{D}$ indicates the data, $p\left(\boldsymbol{\theta} \mid \mathbf{D}^H, \mathbf{D}^C\right)$ is the joint posterior distribution of the model parameters by fusing the historical trajectories $\mathbf{D}^T$ and condition monitoring data $\mathbf{D}^C$, $p(\boldsymbol{\theta})$ is the prior distribution of model parameter $\boldsymbol{\theta}$, $L\left(\mathbf{D}^C, \boldsymbol{\theta}^R \mid \boldsymbol{\theta}^F, \boldsymbol{\theta}^H\right)$ and $L\left(\mathbf{D}^T, \boldsymbol{\theta}^R \mid \boldsymbol{\theta}^F, \boldsymbol{\theta}^H\right)$ are the likelihood function of the data $\mathbf{D}^C$ and $\mathbf{D}^T$, respectively, which can be formulized as follows.

$$\begin{aligned} & L\left(\mathbf{D}, \boldsymbol{\theta}^R \mid \boldsymbol{\theta}^F, \boldsymbol{\theta}^H\right) = \\ & \prod_{i=1}^{n^C} f\left(\boldsymbol{\theta}_i^R \mid \boldsymbol{\theta}^H\right) \prod_{j=1}^{m_i} \int_{\varepsilon_{ij}} \begin{pmatrix} f\left(y_{ij} - \varepsilon_{ij}, t \mid \boldsymbol{\theta}^F, \boldsymbol{\theta}_i^R\right) \\ \times f\left(\varepsilon_{ij} \mid \boldsymbol{\theta}^E\right) \end{pmatrix} d\varepsilon_{ij} \end{aligned} \quad (4)$$

where n^C is the number of units observed, m_i is the number of observations for the ith unit, $f\left(\boldsymbol{\theta}_i^R \mid \boldsymbol{\theta}^H\right)$ is the probability density function (PDF) associated with $G_{\boldsymbol{\theta}}\left(\boldsymbol{\theta}^R \mid \boldsymbol{\theta}^H\right)$, $f\left(y_{ij} - \varepsilon_{ij}, t \mid \boldsymbol{\theta}^F, \boldsymbol{\theta}_i^R\right)$ and $f\left(\varepsilon_{ij} \mid \boldsymbol{\theta}^E\right)$ are the PDFs with $\mathrm{Z}\left(\Delta x_{t+\Delta t}, t, t+\Delta t \mid \boldsymbol{\theta}^F, \boldsymbol{\theta}^R\right)$ and $G_{\varepsilon}\left(\varepsilon_{t+\Delta t} \mid \boldsymbol{\theta}^E\right)$ respectively.

By utilizing the MCMC method, posterior samples can be generated from the posterior distribution given in Eq. (3). These generated posterior samples can be further used to update the parameter and state when newly observed $y_{t+\Delta t}$, for which an auxiliary particle filter is developed. Critical procedures of the auxiliary particle filter are given as follows (Liu & West, 2011).

1. For each $j = 1, \ldots, K$, identify the prior point estimates of $\{x_t, \boldsymbol{\theta}\}$ from the state model based on the posterior samples initially and the previous particles afterward;

2. Sample an auxiliary integer variable from the set $\{1, \ldots, K\}$ with the probabilities proportional to the PDF of the measurement error with newly observed data $y_{t+\Delta t}$ and the particles associated with the *K* groups point estimates. This auxiliary integer is called the sampled index *k*;

3. Sample a new parameter vector from the *k*th normally distributed component of a specified kernel density;

4. Sample a value of the current state vector from the state model based on the newly sampled parameter;

5. Evaluate the corresponding weight of the newly sample parameter and state vector;

6. Repeat step (2)-(5) to produce a group of particles.

4. CONCLUSION

In this paper, degradation based on-line RUL prediction is studied. A generic degradation model and a simulation-based filtering method are introduced in this paper. Three-source variability can be modelled through the generic degradation model. Both historical trajectories and condition monitoring observations can be integrated through the simulation-based filtering method. On-line updating of model parameters and RUL prediction can also be implemented by further utilizing the simulation-based filtering method. Further demonstration of the proposed method will be carried out through simulation study and real case application.

ACKNOWLEDGEMENT

The paper is supported in part by National Natural Science Foundation of China under Grant No. 51605081, and in part by the Fundamental Research Funds for the Central Universities under Grant No. ZYGX2016KYQD119.

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