The Lethargy Coefficient Estimation of the Probabilistic Fatigue Life Model Using the Markov Chain Monte Carlo

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ABSTRACT

Recently, the researchers of prognostics and health management (PHM) have been developed to the field of engineering. In this study, probabilistic fatigue life which based on Zhurkov model is suggested using stochastically and statistically estimated lethargy coefficient. The fatigue life model was derived using Zhurkov life model and it was deterministically validated with the reference of fatigue life data. For this process, firstly, lethargy coefficient which is relative to the failure of materials has to be obtained with rupture time and stress from quasi-static tensile test. These experiments are performed using HS40R steel. However, lethargy coefficient has uncertainties due to inherent uncertainty and the variation of material properties in the experiments. Bayesian approach was employed for estimating the lethargy coefficient of the fatigue life model using Markov Chain Monte Carlo (MCMC) sampling method and considering its uncertainties. Once the samples are obtained, one can proceed to the posterior predictive inference on the fatigue life. This life model is reasonable through comparing with experimental fatigue life data. As a result, predicted fatigue life was observed that it was significantly decreased in accordance with increasing stress conditions relatively. This life model is reasonable through comparing with experimental fatigue life data.

Key Words: Prognostics and health management (PHM), Fatigue life, Lethargy coefficient, Zhurkov model, Markov Chain Monte Carlo (MCMC), Bayesian approach

1. INTRODUCTION

The Prognostics and Health Management (PHM) have been applied to the field of engineering. The fatigue fracture by the repeated load is occurred due to accumulating damage continuously in engineering structures. Therefore, it needs the design of a fatigue life about various structures under operating conditions. The estimation of parameters is required using finite data set in the health management of structures. In the early stage of structural design, material properties are obtained from various experiments. For quantifying the uncertainties of material parameters or model coefficients, stochastic and statistical manners are employed.

Bayesian framework for fatigue model determination, updating and averaging using trans-dimensional Markov Chain Monte Carlo (MCMC) simulation (C. Andrieu et al., 2003) is presented. This is also outlined for the parameter estimation that arises during the uncertainty quantification in the numerical simulation as well as in the prognosis of the structural performance. The parameters are estimated in the form of posterior distribution conditional on the provided data. During the numerical implementation, MCMC method is employed, which is a modern computational technique for the efficient and straightforward estimation of parameters (Choi et al., 2011).

In other research, the parameters of the proposed creep-fatigue model were estimated using a standard Bayesian regression approach. It has been performed Bayesian analysis using the MCMC sampling method. The results have shown a reasonable fit between the experimental data and the proposed probabilistic creep-fatigue life assessment models (F. Ibisoglu et al., 2015).

In this study, probabilistic fatigue life which based on Zhurkov (S. N. Zhurkov, 1965) model is suggested. The fatigue life model was mathematically derived. Using stochastic and statistical methods, we estimated the lethargy coefficient of the fatigue model. It was deterministically validated with the reference of experimental fatigue life data (Park et al., 2011). For this process, the lethargy coefficient which is relative to the failure of materials has to be obtained with rupture time and stress through quasi-static tensile-shear test of HS40R steel (Sin et al., 2011; Park et al., 2011). The lethargy coefficient has uncertainties due to experiment errors and the variation of material properties. We calibrated both the fatigue model and experimental fatigue life data. Bayesian approach was employed for estimating its coefficient of the fatigue life model using MCMC sampling method.

2. EXPERIMENTS

2.1 Quasi-Static Tensile-Shear Test for HS40R

HS40R steel is widely used for the body frame of automobiles and so on. For obtaining the lethargy coefficient, rupture time and stress were obtained through quasi-static tensile-shear test. Using specimens in figure 1 were carried out the tensile test with the controlling displacement method with INSTRON 8516. The tensile velocity was set to 2mm/min and displacements were measured with a contacted strain gauge. Spot welding condition of the specimen is depending on KS B 0850. Chemical compositions and obtained material properties of the HS40R are represented respectively in the table 1 and table 2.

