

# Optimization of Creep Rupture Life Prediction for A231 T91 Alloy Steel Using Kriging Surrogate Model

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## ABSTRACT

Creep rupture life prediction for the A231 T91 alloy steel has been optimized using the Kriging surrogate model. Creep rupture data published by NIMS data sheet was used in this optimization, and isothermal creep rupture life-stress and TTP(Time-Temperature Parameter)-stress curves were optimized with Kriging model. The creep rupture data having the rupture life of less than 10,000 hours was interpolated with the Kriging surrogate model and the Kriging extrapolation to the creep life of 100,000 hours was verified by comparing with experimental data. It was found that the Kriging model could fit to the data within 3% of relative error. Furthermore, the Kriging surrogate model was compared to Wilshire, Manson-Brown, Manson-Haferd, and MCX models. It can be found that the creep rupture stress extrapolated to 100,000 hours by Kriging surrogate showed a relatively good accuracy and uncertainty of 4% ~ 16%.

## 1. INTRODUCTION

In order to develop high temperature power plants or manage the life of the plant, analysis of creep properties has been carried out. In general, most of the creep properties, which are known to be dependent on time, would be determined by accelerated test within a reasonable time frame considering the time and economic reasons. So, to use these properties in design and life management of the plants, reliable data analysis, including modelling the creep rupture data and extrapolation of the data, may be required.

A231 T91(P91) steel was commonly used for steam generator applications in fossil-fuel and nuclear power generating industries. Recently, this steel also is a promising candidate materials for steam generators of Pulverised Coal Combustion (PCC), Circulating Fluidised Bed Combustion (CFBC) plant.

Reliable analysis for obtaining creep properties through optimum creep curve modelling and creep rupture extrapolation has been tried to be established. Classically, the creep data was modelled with the temperature parameters (TTPs), and then fitted to a suitable polynomial stress function. The Larson-Miller and the Manson-Haferd models were the most commonly used ones. Extensive work for the assessment of creep data was carried out in European Creep Collaborative Committee (ECCC) by arranging inter-comparison round robins. The ECCC assessment rules included the least squares (DESA)(Fehér et al., 2009) or maximum likelihood (PD 6605) (BS, 1998) data fitting procedures. In these procedures, three generalized models, such as the Minimum Commitment model (Manson, S. S. and Ensign C. R., 1978) and two Soviet models (Trunin et al., 1971), were supported. These models has been known to be more stable than the polynomial TTPs and not easily produce turn-back and sigmoidal behavior.

In this study, Kriging surrogate model was adopted in optimizing and extrapolating creep rupture data published in NIMS Data sheet for A231 T91(P91) steel. Kriging surrogate model has been designated as the efficient global optimization algorithm. It estimates the deviations between the model and sample data, and the expected improvement is maximized to determine the location of the new data. In this study, the creep rupture data were fitted and optimized with this Kriging surrogate model and the creep rupture strengths at 100,000 and 200,000 hours, respectively, were determined at various operating temperature ranging from 450 °C to 700 °C. Furthermore, the Kriging surrogate models were compared with the Wilshire, Manson-Brown, Manson-Haferd, and MCX models.

## 2. KRIGING SURROGATE MODEL

The kriging surrogate model can be mathematically presented as a linear combination of a global model and departures:

$$y(x) = f(x) + Z(x) \quad (1)$$

,where  $y(x)$  is the unknown deterministic response,  $F(x)$  is a known function of  $x$ , and  $Z(x)$  is a realization of a normally distributed Gaussian random process with zero mean and variance,  $\sigma^2$ , and non-zero covariance. The regression part  $f(x)$  approximates globally the function  $y$ , and  $Z(x)$  localized variances.

The correlation between the random variables in the Kriging model is given by

$$\text{Corr}[Y(x_i), Y(x_j)] = \exp\left(-\sum_{h=1}^k \theta_h |x_h^{(i)} - x_h^{(j)}|^{p_h}\right) \quad (2)$$

Covariance matrix equals to

$$\text{Cov}(Y) = \sigma^2 \mathbf{R} \quad (3)$$

where  $\mathbf{R}$  is a  $pxp$  correlation matrix with  $(i,j)$ . Here,  $\mathbf{R}(x(i), x(j))$  is the correlation function between sampled data points  $x(i)$  and  $x(j)$ .

In order to predict the value of the function at some new points  $x^*$ , the parameters of the likelihood formula is needed to be estimated. Maximizing the log of the likelihood formula, optimal values expressed as function of  $\mathbf{R}$  can be obtained. Then, a concentrated log-likelihood function depending only on  $\mathbf{R}$  is derived. Maximization of this function provides the estimates  $\hat{\mu}$  and  $\hat{\sigma}^2$ . So, at some point  $x^*$ , a function value  $y^*$  can be guessed, and point  $(x^*, y^*)$  would be added to the data as the  $(n+1)^{\text{th}}$ .

The Kriging predictor is defined as

$$\hat{y}(X_{new}) = \hat{\mu} + rR^{-1}(y - 1\hat{\mu}) \quad (4)$$

where  $r$  denotes the vector of correlation  $Y(x_{new})$  with  $Y(x_i)$  for  $i=1, \dots, n$ .

## 3. RESULTS AND DISCUSSION

### 3.1. The creep rupture data

The creep rupture life-stress data for A231 T/P91 published by the National Institute for Materials Science (NIMS), Japan, were used in this study. These data were determined at several testing temperatures ranging from 450 °C to 700 °C from multi-batches of the material. Fig. 1 presented the creep rupture. A total of 211 creep rupture data were selected and those mainly consisted of the tube and pipe products.

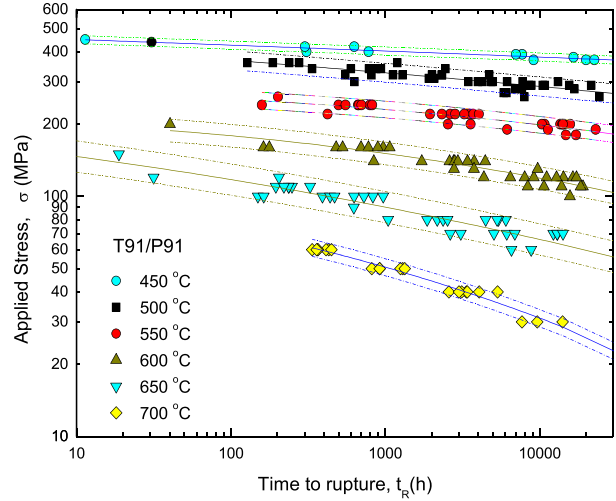


Figure 1. Isothermal creep rupture life-stress curves at various temperatures used in this study (NIMS, 1996.)

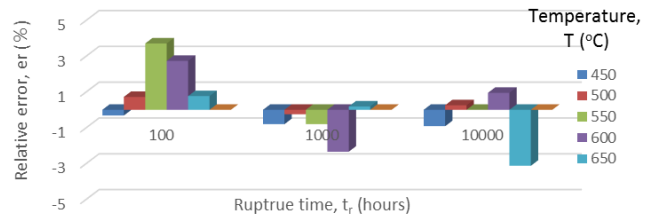


Figure 2. Relative error of interpolation by the Kriging surrogate modelling

### 3.2. Optimization of the creep rupture data with the Kriging surrogate model

The solid lines in Fig. 1 indicated the curves optimized with the Kriging surrogate model. The optimization was carried with a Matlab program developed in this study. Here, the Kriging surrogate model was based on the Gaussian quadratic polynomials. The curves fitted with the Kriging surrogate model at the respective testing temperatures were considerably closely fitted to the measured data. Fig.2 represented a relative error of the Kriging interpolation. Here, the relative error was defined as the ratio of the difference between the interpolated mean value and mean data and mean data. As can be found in Fig. 2, the data was fitted within the error of 3.7%. The biggest error was 3.7% at 100h of rupture time and temperature of 550 °C. However, most of the data were optimized within less than 1% of relative error. In Fig. 1, the Kriging surrogate interpolation was displayed with the prediction and confidence interval with 95% confidence level. The prediction intervals were ranged from 35 MPa to 2 MPa depending on the temperature and rupture time, and the biggest prediction interval was at 500 °C.