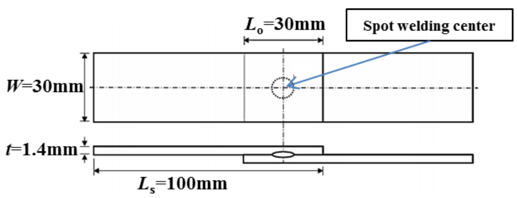

Figure 1. Quasi-static tensile-shear test specimen

Table 1. The chemical composition of HS40R (w_t%)

C	Si	Mn	P
0.0876	0.0065	0.7407	0.1241
S	Ni	Al	Fe
0.0036	0.0091	0.3577	Bal.

Table 2. Mechanical properties of HS40R

Material	σ_t (MPa)	σ_y (MPa)	Elongation (%)
HS40R	416.5	286.1	39

2.2 FATIGUE TEST OF HS40R STEEL

The rupture stress and time were obtained through the quasi static tensile-shear test for calculating lethargy coefficients. In order to compare with predicted life, fatigue test is preceded with dynamic fatigue tester as INSTRON 8516. The frequency of alternative load was set to 10Hz and the cyclic load is controlled under stress ratio condition (R ≒ 0). The behaviors of fatigue crack propagation under the spot welding with 6kA and 2lap are observed by using direct current potential drop method (DCPDM). It use the displacement current by the behavior of fatigue crack propagation. The fatigue life of the HS40R is represented in the table 3

Table 3. The results of experiments of HS40R

HS40R Steel (300K)	Welding current (kA)	σ_r (MPa)	t_r (sec)	Alternate load (MPa)	Experimental fatigue life (Cycle)
2lap	6	229.5	60.6	113.5	52,750
				112.5	162,440
				112.0	291,160
				111.7	408,190

3. FATIGUE LIFE MODEL

Zhurkov drew the empirical fatigue life model under uniform stress and temperature conditions. This model is assumed that the failure occurs when the grid of atoms is removed from stable state. The probability which an atom is removed by thermal vibration for steady time from the position of grid is considered under the mentioned hypothesis. The static fatigue equation of Zhurkov is transformed into the relation of dynamic fatigue. The fatigue life model is represented as equation (1).

$$\int_0^L \frac{dt}{t_0 e^{\frac{U_0-\gamma\sigma(t)}{kT}}} = 1 \qquad (1)$$

Here, The stress $[\sigma(t)]$ is changed with arbitrary time function. U_0 is internal energy as 418.4 kJ/mole; t_0 is time constant as 1e-13sec and k is Boltzman constant as 8.384e-3kJ/mole·k. γ is lethargy coefficient that the material characteristics are involved in accordance with defects and metallography. Generally, fatigue is generated by repeated load under uniform temperature. Therefore, we can consider the relation of stress ($\sigma = \bar{\sigma} + \hat{\sigma}\cos\omega_1 t$) through transforming into time functions. The fatigue life model is derived as follows equation (2). We have to consider the frequency of alternate stress. fatigue life cycle can be obtained by the multiplication of fatigue life (L_f) and frequency (f), $N_f = L_f \cdot f$. We can predict the fatigue life cycle using equation (3) (Yang et al., 1997; Park et al., 2011).

$$L_f = t_0\sqrt{2\pi}\sqrt{\frac{\gamma\hat{\sigma}}{kT}}\, e^{\left(\frac{U_0-\gamma(\bar{\sigma}+\hat{\sigma})}{kT}\right)} \qquad (2)$$

$$N_f = f t_0\sqrt{2\pi}\sqrt{\frac{\gamma\hat{\sigma}}{kT}}\, e^{\left(\frac{U_0-\gamma(\bar{\sigma}+\hat{\sigma})}{kT}\right)} \qquad (3)$$

3.1. Fatigue life using lethargy coefficient

The lethargy coefficient is represented with defect constant of material characteristic; therefore, many tensile tests are needed to determine γ. In this study, for obtaining γ, results from simple quasi-static tensile-shear tests performed by Park et al. (1998) were used. We calculated the lethargy coefficient and fatigue life using the rupture stress and rupture time under uniform temperature and cyclic loading conditions. The lethargy coefficient can be obtained by using equation (4) (Song et al., 2004).

$$\gamma = \frac{U_0}{\sigma_u}(1-\eta) \qquad (4)$$

The lethargy coefficient is proportional to the internal energy over the rupture stress and $(1-\eta)$. η is variable for calculating lethargy coefficient that is expressed as equation (5).