**3.3. Extrapolation of the creep rupture**

Design of the high temperature power components is required to be able to be sustained without creep failures within 100,000 hours at the service temperature. So, the 100,000 hour creep rupture properties are necessary to be determined from the well-designed creep rupture test programme or relatively reliable estimate from the short-term rupture data. The reliable optimal creep rupture data assessment procedure had been proposed by ECCC(European Creep Collaborative Committee). According the procedure, the extrapolation was recommended to be limited in time to three times the longest testing time. In addition, several models were proposed in PD6605 and ECCC procedure. In this study, the isothermal creep rupture stresses were determined by extrapolating the NIMS data with the Kriging surrogate model, as shown in Fig. 1. The stress extrapolated by the Kriging surrogate model were verified with comparison of the creep-rupture data ranging 40,000 to 200,000 hours in rupture time. Fig. 3 represents the stress extrapolated by the Kriging surrogate model determined at the rupture time of 100,000 hours. It can be found that the Kriging extrapolation showed a relatively good accuracy and prediction interval ranging from 4% to 16%. The relative error of the stress extrapolated to 100,000 hour was less than about 7 % at the temperature of less than 600 °C, while the error became 30% at 650 °C of testing temperature. The Kriging extrapolation has been demonstrated to be safer than the other surrogate models. (Zhang et al, 2005)

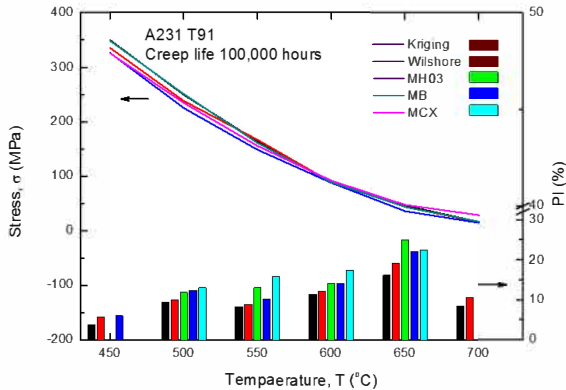


Fig. 3 Comparison of the stresses extrapolated with the Kriging surrogate model and existing rupture models

**3.4. Comparison of the optimization models**

Several models for fitting and extrapolating the short-term creep rupture data were proposed. In this study, the creep rupture models, which were compared to the Kriging surrogate model in this study, are listed in Table. 1. Here, constants and exponents used in the respective models were

determined by fitting the NIMS data into the respective model.

Fig. 4 represents the comparison of the Kriging interpolation

Table 1 Creep rupture models

| Model  | Trend Equation  | Ref  |
|--|---|--|
| Wilshire   | $t_r = \left( \frac{\ln\left(\frac{\sigma}{\sigma_{UTS}}\right)}{-k} \right)^{1/u} \cdot \frac{1}{\exp\left(\frac{-Q_c}{8.314 \cdot T}\right)}$ $u = 7.5 \sim 10.7, k = 0.09 \sim 0.57$ $Q_c^* = 360 \text{ kJ/mol}$              | $k=0.18$   |
| MH03 (Manson-Haferd, 3 <sup>rd</sup> degree)     | $P(\sigma) = B_1 + B_2 \log \sigma + B_3 (\log \sigma)^2 + B_4 (\log \sigma)^3$ $\log(t_r) = B_0 + P(\sigma) \cdot T$   | $B_0 = 80.2534$<br>$B_1 = -0.0694$<br>$B_2 = -0.0146$<br>$B_3 = 0.0163$<br>$B_4 = -0.0058$ |
| MB (Manson-Brown) Model (4 <sup>th</sup> degree) | $P(\sigma) = -22.907 - 0.801\sigma^{0.75} - 0.022(\sigma^{0.75})^2 + 5.1461 \cdot 10^{-4}(\sigma^{0.75})^3 - 5.857 \cdot 10^{-6}(\sigma^{0.75})^3$ $\log(t_r) = P(\sigma) \cdot \left( \frac{T - 505}{1000} \right)^{2.5} + 9.62$ | Stefan H., (2010).   |
| MCX  | $P(\sigma) = B_1 + B_2 \ln \sigma - B_3 \sigma$ $\ln(t_r) = B_0 \cdot T + P(\sigma)$  | $B_0 = -0.093$<br>$B_1 = 117.437$<br>$B_2 = -4.776$<br>$B_3 = 0.339$                       |

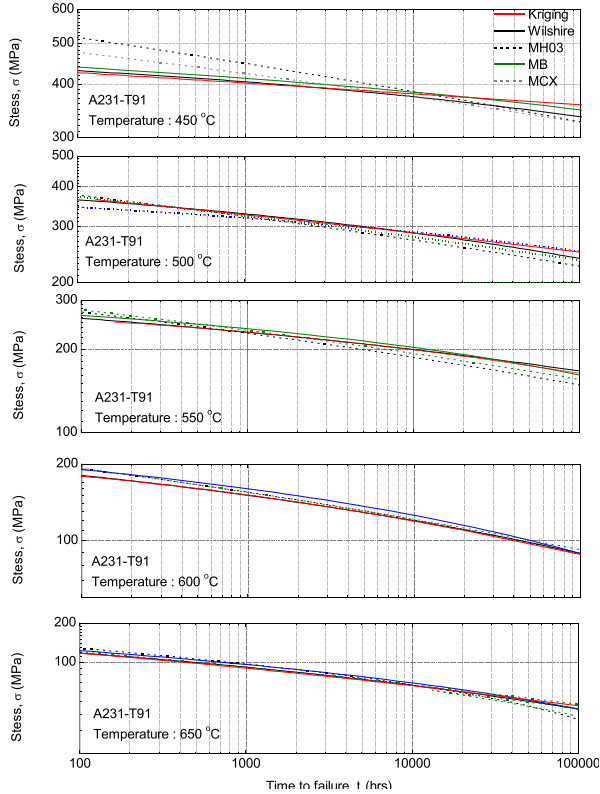


Figure 4. Comparison of the isothermal creep rupture data interpolated with several rupture models with the interpolation carried out by the models listed in Table 1 at the respective temperatures. From Fig. 4, it can be seen that the Wilshire and the Kriging models optimized the data considerably well. The Wilshire interpolation was performed within the relative error of 5.4%, which was compared to the biggest error of 3.7% for the Kriging interpolation. In comparison, MH03, MB and MCX models interpolated the rupture data with relatively large error. Rupture stresses extrapolated with the Kriging surrogate model at the rupture time of 100,000 hours were compared to those obtained by the four models, Wilshire, MH03, MB and MCX. In Fig. 3, the rupture stresses extrapolated by the respective models were plotted with the prediction interval obtained at 95% confidence level. The prediction interval for the Wilshire extrapolation over the temperature range of from 450 °C to 650 °C was within from 5.7 % to 19.1%, which was relatively close to 4~16% for the Kriging extrapolation. However, the prediction intervals for the other models was as high as 25%.

Fig. 5 shows the relative errors for the rupture stress at 100,000 hours extrapolated by the five models. As mentioned previously, the Kriging extrapolation showed a relatively good accuracy. The relative error was less than about 7 % at the temperature of less than 600 °C, while the error became 30% at 650 °C of testing temperature. As can be found in Fig. 5, at 650°C, extrapolation performed by all models except the MB03 model had a relatively large error

of greater than 20%, and the relative errors for the Kriging and MB03 models extrapolation were relatively smaller than those for the Wilshire, MB and MCX models. On the other hand, the prediction interval for the Kriging model was relatively smaller than that for the MB03 model, and the interval for the Kriging model looked to be the smallest. Therefore, it can be said that the Kriging interpolation and extrapolation were relatively more accurate and reliable, compared to the optimization by the other 4 models.

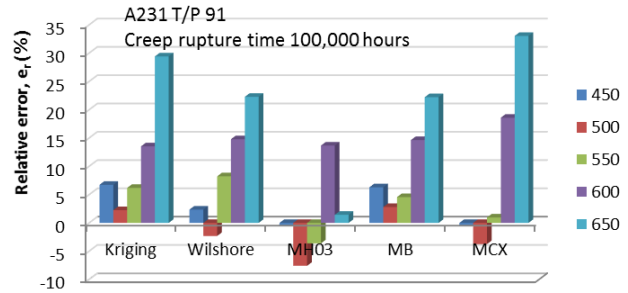


Figure 5. Comparison of relative errors for the rupture stress extrapolated by the several models

4. SUMMARY AND CONCLUSIONS

Creep rupture stress-life data for the A231 T91 alloy steel has been optimized using the Kriging surrogate model. Using the creep rupture data published by NIMS data sheet, isothermal creep rupture life-stress and TTP(Time-Temperature Parameter)-stress curves were optimized with Kriging model.

- 1) The Kriging surrogate model was considerably closely fitted to the creep rupture data, and the optimization was performed within the relative error of 3%.
- 2) The Kriging extrapolation to 100,000 hours of rupture life was carried out for the data with the rupture life of less than 20,000 hours and it was verified with the experimental data. It showed a relatively good accuracy and uncertainty of 4% ~ 16%.
- 3) The Kriging interpolation and extrapolation were compared to those by Wilshire, Manson-Brown, Manson-Haferd, and MCX models. Compared to these existing models, the Kriging surrogate modelling showed a relatively accurate and reliable optimization.

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#### ACKNOWLEDGEMENT

This work was supported by the National Research Council of Science & Technology (NST) grant by the Korea government (MSIP) (No. CRC-15-07-KIER).