Where,

$$\eta = \frac{\ln\left(\frac{t_r}{t_0}\right)}{\frac{U_0}{kT}} \left(1 - \frac{\ln\left(\frac{U_0}{kT} - \ln\left(\frac{t_r}{t_0}\right)\right)}{\ln\left(\frac{t_r}{t_0}\right)\left[1 - \left(\frac{U_0}{kT} - \ln\left(\frac{t_r}{t_0}\right)\right)^{-1}\right]} \right) \tag{5}$$

3.2 Comparison of predicted fatigue life and experiments

Deterministic fatigue life was obtained using Zhurkov life model about HS40R. The lethargy coefficient was calculated as follow table 8. After that, fatigue life was predicted using equation (3) in accordance with alternate stress conditions. One can see that predicted life was approximately corresponded with experimental data. Predicted fatigue life cycle is represented in the table 8.

Table 8. Results of fatigue life for HS40R (2lap / 6kA)

Rupture stress (MPa)	Rupture time (sec)	Lethargy coefficient (kJ/mole · mm^2/N)	Alternate stress (MPa)	Predicted Fatigue life (Cycle)	Experimental Fatigue life (Cycle)
229.5	60.6	1.4501	113.5	51,929	52,750
			112.5	163,770	162,440
			112.0	290,840	291,160
			111.7	410,490	408,190

4. BAYESIAN ESTIMATION OF LETHARGY COEFFICIENTS

For estimation of the lethargy coefficient of the fatigue model, Bayes' rule is used as follows equation (6) (Bayes, 1763).

$$p(\boldsymbol{\theta} \mid \mathbf{y}) \propto L(\mathbf{y} \mid \boldsymbol{\theta})\, p(\boldsymbol{\theta}) \tag{6}$$

Here, $L(\mathbf{y}|\boldsymbol{\theta})$ is the likelihood of observed data $\mathbf{y}$ conditional on the given parameters $\boldsymbol{\theta}$, $p(\boldsymbol{\theta})$ is the prior distribution of $\boldsymbol{\theta}$, and $p(\boldsymbol{\theta}|\mathbf{y})$ is the posterior distribution of $\boldsymbol{\theta}$ conditional on $\mathbf{y}$. The equation states that our degree of belief on the parameter $\boldsymbol{\theta}$ is expressed as posterior *pdf* in right of the given data $\mathbf{y}$. As more data are provided, the posterior distribution is again used as a prior at the next step, and the values are update to more confident information (Leem et al., 2011). For estimating posterior distribution of the lethargy coefficient, Markov model is widely used in various fields in which sequence of the data is very meaningful. Markov chain consists of Markov model defines probability of posterior event given the prior events. Most important technique can be employed in MCMC based Metropolis-Hastings (M-H) algorithm.

In this study, Bayesian method was employed for estimating the lethargy coefficient. Posterior distribution was also estimated through MCMC simulation assuming proposal distribution as normal distribution. The formulation of Bayes' rule is represented as our engineering problem in equation (7).

$$p(\gamma \mid N) \propto L(N \mid \gamma)\, p(\gamma) \tag{7}$$

With 10,000 iterations, the sampling result follows the target distribution quite well. Estimated posterior distributions of lethargy coefficients were corresponded with actual lethargy coefficients as follows figure 2.

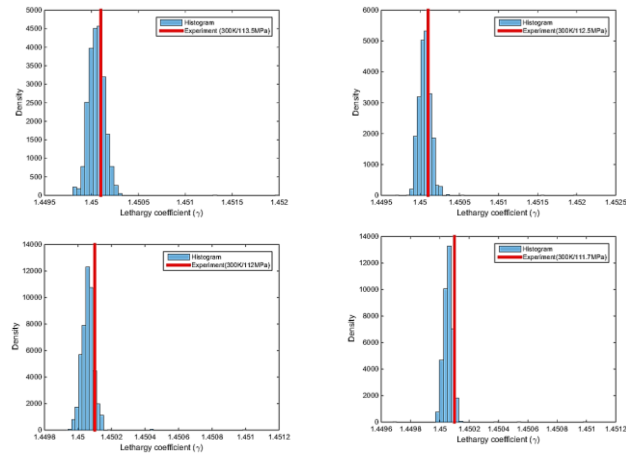

Figure 2. Posterior distribution of γ (Fatigue life)

5. RESULTS OF PROBABILISTIC FATIGUE LIFE

The probability density function (*pdf*) distributions of fatigue life were predicted using estimated the lethargy coefficient by MCMC. In case of fatigue life, the *pdf* distributions were predicted according to alternate stress at room temperature (300K). Results are corresponded with actual life cycle. The *pdf* of estimated fatigue life is shown as narrow deviations because lethargy coefficient is identical according to alternate stress.

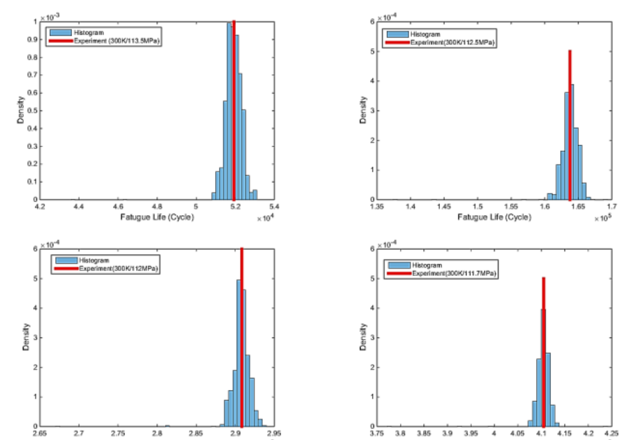

Figure 3. The probabilistic fatigue life

The *pdf* distribution that has the smallest value of the likelihood function was estimated by the maximum likelihood estimation (MLE) method according to alternating stress. The 95% confidence interval (CI) of the fatigue life was obtained by using inverse cumulative density function (ICDF) under each alternating stress conditions. The probabilistic S-N curve can be obtained using the 95% CI as follow figure 4.

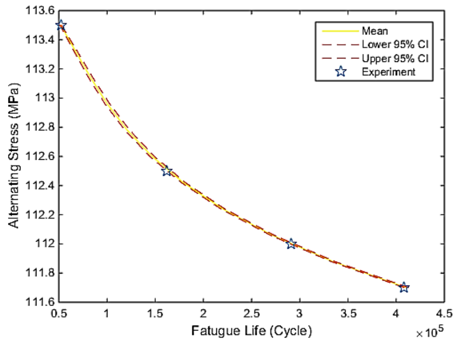

Figure 4. The probabilistic S-N curve under 95% CI

6. CONCLUSION

The Zhurkov model based probabilistic fatigue life model is derived under the cyclic stress. Lethargy coefficients were calculated using experimental data of rupture stress and time. Using this parameter, the life model was deterministically validated with actual life data. For considering the inherent uncertainty of lethargy coefficient of life models, the degree of belief on the model parameters is expressed through a posterior probability distribution in light of the observed data combined with the prior knowledge. Bayesian inference manner was employed and its coefficient was also estimated using MCMC which is used to obtain the posterior predictive distribution. The probabilistic fatigue life using estimated the lethargy coefficient is calibrated with experiment fatigue data and the S-N curve was also obtained considering 95% CI. From this result, Bayesian approach is proved to be useful means for the uncertainty quantification of the unknown parameters in the practical engineering problem. This manner is also useful in the field of prognostics.

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NOMENCLATURE

γ	Lethargy coefficient (kJ/mole • mm^2/N)
σ	Stress (MPa)
σ_t	Tensile stress (MPa)
σ_y	Yield stress (MPa)
σ_r	Rupture stress (MPa)
$\bar{\sigma}$	Mean stress (MPa)
$\hat{\sigma}$	Alternating stress (MPa)
f	Frequency (Hz)
N_f	Life cycle (cycle)
t	Time (sec)
t_r	Rupture time (sec)
T	Absolute temperature (K)

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BIOGRAPHIES

Jongsoo Lee received B.S. in Mechanical Engineering at Yonsei University, Korea in 1988 and Ph.D. in Mechanical Engineering at Rensselaer Polytechnic Institute, Troy, NY in 1996. After a research associate at Rensselaer Rotorcraft Technology Center, he is a professor of Mechanical Engineering at Yonsei University. His research interests include multidisciplinary/multi-physics/multi-scale design optimization and reliability-based robust engineering design with applications to structures, structural dynamics, fluid-structure interactions and flow induced noise and vibration problems.